Absence of thermospin current response of a spin-orbit-coupled two-dimensional electron gas

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We consider the spin current flowing in a two-dimensional electron gas with Rashba and linear Dresselhaus spin-orbit interaction as a linear response to a temperature gradient, taking into account the contribution due to the thermoelectric effect. We derive a relation connecting the electrically and thermally driven spin and charge conductivities, for the Hamiltonians corresponding to samples grown in the main crystallographic directions. Based on this connection, it is shown that the transverse and longitudinal spin currents generated by the temperature gradient vanish exactly for each spin-orbit Hamiltonian case. This result is in contrast to the recently predicted finite thermospin Hall effect for such class of systems.

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Low-dimensional electron systems with spin-orbit interaction (SOI) show a variety of spin-dependent effects arising from the coupling between charge and spin degrees of freedom. The most remarkable examples are the spin Hall effect [1] and the current-induced spin polarization [2], where an electric current can induce a transverse spin current and a nonequilibrium spin accumulation across the sample or near the boundaries. It has been observed in SO-coupled systems such as two-dimensional electron gases (2DEGs) or quantum wells formed in semiconductor heterostructures. In this class of systems the dominant SO contributions are the Rashba (R) and Dresselhaus (D) couplings.

On the other hand, charge and spin currents can be generated not only by electric fields, but also due to temperature gradient [3,4]. One such phenomenon is the spin-Seebeck effect, where longitudinal spin current and spin voltage are generated by a temperature gradient. This effect has been observed in spin-polarized metals [5], semiconductors [6], and insulators [7]. Recently, there is a growing interest in spin related thermoelectric effects in systems with SOI, where a temperature gradient gives rise to a spin polarization [8,9] or a spin current [10]. In addition to the spin-Seebeck effect, an intrinsic thermospin effect called the thermospin Hall current has been predicted, which refers to the creation of a transverse spin current generated by a temperature gradient and the thermoelectric effect in a 2DEG with Rashba SOI [10]. First-principles calculations of this thermospin Hall effect in crystals with impurities [11] and for some metallic alloys [12] have been recently reported.

In this paper we revisit the problem studied in Ref. [10] taking into account the simultaneous presence of both SO contributions, the Rashba and linear Dresselhaus couplings. It is well known that the interplay between these couplings in a 2DEG opens the possibility of new effects such as a nonballistic spin transistor [13], emergence of a persistent spin helix [14], vanishing of interband light absorption [15], SOI-induced anisotropies of plasmon dynamics [16], control of the spin and charge optical conductivities or of the electric-field-induced spin polarization [17,18], just to mention a few. Here, we investigate the longitudinal and transverse spin currents generated by a temperature gradient and the thermoelectric effect, in a 2DEG with both types of SO coupling. We calculate the corresponding thermospin conductivity tensor describing the spin current response to a spatially homogeneous temperature gradient under the usual Seebeck conditions. This thermospin current arises from the direct response to a temperature gradient and from the spin Hall effect induced by the thermoelectric field. We find that these two contributions cancel each other yielding a null thermospin current. This result is in remarkable contrast to the predicted finite effect mentioned above [10].

We consider a two-dimensional free electron gas lying at the $z=0$ plane described by the one-electron Hamiltonian $H_0=\hbar^2k^2/2m+H_{\rm SO}$ with spin-orbit contribution

$$H_{\rm SO}=\frac{\hbar}{2}\,\Omega_i\sigma_i=\sigma_i\mu_{ij}k_j\quad (i,j=x,y,z), \tag{1}$$

where we use the convention of sum over any repeated index, σ_i are the Pauli matrices, and the effective spin-orbit field $(\hbar/2)\mathbf{\Omega}(\mathbf{k})$ is assumed linear in the electron in-plane wave vector $\mathbf{k}=(k_x,k_y,0)$; m is the effective mass. The matrix $\mu_{ij}=(\hbar/2)\partial\Omega_i/\partial k_j$ contains the parameters characterizing the strengths of SO couplings due to structural inversion asymmetry (Rashba coupling) and bulk inversion asymmetry (Dresselhaus coupling). For narrow quantum wells (QW) grown along the [001], [110], and [111] directions, this matrix takes the forms

$$\mu_{ij}=\begin{pmatrix}-\beta_{[001]} & \alpha & 0\\ -\alpha & \beta_{[001]} & 0\\ 0 & 0 & 0\end{pmatrix},\quad \mu_{ij}=\begin{pmatrix}0 & \alpha & 0\\ -\alpha & 0 & 0\\ \beta_{[110]} & 0 & 0\end{pmatrix}$$

$$\mu_{ij}=\begin{pmatrix}0 & \tilde{\alpha} & 0\\ -\tilde{\alpha} & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, \tag{2}$$

respectively, where α is the SO coupling strength of the Rashba interaction, $\beta_{[hkl]}$ is the SO parameter of the Dresselhaus coupling of a sample grown in the crystalloghraphic direction $[hkl]$, and $\tilde{\alpha}=\alpha+\beta_{[111]}$. For a [001]-grown QW, the coordinate system x,y,z is $x\parallel[100], y\parallel[010], z\parallel[001]$; for a [110]-grown QW, it is $z\parallel[110]$, $x\parallel[\bar{1}10]$, $y\parallel\mathbf{z}\times\mathbf{x}\parallel[001]$;

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and for a [111]-grown QW, it is $z \parallel [111]$, $y \parallel [\bar{1}10]$, $x \parallel \mathbf{y} \times \mathbf{z} = [11\bar{2}]$ [19]. The energy spectrum of the above Hamiltonian is $\varepsilon_\lambda(\mathbf{k}) = \hbar k^2/2m + \lambda|\Omega(\mathbf{k})|/2$, where $\lambda = \pm$ specifies the chirality of the spin states $|\mathbf{k}\lambda\rangle$ and the upper (+) and lower (−) parts of the spectrum. The energy spin splitting is determined by $\varepsilon_+(\mathbf{k}) - \varepsilon_-(\mathbf{k}) = \hbar|\mathbf{\Omega}(\mathbf{k})| = 2\sqrt{\mu_{ij}\mu_{il}k_jk_l}$. We shall use the symbol R + D[hkl] to denote each Hamiltonian case.

The ξ-current density generated by an electric field $\mathbf{E}$ and a temperature gradient ∇T is phenomenologically given by

$$J_i^{(\xi)} = L_{ij}^{(\xi e)} E_j + L_{ij}^{(\xi q)}\left(-\frac{\nabla_j T}{T}\right), \tag{3}$$

where $\xi = e,q,s$ denotes an electric charge, heat, or spin current, and the transport coefficients $L_{ij}^{(\xi\nu)}$ are the corresponding conductivity tensors with spatial subscripts $i,j = x,y$. Given that is usual in measurements, we consider open circuit conditions where the electrical current vanishes, $\mathbf{J}^{(e)} = 0$. This implies that there will be an internal electric field $E_i = S_{ij}\nabla_j T$, in accordance to expression (3), related to the externally driving thermal force $-\nabla T/T$ through the thermopower $S_{ij} = [L^{(ee)}]^{-1}_{il} L^{(eq)}_{lj}/T$, where $L_{ij}^{(ee)}$ and $L_{ij}^{(eq)}$ are the electric and thermoelectric conductivity tensors, respectively.

Under this condition, there is a spin current associated with the temperature gradient via the thermoelectric effect

$$J_i^{(s)z} = \Sigma_{ij}^{(s)z}\nabla_j T, \tag{4}$$

flowing along the i direction with the spin polarized in the z direction, where [see Eq. (3)]

$$\Sigma_{ij}^{(s)z} = \frac{1}{T}\left[T L_{il}^{(se)z} S_{lj} - L_{ij}^{(sq)z}\right] \tag{5}$$

$$= L_{il}^{(se)z}\left(S_{lj} - S_{lj}^{(s)z}\right) \tag{6}$$

is the zero charge current thermospin conductivity tensor. The second line writes this tensor in terms of the spin thermopower $S_{ij}^{(s)z} = [L^{(se)z}]^{-1}_{il} L^{(sq)z}_{lj}/T$ [20]. Without loss of generality, we choose the temperature gradient ∇T along the x direction ($\nabla_y T = 0$ and $\nabla_x T \neq 0$). Thus, the only components in (5) are

$$\Sigma_{xx}^{(s)z} = \frac{1}{T}\left[T\left(L_{xx}^{(se)z} S_{xx} + L_{xy}^{(se)z} S_{yx}\right) - L_{xx}^{(sq)z}\right], \tag{7}$$

$$\Sigma_{yx}^{(s)z} = \frac{1}{T}\left[T\left(L_{yx}^{(se)z} S_{xx} + L_{yy}^{(se)z} S_{yx}\right) - L_{yx}^{(sq)z}\right]. \tag{8}$$

The transverse component $\Sigma_{yx}^{(s)z}$, named thermospin Hall conductivity, determines a spin current $J_y^{(s)z}$ that flows perpendicularly to the gradient $\nabla_x T$, and describes a thermospin Hall effect (via thermoelectric effect). The longitudinal conductivity $\Sigma_{xx}^{(s)z}$ is relevant to a spin-Seebeck-type effect which relates an applied thermal gradient to an induced spin current $J_x^{(s)z}$ flowing parallel to it. The transport coefficients $L_{ij}^{(\xi\nu)}$ can be obtained within the linear response theory [21]. Indeed, through a purely Hamiltonian approach with external fields, the response to a thermal perturbation can be found by introducing an auxiliary vector potential $\mathbf{A}^{(q)}$ which works as a driving force for the heat current, analogous to the electrical conductivity $L_{ij}^{(ee)}$ describing the response to an electromagnetic vector potential $\mathbf{A}^{(e)}$ [8]. For a spatially homogeneous electric field and a temperature gradient, the corresponding vector potentials are given by $\mathbf{A}^{(e)} = (c/i\omega)\mathbf{E}e^{-i\omega t}$ and $\mathbf{A}^{(q)} = -(1/i\omega)(\nabla T/T)e^{-i\omega t}$ [10,22]. In the frequency domain [23,24], the Kubo formulas for the ξ-current–ν-current correlation functions $L_{ij}^{(\xi\nu)}(\omega)$ are

$$L_{ij}^{(\xi\nu)}(\omega) = \frac{1}{\hbar\tilde{\omega}}\int_0^\infty dt\, e^{i\tilde{\omega}t}\left\langle\left[\hat{J}_i^{(\xi)}(t),\hat{J}_j^{(\nu)}(0)\right]\right\rangle, \tag{9}$$

with $\tilde{\omega} = \omega + i0^+$. The symbol $\langle[\hat{A}(t),\hat{B}(0)]\rangle = \sum_\lambda \int d^2k\, f[\varepsilon_\lambda(\mathbf{k})]\langle\mathbf{k}\lambda|[\hat{A}(t),\hat{B}(0)]|\mathbf{k}\lambda\rangle$ indicates quantum and thermal averaging of the commutator of the operators $\hat{A}$ and $\hat{B}$, and $f[\varepsilon_\lambda(\mathbf{k})] = (1 + e^{(\varepsilon_\lambda(\mathbf{k})-\mu)/k_BT})^{-1}$ is the Fermi distribution function, where μ is the chemical potential, and k_B is the Boltzmann constant. The charge, heat-, and spin-current operators are defined as $\hat{J}_i^{(e)} = e\hat{v}_i$, $\hat{J}_i^{(q)} = \{H_0,\hat{v}_i\}/2 - \mu\hat{v}_i$, and $\hat{J}_i^{(s)a} = \hbar\{\hat{v}_i,\sigma_a\}/4$, respectively, where $\hat{v}_i = \partial H_0/\hbar\partial k_i$ is the velocity operator, and $\{\cdot,\cdot\}$ means the anticommutator; the spin index $a = x,y,z$ refers to the direction of the spin.

Figure 1 shows the Hall component Re $L_{xy}^{(sq)z}(\omega)$ for a sample with R + D[001] SOI, for several values of the ratio $\beta_{[001]}/\alpha$. The spectral features can be explained in terms of the nonisotropic momentum space available for electric-dipole inter-spin-split subband transitions and the presence of critical points in the joint density of states [17]. The spectra differs notably from the pure Rashba or R + D[111] case which presents an isotropic spin splitting of the subbands $\varepsilon_\lambda(k_x,k_y)$. Besides the temperature dependence and the tunability of the Rashba interaction through electrical gating, the spectral characteristics shown in Fig. 1 suggests new possibilities of spin manipulations via thermal perturbations.

As for the thermospin conductivity components (7) and (8), it is useful to consider relations between some of the coefficients $L_{ij}^{(\xi\nu)}(\omega)$. Several authors have established connections between the electric conductivity $L_{ij}^{(ee)}$ and the spin current conductivity $L_{ij}^{(se)}$ in the presence of SOI [17,18,25,26], even under very general conditions, such as arbitrary (spin-independent) disorder, electric-field strength, and for

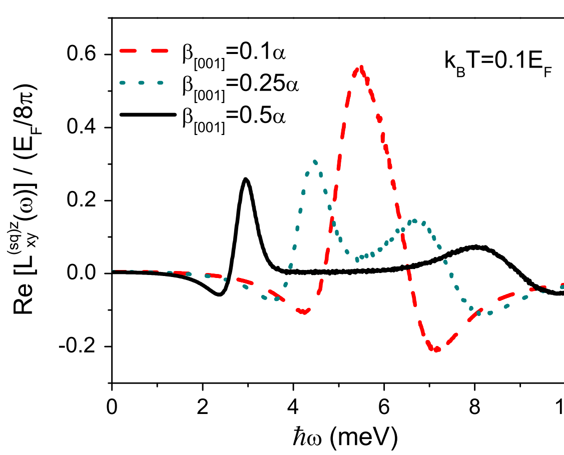

FIG. 1. (Color online) Transverse spin-current–heat-current response function Re $L_{xy}^{(sq)z}(\omega)$ of a [001]-grown quantum well with Rashba and Dresselhaus SO coupling. The parameters used are $m = 0.055m_0$, electron density $n = 5 \times 10^{11}$ cm^{-2}, $\alpha = 160$ mev Å, and $k_BT = 0.1E_F$; the Fermi energy is $E_F = 23.7$ meV.

interacting electrons [27,28]. In the following we derive these connections, including the thermal coefficients $L_{ij}^{(eq)}$ and $L_{ij}^{(sq)}$, and show through them that the thermospin current response [Eq. (5)] vanishes for a 2DEG with intrinsic R + D[hkl] SOI in the collisionless regime.

We start from the coefficient $L_{ij}^{(ev)}(\omega)$,

$$L_{ij}^{(ev)}(\omega) = \frac{i}{\hbar\tilde{\omega}^2}\int_0^\infty dt\, e^{i\tilde{\omega}t}\left\langle\left[\frac{d\hat{J}_i^{(e)}(t)}{dt},\hat{J}_j^{(\nu)}(0)\right]\right\rangle, \tag{10}$$

obtained from (9) after integration by parts. The electric current operator $\hat{J}_i^{(e)}(t)$ has a complicated dynamics determined by the SOI, $i\hbar\, d\hat{J}_i^{(e)}/dt = e[\hat{v}_i^{\rm SO},H_{\rm SO}]$, where $\hat{v}_i^{\rm SO} = \partial H_{\rm SO}/\hbar\partial k_i = \sigma_j\mu_{ji}/\hbar$ is the anomalous velocity,

$$\frac{d\hat{J}_i^{(e)}}{dt} = \frac{2e}{\hbar^2}k_l\boldsymbol{\sigma}\cdot(\boldsymbol{\mu}_i\times\boldsymbol{\mu}_l), \tag{11}$$

where $\boldsymbol{\mu}_j$ is the vector with components μ_{ij} defined by the jth column of the matrix of SO parameters (2). We can use the expression $\hat{J}_i^{(s)a} = \hbar^2k_i\sigma_a/2m + \mu_{ai}/2$ to rewrite (11) in terms of the spin-current operator,

$$\frac{d\hat{J}_i^{(e)}}{dt} = \frac{4em}{\hbar^4}(\boldsymbol{\mu}_i\times\boldsymbol{\mu}_l)_a\hat{J}_l^{(s)a}. \tag{12}$$

The substitution in (10) implies the correlation function $\langle[\hat{J}^{(s)}(t),\hat{J}^{(\nu)}(0)]\rangle$, leading to the desired connection

$$L_{ij}^{(ev)}(\omega) = i\left(\frac{G_0}{\Sigma_0}\right)\left[\frac{m(\boldsymbol{\mu}_i\times\boldsymbol{\mu}_l)_a/\hbar^2}{\hbar\tilde{\omega}}\right]L_{lj}^{(s\nu)a}(\omega), \tag{13}$$

which includes also the thermally driven case ($\nu = q$). Given that the term between square brackets is dimensionless, the final expression is written in terms of the quantum of electrical conductance $G_0 = e^2/2\pi\hbar$ and the unit $\Sigma_0 = e/8\pi$ of the spin Hall conductivity $L^{(se)}$ [17,29]. We can then employ Eqs. (13) and (3) to derive a corresponding relationship between electrical current and spin current

$$J_i^{(e)}(\omega) = i\left(\frac{G_0}{\Sigma_0}\right)\left[\frac{m(\boldsymbol{\mu}_i\times\boldsymbol{\mu}_l)_a/\hbar^2}{\hbar\tilde{\omega}}\right]J_l^{(s)a}(\omega). \tag{14}$$

This formula suggests that under the usual zero charge current condition it follows that the thermospin current (4) vanishes. We shall verify this conclusion and check that the thermospin conductivities (7) and (8) also vanish for each R + D[hkl] Hamiltonian case, although the coefficients $L^{(\xi\nu)}(\omega)$ and $S_{ij}(\omega)$ do not.

For the pure Rashba coupling case, the characteristic SO energy $m(\boldsymbol{\mu}_i\times\boldsymbol{\mu}_l)_a/\hbar^2 = (m\alpha^2/\hbar^2)\epsilon_{ila}\delta_{az}$ implies that $J_x^{(e)}\propto J_y^{(s)z}$ and $J_y^{(e)}\propto -J_x^{(s)z}$, and therefore $J_i^{(e)} = 0$ yields $J_i^{(s)z} = 0$. Since the charge conductivity is diagonal and isotropic, $L_{ij}^{(ev)}(\omega) = L_{xx}^{(ev)}(\omega)\delta_{ij}$, and the spin conductivity antisymmetric, $L_{ij}^{(s\nu)z}(\omega) = L_{xy}^{(s\nu)z}(\omega)\epsilon_{ijz}$, Eq. (13) implies that

$$L_{xx}^{(ev)}(\omega) = i\frac{G_0}{\Sigma_0}\left(\frac{m\alpha^2/\hbar^2}{\hbar\tilde{\omega}}\right)L_{yx}^{(s\nu)z}(\omega). \tag{15}$$

The Seebeck coefficients are $TS_{xx}(\omega) = L_{xx}^{(eq)}(\omega)/L_{xx}^{(ee)}(\omega)$ and $S_{yx}(\omega) = 0$. Using these results in Eqs. (7) and (8) it is obtained that $\Sigma_{xx}^{(s)z}(\omega) = \Sigma_{yx}^{(s)z}(\omega) = 0$. A similar conclusion holds for the R + D[111] Hamiltonian case, with the SO strength being $\tilde{\alpha}$ instead of α. This absence of spin current induced by a temperature gradient is in contrast to the nonvanishing thermospin Hall effect predicted for the pure Rashba case [10].

We now consider the 2DEG with Hamiltonian R + D[001]. For this case $m(\boldsymbol{\mu}_i\times\boldsymbol{\mu}_l)_a/\hbar^2 = [m(\alpha^2-\beta_{[001]}^2)/\hbar^2]\epsilon_{ila}\delta_{az}$, and again $J_x^{(e)}\propto J_y^{(s)z}$ and $J_y^{(e)}\propto -J_x^{(s)z}$, implying that $J_i^{(s)z} = 0$ follows from the condition $J_i^{(e)} = 0$, as before. Now Eq. (13) reads

$$\begin{pmatrix}L_{xj}^{(ev)}(\omega)\\ L_{yj}^{(ev)}(\omega)\end{pmatrix} = i\frac{G_0}{\Sigma_0}\left(\frac{m(\alpha^2-\beta_{[001]}^2)/\hbar^2}{\hbar\tilde{\omega}}\right)\begin{pmatrix}L_{yj}^{(s\nu)z}(\omega)\\ -L_{xj}^{(s\nu)z}(\omega)\end{pmatrix}. \tag{16}$$

Using these connections to rewrite the Seebeck coefficients in terms of $L^{(s\nu)z}$, and the results $L_{xx}^{(ev)} = L_{yy}^{(ev)}$, $L_{xy}^{(ev)} = L_{yx}^{(ev)}$, $L_{xx}^{(s\nu)z} = -L_{yy}^{(s\nu)z}$, and $L_{xy}^{(s\nu)z} = -L_{yx}^{(s\nu)z}$, as obtained from Eq. (9), it is straightforward to find that the longitudinal and transverse components (7) and (8) vanish, $\Sigma_{xx}^{(s)z}(\omega) = \Sigma_{yx}^{(s)z}(\omega) = 0$.

For a [110] direction of growth, the Hamiltonian R + D[110] yields $m(\boldsymbol{\mu}_i\times\boldsymbol{\mu}_l)_a/\hbar^2 = (m\alpha^2/\hbar^2)\epsilon_{ila}\delta_{az} + (m\alpha\beta_{[110]}/\hbar^2)\epsilon_{ilz}\delta_{ay}$, and Eq. (13) becomes

$$L_{ij}^{(ev)}(\omega) = i\frac{G_0}{\Sigma_0}\frac{m\alpha/\hbar^2}{\hbar\tilde{\omega}}\epsilon_{ilz}\left(\alpha L_{lj}^{(s\nu)z} + \beta_{[110]}L_{lj}^{(s\nu)y}\right), \tag{17}$$

where $L_{ij}^{(ev)}(\omega)$ is diagonal but anisotropic, while $L_{ij}^{(s\nu)z}(\omega)$ is off-diagonal with $L_{xy}^{(s\nu)z}\neq L_{yx}^{(s\nu)z}$. From Eq. (9) it is verified that $\beta_{[110]}L_{ij}^{(s\nu)z} = \alpha L_{ij}^{(s\nu)y}$, or equivalently $\beta_{[110]}J_l^{(s)z} = \alpha J_l^{(s)y}$. These results lead again to a connection of the form $J_i^{(e)}\propto\epsilon_{ilz}J_l^{(s)z}$ which implies the vanishing of the thermospin current for a system without electrical current flowing through it. As for the transport coefficients, Eq. (17) simplifies to

$$\begin{pmatrix}L_{xx}^{(ev)}(\omega)\\ L_{yy}^{(ev)}(\omega)\end{pmatrix} = i\frac{G_0}{\Sigma_0}\left(\frac{m(\alpha^2+\beta_{[110]}^2)/\hbar^2}{\hbar\tilde{\omega}}\right)\begin{pmatrix}L_{yx}^{(s\nu)z}(\omega)\\ -L_{xy}^{(s\nu)z}(\omega)\end{pmatrix}; \tag{18}$$

similar relations have already been derived for $\nu = e$ only [28]. Using this, and that in the present case $TS_{xx} = L_{xx}^{(eq)}/L_{xx}^{(ee)}$, $S_{yx} = 0$, it can be verified straightforwardly that the thermospin conductivities (7) and (8) also vanish, $\Sigma_{xx}^{(s)z}(\omega) = \Sigma_{yx}^{(s)z}(\omega) = 0$.

We can condense these results through the simple expression $S_{ij} = S_{ij}^{(s)z}$ for each SO Hamiltonian case, yielding $\Sigma_{ij}^{(s)z} = 0$. The spin current (4) arises from two contributions. There is a spin current arising directly from the action of the temperature gradient, characterized by the coefficient $L^{(sq)z}$, and a spin current generated by the thermoelectric field through the spin Hall effect. We have shown explicitly that these contributions cancel each other, implying the vanishing of the thermospin current. This result ultimately arises from the linear-in-momentum dependence of the SO Hamltonian (1) and the corresponding nature of the spin states.

To our knowledge, no such connections like (13) are known where the coupling is not simply linear in momentum. However, it was recently reported [15] that the joint density of states for the spin-split subbands acquires additional features when cubic Dresselhaus terms are included, in contrast to the purely linear SOI. As a consequence, the optical conductivity no longer vanishes at the condition of persistent spin helix state, where $\alpha = \beta_{[001]}$. Based on this, one might expect a non-null thermospin current response in the presence of cubic Dresselhaus terms, although explicit calculations need to be done.

In summary, we have shown that under the well-known conditions of the Seebeck effect, there is no spin current induced by a temperature gradient in an infinite homogeneous 2DEG with Rashba and Dresselhaus SOI. For each Hamiltonian case $\mathrm{R} + \mathrm{D}[hkl]$, we have checked that the thermospin conductivity tensor is null for all frequencies and values of the SO strength parameters, not only for the special symmetry cases of fixed precession axis $\alpha = \beta_{[001]}$ or symmetric ($\alpha = 0$) [110]-grown quantum well. The transport and Seebeck coefficients are non-null, however. As an example, we calculate the transverse spin-current–heat-current response $L_{xy}^{(sq)z}(\omega)$ as a function of frequency for the anisotropic SO case $\mathrm{R} + \mathrm{D}[001]$, which shows characteristic spectral features, suggesting new possibilities of manipulation through thermal gradients. Within the linear response formalism, we derived connection formulas relating the electrically and thermally driven spin and charge conductivities, characteristic of the linear in $\mathbf{k}$ SO Hamiltonians, which lead to a connection between the charge and spin currents. The derivation of the absence of thermospin current response was based on this connection, without the need for explicit evaluation of the phenomenological transport coefficients $L_{ij}^{(\xi\nu)}(\omega)$. Contrary to the recently predicted intrinsic thermospin Hall effect in a 2DEG with Rashba SO coupling [10], we find that there is no such effect. We hope that this work will stimulate further investigations of this problem under more general conditions, such as the presence of cubic Dresselhaus coupling, electron-electron interaction, and extrinsic or finite-size effects.

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