Dynamical localization in a chain of hard core bosons under periodic driving

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We study the dynamics of a one-dimensional lattice model of hard core bosons which is initially in a superfluid phase with a current being induced by applying a twist at the boundary. Subsequently, the twist is removed, and the system is subjected to periodic δ-function kicks in the staggered on-site potential. We present analytical expressions for the current and work done in the limit of an infinite number of kicks. Using these, we show that the current (work done) exhibits a number of dips (peaks) as a function of the driving frequency and eventually saturates to zero (a finite value) in the limit of large frequency. The vanishing of the current (and the saturation of the work done) can be attributed to a dynamic localization of the hard core bosons occurring as a consequence of the periodic driving. Remarkably, we show that for some specific values of the driving amplitude, the localization occurs for any value of the driving frequency. Moreover, starting from a half-filled lattice of hard core bosons with the particles localized in the central region, we show that the spreading of the particles occurs in a light-cone-like region with a group velocity that vanishes when the system is dynamically localized.

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I. INTRODUCTION

Periodically driven closed quantum systems have been studied extensively in recent years from the viewpoint of quenching dynamics as well as quantum information theory. Some of these systems show dynamical localization (DL) where the energy of the system never exceeds a maximum bound. Systems showing the signature of DL include driven two-level systems [1], classical and quantum kicked rotors [2,3], and the Kapitza pendulum [4]. In parallel, there have been several studies of many-body localization transition which have indicated that disordered interacting systems can behave nonergodically [5–7]. Given the recent interest in quenching dynamics of quantum systems [8–10] driven across a quantum critical point (QCP) [11,12], the dynamics of those systems under a periodic modulation of the field has also been investigated [13,14]; the connection between thermalization and many-body localization has also been explored [15]. In particular, it has been observed that when a quantum many-body system, specifically, an Ising chain in a transverse magnetic field, is periodically driven across a QCP, there is a synchronization to a "periodic" steady state [16].

In this work, we study the dynamics of a chain of hard core bosons (HCBs) which is subjected to a periodic kick in the staggered on-site potential. We address the issue of DL within the framework of Floquet theory applicable to a time-periodic Hamiltonian [17]. Low-dimensional bosonic systems have been realized experimentally by trapping ultracold atoms in optical lattices [18,19], and the quantum phase transition from a superfluid (SF) to a Mott insulator (MI) phase has been observed in three dimensions [20] as well as in one dimension [21]. The HCB system has also been realized experimentally in optical lattices [22,23]. Following these experimental realizations, there have been numerous analytical studies of these systems in recent years; for a review, see Ref. [24]. The integrability of a HCB chain (and its continuum version known as the Tonks-Girardeau gas [25]) has been exploited extensively, for instance, to investigate the surviving current when the HCB chain is quenched from the SF to the MI phase [26], to study the quench dynamics when the system is released from a trap [27], to analyze the origin of superfluidity out of equilibrium [28], and to explore the DL of bosons in an optical lattice [29].

This paper is organized as follows. In Sec. II, we introduce the model, the initial state of the system (which carries a nonzero current), and the periodic driving scheme. We explicitly derive the Floquet operator and its eigenvalues for a single δ-function kick of the staggered potential. In Sec. III, we present analytical and numerical results for the current and work done in the asymptotic limit of an infinite number of kicks. We analyze these results to highlight the light-cone-like propagation of the particles and the phenomenon of dynamical localization which occurs for certain driving amplitudes and for large driving frequencies. We make some concluding remarks in Sec. IV.

II. THE MODEL AND THE FLOQUET OPERATOR

The model we consider here is a chain of HCBs on a lattice at half-filling described by the Hamiltonian

$$\mathcal{H} = -w \sum_l (b_l^\dagger b_{l+1} + b_{l+1}^\dagger b_l) \tag{1}$$

where b_l's are bosonic operators satisfying the commutation relations $[b_l, b_{l'}^\dagger] = \delta_{l,l'}$ and the hard core condition $(b_l)^2 = (b_l^\dagger)^2 = 0$, and w (assumed to be positive) is the hopping amplitude. Using the Jordan-Wigner transformation from HCBs to spinless fermions [30], the Hamiltonian in (1) can be mapped to a system of non-interacting fermions, which, in momentum space, gets decoupled into 2×2 Hamiltonians in terms of the momenta k and $k + \pi$, where $-\pi/2 \leqslant k \leqslant \pi/2$. Using the basis vector $|k\rangle = (1\ 0)^T$ and $|k + \pi\rangle = (0\ 1)^T$, one can rewrite the 2×2 Hamiltonians as

$$\mathcal{H}_k = -2w \cos k \sigma^z, \tag{2}$$

where σ's denote the Pauli matrices. At half filling, all the k values from $-\pi/2$ to $+\pi/2$ are filled; the ground state for every k mode is the pseudo-spin-up state of the operator σ^z denoted by $(1\ 0)^T$.

When a staggered on-site potential (in real space) of the form $V\sum_l(-1)^l b_l^\dagger b_l$ is added to the Hamiltonian in (1), a coupling is generated between the modes with momenta k and $k+\pi$. Consequently, an energy gap opens up at $k=\pm\pi/2$; hence, the system is in the Mott insulator phase for any finite value of V. There is a quantum phase transition separating the gapped MI phase from the gapless SF phase when $V\to 0$.

We now consider a boosted Hamiltonian, $\mathcal{H}_\nu = -w\sum_l(b_l^\dagger b_{l+1}e^{-i\nu}+b_{l+1}^\dagger b_l e^{i\nu})$, whose ground state has a nonzero current. (We call ν a boost because it effectively shifts the momentum from k to $k-\nu$. Assuming periodic boundary conditions, one can perform certain phase transformations on the b_l to remove ν from each of the terms in the Hamiltonian except for the last term, which hops from site L to site 1; the phase of this hopping amplitude then becomes $L\nu$, where L is the number of sites. Hence, ν also describes a twist in the boundary condition.) With the twist, the ground state for the modes in the range $-\pi/2<k<-\pi/2+\nu$ is given by $(0\ 1)^T$ (i.e., the $k+\pi$ modes are occupied), while for the modes in the range $-\pi/2+\nu<k<\pi/2$ the ground state is $(1\ 0)^T$. We define the current operator $\hat{j}=-(1/L)(\partial H_\nu/\partial\nu)_{\nu=0}=(iw/L)\sum_l(b_{l+1}^\dagger b_l - b_l^\dagger b_{l+1})$, which takes the form $\hat{j}_k=(2w/L)\sin k\sigma^z$ in the space of momenta $(k,k+\pi)$. In the limit $L\to\infty$, we find the initial current to be $j=(2w/\pi)\sin\nu$. (We will set $w=1$ below.)

We will now remove the twist (at $t=0$) and study what happens to the initial current-carrying state when a periodic perturbation is applied (that starts at $t=T$ with a period T) to the staggered potential. More specifically, we will focus on the situation when the Hamiltonian in (1) is subjected to a periodic staggered on-site potential of the form of a Dirac δ-function kick of amplitude α, applied at regular intervals of time denoted by T,

$$V(t) = -\alpha\sum_{n=1}^{\infty}\delta(t-nT). \tag{3}$$

The main question that we will address here is whether the initial current generated by the twist survives in the asymptotic limit ($t\to\infty$) under these periodic perturbations even when the HCB chain is always in the superfluid state (except for the δ-function kicks at $t=nT$). The interesting result we would like to emphasize at the outset is that following an infinite number of kicks ($n\to\infty$), the current vanishes in the limit of large driving frequency $\omega_0=2\pi/T$, while the excess energy energy saturates to a nonzero finite value. As will be discussed below, the vanishing of the current can be attributed to a DL due to a decoherence which leads to a mixed density matrix at large times; we will see that the probability of finding a boson at any site becomes equal to 1/2 for $\omega_0\to\infty$, for all values of α. At the same time, the work done W_d saturates to a finite value. We will also show that for values of the kick amplitude α for which $\cos\alpha=0$, the current vanishes for $n\to\infty$ for all values of ω_0.

We now recall the Floquet theory for a generic time-periodic Hamiltonian, $H(t)=H(t+T)$. One can construct a Floquet operator $\mathcal{F}=\mathcal{T}e^{-i\int_0^T H(t)dt}$, where $\mathcal{T}$ denotes time ordering. The solution of the Schrödinger equation for the jth state in the Floquet basis [$|\Phi_j(t)\rangle$, which are eigenstates of $\mathcal{F}$] can be written in the form $|\Psi_j(t)\rangle = e^{-i\mu_j t}|\Phi_j(t)\rangle$. The states $|\Phi_j(t)\rangle$ are time periodic [$|\Phi_j(t)\rangle = |\Phi_j(t+T)\rangle$], and $e^{-i\mu_j T}$ are the corresponding eigenvalues of $\mathcal{F}$; μ_j are called Floquet quasienergies. To study the dynamics of the Hamiltonian in (1) under the periodic kicks, we note that the Floquet operator in momentum space is given by

$$\mathcal{F}_k = \exp(-iP_k)\exp(-i\mathcal{H}_k T), \tag{4}$$

where $P_k=-\alpha\sigma^x$ and $\mathcal{H}_k=-2\cos k\sigma^z$. The first term in (4) represents time evolution due to the δ-function kick at time $t=T$, while the second term denotes the time evolution of the system dictated by the Hamiltonian in (2) for an interval of time T. Looking at the form of the Floquet operator, one immediately finds some specific values of α given by $\alpha=m\pi$, where $m=0,1,2,\ldots$, for which the δ-function kicks do not affect the temporal evolution of the HCB chain; the ground state remains frozen in its initial state.

The expression for the Floquet operator for a single kick can be obtained exactly [31]:

$$\mathcal{F}_k = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}, \tag{5}$$

where $a=\cos\alpha\cos(2T\cos k)+i\cos\alpha\sin(2T\cos k)$, $b=\sin\alpha\sin(2T\cos k)+i\sin\alpha\cos(2T\cos k)$. The eigenvalues of the operator in (5) are $e^{i\mu_k^\pm T}$, where

$$\mu_k^\pm T = \pm\arccos[\cos\alpha\cos(2T\cos k)] \tag{6}$$

lie in the range $[-\pi,\pi]$. The Floquet quasistates $|\Phi_k^\pm\rangle$ are given by the eigenstates of $\mathcal{F}_k$. It is clear from the structure of (5) that it is sufficient to consider values of α lying in the range $[0,\pi]$. Further, $\mathcal{F}_k$ for $\alpha=0$ and $\alpha=\pi$ only differ by a minus sign; hence all the physical properties of the system are the same at these two values of α, as we will show below.

Under the periodic driving, the time-evolved state at time $t=nT$ can be obtained by n applications of the Floquet operator, namely, $|\Psi_k(nT)\rangle = c_k^+ \mathrm{e}^{-i\mu_k^+ nT}|\Phi_k^+\rangle + c_k^- \mathrm{e}^{-i\mu_k^- nT}|\Phi_k^-\rangle$, where $c_k^\pm = \langle\Phi_k^\pm|\Psi_k(0)\rangle$, with $|\Psi_k(0)\rangle$ being the ground state of the Hamiltonian in (2). We can then compute the current $J(nT)=\sum_k J_k(nT)\equiv\sum_k\langle\Psi_k(nT)|\hat{j}_k|\Psi_k(nT)\rangle$ and the work done $W_d=(1/L)\sum_k W_k\equiv(1/L)\sum_k[e_k(nT)-e_k(0)]$, where $e_k(nT)$ is the energy of the kth mode measured after n kicks, given by $e_k(nT)=\langle\Psi_k(nT)|\mathcal{H}_k|\Psi_k(nT)\rangle = -2\langle\Psi_k(nT)|\sigma^z|\Psi_k(nT)\rangle\cos k$, and $e_k(0)=-2\langle\Psi_k(0)|\sigma^z|\Psi_k(0)\rangle\cos k$ is the initial ground-state energy.

III. THE $n\to\infty$ LIMIT: RESULTS AND IMPLICATIONS

We now consider the limit $n\to\infty$ when $\langle\Psi_k(nT)|\sigma^z|\Psi_k(nT)\rangle = \sum_{m=\pm}|c_k^m|^2\langle\Phi_k^m|\sigma^z|\Phi_k^m\rangle$, where we have dropped rapidly oscillating cross terms (with coefficients $c_k^{+*}c_k^-$ and $c_k^{-*}c_k^+$) which decay to zero in the limit $t\to\infty$ when integrated over a large number of momenta modes. Given the initial ground state with a twist, we find that

$$\begin{aligned}\sum_{m=\pm} |c_k^m|^2 \langle\Phi_k^m|\sigma^z|\Phi_k^m\rangle \\ &= -f(k) \text{ for } -\pi/2 \leqslant k \leqslant -\pi/2+\nu, \\ &= f(k) \text{ for } -\pi/2+\nu \leqslant k \leqslant \pi/2, \\ f(k) &= \frac{\cos^2\alpha \sin^2(2T\cos k)}{\sin^2\alpha + \cos^2\alpha\sin^2(2T\cos k)}.\end{aligned} \tag{7}$$

These expressions imply that the properties of the system will remain the same if we change $\alpha \to \alpha+\pi$ or $\pi-\alpha$.

We will eventually be interested in the thermodynamic limit $L\to\infty$, where we replace $(2\pi/L)\sum_k \to \int dk$. We then obtain the following expressions for the current and work as $n,L\to\infty$:

$$\begin{aligned}J(\infty) &= \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} dk \sum_{m=\pm} |c_k^m|^2\langle\Phi_k^m|\sigma^z|\Phi_k^m\rangle \sin k \\ &= -\frac{2}{\pi}\int_{-\pi/2}^{-\pi/2+\nu} dk f(k)\sin k, \\ W_d(\infty) &= \frac{2}{\pi}\cos\nu - \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} dk \sum_{m=\pm}|c_k^m|^2\langle\Phi_k^m|\sigma^z|\Phi_k^m\rangle\cos k \\ &= \frac{2}{\pi}\cos\nu - \frac{1}{\pi}\int_{-\pi/2+\nu}^{\pi/2+\nu} dk f(k)\cos k,\end{aligned} \tag{8}$$

where the first term in the last two equations comes from $-(1/2\pi)\int_{-\pi/2}^{\pi/2} dk \langle\Psi_k(0)|\sigma^z|\Psi_k(0)\rangle \cos k = (2/\pi)\cos\nu$. We will denote $J(\infty)$ and $W_d(\infty)$ by J and W_d below.

The expressions in (7) vanish in two cases: (i) $T\to 0$, i.e., the driving frequency $\omega_0\to\infty$, while α may take any value, and (ii) $\cos\alpha = 0$, i.e., $\alpha = (m+1/2)\pi$, while T may take any value. In these two cases, we obtain $J=0$ and $W_d = (2/\pi)\cos\nu$.

The neglect of the cross terms in the limit $n\to\infty$ as discussed earlier implies that we have a decohered density matrix. The special feature of the two cases $\omega_0\to\infty$ (and any α) and $\cos\alpha = 0$ (and any ω_0) is that $|c_k^+|^2 = |c_k^-|^2 = 1/2$ for all k; namely, the density matrix is given by 1/2 times the identity matrix in the space of momenta $(k,k+\pi)$ for all k. Since the density matrix is proportional to the identity, it is invariant under all unitary transformations. In particular, we can transform to the position basis and conclude that the system is described by a mixed density matrix in which the probability of finding a boson at any site is equal to 1/2. This corresponds to a completely localized state; this is like a classical state in which the bosons have a probability of 1/2 of being at each site. This explains the vanishing of the current and the saturation of the work done in these two cases.

Using Eq. (7), we can evaluate the leading-order behaviors of the quantities in (8) in various limits. In the limit $\omega_0 = 2\pi/T\to\infty$, we find that

$$J \to \frac{32\pi}{3}\frac{\cot^2\alpha\sin^3\nu}{\omega_0^2}, \quad W_d \to \frac{2}{\pi}\cos\nu. \tag{9}$$

Two other limits are of interest. For $\omega_0\to 0$, we find that $J\to(2/\pi)\sin\nu(1-|\sin\alpha|)$, while for $\nu\to 0$, we find $J\to(32\pi/3)\nu^3\cot^2\alpha/\omega_0^2$. The latter behavior has been called the ν^3 law in Ref. [26].

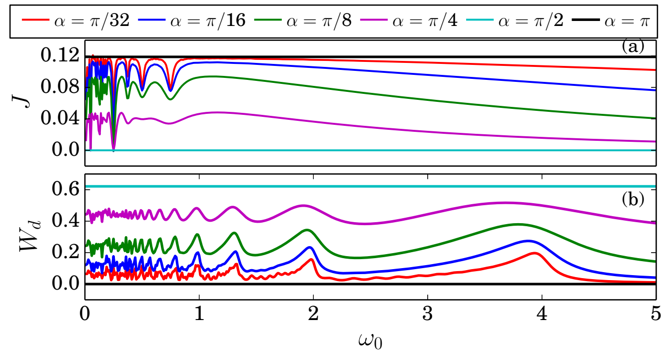

FIG. 1. (Color online) Plots of (a) current J and (b) work done W_d as functions of ω_0 for small values of ω_0 and several values of α, with $L=100$ and $\nu=0.2$. W_d has peaks at some specific values of ω_0 given by $4/n$, where n is an integer, which are the quasidegeneracy points of the Floquet spectrum. The positions of the dips in J are different from the peak positions in W_d as explained in the text.

Next, we investigate the current J and work done W_d as functions of ω_0 for a wide range of ω_0, with different values of α. An examination of Figs. 1 and 2 shows three distinct regions where the current and work done show three different behaviors. (i) For smaller values of ω_0, J shows dips at some specific values of ω_0, while W_d exhibits peaks at ω_0 which are different from the positions of the dips in the current. (ii) In an intermediate region of frequency, J decreases monotonically with increasing α up to $\alpha<\pi/2$, while W_d increases in a similar fashion. (iii) Both quantities saturate asymptotically at some specific values in the large frequency limit.

We now discuss the positions of the peaks in W_d and dips in J in the small ω_0 regime, as shown in Figs. 1(a) and 1(b), obtained through numerical studies of Eq. (7). We will argue that the positions of the peaks in W_d are related to the quasidegeneracy of the Floquet spectrum near $k=0$. Since W_k is proportional to $\cos k$, W_d receives its largest contribution from the region near $k=0$. For small values of α, the positions

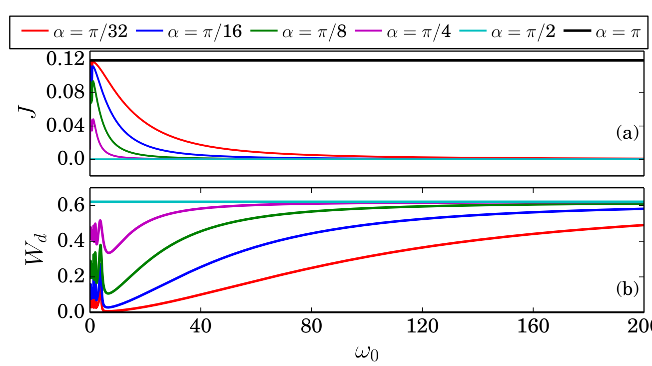

FIG. 2. (Color online) Plots of (a) current J and (b) work done W_d as functions of ω_0 for large values of ω_0 and several values of α, with $L=100$ and $\nu=0.2$. J (W_d) stays at a higher (lower) value for small values of α. J and W_d asymptotically saturate to zero and a finite value, respectively. For the special value $\alpha=\pi$, J sticks to the initial value as ω_0 is varied, while for $\alpha=\pi/2$, J always stays at zero.

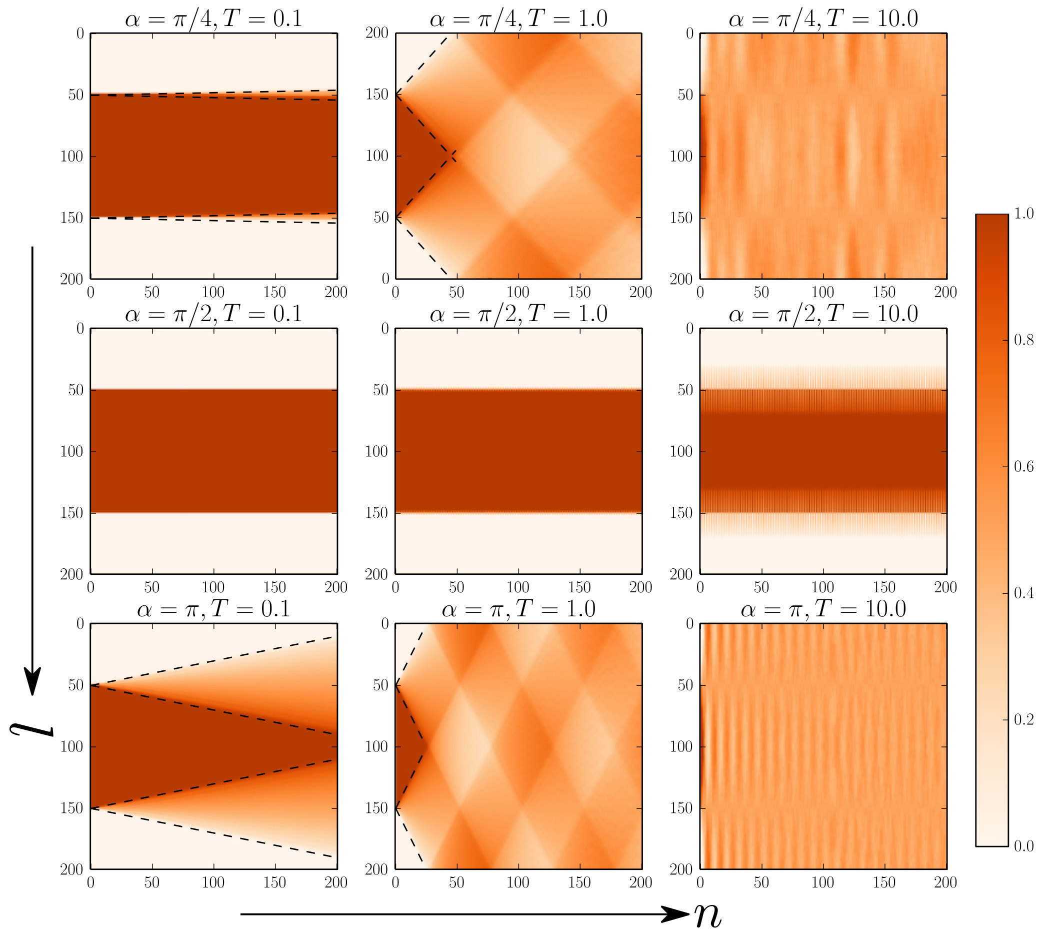

FIG. 3. (Color online) Pictures showing the density of particles in a 200-site system as a function of the stroboscopic time $t = nT$ (on the x axis) and the location l (on the y axis) for various values of T and α. See text for details.

of the maxima in W_d are therefore determined by the condition $2T(\cos k)|_{k=0} = m\pi$, i.e., $\omega_0 = 4/m$, where m is an integer. Indeed, we see that W_d has peaks around $\omega_0 = 4,2,1.3,\ldots$ in Fig. 1(b). We now turn to the dips in J. For small values of ν, we see from Eq. (8) that the integral expression for J goes over a small range from $-\pi/2$ to $-\pi/2+\nu$. The integrand $f(k)\sin k$ vanishes at the lower limit $k=-\pi/2$; it also vanishes at the upper limit if $2T\cos(-\pi/2+\nu) = (4\pi/\omega_0)\sin\nu = m\pi$, where m is an integer. We therefore expect that the entire integral will show a dip as a function of ω_0 if $\omega_0 = (4\sin\nu)/m$. For $\nu = 0.2$, we expect J to show dips around $\omega_0 = 0.8,0.4,0.26,\ldots$, as shown in Fig. 1(a). For large ω_0, J (W_d) asymptotically saturate to zero [$(2/\pi)\cos\nu$]; see Figs. 2(a) and 2(b), where we show the variation of J and W for the entire range of ω_0. We also see that J approaches zero for smaller values of ω_0 as α increases; this is in accordance with Eq. (9) since $\cot\alpha$ decreases as α increases from zero to $\pi/2$.

Finally, we summarize our observations on the dependences of J and W_d on α. (i) J and W_d remain at the constant values $(2/\pi)\sin\nu$ and zero for all ω_0 for the special values $\alpha = m\pi$. (ii) J (W_d) remains at zero [$(2/\pi)\cos\nu$] for any ω_0 for $\alpha = (m+1/2)\pi$. In this case, the Floquet quasistates $|\Phi_k^{\pm}\rangle$ have zero expectation values for the matrix σ^z appearing in the expression for the current. (iii) The magnitude of J (W_d) decreases (increases) as α increases from zero to $\pi/2$.

The DL which occurs in either of the limits $T \to 0$ or $\alpha = \pi/2$ is illustrated in Fig. 3. The panels show the density of particles in a 200-site system as a function of the time $t = nT$ (along the x axis) and the location l (along the y axis) for various values of T and α. The initial state at $t = 0$ is one in which sites 51 to 150 have one particle each (shown by dark regions) and the remaining sites are empty (shown by light regions). As t increases, the particles spread out with group velocities given by $v_k^{\pm} = d\mu_k^{\pm}/dk$. The spreading occurs in light-cone-like regions whose slopes $dl/dt = (1/T)dl/dn$ are given by the maximum value of $|v_k^{\pm}|$ as a function of k; these are shown by the black dashed lines. It can be shown from Eq. (6) that the maximum velocity goes to zero as either $T \to 0$ or $\alpha \to \pi/2$. [For instance, if $\alpha = \pi/2$, we find that $\mu_k^{\pm} = \pm\pi/(2T)$, so that $v_k^{\pm} = 0$ for all k.] This clearly demonstrates the DL. While light-cone-like effects have been studied following a quantum quench both theoretically [32] and experimentally [33], our work appears to be the first to study this in the context of periodic driving. (We remark that the ripples appearing in Fig. 3 in the panel for $\alpha = \pi/2, T = 10.0$ are finite-size effects).

IV. CONCLUSIONS

To summarize, we have explored the consequences of applying periodic δ-function kicks in the staggered on-site potential on the current-carrying ground state of a HCB chain. In the long-time limit ($n \to \infty$), there is an onset of DL if either the frequency of driving is large or the driving amplitude takes some particular values. We conclude with the remark that the DL occurring as a result of the periodic driving is not special to the one-dimensional model of hard core bosons discussed here; it can also be shown to occur in models of noninteracting fermions on a variety of higher-dimensional lattices (such as square and cubic lattices) with a periodic driving of a staggered on-site potential. In the future it may be interesting to study the effect of interactions between fermions on DL.

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