Complete band gaps in one-dimensional photonic crystals with negative refraction arising from strong chirality

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In this paper, we propose the idea of using chiral negatively refractive media to realize three-dimensional (3D) complete band gaps in the one-dimensional (1D) photonic crystals with a two-layer unit cell. We explain why at least a three-layer unit cell is necessary for realizing the 3D complete band gaps in 1D achiral photonic crystals, and show how we can reduce the number of layers of the unit cell to two if chirality is introduced. Although the media with strong chirality supply negative refraction only for one of the two eigenwaves, the complete band gaps could be still obtained with an even simpler lattice structure compared to the achiral counterpart.

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Photonic crystals are materials with periodical spatial modulation of electromagnetic parameters that may include permittivity, permeability, conductivity, and chirality. The exciting applications of photonic crystals, such as photonic microcavity and photonic crystal waveguide, rely on the existence of photonic band gaps in such materials [1,2]. The photonic crystals with complete band gaps prevent the electromagnetic waves of any polarization from propagating in any direction for the frequencies lying in the gaps. According to the conventional concept of photonic crystals, three-dimensional (3D) photonic crystals are usually necessary for the creation of complete 3D band gaps. However, when double-negative media [3,4] are considered as the constituents of photonic crystals, many new concepts emerge, such as the discrete modes and associated zero-$\bar{n}$ band gaps [5–9]. Moreover, complete 3D band gaps could exist in the one-dimensional (1D) photonic crystals containing double-negative media [10], which is in sharp contrast with conventional knowledge.

However, not only the double-negative medium but also a three-layer unit cell is necessary to realize the complete 3D band gaps in 1D photonic crystals. Double-negative medium is necessary to suppress the fundamental modes guided along the off-axis direction [10]. A three-layer unit cell is also necessary, simply because of the unavoidable existence of Brewster angle, at which incident plane waves have no reflection at the interface between two different media. The Brewster angle (if it exists) of s-polarized waves θ_{Bs} satisfies [11]

$$\tan^2\theta_{Bs} = -\frac{\mu_2}{\mu_1}\frac{\epsilon_1\mu_2 - \epsilon_2\mu_1}{\epsilon_1\mu_1 - \epsilon_2\mu_2}. \tag{1}$$

Replacing ϵ by $-\mu$ and μ by $-\epsilon$, we get the condition for the Brewster angle (if it exists) of p-polarized waves as follows:

$$\tan^2\theta_{Bp} = \frac{\epsilon_2}{\epsilon_1}\frac{\epsilon_1\mu_2 - \epsilon_2\mu_1}{\epsilon_1\mu_1 - \epsilon_2\mu_2}. \tag{2}$$

Clearly there is

$$\frac{\tan^2\theta_{Bs}}{\tan^2\theta_{Bp}} = -\frac{\epsilon_1}{\mu_1}\frac{\mu_2}{\epsilon_2} < 0, \tag{3}$$

provided the medium is either double positive ($\epsilon > 0, \mu > 0$) or double negative ($\epsilon < 0, \mu < 0$). Thus, if s-polarized waves have no Brewster angle, p-polarized waves must have one, and vice versa. According to the principle of reversibility, a plane wave with incident angle equal to Brewster angle will pass through the whole 1D two-layer unit-cell photonic crystals without reflection, which prevents the opening of complete band gaps.

Up to here, the discussions are restricted to the 1D photonic crystals composed of achiral media. Realization of complete 3D band gaps in 1D photonic crystals with two-layer unit cell, however, is still possible if we enlarge our scope to the more general chiral media. Chiral media are well known for their optical activity due to the lacking of mirror symmetry. Double-negative media are exotic because of negative refraction. Apart from driving both permittivity and permeability to negative by simultaneous electric and magnetic resonances, introduction of strong chirality is another important mechanism for negative refraction [12–24], in which only one resonance with respect to chirality is sufficient and double-negative condition is not necessary any more [12].

A bi-isotropic reciprocal chiral medium could be described by the constitutive relations

$$\begin{aligned}\mathbf{D} &= \epsilon\mathbf{E} + i\xi\mathbf{H},\\ \mathbf{B} &= \mu\mathbf{H} - i\xi\mathbf{E},\end{aligned} \tag{4}$$

where ξ denotes the chiral parameter. Application of the so-called Bohren decompositions $\mathbf{F}_\pm = \mathbf{E} \pm i\eta\mathbf{H}$ [25] is always convenient in the treatment of homogeneous chiral media, where the impedance η of a chiral medium is defined by $\eta = \sqrt{\mu/\epsilon}$. Corresponding field equations are derived as

$$\nabla \times \mathbf{F}_\pm = \pm k_\pm \mathbf{F}_\pm, \tag{5}$$

where $k_\pm = \omega(s\sqrt{\epsilon\mu} \pm \xi)$, $s = 1$ for double-positive media, and $s = -1$ for double-negative media. A time dependence $\exp(-i\omega t)$ is implicitly assumed for all field quantities. The two eigenwaves in an unbounded chiral medium are separately right-handed circular polarized (RCP) and left-handed circular polarized (LCP), with k_+ and k_- as eigenwave numbers, respectively. If the chirality is strong enough such that $|\xi| > \sqrt{\epsilon\mu}$, the medium is negatively refractive for one eigenwave and positively refractive for the other eigenwave.

Consider a 1D photonic crystal as depicted by Fig. 1(a), where both medium 1 and medium 2 are chiral and obey the constitutive relations (4). The field solution in either

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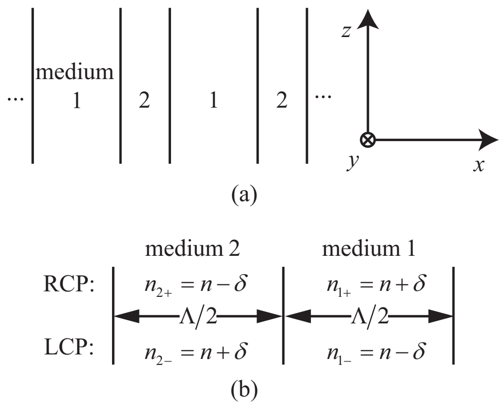

FIG. 1. (a) Sketch of 1D photonic crystals and layout of Cartesian coordinate system. (b) The unit cell of a 1D photonic crystal composed of chiral media is constructed so that the lattices seen by RCP and LCP waves are equivalent.

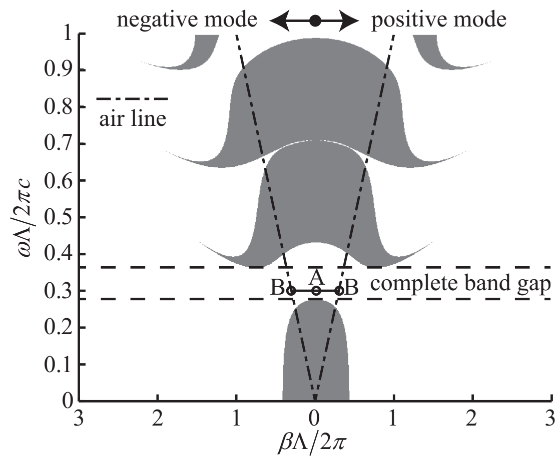

FIG. 2. Projected band structure of a 1D chiral photonic crystal with $\epsilon_1 = \epsilon_0$, $\epsilon_2 = 2\epsilon_0$, $\mu_1 = 2\mu_0$, $\mu_2 = \mu_0$, $d_1 = d_2$, and $\delta = 3$.

homogeneous layer is expressed as

$$\begin{aligned} F_{j\pm,x} &= -\frac{i\beta}{\pm k_{j\pm}}(A_{j\pm}e^{i\sigma_{j\pm}x} + B_{j\pm}e^{-i\sigma_{j\pm}x})e^{i\beta z}, \\ F_{j\pm,y} &= (A_{j\pm}e^{i\sigma_{j\pm}x} + B_{j\pm}e^{-i\sigma_{j\pm}x})e^{i\beta z}, \\ F_{j\pm,z} &= \frac{i\sigma_{j\pm}}{\pm k_{j\pm}}(A_{j\pm}e^{i\sigma_{j\pm}x} - B_{j\pm}e^{-i\sigma_{j\pm}x})e^{i\beta z}, \end{aligned} \tag{6}$$

where $\sigma_{j\pm} = (k_{j\pm}^2 - \beta^2)^{1/2}$ and j represents 1 or 2. Applying the continuity of tangential components of $\mathbf{E}$ and $\mathbf{H}$ at the interfaces and the Bloch condition that $\mathbf{G}(x) = \mathbf{G}(x + \Lambda)\exp(iK\Lambda)$, where $\mathbf{G}$ represents $\mathbf{E}$ or $\mathbf{H}$, K is the Bloch wave number and Λ the period of 1D photonic crystals, we finally achieve the following dispersion relations [26]:

$$\cos(K\Lambda) = \Sigma \pm \sqrt{\Sigma^2 - \Gamma}, \tag{7}$$

where

$$\Sigma = \frac{(\eta_1+\eta_2)^2(\Pi_{++}+\Pi_{--}) - (\eta_1-\eta_2)^2(\Pi_{+-}+\Pi_{-+})}{8\eta_1\eta_2},$$

$$\Gamma = \frac{(\eta_1+\eta_2)^2\Pi_{++}\Pi_{--} - (\eta_1-\eta_2)^2\Pi_{+-}\Pi_{-+}}{4\eta_1\eta_2} + \frac{1}{16}\left(\frac{\eta_1}{\eta_2} - \frac{\eta_2}{\eta_1}\right)^2(\Omega_1 - 1)(\Omega_2 - 1),$$

$$\Pi_{pq} = \cos\phi_{1p}\cos\phi_{2q} - \frac{pq}{2}\left(\frac{\tau_{1p}}{\tau_{2q}} + \frac{\tau_{2q}}{\tau_{1p}}\right)\sin\phi_{1p}\sin\phi_{2q},$$

$$\Omega_j = \cos\phi_{j+}\cos\phi_{j-} - \frac{1}{2}\left(\frac{\tau_{j+}}{\tau_{j-}} + \frac{\tau_{j-}}{\tau_{j+}}\right)\sin\phi_{j+}\sin\phi_{j-},$$

in which p and q represent $+$ or $-$. $\tau_{j\pm}$ and $\phi_{j\pm}$ are defined through $\tau_{j\pm} = k_{j\pm}/\sigma_{j\pm}$ and $\phi_{j\pm} = \sigma_{j\pm}d_j$, where d_j denotes the width of layer j.

Achiral electromagnetism should be included as a special example of the chiral counterpart, and indeed Eq. (7) reduces to the well-known dispersion relations [5] of s- and p-polarized waves when the chirality is removed. However, if we name the modes corresponding to the upper sign and lower sign of $\pm$ in Eq. (7) positive mode and negative mode, respectively, there is no simple correspondence between the positive and negative modes and the p- and s-polarized waves. Depending on the concrete structures of photonic crystals and the wave numbers β and $k_\pm$ of fields, the positive mode could degenerate to either s- or p-polarized wave, which also holds true for the negative mode. But it is always that if the positive mode degenerates to s-polarized wave, under the same condition, the negative mode must degenerate to p-polarized wave, and vice versa.

A complete band gap requires that for a band of frequencies both the positive and negative modes have no real-valued solution of K for all possible real-valued β. Though not necessary, we will design the structure so that the positive and negative modes have the same dispersion relation. We could reach this goal when the unit cell is constructed so that the RCP wave and LCP wave see the equivalent lattice structure, namely, $k_{2+} = k_{1-}$, $k_{2-} = k_{1+}$, and $d_1 = d_2$, as demonstrated by Fig. 1(b). We can verify that these conditions truly lead to $\Sigma^2 - \Gamma = 0$ in Eq. (7). If we define the mean index of chiral medium as $n = (k_+ + k_-)/2k_0 = s(\epsilon\mu)^{1/2}c$ and the relative chiral parameter $\delta = \xi c$, $k_{2+} = k_{1-}$ and $k_{2-} = k_{1+}$ are equivalent to $n_1 = n_2$ and $\delta_1 = -\delta_2$, where $k_0 = \omega/c$ and c represent the wave number and light speed in vacuum, respectively. We assume that $n_1 = n_2 = n > 0$ and $\delta_1 = -\delta_2 = \delta > n$, namely, both media are double positive and strongly chiral, with equal mean indices and opposite chiral parameters. It is found that a complete band gap opens between the first band and second band, if the impedances of two kinds of chiral media composing the photonic crystals are not equal and n and δ are appropriately chosen. An example is demonstrated in Fig. 2, where the material parameters are $\epsilon_1 = \epsilon_0$, $\epsilon_2 = 2\epsilon_0$, $\mu_1 = 2\mu_0$, $\mu_2 = \mu_0$, and $\delta = 3$, and a complete band gap emerges between the normalized frequency 0.277 and 0.362.

We should emphasize that the complete gap exists robustly for a range of material parameters, not just limited to some special ones. To show this, we fix $\epsilon_1/\epsilon_0 = \mu_2/\mu_0 = 1$, and calculate the gap size (gap-midgap ratio) for different n and δ; the result is given in Fig. 3. It is definite that if the two constituting media are impedance matching, it is impossible

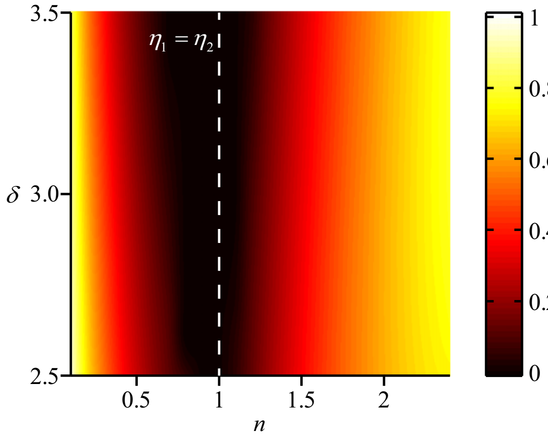

FIG. 3. (Color online) Dependence of gap size (gap-midgap ratio) on n and δ, where we fix $\epsilon_1/\epsilon_0 = \mu_2/\mu_0 = 1$, and the dashed line indicates the impedance match of two constituting media.

for the complete band gap to exist, since at least for the on-axis propagation ($\beta = 0$), as seen below, the wave is not reflected by the interfaces.

As mentioned in the beginning, the Brewster angle prevents us from obtaining 3D complete band gaps in 1D achiral photonic crystals with a two-layer unit cell. Now we prove that for the structure of 1D chiral photonic crystals proposed above (with $n_1 = n_2 = n > 0$, $\delta_1 = -\delta_2 > n$, and $\eta_1 \neq \eta_2$), there is no similar problematic angle. Consider the refraction and reflection at some interface, as shown in Fig. 4, where A_{1+}, B_{1-}, A_{2+}, and B_{2-} represent the complex amplitudes of incident waves with energy flowing toward the interface. Suppose that under some special condition no portion of incident waves is reflected, namely, if the waves are incident from the left (at least one of A_{1+} and B_{1-} is nonzero), there is $A_{2+} = B_{2-} = 0$ (no waves incident from the right), and $A_{1-} = B_{1+} = 0$ (no reflected waves), with at least one of A_{2-} and B_{2+} being nonzero. Applying the boundary conditions, we get

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ \frac{1}{\eta_1} & -\frac{1}{\eta_1} & -\frac{1}{\eta_2} & \frac{1}{\eta_2} \\ \frac{\sigma_{1+}}{k_{1+}} & \frac{\sigma_{1-}}{k_{1-}} & \frac{\sigma_{1-}}{k_{1-}} & \frac{\sigma_{1+}}{k_{1+}} \\ \frac{1}{\eta_1}\frac{\sigma_{1+}}{k_{1+}} & -\frac{1}{\eta_1}\frac{\sigma_{1-}}{k_{1-}} & \frac{1}{\eta_2}\frac{\sigma_{1-}}{k_{1-}} & -\frac{1}{\eta_2}\frac{\sigma_{1+}}{k_{1+}} \end{bmatrix} \begin{bmatrix} A_{1+} \\ B_{1-} \\ B_{2+} \\ A_{2-} \end{bmatrix} = 0, \tag{8}$$

where $k_{2+} = k_{1-}$ and $k_{2-} = k_{1+}$ have been applied. The determinant of the coefficient matrix $\mathbf{M}$ of Eq. (8),

$$|\mathbf{M}| = \left[\left(\frac{1}{\eta_1} + \frac{1}{\eta_2}\right)\left(\frac{\sigma_{1+}}{k_{1+}} + \frac{\sigma_{1-}}{-k_{1-}}\right)\right]^2 - \frac{16}{\eta_1\eta_2}\frac{\sigma_{1+}}{k_{1+}}\frac{\sigma_{1-}}{-k_{1-}},$$

is no less than zero (note that k_{1-} is negative). For oblique incidence ($\beta \neq 0$), $|\mathbf{M}|$ acquires zero when both $k_{1+} = -k_{1-}$ and $\eta_1 = \eta_2$ are satisfied. For normal incidence ($\beta = 0$), we have $\sigma_{1+}/k_{1+} = \sigma_{1-}/(-k_{1-}) = 1$; the expression of $|\mathbf{M}|$ is simplified to

$$|\mathbf{M}| = 4\left[\left(\frac{1}{\eta_1} + \frac{1}{\eta_2}\right)^2 - \frac{4}{\eta_1\eta_2}\right] = 4\left(\frac{1}{\eta_1} - \frac{1}{\eta_2}\right)^2,$$

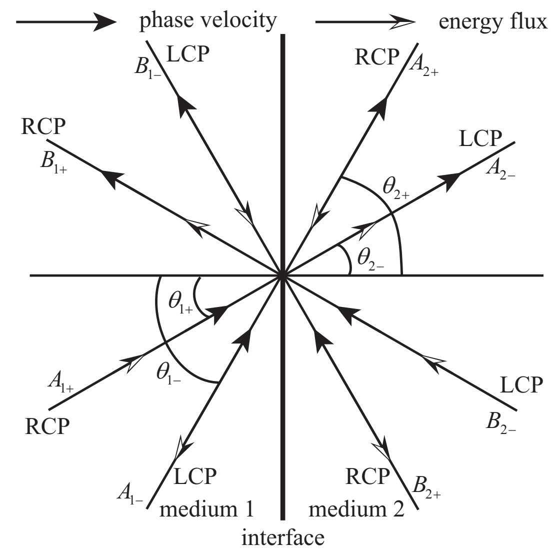

FIG. 4. Refraction and reflection at some interface of the 1D chiral photonic crystal with $n_1 = n_2 > 0$ and $\delta_1 = -\delta_2 > n$. According to the chiro-Snell's law [27], $k_{j\pm}\sin(\theta_{j\pm})$ is a conserved quantity; thus the wave vector with larger absolute value has a smaller angle with the on-axis direction. Note the negative *reflection* [28] indicated by the reflection from A_{j+} to A_{j-} and from B_{j-} to B_{j+}.

which acquires zero when $\eta_1 = \eta_2$. Thus, as long as the two media composing the photonic structure have unequal impedances, Eq. (8) only has a trivial solution, namely, there is no possibility that waves can transmit the interface without reflection.

In general, media with strong chirality are strongly dispersive. Consequently, the band diagram given in Fig. 2 does not sever for one actual photonic crystal, because the material parameters are set invariant with respect to the frequency;

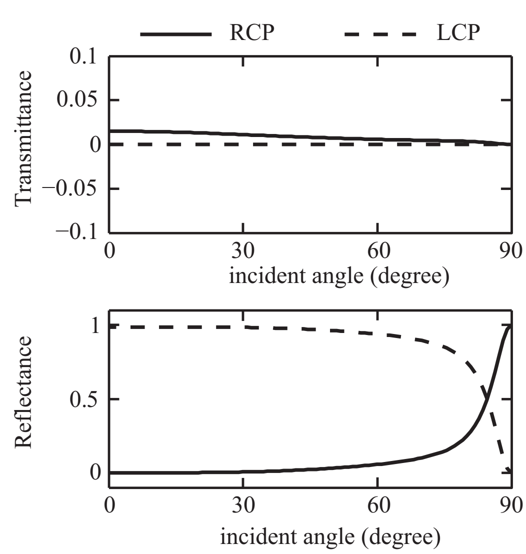

FIG. 5. Transmittance and reflectance of the RCP plane waves with normalized frequency 0.3, incident from air into the 1D multilayer reflector with the same lattice structure of Fig. 2. The reflector contains five periods. In general, the reflected and refracted waves contain both RCP (solid line) and LCP (dashed line) components.

but, it provides the information about how we can design the period Λ of the photonic crystal, such that the desired narrow range of frequencies where the material parameters corresponding to Fig. 2 could be obtained in practice lie in the complete band gap region. On the other hand, the fascinating applications of negative refraction can be implemented only when the material absorption is reduced to be sufficiently low, and the 3D complete gap in 1D photonic crystals is no exception. For the achiral realization of negative refraction, low material absorption is very difficult to achieve, as both electric and magnetic resonances are necessary near the objective frequency band to produce simultaneously negative permittivity and permeability. However, recent theoretical investigations [16,22] have shown that the chiral approach may lead to negative refraction with ultralow material absorption and tunable electromagnetic parameters. The 3D complete gap in 1D chiral negatively refractive photonic crystals proposed here should be promisingly applicable in the future, with the advance of chiral metamaterials.

The complete band gap obviously promises omnidirectional reflection. As an example, Fig. 5 gives the dependence of transmittance and reflectance[1] on the incident angle for the RCP plane waves with normalized frequency 0.3, incident from air into the corresponding multilayer reflector with five periods. The omnidirectional reflection is clearly seen from the transmittance diagram. The very low but not zero transmittance of RCP component is due to the fact that the reflector contains only a finite number of periods, rather than an infinite number of periods for the ideal photonic crystals. The transmittance of RCP component decreases as the incident angle increases, which is understandable, since as seen from Fig. 2, for frequency 0.3 the larger the incident angle the further the equal-frequency line away from the edge of first band, corresponding to the movement from point A to point B in Fig. 2. For the normal incidence the reflected wave is LCP and for the glancing incidence the reflected wave is RCP, which is the result of conservation of angular momentum of the photon. For the incidence of LCP waves, we just need to exchange the two labels "RCP" and "LCP" of Fig. 5. Omnidirectional reflection of both RCP and LCP waves indicates the omnidirectional reflection of waves with any polarization, which is the expected property of complete band gaps.

Strong chirality is one of the important ways to realize negative refraction. However, negative refraction occurs for only one of the two eigenwaves of chiral media, so at first glance it may be thought that the interesting properties of negative refraction could be applicable to only one eigenwave, namely either RCP wave or LCP wave. But we demonstrate that the complete band gap in 1D achiral photonic crystals, which is associated with the simultaneous negative refraction for both the s- and p-polarized waves in the same medium layer, could be realized in 1D chiral photonic crystals as well, with even simpler lattice structure, where in each medium layer only one eigenwave is negatively refractive. We believe that, in addition to the mechanism realizing negative refraction due to strong chirality, the applications of chiral negatively refractive media in photonic devices also deserve more attention.

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[1]Transmittance and reflectance are calculated by the transfer matrix formulation, which can be easily derived under our theoretical framework. However, we should point out that the transfer matrix formulation presented in Ref. [27] is also applicable to the chiral multilayer system with negative refraction, if we generalize the definition of $k_{\pm}$ therein to $k_{\pm} = \omega(s\sqrt{\epsilon\mu} \pm \xi)$.

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