Two-electron state from the Floquet scattering matrix perspective

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Two single-particle sources coupled in series to a chiral electronic waveguide can serve as a probabilistic source of two-particle excitations with tunable properties. The second-order correlation function, characterizing the state of emitted electrons in space-time, is expressed in terms of the Floquet scattering matrix of a source. It is shown that the Fourier transform of the correlation function, characterizing the emitted state in energy space, can be accessed with the help of an energy resolved shot-noise measurement. The two-electron state emitted adiabatically is discussed in detail. In particular, the two-electron wave function is represented via two different sets of single-particle wave functions accessible experimentally.

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I. INTRODUCTION

The realization of a high-speed on-demand single-electron source [1–3] has marked the birth of a new field focused on operations with electron wave packets containing one to few particles propagating in a ballistic conductor. Inspired by quantum optics the several experiments demonstrating a single-particle nature of emitted electron wave packets were reported [4–6]. The dynamical switching into different paths of individual electrons propagating ballistically was reported in Ref. [7]. Provided the single-electron source is available, the engineering of few-electron states becomes possible. Controlled emission of few electron wave packets in mesoscopic conductor was already realized experimentally in Refs. [1,8–12] using a dynamic quantum dot and in Ref. [3] using a voltage pulse with quantized flux as suggested in Refs. [13,14].

The aim of this paper is to analyze a dynamical two-electron source composed of two periodically driven single-particle emitters attached to a chiral electronic waveguide, see Fig. 1, as suggested in Ref. [15]. The advantage of such a two-particle emitter is the possibility to vary the times when single electrons are emitted by individual sources and, therefore, continuously switch from the single-electron emission to emission of a pair of electrons. The closely related source, addressed in Refs. [3,13,14,16–24], would utilize quantized Lorentzian voltage pulses with variable center position.

In order to characterize the emitted two-particle state I extend the approach of Ref. [25] and introduce the second-order correlation function for emitted electrons. Within the scattering matrix formalism for noninteracting electrons [26], which describes well the single-particle source of Ref. [2], see Ref. [27], as well as the one used in Ref. [3], see Ref. [24], the source of electrons is described by the corresponding Floquet scattering matrix. I express the second-order correlation function in terms of the Floquet scattering matrix of the source and isolate the contribution due to emitted particles. This contribution can be represented as the Slater determinant composed of the first-order correlation functions, as it should be for noninteracting fermions [28], that justifies the decomposition into emitted electrons and electrons of the underlying Fermi sea we use. This procedure can be readily extended to describe an n-particle emitter and n-electron states. A closely related approach to the first-order correlation function (single-electron coherence) is developed in Refs. [29,30] and applied to the analysis of n-electron Lorentzian pulses in Ref. [31]. A Wigner function representation of the first-order electronic coherence is discussed in Ref. [32].

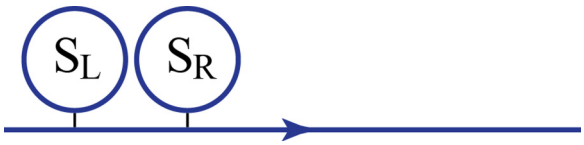

FIG. 1. (Color online) Two single-electron emitters S_L and S_R attached to the same chiral electronic waveguide serve as a two-electron source if they emit electrons at close times. The arrow shows the direction of movement of electrons.

The correlation function fully characterizes an emitted state. However, presently it is a challenge to access experimentally a correlation function on a single-electron level. On the other hand the first steps in this direction are already done. A time-resolved current profile on a single-electron level [2,33] and a single-electron wave-packet probability profile [3,7] were reported. This inspires hope that the full quantum characterization of an emitted single- to few-electron state, like it is done for single photons in optics [34,35], is coming soon.

Another interesting object to look at is the energy distribution function, which is easier to access experimentally. The nonequilibrium single-particle distribution function was already measured in Refs. [36–38] via the energy resolved dc current and in Refs. [3,39] via the low-frequency shot-noise spectroscopy. In this paper we discuss how to measure a two-particle distribution function via the energy resolved shot noise.

Though the distribution function provides only partial information on the emitted two-particle state, nevertheless, it already demonstrates an essential feature of the state of two fermions propagating together, namely, an increase of the energy compared to the case when they propagate separately [3,24,40,41]. We demonstrate this explicitly analyzing the evolution of the state emitted adiabatically. We calculate a two-particle wave function and show that with decreasing

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the time difference between the emission of two electrons it evolves from the product of single-electron wave functions to the Slater determinant composed of them. The single-electron wave functions in turn evolve from the bare ones emitted by the single-electron sources to the mutually orthogonal functions. The result of orthogonalization can be interpreted as if one bare single-electron wave function remains unchanged while the other one is adapted accordingly. Depending on which of two wave functions is kept unchanged, there are two bases for the representation of a two-electron wave function. Interestingly, performing measurements on the system with either one or two single-particle sources being switched on, one can access all four single-particle wave functions mentioned above.

The paper is organized as follows: In Sec. II the second-order correlation function for emitted particles is calculated in terms of the Floquet scattering matrix of a periodically driven electron source. In Sec. II C we discuss how the Fourier transform of a correlation function, the energy distribution function, can be accessed via current cross-correlation measurement. In Sec. III the two-electron state emitted adiabatically is analyzed in detail. A short conclusion is in Sec. IV. Some details of calculations are given in the Appendixes.

II. GENERAL FORMALISM

A. Second-order correlation function for emitted particles

For a start let us define the second-order electronic correlation function, in full analogy with how it is defined in optics [42],

$$\mathcal{G}^{(2)}(1,2,3,4) = \langle \hat{\Psi}^\dagger(1)\hat{\Psi}^\dagger(2)\hat{\Psi}(3)\hat{\Psi}(4)\rangle, \tag{1}$$

where $\hat{\Psi}(j) \equiv \hat{\Psi}\left(x_j t_j\right)$ is a single-particle electron field operator in second quantization evaluated at point x_j and time t_j, $j = 1,2,3,4$. The quantum-statistical average $\langle \dots \rangle$ is taken over the equilibrium state of electrons incoming to the source. To access information about emitted particles let us evaluate $\mathcal{G}^{(2)}$ behind the source, where the field operator in second quantization for chiral electrons reads [43]

$$\hat{\Psi}\left(x_j t_j\right) = \int \frac{dE}{\sqrt{hv(E)}} e^{i\phi_j(E)} \hat{b}(E). \tag{2}$$

Here $1/[hv(E)]$ is a one-dimensional density of states at energy E, $\hat{b}(E)$ is an operator for electrons passed by (scattered off) the source, and the phase $\phi_j(E) = -Et_j/\hbar + k(E)x_j$.

The electronic source driven periodically with frequency Ω is characterized by the Floquet scattering matrix with elements $S_F(E_n, E)$, $E_n = E + n\hbar\Omega$, being amplitudes for an electron with energy E to exchange n energy quanta $\hbar\Omega$ with the scatterer. In such a case we can write [44]

$$\hat{b}(E) = \sum_n S_F(E, E_n)\, \hat{a}(E_n), \tag{3}$$

where $\hat{a}(E)$ is an operator for equilibrium electrons incoming to the scatterer. We suppose that incoming electrons are emanated by the equilibrium reservoir characterized by the Fermi distribution function $f_0(E)$ with temperature T_0 and the chemical potential μ. A quantum statistical average over the equilibrium state of incoming electrons for the product of two operators is calculated as follows:

$$\langle \hat{a}^\dagger(E)\hat{a}(E')\rangle = f_0(E)\delta\left(E - E'\right). \tag{4}$$

The average of the product of more than two operators is calculated using the well-known Wick's theorem.

Using the quantities introduced above one can represent the correlation function $\mathcal{G}^{(2)}$ in terms of the Floquet scattering matrix of the source,

$$\begin{aligned}\mathcal{G}^{(2)}&(1,2,3,4)\\ &= \frac{1}{2}\sum_{n,m,p,q}\iint \frac{dE\,dE'\, f_0(E_n)\, f_0\left(E'_m\right)}{h^2 v(E) v(E')}\\ &\times S_F^*(E,E_n)\, S_F^*\left(E',E'_m\right) S_F\left(E_p,E_n\right) S_F\left(E'_q,E'_m\right)\\ &\times \det\begin{pmatrix} e^{i\phi_1(E)} & e^{i\phi_2(E)} \\ e^{i\phi_1(E')} & e^{i\phi_2(E')}\end{pmatrix}^* \det\begin{pmatrix} e^{i\phi_4(E_p)} & e^{i\phi_3(E_p)} \\ e^{i\phi_4(E'_q)} & e^{i\phi_3(E'_q)}\end{pmatrix}.\end{aligned} \tag{5}$$

In the equation above I use a wide band approximation and neglect the variation of the density of states on the scale of $\hbar\Omega$: $1/[hv(E_n)] \approx 1/[hv(E)]$. The structure of Eq. (5) tells us that the correlation function $\mathcal{G}^{(2)}$ is composed of elementary two-particle propagators describing the transfer of two electrons to points 4 and 3 from points 1 and 2. The action of the driven scatterer is to change energies of electrons, E_n and E'_m, as at destination points (to energies E_p, E'_q) and as at initial points (to energies E, E'). The Fermi functions $f_0(E_n)$ and $f_0\left(E'_m\right)$ describe whether the states with original energies E_n and E'_m are occupied. To understand the content of $\mathcal{G}^{(2)}$ even better let us use the following identity:

$$\begin{aligned}f_0(E_n)f_0(E'_m) = {}& f_0(E)f_0(E') + f_0(E)[f_0(E'_m) - f_0(E')]\\ &+ [f_0(E_n) - f_0(E)]f_0(E')\\ &+ [f_0(E_n) - f_0(E)][f_0(E'_m) - f_0(E')].\end{aligned} \tag{6}$$

Four terms on the right-hand side (RHS) of Eq. (6) results in four terms in $\mathcal{G}^{(2)}$, Eq. (5). The first term, $f_0(E)f_0\left(E'\right)$, results in the second-order correlation function for the Fermi sea incoming from the reservoir unperturbed by the driven scatterer. This is so, since the Floquet matrix elements drop out from the corresponding equation in force of the unitarity condition [44],

$$\sum_n S_F^*(E,E_n)S_F(E_p,E_n) = \delta_{p,0}. \tag{7}$$

The next two terms result in contributions dependent only on two Floquet scattering elements and, therefore, can be interpreted as describing correlations between one unperturbed electron of the Fermi sea and one excited electron. And finally the last term on the RHS of Eq. (6) describes correlations between two excited electrons. This last contribution is of our interest here and we denote it as $G^{(2)}$. Therefore, the quantity $G^{(2)}$ is referred to as *the second-order correlation function for emitted particles*. As expected it can be represented as the determinant,

$$G^{(2)}(1,2,3,4) = \det\begin{pmatrix} G^{(1)}(1,4) & G^{(1)}(1,3) \\ G^{(1)}(2,4) & G^{(1)}(2,3)\end{pmatrix}, \tag{8}$$

composed of the first-order correlation functions for emitted particles [25],

$$G^{(1)}(j,j') = \sum_{n,m=-\infty}^{\infty} \int \frac{dE\{f_0(E_n) - f_0(E)\}}{hv(E)} \times S_F^*(E,E_n) S_F(E_m,E_n) e^{-i\phi_j(E)} e^{i\phi_{j'}(E_m)}. \quad (9)$$

The fact that $G^{(2)}$ is expressed in terms of $G^{(1)}$ is a mere consequence of the well-known Wick theorem. Note that $G^{(1)}$ is the first-order correlation function for the combined emitter. It has no simple relation to the states emitted by the single-particle sources working independently.

B. Distribution functions for emitted particles

To characterize the state of emitted particles in the energy space it is convenient to introduce distribution functions.

1. Single-particle distribution function

The single-particle distribution function for emitted particles is defined as follows:

$$f(E)\delta(E - E') = \langle \hat{b}^\dagger(E)\hat{b}(E')\rangle|_{E'=E} - \langle \hat{a}^\dagger(E)\hat{a}(E')\rangle. \quad (10)$$

It is a probability (density) that one can detect one particle in the state with energy E. According to Eq. (2) such a state is a plane-wave state. In terms of the Floquet scattering matrix of the source the single-particle distribution function reads [44]

$$f(E) = \sum_{n=-\infty}^{\infty} |S_F(E,E_n)|^2 \{f_0(E_n) - f_0(E)\}. \quad (11)$$

The difference of the Fermi functions, entering the equation above, emphasizes that what is calculated is related to excitations, not to the Fermi sea.

It is easy to see that $f(E)$ can also be calculated as the Fourier transform of the first-order correlation function for emitted particles $G^{(1)}(j,j')$, Eq. (9), taken at $x_j = x_{j'} \equiv x$; see also Ref. [31]. Note that $G^{(1)}$ is periodic in $t_{j'}$ when the difference $t_j - t_{j'}$ is kept constant. Therefore, performing a continuous Fourier transformation with respect to $\tau_{jj'} = t_j - t_{j'}$ and afterwards averaging over period $\mathcal{T} = 2\pi/\Omega$ the resulting function of $t_{j'}$ we obtain the desired relation

$$G^{(1)}(E) = \int_0^{\mathcal{T}} \frac{dt_{j'}}{\mathcal{T}} \int_{-\infty}^{\infty} d\tau_{jj'} e^{-i(E/\hbar)\tau_{jj'}} G^{(1)}(t_j x, t_{j'} x) = \frac{f(E)}{v(E)}. \quad (12)$$

2. Two-particle distribution function

One can derive a similar equation relating the two-particle distribution function for emitted particles $f(E,E')$ and the Fourier transform of the second-order correlation function $G^{(2)}$. In the relevant for the present paper case, when $E' - E = \ell\hbar\Omega$, (ℓ is an integer), the distribution function reads (for details, see Appendix A)

$$f(E,E_\ell) = f(E)f(E_\ell) + \delta f(E,E_\ell),$$
$$\delta f(E,E_\ell) = (-1)\left| \sum_{n=-\infty}^{\infty} \{f_0(E_n) - f_0(E)\} \times S_F^*(E,E_n) S_F(E_\ell,E_n)\right|^2. \quad (13)$$

The quantity $f(E,E_\ell)$ admits an interpretation as a joint detection probability to find one electron in the state with energy E and the other electron in the state with energy $E_\ell = E + \ell\hbar\Omega$. For $\ell = 0$ we find $f(E,E) = 0$. That is a consequence of the Pauli exclusion principle, according to which two electrons (fermions) cannot be in the same state, i.e., cannot have the same energy in our case. This feature becomes manifest if the distribution function is rewritten in terms of determinants,

$$f(E,E_\ell) = \frac{1}{2} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \{f_0(E_n) - f_0(E)\}\{f_0(E_m) - f_0(E)\} \times \left| \det \begin{pmatrix} S_F(E,E_n) & S_F(E,E_m) \\ S_F(E_\ell,E_n) & S_F(E_\ell,E_m) \end{pmatrix} \right|^2. \quad (14)$$

The equation above is valid for arbitrary periodic driving. A similar equation but valid for adiabatic driving only was derived in Ref. [45].

C. How to measure distribution functions for emitted particles

The single-particle distribution function, as it was demonstrated experimentally [36–38], is related to the dc current through an energy filter, a quantum dot with a single conducting resonant level. By analogy the two-particle distribution function can be accessed via the correlator of currents through two energy filters.

The cartoon of a possible quantum coherent electronic circuit is shown in Fig. 2. The two-particle source, composed of two single-electron sources S_L and S_R, emits electrons in pairs (possibly electrons and holes). All the metallic contacts 1–6 have the same potentials and the same temperatures. Electrons

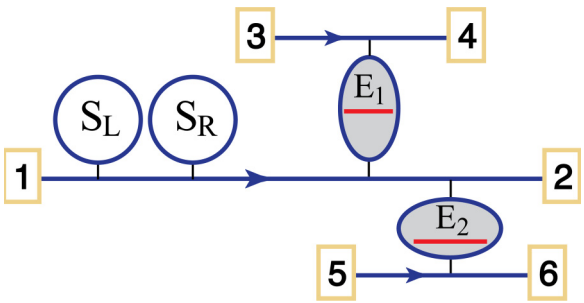

FIG. 2. (Color online) Measurement of a two-particle energy distribution function. 1–6 are metallic contacts. Blue lines with arrows are electronic chiral waveguides. Single-electron emitters S_L and S_R comprise a two-particle emitter. The quantum dots with active levels E_1 and E_2 serve as energy filters.

in contacts are in equilibrium and are characterized by the same Fermi distribution function $f_0(E)$. Blue solid straight lines are chiral electronic waveguides connecting metallic contacts. Such a waveguide can be, for instance, the edge state in the quantum Hall regime. The currents at contacts 4 and 6 and their correlation function are of our interest.

The dynamical periodic source is characterized by the Floquet scattering amplitudes $S_F(E_n,E)$, $E_n = E + n\hbar\Omega$, where n is an integer, and $\Omega = 2\pi/\mathcal{T}$ with $\mathcal{T}$ the period of a drive. Two energy filters $\kappa = 1,2$, quantum dots, having one resonant level, E_κ, each are attached to the central waveguide. The emitted electrons can escape to the contact $\beta = 4,6$ if they pass through the filter $\kappa = 1,2$, respectively.

To calculate the current, I_β, flowing to the contact $\beta = 4,6$, let us start from the current operator in second quantization [43],

$$\hat{I}_\beta(t) = \frac{e}{h}\iint dE dE' e^{it(E-E')/h} \times \{\hat{b}^\dagger_\beta(E)\hat{b}_\beta(E') - \hat{a}^\dagger_\beta(E)\hat{a}_\beta(E')\}. \quad (15)$$

In the equation above the first term in curly brackets describes particles entering the contact β while the second term describes particles leaving the contact β. The path for the latter particles is not shown in Fig. 2.

In the scattering matrix formalism the operators $\hat{b}_\beta$ for particles leaving the circuit to the contact β are expressed in terms of operators $\hat{a}_\gamma$ for particles entering the circuit from the contact γ. For a dynamical circuit these operators are related via the Floquet scattering matrix of the circuit, $S_F^{\rm cir}$ [44]:

$$\hat{b}_\beta(E) = \sum_\gamma \sum_{n=-\infty}^{\infty} S^{\rm cir}_{F,\beta\gamma}(E,E_n)\,\hat{a}_\gamma(E_n), \quad (16)$$

where γ counts all the contacts where electrons can enter the circuit. For the circuit shown in Fig. 2, $\gamma = 1,3,5$. The Floquet scattering matrix elements, $S^{\rm cir}_{F,\beta\gamma}(E,E_n)$, relevant for us here, are listed in Appendix B.

1. dc current

The dc current flowing into the contact $\beta = 4,6$ is calculated as a quantum statistical average over the (equilibrium) state of electrons incoming to the circuit,

$$I_\beta = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \langle \hat{I}_\beta(t)\rangle. \quad (17)$$

We use Eqs. (4), (15), and (16) and find

$$I_\beta = \frac{e}{h}\int dE\, F_\beta(E) \sum_{n=-\infty}^{\infty} |S_F(E,E_n)|^2 \{f_0(E_n) - f_0(E)\}, \quad (18)$$

where $F_4(E) = T_1(E)$ and $F_6(E) = R_1(E)T_2(E)$ with $T_\kappa = |t_\kappa|^2$, $\kappa = 1,2$ and $R_1 = 1 - T_1$; t_κ/r_κ is the transmission/reflection amplitude of the energy filter $\kappa = 1,2$. While deriving the equation above I used the relation $t_\kappa r_\kappa = -t^*_\kappa r^*_\kappa$, which follows from the unitarity of the scattering matrices describing the energy filters. Comparing Eqs. (18) and (11) one can relate the dc current and the distribution function for emitted particles [46],

$$I_\beta = \frac{e}{h}\int dE\, F_\beta(E) f(E). \quad (19)$$

This yields the known possibility of measuring distribution function with quantum dots as energy filters.

2. Zero-frequency noise

The zero-frequency cross-correlation function $\mathcal{P}_{46}$ for currents flowing into contacts 4 and 6 reads [47]

$$\mathcal{P}_{46} = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} \int dt' \frac{\langle \delta\hat{I}_4(t)\delta\hat{I}_6(t+t') + \delta\hat{I}_6(t+t')\delta\hat{I}_4(t)\rangle}{2}. \quad (20)$$

Here $\delta\hat{I}_\beta = \hat{I}_\beta - \langle\hat{I}_\beta\rangle$ is an operator of current fluctuations. After some calculations, see Appendix C, we find

$$\mathcal{P}_{46} = \frac{e^2}{h}\int dE T_1(E) \sum_{p=-\infty}^{\infty} R_1(E_p)T_2(E_p)\delta f(E,E_p). \quad (21)$$

The two Eqs. (19) and (21) allow us to reconstruct the two-particle distribution function, $f(E,E') = f(E)f(E') + \delta f(E,E')$, from all-electric measurements. To illustrate it let us consider ideal energy filters with $T_\kappa(E) = \gamma^2/([E - E_\kappa]^2 + \gamma^2)$. The width of the resonance γ is assumed to be large compared to the energy quantum $\hbar\Omega$ but small compared to the energy scale over which the Floquet scattering matrix changes [46]. The latter requirement is essential to keep f in Eq. (19) and δf in Eq. (21) at the resonant energies only. The former requirement allows us to simplify calculations and to replace $\sum_n \to \int d\omega_n/(\hbar\Omega)$ with $\omega_n = n\hbar\Omega$ and whenever necessary the Kronecker δ is replaced by the Dirac δ, $\delta_{nm} \to \hbar\Omega\delta(\omega_n - \omega_m)$.

After simple calculations one can find $I_4 = eC_1 f(E_1)/\mathcal{T}$, $I_6 = eC_2 f(E_2)/\mathcal{T}$, and $\mathcal{P}_{46} = e^2C_1C_2\delta f(E_1,E_2)$, where $C_1 = \pi\gamma/(\hbar\Omega)$ and $C_2 = C_1(x^2+2)/(x^2+4)$ with $x = (E_1 - E_2)/\gamma$. Combining these equations together we finally arrive at the following relation:

$$f(E_1,E_2) = \frac{\mathcal{T}^2}{e^2C_1C_2}\left\{I_4(E_1)I_6(E_2) + \frac{\mathcal{P}_{46}(E_1,E_2)}{\mathcal{T}}\right\}. \quad (22)$$

The equation above is derived in the case of a dynamical emission of electrons if all the contacts are grounded. In the stationary case but with biased contacts the analogous relation is used in mesoscopics; see, e.g., Refs. [48] and [49].

Equation (22) is calculated within a noninteracting theory. The Coulomb interaction, generally important for quantum dots serving as energy filters, can be screened by the top gates by analogy with how it is done in Ref. [2].

III. ADIABATIC EMISSION

As an example, showing how the general formalism developed above can be used to analyze the emitted state, here I consider in detail the case when particles are emitted adiabatically into an electronic chiral waveguide. Such emission can be realized, for example, with the help of a slow driven

mesoscopic capacitor [50,51], such as the one used in Ref. [2], or using Lorentzian voltage pulses such as in Ref. [3]. In the former case the source emits a stream of alternating electrons and holes, while in the latter case an electron stream is emitted. The quantities related to adiabatic regime will be marked by the subscript "*ad*."

I present results for the low-temperature limit,

$$k_B T_0 \ll \hbar\Omega_0. \tag{23}$$

The generalization to finite temperatures is rather straightforward.

A. Wave functions

1. First-order correlation function and two single-particle bases

We calculate $G^{(1)}$, Eq. (9), for the source emitting particles (electrons or electrons and holes) adiabatically and denote it as $G^{(1)}_{ad}$. The adiabatic regime implies that the scattering amplitudes can be kept almost constant over the energy interval of order $\hbar\Omega$ [44]. This allows us, first, to linearize the dispersion relation, for instance, $k(E_n) \approx k(E) + n\Omega/v(E)$. And, second, to calculate the Floquet scattering amplitude as the corresponding Fourier coefficient,

$$S_F(E_n, E) = S_n(E) \equiv \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} S(t,E)\, e^{in\Omega t}, \tag{24}$$

of the frozen scattering amplitude $S(t,E)$, which is the stationary scattering amplitude parametrically dependent on time. For low temperatures, $k_B T_0 \ll \hbar\Omega$, let us make in Eq. (9) the following substitution: $f_0(E_n) - f_0(E) \approx \delta(E-\mu)\, n\hbar\Omega$, and get [52]

$$G^{(1)}_{ad}(j,j') = \frac{i e^{-i[\phi_j(\mu)-\phi_{j'}(\mu)]}}{2\pi v_\mu} \frac{1 - S^*(\tau_j) S(\tau_{j'})}{\tau_j - \tau_{j'}}. \tag{25}$$

Here I introduced a reduced time $\tau_j = t_j - x_j/v_\mu$, denote $v_\mu \equiv v(\mu)$, and omit the energy argument, $S(\tau) \equiv S(\tau,\mu)$.

Here we are interested in the regime when the source emits wave packets comprising two particles. For definiteness we consider a source composed of two capacitors, S_L and S_R, attached in series to the same chiral electronic waveguide, see Fig. 1, and emitting particles at close times, t^-_L and t^-_R, respectively. To be precise, let us concentrate on a two-electron emission. A two-hole emission can be analyzed in the same way. An electron-hole pair emission in the adiabatic regime is trivial [17] and we do not address it here. In particular, in the case of two identical capacitors there is the reabsorption effect [15,41]: an electron (a hole) emitted by the first capacitor is reabsorbed by the second capacitor attempting to emit a hole (an electron) at the same time. As a consequence nothing is emitted. In contrast, in the nonadiabatic regime an electron-hole pair is emitted [41]. The closely related case of n-particle Lorentzian wave packets is discussed in detail in Refs. [24,31].

In the adiabatic regime the scattering amplitude of the entire source is the product of scattering amplitudes of its constituents, capacitors, $S(\tau) = S_L(\tau) S_R(\tau)$. Close to the time of emission of an electron, t^-_α, the scattering amplitude of the capacitor α can be represented in the Breit-Wigner form [18,53],

$$S_\alpha(\tau) = \frac{\tau - \tau^-_\alpha + i\Gamma_\alpha}{\tau - \tau^-_\alpha - i\Gamma_\alpha}. \tag{26}$$

Here $\tau^-_\alpha = t^-_\alpha - x_\alpha/v_\mu$ is the reduced emission time with x_α the coordinate of the source α; $\Gamma_\alpha \ll \mathcal{T}$ is the half width of the density profile [53] and, correspondingly, the coherence time [25,52] of the single-electron state emitted by the capacitor $\alpha = L,R$. The equation above is given for a single period, $0 < \tau < \mathcal{T}$. To other times it should be extended periodically, $S_\alpha(\tau) = S_\alpha(\tau + \mathcal{T})$.

a. Single-electron emission. If each capacitor would work independently then it would emit an electron on the top of the Fermi sea in the state with the following wave function:

$$\begin{aligned} \Phi_\alpha(j) &\equiv \Phi_\alpha(x_j t_j) = A_\alpha(\tau_j) e^{-i\phi_j(\mu)}, \\ A_\alpha(\tau_j) &= \sqrt{\frac{\Gamma_\alpha}{\pi v_\mu}} \frac{1}{\tau_j - \tau^-_\alpha - i\Gamma_\alpha}. \end{aligned} \tag{27}$$

The wave function given above can be inferred from the first-order correlation function, Eq. (25), with $S = S_\alpha$, which is factorized [31],

$$G^{(1)}_{ad,\alpha}(j,j') = \Phi^*_\alpha(j)\Phi_\alpha(j'). \tag{28}$$

See also Ref. [17] for an alternative derivation.

Straightforward calculations show that the corresponding second-order correlation function, see Eq. (8), is zero, $G^{(2)}_{ad} = 0$, witnessing a single-particle emission. The times τ_j and $\tau_{j'}$ belong to the same period, $|\tau_j - \tau_{j'}| \leqslant \mathcal{T}$. Therefore, the particles emitted during the different periods do not contribute to $G^{(2)}_{ad}$.

b. Two-electron emission. To calculate $G^{(1)}_{ad}$ for a two-particle source with $S(\tau) = S_L(\tau) S_R(\tau)$, I use in Eq. (25) the following identity: $1 - ab = 0.5[1-a][1+b] + 0.5[1-b][1+a]$ with $a = S^*_L(\tau_j) S_L(\tau'_j)$, $b = S^*_R(\tau_j) S_R(\tau'_j)$ and find

$$G^{(1)}_{ad}(j,j') = \frac{1}{2} \sum_{\alpha=L,R} \{\Phi^*_\alpha(j)\Phi_\alpha(j') + \Phi^*_{\alpha\bar\alpha}(j)\Phi_{\alpha\bar\alpha}(j')\}, \tag{29}$$

where $\bar\alpha = L(R)$ for $\alpha = R(L)$ and

$$\begin{aligned} \Phi_{\alpha\bar\alpha}(j) &\equiv \Phi_{\alpha\bar\alpha}(x_j t_j) = A_{\alpha\bar\alpha}\left(\tau_j\right) e^{-i\phi_j(\mu)}, \\ A_{\alpha\bar\alpha}(\tau_j) &= S_\alpha(\tau_j) A_{\bar\alpha}(\tau_j). \end{aligned} \tag{30}$$

The pair of indices $\alpha\bar\alpha$ can be either RL or LR.

The functions Φ_α and $\Phi_{\alpha\bar\alpha}$ (correspondingly, the envelope functions A_α and $A_{\alpha\bar\alpha}$) are mutually orthogonal,

$$\int dx \Phi_\alpha(xt)\Phi^*_{\alpha\bar\alpha}(xt) = 0, \tag{31}$$

and normalized. Therefore, they can serve as a basis for the representation of a two-particle state of emitted electrons. Note that this basis is time dependent. The unitary rotation from one basis to the other is also time dependent. As we will see later on, this results in a basis-dependent energy distribution for each of the electrons, while the energy distribution for two electrons is basis independent.

Let us choose the basis corresponding to some α. Then representing $\Phi_{\bar\alpha}$ and $\Phi_{\bar\alpha\alpha}$ in terms of the basis functions Φ_α

and $\Phi_{\alpha\bar{\alpha}}$ one can rewrite Eq. (29) as the sum of two terms,

$$G^{(1)}_{ad}(j,j') = \Phi^*_{\alpha}(j)\Phi_{\alpha}(j') + \Phi^*_{\alpha\bar{\alpha}}(j)\Phi_{\alpha\bar{\alpha}}(j'). \quad (32)$$

Note that here $\alpha = L$ or $\alpha = R$. There is no a summation over α on the right-hand side in the equation above.

The two terms on the right-hand side of Eq. (32) are single-particle propagators for one electron emitted in the state with the wave function Φ_{α} and the other one with the wave function $\Phi_{\alpha\bar{\alpha}}$, respectively. A closely related representation but for a pulse comprising n identical (with the same Γ and emitted at the same time) particles is given in Ref. [31].

If in a waveguide the electrons propagate to the right, see Fig. 1, then it is natural to choose the basis corresponding to $\alpha = R$, i.e., the basis functions are Φ_R and Φ_{RL}. Then the equation above admits an intuitive and transparent interpretation. An electron in the state with a wave function $\Phi_R(xt)$ is emitted by the rightmost single-electron source and propagates away. Another electron is emitted by the leftmost single-electron source and it passes by the second source. If the latter source would not work the wave function of a second electron would be $\Phi_L(xt)$ (times an irrelevant constant phase factor $S_R(\tau) = \text{const}$, $|S_R|^2 = 1$). However, the working second source adds an extra nontrivial (i.e., time-dependent) factor $S_R(\tau)$ to the wave function of an electron passing it [54]. Therefore, the corresponding wave function becomes $\Phi_{RL}(xt) = S_R(\tau)\Phi_L(xt)$, $\tau = t - x/v_{\mu}$. The extra factor $S_R(\tau)$ in Φ_{RL} is responsible for the orthogonalization of single-particle states that is necessary for two electrons (fermions) to propagate in close vicinity to each other. An intuitive interpretation presented here is possible due to a properly chosen single-particle wave function basis. If we would use another basis the interpretation would be less transparent, while the description would still be correct.

The effect of one source on the electron emitted by the other source depends essentially on the difference of times, $\Delta\tau = \tau^-_R - \tau^-_L$, when electrons are emitted. As an illustration, in Fig. 3 the envelope functions A_R (black solid line) and A_{RL} are contrasted in the case of equal coherence times, $\Gamma_L = \Gamma_R \equiv \Gamma$. If the two electrons are emitted at the same time, $\Delta\tau = 0$, the envelope function A_{RL} (red dashed line) differs substantially from A_R. While if the two electrons are emitted with a long time delay, $\Delta\tau = 10\Gamma$, the envelop function A_{RL} (blue dotted line) resembles essentially A_R.

2. Two-particle wave function

Substituting Eq. (32) into Eq. (8) one can factorize the second-order correlation function,

$$G^{(2)}_{ad}(1,2,3,4) = \left(\Phi^{(2)}_{\alpha}(1,2)\right)^* \Phi^{(2)}_{\alpha}(4,3), \quad (33)$$

and correspondingly find a two-particle wave function,

$$\Phi^{(2)}_{\alpha}(j,j') = A^{(2)}_{\alpha}(\tau_j,\tau_{j'})e^{-i[\phi_j(\mu)+\phi_{j'}(\mu)]},$$
$$A^{(2)}_{\alpha}(\tau_j,\tau_{j'}) = \det\begin{pmatrix} A_{\alpha}(\tau_j) & A_{\alpha\bar{\alpha}}(\tau_j) \\ A_{\alpha}(\tau_{j'}) & A_{\alpha\bar{\alpha}}(\tau_{j'}) \end{pmatrix}. \quad (34)$$

Remember that the reduced time is $\tau = t - x/v_{\mu}$. This wave function is the Slater determinant composed of single-particle wave functions Φ_{α} and $\Phi_{\alpha\bar{\alpha}}$ constituting the basis [31]. Therefore, the Pauli exclusion principle for fermions is satisfied manifestly, $\Phi^{(2)}_{\alpha}(j,j) = 0$.

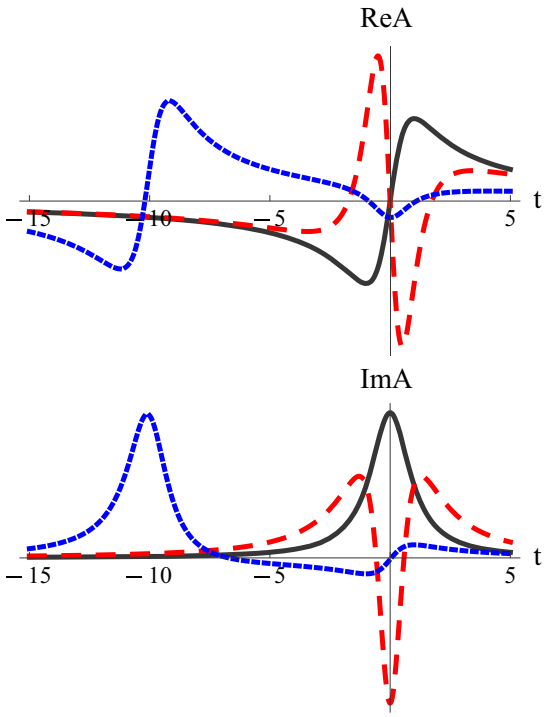

FIG. 3. (Color online) Single-particle wave functions for electrons comprising a pair. The real part (upper panel) and imaginary part (lower panel) of the envelope functions A_R, Eq. (27) (black solid line), and A_{RL}, Eq. (30), are shown as a function of time. The earlier times correspond to events happening first. At simultaneous emission, $t^-_L = t^-_R$, A_{RL} (red dashed line) is quite different from A_R. At successive emission, $t^-_R - t^-_L = 10\Gamma_R$, A_{RL} (blue dotted line) and A_L are essentially the same. The parameters are the following: emission time $t^-_R = 0$; coherence times of single-electron emitters $\Gamma_R = \Gamma_L = 1$.

Note depending on the base wave functions chosen, $\alpha = L$ or $\alpha = R$, the time profile of a two-particle wave function $\Phi^{(2)}_{\alpha}$ will be different, see insets to Fig. 4, while the propagator $G^{(2)}_{ad}$, Eq. (33), remains the same. Note also that the two-particle density profiles is basis independent, $|\Phi^{(2)}_L(j,j')|^2 = |\Phi^{(2)}_R(j,j')|^2$.

When increasing the difference of emission times, $\Delta\tau = \tau^-_R - \tau^-_L \gg \Gamma_{\Sigma}$, the two-particle wave function $\Phi^{(2)}_{\alpha}(j,j')$ is noticeable only for $\tau_j \approx \tau^-_L$, $\tau_{j'} \approx \tau^-_R$, or for $\tau_j \approx \tau^-_R$, $\tau_{j'} \approx \tau^-_L$. In any of these cases the matrix in Eq. (34) has only two nonzero entries, either along the main diagonal or the other two. Apparently the two-particle state is now the product of two single-particle states.

3. Two bases: How to measure

One of the possibilities to access the basis wave functions is to measure the first-order correlation function and utilize its additivity property; see Eq. (32). Two protocols, a single-electron quantum tomography [29,30] and a time-resolved single-electron interferometry [25,52], are already proposed for such a measurement.

First, let us switch on only one single-particle source. Then, according to Eq. (28), if only the capacitor S_L works, we

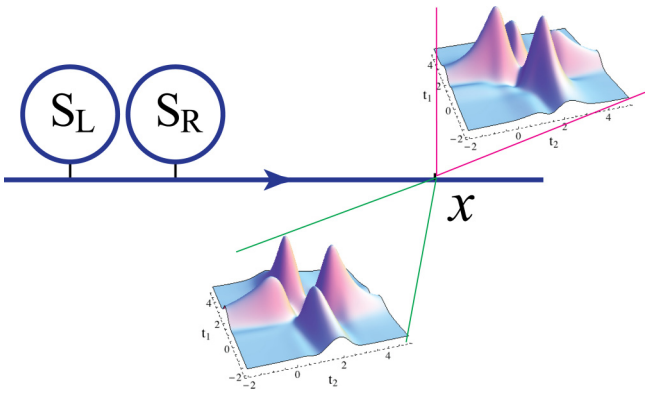

FIG. 4. (Color online) The real part of the envelope of a two-electron wave function emitted adiabatically. It is shown at some position x after the two-particle source and is represented in two different bases, $A_L^{(2)}(t_1x,t_2x)$ (upper inset) and $A_R^{(2)}(t_1x,t_2x)$ (lower inset); see Eq. (34). The parameters are the following: emission times $t_L^- = 2$, $t_R^- = 1$; coherence times of single-electron emitters $\Gamma_R = \Gamma_L = 1$.

measure a single-particle propagator $G^{(1)}_{ad,L}$ and derive the wave function Φ_L. In the same way, if only the capacitor S_R works, we measure $G^{(1)}_{ad,R}$ and obtain the wave function Φ_R. As the next step let us perform a measurement with two single-particle sources switched on. The corresponding measurement provides us with the first-order correlation function, $G^{(1)}_{ad}$, Eq. (32). This correlation function can be represented either as $G^{(1)}_{ad} = G^{(1)}_{ad,L} + G^{(1)}_{ad,LR}$ or as $G^{(1)}_{ad} = G^{(1)}_{ad,R} + G^{(1)}_{ad,RL}$. Therefore, combining the latter measurement with one of the former measurements one can derive $G^{(1)}_{ad,RL}(j,j') = \Phi^*_{RL}(j)\Phi_{RL}(j')$ and $G^{(1)}_{ad,LR}(j,j') = \Phi^*_{LR}(j)\Phi_{LR}(j')$, correspondingly. Combining together Φ_R and Φ_{RL} or Φ_L and Φ_{LR} we, according to Eq. (34) reconstruct a two-particle wave function $\Phi^{(2)}_R$ or $\Phi^{(2)}_L$, respectively. Therefore, the use of different measurement setups allows us to explore different bases for a two-particle wave-function representation.

B. Distribution functions

Electrons emitted by the source on the top of the Fermi sea at low temperatures, Eq. (23), have energies larger than the Fermi energy μ. It is convenient to count the energy E from μ and to introduce the Floquet energy $-\hbar\Omega < \epsilon < 0$ and $\epsilon_n = \epsilon + n\hbar\Omega$ with integer n. Then any energy $E > \mu$ can be represented as $E = \mu + \epsilon_n$ with some $n \geqslant 1$. For any function of one energy, $X(E)$, and two energies, $Y(E,E')$, let us use the following notations: $X(\epsilon_n) \equiv X(E)$ and $Y(\epsilon_r,\epsilon'_s) \equiv Y(E,E')$, where n, r, s are some integers.

1. Single-particle distribution function

For adiabatic emission we use Eq. (24) in Eq. (11) and at low temperatures we arrive at the following equation for a single-particle distribution function for electrons ($\epsilon_n > 0$) [45]:

$$f_{ad}(\epsilon_n) = \sum_{m=0}^{\infty} |S_{n+m}|^2 . \tag{35}$$

Remember that the Fourier coefficients of the scattering matrix, S_{n+m}, are calculated at the Fermi energy μ.

a. Single-electron emission. If the single-particle source α works alone, we have to use $S = S_\alpha$. Calculating the Fourier coefficients for the function $S_\alpha(\tau)$ given in Eq. (26) and substituting them into Eq. (35) one can find [17,55]

$$f_{ad,\alpha}(\epsilon_n) = 2\Omega\Gamma_\alpha e^{-2n\Omega\Gamma_\alpha}. \tag{36}$$

The mean energy of an emitted electron (counted from the Fermi energy) is

$$\langle\epsilon\rangle_\alpha \equiv \sum_{n=1}^{\infty} \epsilon_n f_{ad,\alpha}(\epsilon_n) = \frac{\hbar}{2\Gamma_\alpha}. \tag{37}$$

This is compatible with dc heat calculations [40]. Note that in the equation above we neglected $|\epsilon| \sim \hbar\Omega$ compared to the rest, since $1/\Gamma_\alpha \gg \Omega$.

Alternatively the distribution function $f_{ad,\alpha}(\epsilon_n)$, Eq. (36), can also be calculated via the Fourier transform of the envelope function. Using Eq. (27) we find

$$\begin{aligned} f_{ad,\alpha}(\epsilon_n) &= v_\mu \mathcal{T} |A_{\alpha,n}|^2, \\ A_{\alpha,n} &= \int_0^{\mathcal{T}} \frac{d\tau}{\mathcal{T}} A_\alpha(\tau) e^{in\Omega\tau}. \end{aligned} \tag{38}$$

The relation above tells us that the mean energy $\langle\epsilon\rangle_\alpha$ can be directly expressed in terms of the wave function Φ_α (or in terms of the envelop function A_α) as follows:

$$\begin{aligned} \langle\epsilon\rangle_\alpha &= \int dx \Phi^*_\alpha(xt) \left[i\hbar\frac{\partial}{\partial t} - \mu \right] \Phi_\alpha(xt) \\ &= \int dx A^*_\alpha(\tau) \left[i\hbar\frac{\partial}{\partial \tau} \right] A_\alpha(\tau), \end{aligned} \tag{39}$$

where the expression in the square brackets is nothing but the energy operator for emitted particles.

b. Two-electron emission. For a two-particle source composed of two capacitors, $S = S_L S_R$, with S_α given in Eq. (26) we calculate from Eq. (35)

$$\begin{aligned} f_{ad}(\epsilon_n) = {} & \frac{2\Omega\left(\Delta\tau^2 + \Gamma_\Sigma^2\right)}{\Delta\tau^2 + \Delta\Gamma^2} \\ & \times \left\{ \Gamma_L e^{-2n\Omega\Gamma_L} + \Gamma_R e^{-2n\Omega\Gamma_R} - \frac{4\Gamma_L\Gamma_R e^{-n\Omega\Gamma_\Sigma}}{\Delta\tau^2 + \Gamma_\Sigma^2} \right. \\ & \left. \times \left[\Gamma_\Sigma \cos(n\Omega\Delta\tau) + \Delta\tau \sin(n\Omega\Delta\tau) \right] \right\}. \end{aligned} \tag{40}$$

Here $\Delta\tau = \tau_R^- - \tau_L^-$, $\Delta\Gamma = \Gamma_R - \Gamma_L$, and $\Gamma_\Sigma = \Gamma_R + \Gamma_L$. Remember that $\tau_\alpha^- = t_\alpha^- - x_\alpha/v_\mu$ is a reduced emission time, which accounts for the position x_α of source α. The distribution function $f_{ad}(\epsilon_n)$ is normalized as follows:

$$\sum_{n=1}^{\infty} f_{ad}(\epsilon_n) = 2, \tag{41}$$

indicating that there are altogether two electrons (emitted per period) in the state of interest.

If the two sources would work independently they would emit a particle stream which is characterized by the distribution function $\tilde{f}_{ad}(\epsilon_n) = f_{ad,L}(\epsilon_n) + f_{ad,R}(\epsilon_n)$. This is the

asymptotics of Eq. (40) when two emitted particles do not feel each other, i.e., do not overlap, $|\Delta\tau| \gg \Gamma_\Sigma$. In contrast, at closer emission times, $|\tau_R^- - \tau_L^-| \sim \Gamma_\Sigma$, the wave function's orthogonalization results in an increase of the energy of emitted particles. For instance the mean energy $\langle\epsilon\rangle$ of two emitted particles exceeds the sum $\langle\epsilon\rangle_\Sigma = \langle\epsilon\rangle_L + \langle\epsilon\rangle_R$. Calculating $\langle\epsilon\rangle$ with the help of the distribution function $f_{ad}(\epsilon_n)$, Eq. (40), we find

$$\frac{\langle\epsilon\rangle}{\langle\epsilon\rangle_\Sigma} = 1 + |J(\Delta\tau)|^2, \tag{42}$$

where $J = \int dx \Phi_L^*(xt)\Phi_R(xt) = 2i\sqrt{\Gamma_L\Gamma_R}/(\Delta\tau + i\Gamma_\Sigma)$ is the overlap integral of the wave-functions Φ_L and Φ_R; see Eq. (27). The same overlap integral appears [24] in the problem of the shot-noise suppression for electrons colliding at the quantum point contact [22,41,53]. Note that the mean energy increase shown in Eq. (42) agrees with an enhanced dc heat production of the two-particle emitter [40].

Since the two sources work jointly, the two emitted particles are in states with wave functions Φ_α and $\Phi_{\alpha\bar\alpha}$, see Eq. (32), rather than with Φ_L and Φ_R. Correspondingly, the distribution function given in Eq. (40) can be represented as the sum of two contributions,

$$f_{ad}(\epsilon_n) = f_{ad,\alpha}(\epsilon_n) + f_{ad,\alpha\bar\alpha}(\epsilon_n). \tag{43}$$

The first one, $f_{ad,\alpha}(\epsilon_n)$, is due to a particle in the state with the wave function $\Phi_\alpha(xt)$. It is given in Eqs. (36) and (38). The second one, $f_{ad,\alpha\bar\alpha}(\epsilon_n)$, is due to a particle in the state with the wave function $\Phi_{\alpha\bar\alpha}(xt)$; Eq. (30). It can be calculated by analogy with Eq. (38) as follows:

$$f_{ad,\alpha\bar\alpha}(\epsilon_n) = v_\mu \mathcal{T}\left|\int_0^{\mathcal{T}} \frac{d\tau}{\mathcal{T}} A_{\alpha\bar\alpha}(\tau) e^{in\Omega\tau}\right|^2. \tag{44}$$

For instance, for $\Delta\tau = 0$ and $\Delta\Gamma = 0$ we have

$$f_{ad,\alpha\bar\alpha}(\epsilon_n) = 2\Omega\Gamma\,(1 - 2n\Omega\Gamma)^2\, e^{-2n\Omega\Gamma}. \tag{45}$$

where $\Gamma = \Gamma_L = \Gamma_R$. The energy-dependent prefactor in the equation above is formally responsible for the increase of the mean energy per particle; see Eq. (42). Using $f_{ad,\alpha\bar\alpha}(\epsilon_n)$ we find the mean energy of an electron to be $\langle\epsilon\rangle_{\alpha\bar\alpha} = 3\hbar/(2\Gamma)$; compare to Eq. (37). The energy-dependent prefactor also modifies energy fluctuations $\langle\delta^2\epsilon\rangle = \langle\epsilon^2\rangle - \langle\epsilon\rangle^2$ of emitted electrons [56]. Using Eqs. (36) and (45) we find correspondingly,

$$\langle\delta^2\epsilon\rangle_\alpha = \left(\frac{\hbar}{2\Gamma}\right)^2, \quad \langle\delta^2\epsilon\rangle_{\alpha\bar\alpha} = 5\left(\frac{\hbar}{2\Gamma}\right)^2. \tag{46}$$

The absolute value of fluctuations increases for an electron in the state $\Phi_{\alpha\bar\alpha}$ compared to that of an electron in the state Φ_α. However the relative strength of fluctuations, i.e., compared to the mean energy, decreases: $\langle\delta^2\epsilon\rangle_\alpha/\langle\epsilon\rangle_\alpha^2 = 1$ while $\langle\delta^2\epsilon\rangle_{\alpha\bar\alpha}/\langle\epsilon\rangle_{\alpha\bar\alpha}^2 = 5/9 < 1$.

The decomposition given in Eq. (43) depends on the basis used, $\alpha = L$ or $\alpha = R$. Therefore, one cannot attribute any definite distribution function to a single electron, only to two of them together. Generally this is due to indistinguishability of particles caused by the overlap of their original wave functions, Φ_R and Φ_L, and a subsequent orthogonalization of their actual wave functions, Φ_R and Φ_{RL} (or Φ_L and Φ_{LR}). However, there is also a particular reason why the energy distribution for a single particle is not well defined. It is so since the unitary rotation from one basis to another one is time dependent. Hence the energy distributions for basis wave functions are changed during rotation making meaningless the question about energy properties of a separate electron. In the limit when two electrons are emitted with a long time delay, $|\tau_R^- - \tau_L^-| \gg \Gamma_\Sigma$, the two bases converge to each other and the emitted particles become distinguishable. In this case one can say which electron is characterized by which distribution function, $f_L(E)$ or $f_R(E)$.

All four distribution functions, f_L, f_R, f_{LR}, and f_{RL}, can be accessed experimentally using the energy resolved dc current measurement, see Sec. II C 1, with one or two sources being switched on by analogy with what is sketched in Sec. III A 3 for a wave-function measurement.

2. *Two-particle distribution function*

Let us use the adiabatic approximation, Eq. (24), and represent the two-particle distribution function, Eq. (14), with $E = \mu + \epsilon_r$ and $s = r + \ell$ as follows [45]:

$$f_{ad}(\epsilon_r,\epsilon_s) = \frac{1}{2}\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\left|\det\begin{pmatrix} S_{r+p} & S_{r+q} \\ S_{s+p} & S_{s+q}\end{pmatrix}\right|^2. \tag{47}$$

Remember here all the scattering matrix elements are calculated at the Fermi energy, $E = \mu$. In the equation above the low-temperature limit, Eq. (23), is taken. It is supposed that $\epsilon_r > 0$ and $\epsilon_s > 0$ since the electronic excitations above the Fermi sea are of interest here.

a. Single-electron emission. If only one source α works, we use $S = S_\alpha$. Taking into account Eq. (26) we find the corresponding Fourier coefficients, $S_{\alpha,n>0} = -2\Omega\Gamma_\alpha e^{-n\Omega\Gamma_\alpha} e^{in\Omega t_\alpha^-}$. Direct substitution into Eq. (47) gives $f_{ad}(\epsilon_r,\epsilon_s) = 0$ ($r > 0$, $s > 0$), as it should be for a genuine single-particle state. From Eq. (13) we find in this case $\delta f(\epsilon_r,\epsilon_s) = -f(\epsilon_r)f(\epsilon_s)$.

b. Two-electron emission. In the case when two sources work we have $S(\tau) = S_L(\tau)S_R(\tau)$. With $S_\alpha(\tau)$ from Eq. (26) one can calculate

$$f_{ad}^{(2)}(\epsilon_r,\epsilon_s) = \frac{8\Omega^2\Gamma_L\Gamma_R(\Delta\tau^2 + \Gamma_\Sigma^2)}{\Delta\tau^2 + \Delta\Gamma^2} e^{-\Omega\Gamma_\Sigma(r+s)} \times \{\cosh([r-s]\Omega\Delta\Gamma) - \cos([r-s]\Omega\Delta\tau)\}. \tag{48}$$

Remember $\epsilon_r = \epsilon + r\hbar\Omega$ with integer $r \geqslant 1$ and the Floquet energy $-\hbar\Omega_0 < \epsilon < 0$; $\Delta\Gamma = \Gamma_R - \Gamma_L$ is the difference of coherence times of two sources; $\Delta\tau = \tau_R^- - \tau_L^-$ is the difference of (reduced) emission times, $\tau_\alpha^- = t_\alpha^- - x_\alpha/v_\mu$, with t_α^- an emission time and x_α a position of the source α.

Alternatively the distribution function $f_{ad}^{(2)}(\epsilon_r,\epsilon_s)$, Eq. (48), can be calculated via the double-Fourier transform of the (envelope of the) two-particle wave function, Eq. (34):

$$f_{ad}^{(2)}(\epsilon_r,\epsilon_s) = v_\mu^2\mathcal{T}^2\left|\int_0^{\mathcal{T}}\int \frac{d\tau d\tau'}{\mathcal{T}^2} e^{ir\Omega\tau} e^{is\Omega\tau'} A_\alpha^{(2)}(\tau,\tau')\right|^2 \tag{49}$$

for either $\alpha = L$ or $\alpha = R$.

If we sum up $f^{(2)}_{ad}(\epsilon_r,\epsilon_s)$, Eq. (48), over one energy, ϵ_s or ϵ_r, then we arrive at the single-particle distribution function, Eq. (40), either $f_{ad}(\epsilon_r)$ or $f_{ad}(\epsilon_s)$, respectively, e.g., $\sum_{r=1}^{\infty} f^{(2)}_{ad}(\epsilon_r,\epsilon_s) = f_{ad}(\epsilon_s)$. If we put $r = s$ in Eq. (48) we find $f^{(2)}_{ad}(\epsilon_r,\epsilon_r) = 0$ as it should be, since two electrons cannot be found in the same state (i.e., with the same energy).

How to measure a two-particle distribution via an energy-resolved shot noise was discussed in Sec. II C 2. Here let us estimate a feasibility of such a proposal for an adiabatic emission regime. The energy scale, over which the distribution function, Eq. (48), changes, is $\sim\hbar/(\Gamma_L+\Gamma_R)$. For the adiabatic regime of the source of Ref. [2] the coherence time $\Gamma_\alpha \sim T_\alpha/(2\pi\Omega)$, where T_α is the transmission probability of the quantum point contact connecting the capacitor and the waveguide [53]. At $T_\alpha \sim 0.2$ and $\Omega \sim 2\pi\cdot 500$ MHz we find $\Gamma_\alpha \sim 10$ ps. The width γ of the resonance level of an energy filter should satisfy the following inequality: $\hbar\Omega \ll \gamma \ll \hbar/(2\Gamma_\alpha)$. For the parameter chosen it becomes (in temperature units) 24 mK $\ll \gamma/k_B \ll$ 380 mK. The energy filter used in Refs. [36–38] is characterized by $\gamma/k_B \sim 50$ mK. That is quite reasonable for the purposes we are discussing.

IV. CONCLUSION

I analyzed a two-particle state emitted by two uncorrelated but synchronized single-electron sources (e.g., periodically driven quantum capacitors) coupled in series to the same chiral electronic waveguide. The two-particle correlation function for the emitted state is expressed in terms of the Floquet scattering matrix of a combined two-particle source. The Fourier transform of the correlation function, the two-particle distribution function, is calculated and related to a cross-correlation function of currents flowing through the energy filters, quantum dots with a single conductive level each; see Fig. 2.

In the case of emitters working in the adiabatic regime, the two-particle wave function is calculated and represented in two equivalent but different forms depending on the single-particle wave functions used as a basis; see Fig. 4. The existence of these two bases is rooted in the presence of two single-particle emitters, which affect each other. Let us denote as Ψ_L and Ψ_R the wave functions of a single electron emitted by one or another source if they would work independently. The presence in a waveguide of an electron emitted by one source affects the emission of an electron by the other source such that the actual single-particle wave functions become orthogonal and hence cannot be just Ψ_L and Ψ_R, which in general are not orthogonal. The simplest way to construct orthogonal single-electron wave functions is to take one of them, say Ψ_L, unperturbed and to orthogonalize the other, denote it as Ψ_{LR}. Alternatively one can keep Ψ_R unperturbed and orthogonalize the other, Ψ_{RL}. These two bases, Ψ_L, Ψ_{LR} and Ψ_R, Ψ_{RL}, are exactly what appears naturally when the first-order correlation function for the state emitted by the two-particle source is considered; see Eqs. (29) and (32). In particular, when the electrons are emitted with a long time delay such that they do not overlap, the wave functions Ψ_{LR} and Ψ_{RL} approach Ψ_R and Ψ_L, respectively. What is important is that in the general case all four single-electron wave functions are accessible experimentally.

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APPENDIX A: TWO-PARTICLE DISTRIBUTION FUNCTION

The two-particle distribution function for emitted particles $f(E,E')$ is related to the Fourier transform of the second-order correlation function $G^{(2)}(1,2,3,4)$, Eq. (8), taken at $x_1 = x_4$ and $x_2 = x_3$. We perform a continuous Fourier transformation with respect to $\tau_{14} = t_1 - t_4$ and $\tau_{23} = t_2 - t_3$ and average over t_4 and t_3 and obtain

$$f(E,E';x_1,x_2) = v(E)v(E')\int_0^{\mathcal{T}}\frac{dt_4}{\mathcal{T}}\int_0^{\mathcal{T}}\frac{dt_3}{\mathcal{T}}\int_{-\infty}^{\infty}d\tau_{14}e^{-i(E/\hbar)\tau_{14}}\int_{-\infty}^{\infty}d\tau_{23}e^{-i(E'/\hbar)\tau_{23}}G^{(2)}(t_1x_1,t_2x_2,t_3x_2,t_4x_1)$$
$$= f(E)f(E') + \delta f(E,E';x_1,x_2), \tag{A1}$$

where the irreducible part is

$$\delta f(E,E';x_1,x_2) = (-1)\sum_{n=-\infty}^{\infty}\{f_0(E_n)-f_0(E)\}\sum_{m=-\infty}^{\infty}\{f_0(E'_m)-f_0(E')\}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}e^{ix_1[k(E'_q)-k(E)]}$$
$$\times e^{-ix_2[k(E')-k(E_p)]}\frac{(\hbar\Omega/\pi)^2\sin^2\left(\pi\frac{E'-E}{\hbar\Omega}\right)}{(E'-E+q\hbar\Omega)(E'-E-p\hbar\Omega)}S^*_F(E,E_n)S_F(E_p,E_n)S^*_F(E',E'_m)S_F(E'_q,E'_m). \tag{A2}$$

Here $E_n = E + n\hbar\Omega$. In general the equation above is complex. However, if the energy difference is a multiple of the energy quantum $\hbar\Omega$, i.e., $E'-E = \ell\hbar\Omega$, (ℓ is an integer) then Eq. (A2) becomes manifestly real and loses its dependence on spatial coordinates x_1 and x_2. Taking into account that now only $p = -q = \ell$ contribute to Eq. (A2), we arrive at Eq. (13). Note that namely Eq. (13) not Eq. (A2) is relevant for measurable quantities considered in this paper; see, e.g., Eq. (21).

APPENDIX B: SCATTERING MATRIX ELEMENTS RELEVANT FOR THE CIRCUIT SHOWN IN FIG. 2

In order to calculate the operators $\hat{b}_\beta(E)$, Eq. (16), and, correspondingly, Eqs. (17) and (20), we need the following scattering matrix elements:

$$S^{\rm cir}_{F,41}(E,E_n)=e^{i\varphi_{41}(E)}t_1(E)S_F(E,E_n),\quad S^{\rm cir}_{F,43}(E,E_n)=e^{i\varphi_{43}(E)}r_1(E)\delta_{n0},\quad S^{\rm cir}_{F,61}(E,E_n)=e^{i\varphi_{61}(E)}t_2(E)r_1(E)S_F(E,E_n),$$
$$S^{\rm cir}_{F,65}(E,E_n)=e^{i\varphi_{65}(E)}r_2(E)\delta_{n0},\quad S^{\rm cir}_{F,63}(E,E_n)=e^{i\varphi_{63}(E)}t_2(E)t_1(E)\delta_{n0}, \tag{B1}$$

where t_κ/r_κ is the transmission/reflection amplitude of the energy filter $\kappa=1,2$, δ_{n0} is the Kronecker δ, $\varphi_{\beta\gamma}(E)$ is the phase of the free propagation through the circuit on the way from contact α to contact β. Since the circuit under consideration has no loops (it is single connected), the phases $\varphi_{\beta\gamma}$ are irrelevant.

APPENDIX C: ZERO-FREQUENCY CURRENT CORRELATION FUNCTION

Let us substitute Eqs. (15), (16), and (B1) into Eq. (20) and calculate

$$\mathcal{P}_{46}=\frac{e^2}{2h}\int dE\,T_1(E)\Bigg[-2R_1(E)T_2(E)\sum_{n=-\infty}^{\infty}\{f_0(E_n)-f_0(E)\}^2|S_F(E,E_n)|^2+\sum_{p=-\infty}^{\infty}R_1(E_p)T_2(E_p)$$
$$\times\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\{f_0(E_n)-f_0(E_m)\}^2S_F^*(E,E_n)S_F(E,E_m)S_F^*(E_p,E_m)S_F(E_p,E_n)\Bigg]. \tag{C1}$$

Since there is no direct path between contacts 4 and 6 in the circuit shown in Fig. 2, the thermal noise does not appear in the equation above. To simplify Eq. (C1) let us represent

$$\{f_0(E_n)-f_0(E_m)\}^2=\{f_0(E_n)-f_0(E)\}^2+\{f_0(E_m)-f_0(E)\}^2-2\{f_0(E_n)-f_0(E)\}\{f_0(E_m)-f_0(E)\}. \tag{C2}$$

One can use the unitarity of the Floquet scattering matrix, Eq. (7), and show that the two first terms on the right-hand side of Eq. (C2) after substitution into Eq. (C1) cancel the first term on the right-hand side of Eq. (C1). What remains is an equation of interest, Eq. (21), relating $\mathcal{P}_{46}$ and the irreducible part of the two-particle distribution function δf, Eq. (13).

[1] M. D. Blumenthal, B. Kaestner, L. Li, S. P. Giblin, T. J. B. M. Janssen, M. Pepper, D. Anderson, G. A. C. Jones, and D. A. Ritchie, Nat. Phys. **3**, 343 (2007).

[2] G. Fève, A. Mahé, J.-M. Berroir, T. Kontos, B. Plaçais, D. C. Glattli, A. Cavanna, B. Etienne, and Y. Jin, Science **316**, 1169 (2007).

[3] J. Dubois, T. Jullien, F. Portier, P. Roche, A. Cavanna, Y. Jin, W. Wegscheider, P. Roulleau, and D. C. Glattli, Nature (London) **502**, 659 (2013).

[4] A. Mahé, F. D. Parmentier, E. Bocquillon, J.-M. Berroir, D. C. Glattli, T. Kontos, B. Plaçais, G. Fève, A. Cavanna, and Y. Jin, Phys. Rev. B **82**, 201309(R) (2010).

[5] E. Bocquillon, F. D. Parmentier, C. Grenier, J.-M. Berroir, P. Degiovanni, D. C. Glattli, B. Plaçais, A. Cavanna, Y. Jin, and G. Fève, Phys. Rev. Lett. **108**, 196803 (2012).

[6] E. Bocquillon, V. Freulon, J.-M. Berroir, P. Degiovanni, B. Plaçais, A. Cavanna, Y. Jin, and G. Fève, Science **339**, 1054 (2013).

[7] J. D. Fletcher, P. See, H. Howe, M. Pepper, S. P. Giblin, J. P. Griffiths, G. A. C. Jones, I. Farrer, D. A. Ritchie, T. J. B. M. Janssen, and M. Kataoka, Phys. Rev. Lett. **111**, 216807 (2013).

[8] B. Kaestner, V. Kashcheyevs, S. Amakawa, M. D. Blumenthal, L. Li, T. J. B. M. Janssen, G. Hein, K. Pierz, T. Weimann, U. Siegner, and H. W. Schumacher, Phys. Rev. B **77**, 153301 (2008).

[9] A. Fujiwara, K. Nishiguchi, and Y. Ono, Appl. Phys. Lett. **92**, 042102 (2008).

[10] C. Leicht, P. Mirovsky, B. Kaestner, F. Hohls, V. Kashcheyevs, E. V. Kurganova, U. Zeitler, T. Weimann, K. Pierz, and H. W. Schumacher, Semicond. Sci. Technol. **26**, 055010 (2011).

[11] M. Kataoka, J. D. Fletcher, P. See, S. P. Giblin, T. J. B. M. Janssen, J. P. Griffiths, G. A. C. Jones, I. Farrer, and D. A. Ritchie, Phys. Rev. Lett. **106**, 126801 (2011).

[12] L. Fricke, M. Wulf, B. Kaestner, V. Kashcheyevs, J. Timoshenko, P. Nazarov, F. Hohls, P. Mirovsky, B. Mackrodt, R. Dolata, T. Weimann, K. Pierz, and H. W. Schumacher, Phys. Rev. Lett. **110**, 126803 (2013).

[13] L. S. Levitov, H. Lee, and G. B. Lesovik, J. Math. Phys. **37**, 4845 (1996).

[14] D. A. Ivanov, H. W. Lee, and L. S. Levitov, Phys. Rev. B **56**, 6839 (1997).

[15] J. Splettstoesser, S. Ol'khovskaya, M. Moskalets, and M. Büttiker, Phys. Rev. B **78**, 205110 (2008).

[16] A. V. Lebedev, G. B. Lesovik, and G. Blatter, Phys. Rev. B **72**, 245314 (2005).

[17] J. Keeling, I. Klich, and L. S. Levitov, Phys. Rev. Lett. **97**, 116403 (2006).

[18] J. Keeling, A. V. Shytov, and L. S. Levitov, Phys. Rev. Lett. **101**, 196404 (2008).

[19] M. Vanević, Y. V. Nazarov, and W. Belzig, Phys. Rev. B **78**, 245308 (2008).

[20] F. Hassler, B. Küng, G. B. Lesovik, G. Blatter, V. Lebedev, and M. Feigelman, in *Advances in Theoretical Physics: Landau Memorial Conference* (AIP, Melville, NY, 2009), pp. 113–119.

[21] Y. Sherkunov, J. Zhang, N. d'Ambrumenil, and B. Muzykantskii, Phys. Rev. B **80**, 041313 (2009).
[22] M. Moskalets and M. Büttiker, Phys. Rev. B **83**, 035316 (2011).
[23] Y. Sherkunov, N. d'Ambrumenil, P. Samuelsson, and M. Büttiker, Phys. Rev. B **85**, 081108 (2012).
[24] J. Dubois, T. Jullien, C. Grenier, P. Degiovanni, P. Roulleau, and D. C. Glattli, Phys. Rev. B **88**, 085301 (2013).
[25] G. Haack, M. Moskalets, and M. Büttiker, Phys. Rev. B **87**, 201302 (2013).
[26] M. V. Moskalets, *Scattering Matrix Approach to Non-Stationary Quantum Transport* (Imperial College Press, London, 2011).
[27] F. D. Parmentier, E. Bocquillon, J.-M. Berroir, D. C. Glattli, B. Plaçais, G. Fève, M. Albert, C. Flindt, and M. Büttiker, Phys. Rev. B **85**, 165438 (2012).
[28] K. E. Cahill and R. J. Glauber, Phys. Rev. A **59**, 1538 (1999).
[29] C. Grenier, R. Hervé, G. Fève, and P. Degiovanni, Mod. Phys. Lett. B **25**, 1053 (2011).
[30] C. Grenier, R. Hervé, E. Bocquillon, F. D. Parmentier, B. Plaçais, J.-M. Berroir, G. Fève, and P. Degiovanni, New J. Phys. **13**, 093007 (2011).
[31] C. Grenier, J. Dubois, T. Jullien, P. Roulleau, D. C. Glattli, and P. Degiovanni, Phys. Rev. B **88**, 085302 (2013).
[32] D. Ferraro, A. Feller, A. Ghibaudo, E. Thibierge, E. Bocquillon, G. Fève, C. Grenier, and P. Degiovanni, Phys. Rev. B **88**, 205303 (2013).
[33] A. Mahé, F. D. Parmentier, G. Fève, J.-M. Berroir, T. Kontos, A. Cavanna, B. Etienne, Y. Jin, D. C. Glattli, and B. Plaçais, J. Low Temp. Phys. **153**, 339 (2008).
[34] J. S. Lundeen, B. Sutherland, A. Patel, C. Stewart, and C. Bamber, Nature (London) **474**, 188 (2011).
[35] C. Polycarpou, K. N. Cassemiro, G. Venturi, A. Zavatta, and M. Bellini, Phys. Rev. Lett. **109**, 053602 (2012).
[36] C. Altimiras, H. le Sueur, U. Gennser, A. Cavanna, D. Mailly, and F. Pierre, Nat. Phys. **6**, 34 (2010).
[37] H. le Sueur, C. Altimiras, U. Gennser, A. Cavanna, D. Mailly, and F. Pierre, Phys. Rev. Lett. **105**, 056803 (2010).
[38] C. Altimiras, H. le Sueur, U. Gennser, A. Cavanna, D. Mailly, and F. Pierre, Phys. Rev. Lett. **105**, 226804 (2010).
[39] J. Gabelli and B. Reulet, Phys. Rev. B **87**, 075403 (2013).
[40] M. Moskalets and M. Büttiker, Phys. Rev. B **80**, 081302(R) (2009).
[41] M. Moskalets, G. Haack, and M. Büttiker, Phys. Rev. B **87**, 125429 (2013).
[42] R. Glauber, Rev. Mod. Phys. **78**, 1267 (2006).
[43] M. Büttiker, Phys. Rev. B **46**, 12485 (1992).
[44] M. Moskalets and M. Büttiker, Phys. Rev. B **66**, 205320 (2002).
[45] M. Moskalets and M. Büttiker, Phys. Rev. B **73**, 125315 (2006).
[46] F. Battista and P. Samuelsson, Phys. Rev. B **85**, 075428 (2012).
[47] Y. M. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).
[48] N. M. Chtchelkatchev, G. Blatter, G. B. Lesovik, and T. Martin, Phys. Rev. B **66**, 161320 (2002).
[49] P. Samuelsson and M. Büttiker, Phys. Rev. B **73**, 041305 (2006).
[50] M. Büttiker, H. Thomas, and A. Prêtre, Phys. Lett. A **180**, 364 (1993).
[51] J. Gabelli, G. Fève, J.-M. Berroir, B. Plaçais, A. Cavanna, B. Etienne, Y. Jin, and D. C. Glattli, Science **313**, 499 (2006).
[52] G. Haack, M. Moskalets, J. Splettstoesser, and M. Büttiker, Phys. Rev. B **84**, 081303 (2011).
[53] S. Ol'khovskaya, J. Splettstoesser, M. Moskalets, and M. Büttiker, Phys. Rev. Lett. **101**, 166802 (2008).
[54] S. Juergens, J. Splettstoesser, and M. Moskalets, Europhys. Lett. **96**, 37011 (2011).
[55] M. Moskalets, J. Comput. Electron. **12**, 397 (2013).
[56] F. Battista, M. Moskalets, M. Albert, and P. Samuelsson, Phys. Rev. Lett. **110**, 126602 (2013).