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Dissipationless kinetics of one-dimensional interacting fermions

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We study the problem of evolution of a density pulse of one-dimensional interacting fermions with a nonlinear single-particle spectrum. We show that, despite the non-Fermi-liquid nature of the problem, nonequilibrium phenomena can be described in terms of a kinetic equation for certain quasiparticles related to the original fermions by a nonlinear transformation which decouples the left- and right-moving excitations. Employing this approach, we investigate the kinetics of the phase-space distribution of the quasiparticles and thus determine the time evolution of the density pulse. This allows us to explore a crossover from the essentially free-fermion evolution for weak or short-range interaction to hydrodynamics emerging in the case of sufficiently strong, long-range interaction.

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Understanding nonequilibrium phenomena is one of the central themes in condensed matter physics. For Fermi-liquid systems (e.g., electrons in metals) such phenomena are conventionally described in the framework of a quantum kinetic equation for quasiparticle excitations. According to Landau Fermi-liquid theory, it has the same form as for weakly interacting particles up to a renormalization of parameters (effective mass, interaction constants, and scattering integral). This equation governs the evolution of a single-particle density matrix (characterizing the quasiparticle phase space distribution) and readily yields various physical observables [1–3].

For a variety of strongly interacting fermionic systems, the Fermi-liquid theory (at least, in its standard form) is not applicable: interaction destroys the quasiparticle pole. In these cases one has to find an alternative way to describe transport and nonequilibrium phenomena. This is usually done by formulating effective theories in terms of some collective degrees of freedom. A famous realization of a non-Fermi-liquid state is provided by one-dimensional (1D) interacting fermions. This system is characterized by a strongly correlated ground state—Luttinger liquid (LL) [4–8]—which exhibits an infrared divergence of an electronic self-energy, eliminating the quasiparticle pole from the spectral function. This manifests itself in a power-law suppression of the tunneling (zero-bias anomaly) and indicates that quasiparticle excitations are ill defined. A well-known tool for dealing with such correlated 1D systems is bosonization [4–8]. After linearization of the fermionic spectrum, it allows one to map the problem onto one of noninteracting bosons. For arbitrary distribution functions, the nonequilibrium bosonization yields results for LL correlation functions in terms of singular Fredholm determinants [9,10].

In this work we explore the kinetics of interacting 1D fermions, having in mind the following model setup. Initially, a density perturbation (hump or dip of amplitude $\Delta\rho$ and width Δx) is created by an external potential. At time $t = 0$ the potential is switched off, and electronic pulses start to propagate to the right and to the left. The evolution of the electronic density as a function of time is measured. While experiments of this type are particularly natural in the context of cold atomic gases [11,12], we expect them to also be feasible for electronic systems [13]. Since for a linearized spectrum the pulse moves without changing its form, a curvature $1/m$ of the single-particle spectrum is absolutely essential for the problem under consideration. Specifically, due to the combined effect of Fermi statistics and curvature, the top of the pulse moves faster than its bottom. Thus, a tendency to "overturn" of the pulse develops at a certain time $t_c \sim m\Delta x/\Delta\rho$, making the pulse evolution for times $t > t_c$ a challenging problem [14].

The nonlinearity of a fermionic spectrum induces an interaction between bosonic collective modes [15–21], giving rise to a quantum hydrodynamic theory. Such "nonlinear Luttinger liquids" arise in a variety of fermionic, bosonic, and spin systems and have recently attracted considerable attention [22,23].

A natural idea is to try to tackle the interaction between the bosonic modes perturbatively [24]. As it turns out, the 1D character of the problem induces infrared singularities invalidating the naive perturbative expansion. The bosonized theory is treatable only in the limit of strong and long-ranged interaction, which justifies the saddle-point approximation, as was done in Ref. [14] for the Calogero model and in Ref. [25] for a generic interaction. Equations of motion obtained in this way can be viewed as Euler and continuity equations for an ideal fluid, and therefore the system is described by a nondissipative classical hydrodynamics. Depending on the sign of the initial pulse, an interplay between nonlinearity and dispersion leads to the emergence of strong density oscillations or of solitons after the shock [25].

The problem has been also studied in the opposite limit of free fermions [25,26], where the evolution of the Wigner function is described by a simple kinetic equation. For sufficiently long times, $t > t_c$, a population inversion occurs [27], leading to density oscillations that can be viewed as Friedel-type oscillations between different Fermi edges.

Thus, the pulse evolution was analyzed in two opposite limits (no vs strong long-range interaction) by different means (fermionic vs bosonic), and within different physical pictures (inverted population vs hydrodynamic waves). We now address this problem for an arbitrary interaction. By bosonizing the system, performing a certain unitary transformation and refermionizing it, we explicitly build corresponding quasiparticle operators and formulate a kinetic description in their terms. The latter describes, in particular, the sought density evolution.

The problem is characterized by a Hamiltonian $H = H_0 + H_{\rm int}$, where the kinetic part H_0 describes two spinless chiral modes (labeled by subscript $\eta = R,L$ or, occasionally, $\eta = \pm 1$) with a nonlinear spectrum

$$H_0 = \sum_{\eta,k} \eta k v_F : a^+_{\eta k} a_{\eta k} : + (1/2m) \sum_{\eta,k} k^2 : a^+_{\eta k} a_{\eta k} : . \quad (1)$$

The interaction part reads

$$H_{\rm int} = (1/2) \int dx_1 dx_2 g(x_1 - x_2) \rho(x_1) \rho(x_2), \quad (2)$$

where $\rho = \rho_L + \rho_R$ is the density. The kinetic term can be bosonized as follows [15]:

$$H_0 = \pi v_F \int dx \left(\rho_R^2 + \rho_L^2\right) + (4\pi^2/6m) \int \left(\rho_R^3 + \rho_L^3\right) \quad (3)$$

with Fourier components of the densities satisfying the standard commutation relations (L is the system length) $[\rho_{\eta,q}, \rho_{\eta',-q'}] = \eta \delta_{\eta,\eta'} \delta_{q,q'} Lq/2\pi$. The interaction mixes the chiral sectors. On the quadratic level, this coupling can be eliminated by a canonical transformation $R_q = U_2 \rho_{R,q} U_2^\dagger$, $L_q = U_2 \rho_{L,q} U_2^\dagger$ of the standard Bogoliubov form

$$\rho_{R,q} = \cosh \kappa_q R_q - \sinh \kappa_q L_q, \quad (4)$$

$$\rho_{L,q} = -\sinh \kappa_q R_q + \cosh \kappa_q L_q, \quad (5)$$

where $\tanh 2\kappa_q = g_q/(2\pi v_F + g_q)$. In terms of new fields, the quadratic part is

$$H^{(2)} = (\pi/L) \sum_q u_q (R_q R_{-q} + L_q L_{-q}) \quad (6)$$

with a sound velocity $u_q = v_F (1 + g_q/\pi v_F)^{1/2} = v_F/K_q$.

As a side effect of Bogoliubov transformation, the cubic part of the Hamiltonian acquires a form that mixes the right and left movers:

$$H^{(3)} = (2\pi^2/3mL^2) \sum_{\mathbf{q}} \Gamma_{\mathbf{q}} [(R_1 R_2 R_3 + L_1 L_2 L_3) + 3\Gamma'_{\mathbf{q}} (R_1 R_2 L_3 + L_1 L_2 R_3)]. \quad (7)$$

Here we have introduced notations $\mathbf{q} \equiv \{q_1, q_2, q_3\}$, $R_i = R_{q_i}$, $L_i = L_{q_i}$; the summation over $\mathbf{q}$ is restricted to $q_1 + q_2 + q_3 = 0$ and we have defined vertices ($\kappa_i \equiv \kappa_{q_i}$)

$$\begin{aligned} \Gamma_{\mathbf{q}} &= \operatorname{ch}\kappa_1 \operatorname{ch}\kappa_2 \operatorname{ch}\kappa_3 - \operatorname{sh}\kappa_1 \operatorname{sh}\kappa_2 \operatorname{sh}\kappa_3, \\ \Gamma'_{\mathbf{q}} &= \operatorname{sh}\kappa_1 \operatorname{sh}\kappa_2 \operatorname{ch}\kappa_3 - \operatorname{ch}\kappa_1 \operatorname{ch}\kappa_2 \operatorname{sh}\kappa_3 . \end{aligned} \quad (8)$$

The decoupling of the right and left sectors of the theory can be extended to the cubic level. To this end, we perform an additional unitary transformation $\tilde{\rho}_R = U_3 R U_3^\dagger$ and $\tilde{\rho}_L = U_3 L U_3^\dagger$, determined by the operator

$$U_3 = \exp \sum_{\mathbf{q}} [f_{\mathbf{q}} R_1 R_2 L_3 - (L \leftrightarrow R)], \quad (9)$$

where

$$f_{\mathbf{q}} = \frac{2\pi^2}{mL^2} \frac{\Gamma'_{\mathbf{q}}}{u_{q_1} q_1 + u_{q_2} q_2 - u_{q_3} q_3} . \quad (10)$$

After this transformation, the Hamiltonian H mixes the left and right modes only due to the terms quartic in the density

$$H = (\pi/L) \sum_{\eta,q} u_q \tilde{\rho}_{\eta,q} \tilde{\rho}_{\eta,-q} + (2\pi^2/3mL^2) \sum_{\eta,\mathbf{q}} \Gamma_{\mathbf{q}} \tilde{\rho}_{\eta,1} \tilde{\rho}_{\eta,2} \tilde{\rho}_{\eta,3} + O(\tilde{\rho}^4) . \quad (11)$$

The $O(\tilde{\rho}^4)$ terms are small and will be neglected.

We have thus obtained a chiral bosonic theory (11), with interaction originating from the nonlinearity of the fermionic spectrum and a q-dependent sound velocity originating from the electron-electron interaction. We now proceed by refermionizing this theory, following the idea put forward in Ref. [28] (see also [22,29]), where such a mapping was performed after the conventional Bogoliubov transformation U_2. It is crucial for our problem that we also carry out the transformation U_3, decoupling the chiral sectors, and only then refermionize. More specifically, we define "composite fermion" operators that are built from the original ones by consecutive rotations

$$\tilde{\Psi}_\eta = U_3 U_2 \Psi_\eta U_2^\dagger U_3^\dagger . \quad (12)$$

Since the rotation is exponential in the density fields, this somewhat resembles the composite-fermion transformation in the fractional quantum Hall regime. In terms of the new operators, the Hamiltonian is given by

$$H = \sum_{\eta,k} \tilde{\Psi}^\dagger_{\eta,k} \left(\eta u_0 k + \frac{k^2}{2m^*} \right) \tilde{\Psi}_{\eta,k} + \frac{1}{2L} \sum_{\eta,q} V_q \tilde{\rho}_{\eta,q} \tilde{\rho}_{\eta,-q} + \frac{2\pi^2}{3mL^2} \sum_{\eta,\mathbf{q}} \gamma_{\mathbf{q}} \tilde{\rho}_{\eta,q_1} \tilde{\rho}_{\eta,q_2} \tilde{\rho}_{\eta,q_3} . \quad (13)$$

The quadratic part of the Hamiltonian (13) is parametrized by the renormalized Fermi velocity $u_0 \equiv u_{q=0}$ and the spectral curvature

$$1/m^* \simeq \Gamma_{\mathbf{q}=0}/m . \quad (14)$$

There is also a residual interaction between particles represented by two-particle and three-particle vertices

$$V_q = 2\pi(u_q - u_0), \qquad \gamma_{\mathbf{q}} = \Gamma_{\mathbf{q}} - \Gamma_{\mathbf{q}=0} . \quad (15)$$

The residual interaction V_q vanishes at low momenta ($V_q \propto q^2$ for a generic finite-range interaction) and is irrelevant in the renormalization-group sense. The three-body interaction is still weaker ($\gamma_{\mathbf{q}} \propto \mathbf{q}^2$ and, in addition, contains the factor $\rho/m^* u \ll 1$) and we neglect it from now on [30]. The disappearance of the interaction at small momenta makes perturbation theory for the composite fermions regular in the

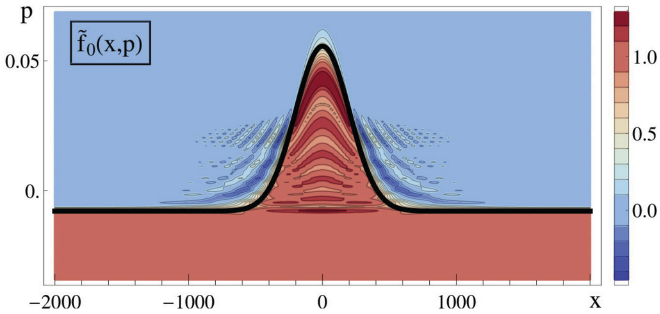

FIG. 1. (Color online) The initial quasiparticle Wigner function $\tilde{f}_0(x,p)$. The thick black line shows the classical Fermi surface $p_F(x) = 2\pi\rho_0(x)$.

infrared limit, and the system behaves as a weakly interacting Fermi gas.

We define the quasiparticle density matrix

$$\tilde{f}_\eta(x,y,t) = \langle \tilde{\Psi}^\dagger_\eta(x - y/2,t)\tilde{\Psi}_\eta(x + y/2,t)\rangle = \int (dp/2\pi) e^{ipy} f_\eta(x,p,t) \tag{16}$$

that within the Hartree approximation satisfies the collisionless quantum kinetic equation

$$\partial_t \tilde{f}_\eta(p,x,t) + (p/m^*)\partial_x \tilde{f}_\eta(p,x,t) + \int (dp/2\pi) e^{-ipy} \times \tilde{f}_\eta(x,y,t)[\tilde{\phi}_\eta(x + y/2) - \tilde{\phi}_\eta(x - y/2)] = 0, \tag{17}$$

with the self-consistent electric field

$$\tilde{\phi}_\eta(x,t) = \int dx' V(x - x')\tilde{\rho}_\eta(x',t). \tag{18}$$

To obtain the physical density ρ out of the solution $\tilde{\rho}$ one needs to use the relation between the densities; in the leading order $\rho \simeq \sqrt{K}\tilde{\rho}$ (see [31]). Note that Eq. (17) is exact in the limits of noninteracting electrons and of a harmonic LL ($m \to \infty$, arbitrary electron interaction) (see [31]).

In order to analyze the pulse dynamics, we solve Eq. (17) numerically (see [31]), focusing on times exceeding the "shock formation time" $t_c \sim m\Delta x/\Delta\rho$ when the phase-space distribution of noninteracting fermions develops an inverse population. The Wigner function in the initial state was discussed in Refs. [25,32] (see also [31]). We plot it in Fig. 1 for a Gaussian density hump [$\tilde{\rho}_0(x) = \Delta\rho \exp(-x^2/2\sigma^2)$ with $\sigma = 200/mv_F$ and $\Delta\rho = 0.01mv_F$] in the initial state. Besides changing from 0 to 1 at classical Fermi surface $p_F(x) = 2\pi\tilde{\rho}_0(x)$, the Wigner function exhibits phase-space oscillations (that do not manifest themselves in the total density for a spatially smooth hump).

While our approach is very general, we now focus on a model of finite-range interaction

$$g(q) = (1/l_0 m)\exp\left(-q^2 l_{\text{int}}^2\right), \tag{19}$$

with two lengths l_0 and l_{int} parametrizing its strength and range. The classical hydrodynamics emerges if two conditions are fulfilled:

$$l_0\Delta\rho \ll 1, \qquad l_{\text{int}}^2\Delta\rho/l_0 \gg 1. \tag{20}$$

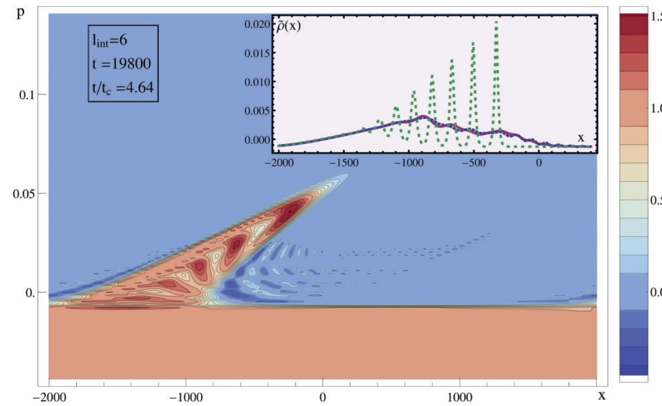

FIG. 2. (Color online) Quasiparticle phase-space distribution (Wigner function) for a short interaction range, $l_{\text{int}} = 6/mv_F$, at $t = 4.6t_c$. Inset: corresponding density (solid red line) in comparison to the density of noninteracting fermions (blue dotted) and the predictions of classical hydrodynamic theory (dashed green).

In the opposite limit (if at least one of the inequalities $l_0\Delta\rho \gg 1$ and $l_{\text{int}}^2\Delta\rho/l_0 \ll 1$ is fulfilled) the solution remains close to that for free fermions. To illustrate the behavior of the solution of the kinetic equation (17) in both regimes and in a crossover between them, we fix $l_0 = 1/mv_F$ and $\Delta\rho = 0.01mv_F$ such that the first of the conditions (20) is well fulfilled and vary l_{int}.

For a sufficiently short-range interaction, an inverse population develops for $t > t_c$. This is demonstrated in Fig. 2, where a snapshot of the phase space at time $t = 4.64t_c$ is shown for interaction range $l_{\text{int}} = 6/mv_F$. In this case the second parameter of Eq. (20) is relatively small, i.e., $l_{\text{int}}^2\Delta\rho/l_0 = 0.36$. The inset shows the corresponding density in comparison to that of noninteracting fermions and the predictions of hydrodynamic theory. As one sees, the interacting density is close to that of free fermions, meaning that the composite fermion interaction effects are weak, as expected. It should be emphasized that the original electron interaction may well be strong in this regime; i.e., the parameter $1/l_0 mv_F$ does not need to be small. (In our modeling it is equal to unity and can also be larger.) As for free fermions, one observes oscillations of the total density that originate from the phase-space oscillations in the initial state and develop in the region where the inverse population is formed [25,26]. We also provide a comparison with the density calculated by using a classical hydrodynamic equation (obtained as a saddle point of the bosonic theory). Clearly, the classical hydrodynamics, which yields much stronger oscillations, is not a proper way to describe the system in this regime of weakly interacting quasiparticles.

As the quasiparticle interaction becomes stronger ($l_{\text{int}} = 20/mv_F$), the density significantly deviates from the free-fermion limit and the agreement with the hydrodynamics improves (see Fig. 3). However, the system still shows clear traces of the population inversion leading to deviations from the hydrodynamic solution that proliferate with time and become quite substantial at $t = 4.6t_c$. In this intermediate regime neither the free-fermion model nor the hydrodynamic approximations are valid, and the kinetic approach is the only adequate tool to controllably address the problem.

With a further increase of the interaction strength ($l_{\text{int}} = 40/mv_F$) the agreement between hydrodynamic and kinetic

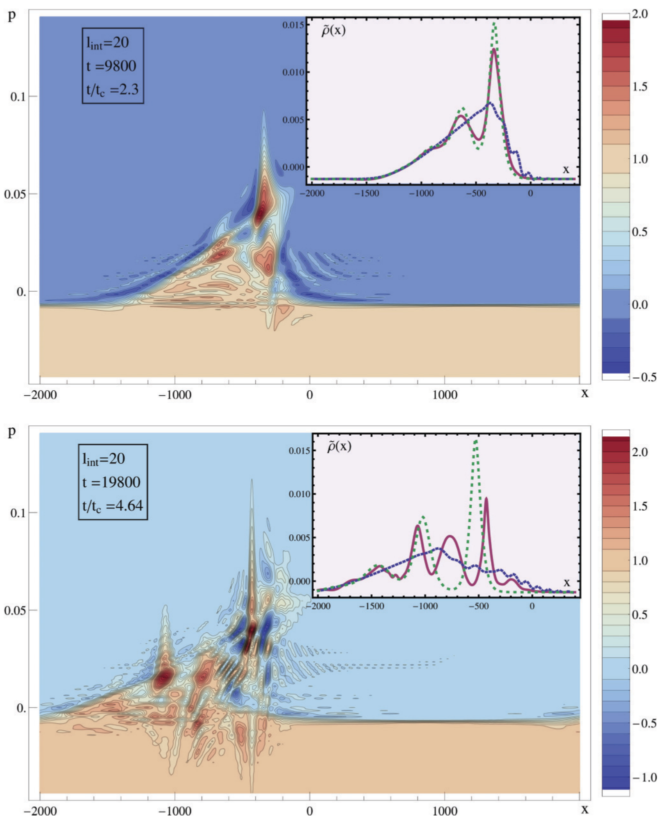

FIG. 3. (Color online) Same as in Fig. 2 but for a medium-range interaction, $l_{\rm int} = 20/mv_F$, at $t = 2.3t_c$ (top) and $t = 4.64t_c$ (bottom).

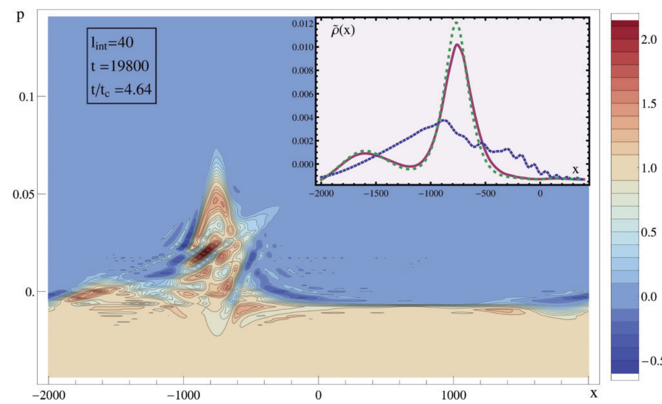

FIG. 4. (Color online) Same as in Fig. 2 but for a long-range interaction, $l_{\rm int} = 40/mv_F$, at time $t = 4.6t_c$.

approaches is reached (see Fig. 4). In this regime the phase-space distribution is approximately given by a Fermi function with a position-dependent Fermi momentum $p_F(x)$, determined by the classical hydrodynamic equation. On top of the sharp Fermi surface, we observe an additional "fine structure" in the phase-space distribution, shown in Fig. 4. It remains to be seen whether these details of the quantum state, which are beyond the hydrodynamic picture, lead to strong deviations from the hydrodynamic solution at the longer times [33].

In addition to the self-consistent electric field, the quasiparticle interaction in Eq. (13) causes inelastic quasiparticle scattering. When taken into account, these processes generate a collision integral in the kinetic equation (17). Dominant contributions originate from triple collisions [34–40] and from the $\tilde{\rho}^3$ term in Eq. (13). A quick estimate shows that the rate $1/\tau_{\rm in}$ of such processes is proportional to a high power of a small parameter $\Delta\rho/mv_F$ (or of T/mv_F at finite temperature T) and is thus very small. Therefore there is a parametrically broad range of times, $t < \tau_{\rm in}$, for which the collisionless kinetic equation studied in this work is applicable. A detailed analysis of the inelastic relaxation leading to a viscous hydrodynamics at $t > \tau_{\rm in}$ will be presented elsewhere.

To summarize, we have studied the evolution of a density pulse of 1D interacting fermions with a nonlinear single-particle spectrum. We identified excitations that play a role of weakly interacting quasiparticles for nonequilibrium phenomena inside the wire and described their dynamics by a quantum kinetic equation. The evolution of the corresponding phase-space distribution is determined by two competing effects: the dispersion that tends to overturn the Fermi surface, and the quasiparticle interaction that tends to stabilize it. By numerically solving the kinetic equation, we have demonstrated a crossover from the free-fermion-like evolution for weak or short-range interaction to hydrodynamics emerging in the case of sufficiently strong, long-range interaction.

Our work shows that while 1D interacting systems are not Fermi liquids in the conventional sense, kinetic phenomena in such systems can be cast into the Landau paradigm of weakly interacting fermionic quasiparticles. We foresee numerous extensions and applications of our formalism, including other types of interaction, relaxation phenomena (also in the presence of disorder), and edge states of integer and fractional quantum Hall systems and topological insulators.

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[1] L. D. Landau, Zh. Eksp. Teor. Fiz. **30**, 1058 (1956) [Sov. Phys. JETP **3**, 920 (1957)].

[2] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics, Part 2*, (Pergamon Press, Oxford, 1980).

[3] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962).

[4] M. Stone, *Bosonization* (World Scientific, Singapore, 1994).

[5] J. von Delft and H. Schoeller, Annalen Phys. **7**, 225 (1998).

[6] A. O. Gogolin, A. A. Nersesyan, and A. M. Tsvelik, *Bosonization in Strongly Correlated Systems* (Cambridge University Press, Cambridge, 1998).

[7] T. Giamarchi, *Quantum Physics in One Dimension* (Claverdon Press, Oxford, 2004).

[8] D. L. Maslov, in *Nanophysics: Coherence and Transport*, edited by H. Bouchiat, Y. Gefen, G. Montambaux, and J. Dalibard (Elsevier, New York, 2005), p. 1.

[9] I. V. Protopopov, D. B. Gutman, and A. D. Mirlin, Phys. Rev. Lett. **110**, 216404 (2013); Lith. J. Phys. **52**, 165 (2012).

[10] D. B. Gutman, Y. Gefen, and A. D. Mirlin, J. Phys. A: Math. Theor. **44**, 165003 (2011); Phys. Rev. B **81**, 085436 (2010).

[11] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).

[12] T. Lahaye, C. Menotti, L. Santos, M. Lewenstein, and T. Pfau, Rep. Prog. Phys. **72**, 126401 (2009).

[13] The technique for fast time-resolved (with resolution of tens of picoseconds) transport measurements in quantum Hall edges was developed long ago; see R. C. Ashoori, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. West, Phys. Rev. B **45**, 3894 (1992); It was used recently to measure charge fractionalization in counterpropagating edge states; see H. Kamata, N. Kumada, M. Hashisaka, K. Muraki, and T. Fujisawa, Nat. Nanotech. **9**, 177 (2014).

[14] E. Bettelheim, A. G. Abanov, and P. Wiegmann, Phys. Rev. Lett. **97**, 246401 (2006).

[15] M. Schick, Phys. Rev. **166**, 404 (1968).

[16] A. Jevicki and B. Sakita, Nucl. Phys. B **165**, 511 (1980).

[17] F. D. M. Haldane, Phys. Rev. Lett. **47**, 1840 (1981).

[18] B. Sakita, *Quantum Theory of Many-Variable Systems and Fields* (Wolrd Scientific, Singapore, 1985).

[19] A. P. Polychronakos, Phys. Rev. Lett. **74**, 5153 (1995).

[20] A. G. Abanov and P. B. Wiegmann, Phys. Rev. Lett. **95**, 076402 (2005).

[21] M. Stone, I. Anduaga, and L. Xing, J. Phys. A: Math. Theor. **41**, 275401 (2008).

[22] A. Imambekov and L. I. Glazman, Science **323**, 228 (2009); Phys. Rev. Lett. **102**, 126405 (2009).

[23] A. Imambekov, T. L. Schmidt, and L. I. Glazman, Rev. Mod. Phys **84**, 1253 (2012).

[24] D. N. Aristov, Phys. Rev. B **76**, 085327 (2007).

[25] I. V. Protopopov, D. B. Gutman, P. Schmitteckert, and A. D. Mirlin, Phys. Rev. B **87**, 045112 (2013).

[26] E. Bettelheim and L. I. Glazman, Phys. Rev. Lett. **109**, 260602 (2012).

[27] Specifically, the semiclassical Fermi surface $p_F(x,t)$ becomes three valued, $p_{F1} < p_{F2} < p_{F3}$, at some space-time points (x,t). The semiclassical distribution is then $f(p,x,t) = \Theta(p_{F_1} - p) + \Theta(p_{F_3} - p)\Theta(p - p_{F_2})$ (cf. Fig. 2).

[28] D. C. Mattis and E. H. Lieb, J. Math. Phys. **6**, 304 (1965).

[29] A. V. Rozhkov, Eur. Phys. J. **47**, 193 (2005); Phys. Rev. B **74**, 245123 (2006); **77**, 125109 (2008).

[30] The three-body interaction turns out to be important for the calculation of the inelastic relaxation time and will be discussed in detail in a separate publication.

[31] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.89.161104 for the discussion of the connection between the densities ρ and $\tilde{\rho}$ and various aspects of the kinetic equation (17).

[32] E. Bettelheim and P. B. Wiegmann, Phys. Rev. B **84**, 085102 (2011).

[33] Our numerical data make plausible the existence of a certain time $t_c' \sim t_c f(l_{\rm int}^2 \Delta\rho / l_0) \gg t_c$ where the hydrodynamics breaks down. Further investigation is required, however, to establish the form of the function $f(x)$.

[34] M. Khodas, M. Pustilnik, A. Kamenev, and L. I. Glazman, Phys. Rev. B **76**, 155402 (2007).

[35] A. M. Lunde, K. Flensberg, and L. I. Glazman, Phys. Rev. B **75**, 245418 (2007).

[36] T. Karzig, L. I. Glazman, and F. von Oppen, Phys. Rev. Lett. **105**, 226407 (2010).

[37] T. Micklitz and A. Levchenko, Phys. Rev. Lett. **106**, 196402 (2011).

[38] T. Micklitz, A. Levchenko, and A. Rosch, Phys. Rev. Lett. **109**, 036405 (2012).

[39] A. P. Dmitriev, I. V. Gornyi, and D. G. Polyakov, Phys. Rev. B **86**, 245402 (2012).

[40] Z. Ristivojevic and K. A. Matveev, Phys. Rev. B **87**, 165108 (2013).