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Probing dynamics of Majorana fermions in quantum impurity systems

Christophe Mora[1] and Karyn Le Hur[2]

[1]Laboratoire Pierre Aigrain, École Normale Supérieure, Université Paris 7 Diderot, CNRS; 24 rue Lhomond, 75005 Paris, France

[2]Center for Theoretical Physics, Ecole Polytechnique, CNRS, 91128 Palaiseau Cedex, France

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We investigate the admittance of a metallic quantum *RC* circuit with a spinful single-channel lead or equally with two conducting spin-polarized channels, in which Majorana fermions play a crucial role in the charge dynamics. We address how the two-channel Kondo physics and its emergent Majoranas arise. The existence of a single unscreened Majorana mode results in non-Fermi-liquid features and we determine the universal crossover function describing the Fermi-liquid to non-Fermi-liquid region. Remarkably, the same universal form emerges both at weak transmission and large transmission. We find that the charge relaxation resistance strongly increases in the non-Fermi-liquid realm. Our findings can be measured using current technology assuming a large cavity.

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The need for fast manipulation and readout of quantum coherent circuits, notably in the perspective of quantum computation, has been a strong motivation to investigate the dynamical response of nanoconductors.[1] Excited at frequencies ω in the quantum regime, $\hbar\omega \gg k_B T$, the systems evolve due to the intriguing interplay of correlation and quantum coherence effects. The quantum *RC* circuit,[2] a quantum dot attached to a single lead and polarized by an external ac gate voltage, has emerged as the archetypical system for studying the dynamics of coherent circuits.[3–11] Recent experiments[12] on quantum hybrid structures combining microwave resonators[13] with semiconductor or nanotube quantum dots offer an alternative perspective to measure the admittance of quantum circuits.[14] The existence of a quantized[15] charge relaxation (ac) resistance $R_q = h/(2e^2)$ in the quantum *RC* circuit has been shown[16] (and measured[2]) to originate from the Fermi-liquid (FL) nature of low-energy excitations where the elementary quasiparticles are noninteracting fermions. A deep connection[3] has also been drawn between the quantized resistance $R_q = h/e^2$ for large dots, the Shiba relation, and the one-channel Kondo model in *RC* circuits[17] close to the charge degeneracy points.

In this Rapid Communication, we investigate the non-Fermi-liquid (NFL) situation where the elementary quasiparticles are Majorana fermions.[18] This description naturally applies to the quantum *RC* circuit with spinful (spin unpolarized) electrons and a large cavity (dot)[19–21] and is associated to the two-channel Kondo model.[22] It could be extended to the case of the helical edges of quantum spin Hall states[23–25] since the model is invariant upon reversing the direction of one of the spin species.[26] As discussed below, the corresponding low-energy effective theory involves eight chiral Majorana fermions[27] and a local Majorana fermion (Klein factor) representing the residual spin of the impurity. Although the local Majorana cannot be manipulated as a separate object and used for quantum computation, its presence is fundamental in the emergence of NFL physics.[28]

The search for the existence of Majorana fermions has engendered a spurt of experimental efforts in condensed-matter systems.[29–35] In our case, the local Majorana is a remnant spin degree of freedom and not a composite object resulting from superconductivity as in topological wires.[36] Nevertheless, our system is described at low energy by a Majorana resonant level model, or the Emery-Kivelson model,[37] which also describes the coupling of a local Majorana fermion to a normal lead in a topological superconducting wire.

In the quantum *RC* circuit with two conducting (spin) modes, the local Majorana fermion acquires a spectral width Γ, due to its coupling to the leads, which sets a crossover energy scale. Below Γ, the dynamics of the local Majorana is quenched and FL physics dominates while NFL behavior[38] emerges at energies above Γ. The crossover energy scale Γ vanishes at the charge degeneracy points.[39] Here, we provide an analytical expression for charge fluctuations along this universal crossover as a function of frequency: the charge relaxation resistance starts at $R_q = h/(2e^2)$ for a "2-mode" large cavity when $\omega = 0$ and rapidly increases with frequency towards the NFL region.

The system under study comprises a large (metallic) quantum dot attached to a lead via a quantum point contact (QPC) with a single spin-unpolarized channel.[19,20,39,40] The quantum *RC* circuit could be equally built at the helical edges of quantum spin Hall insulators.[11] Electron confinement implies a charging energy $E_C = e^2/(2C_g)$, where C_g is the capacitance of the dot, and the interaction term $H_C = E_C(\hat{N} - N_0)^2$ in the Hamiltonian. N_0 is the dimensionless gate voltage, and the operator $e\hat{N}$ gives the electron charge on the dot. Below, we address the extreme cases of almost transparent and weakly transmitting QPC.

We consider first an almost open dot with weak charge quantization, i.e., charge quantization is strongly smeared out by the large dot-lead coupling. The model can be reduced to a one-dimensional form with coordinate x, the region $x < 0$ defining the lead and $x > 0$ the (infinite) dot. Electrons are weakly backscattered, with amplitude $r \ll 1$, at the boundary $x = 0$. In this regime, spin and charge excitations occur at well-separated energy scales, $r^2 E_C \ll E_C$, and the system is conveniently described using bosonization[41,42] in the spin and charge sectors. Following a standard sequence[39,43] of bosonization and refermionization (see Supplemental Material[44]), we find the exact action describing the system $S = S_F + S_c + S_{BS}$, with

$$S_c = \sum_{\omega_m} |\phi_c(\omega_m)|^2 \left(|\omega_m| + \frac{2E_C}{\pi} \right), \tag{1a}$$

$$S_{BS} = i r_0 \int_0^\beta d\tau\, \eta(\tau)\, \hat{a}(\tau) \cos\left(\sqrt{2}\phi_c(\tau) + \pi N_0\right), \tag{1b}$$

where $r_0 = 2v_F\, r\, \sqrt{2/\pi a_0}$, $\eta(\tau) \equiv \eta(x=0,\tau)$, and $\phi_c(\tau) \equiv \phi_c(x=0,\tau)$. The charge bosonic field at $x=0$ is related to the charge on the dot $\phi_c = (\pi/\sqrt{2})\hat{N}$. The bosonization procedure introduces a boson field $\phi(x)$ which embodies spin excitations along the one-dimensional fermionic line. Refermionization of ϕ

$$\frac{1}{2\sqrt{\pi a_0}} e^{i\phi(x)} = \hat{a}\, \psi(x), \tag{2}$$

defines a (Klein factor) Majorana fermion $\hat{a} = \hat{a}^\dagger$. In Eq. (1a), S_F is the free part for the chiral Majorana fermion $\eta(x) = [\psi(x) - \psi^\dagger(x)]/(i\sqrt{2})$. Here the Majorana $\hat{a}$ has nothing to do with superconductivity but rather describes the residual spin-1/2 degree of freedom emerging after the Kondo screening of the original spin-1/2 at the dot-lead interface.[39,43] The existence of this unscreened degree of freedom is responsible, as discussed below, for the emergence of NFL features.

Integrating the massive charge field ϕ_c in Eq. (1a) yields, to leading order in $r \ll 1$, an exactly solvable Majorana resonant level model.[39] The Majorana fermion $\hat{a}$ acquires the spectral width $\Gamma = (8E_C\gamma/\pi^2)\, r^2 \cos^2(\pi N_0)$ with $\Gamma \sim r^2 E_C \ll E_C$ and $\ln\gamma = \mathbf{C} \simeq 0.5772$ is the Euler constant. Below the energy scale Γ, spin excitations are quenched. NFL features arise due to the combined effect of spin excitation, described by $\hat{a}$, with the quenching of charge excitation, that is for energies between Γ and E_c. Below Γ, a crossover to a Fermi-liquid regime is established.[39]

We are interested in the charge susceptibility $\chi_C(t) = i\theta(t)\langle[\hat{N}(t),\hat{N}]\rangle$. The low-frequency expansion of the related admittance,

$$G(\omega) = -i\omega e^2 \chi_C(\omega) \equiv -i\omega C_0(1 + i\omega C_0 R_q), \tag{3}$$

defines the differential capacitance C_0 and the charge relaxation resistance R_q. In order to compute χ_C, the charge field ϕ_c should not be fully integrated and we need to extend the analysis of Refs. 39 and 45: this is discussed in the Supplemental Material. Following a lengthy but straightforward perturbative calculation, for $r \ll 1$, we obtain at zero temperature[46] the result $\chi_C = K_0 + K_1 + K_2$,

$$e^2 K_0(\omega)/C_g = \alpha(\omega), \tag{4a}$$

$$e^2 K_1(\omega)/C_g = -8\gamma v_F\, r^2 \sin^2(\pi N_0)\, \alpha(\omega)^2\, \Pi_{a\eta}(\omega), \tag{4b}$$

$$e^2 K_2(\omega)/C_g = -8\gamma v_F\, r^2 \cos^2(\pi N_0)\, \alpha(\omega)^2\, \frac{\ln(E_C/\Gamma)}{2\pi} \tag{4c}$$

with $\alpha(\omega) = (1 - i\omega\pi/2E_C)^{-1}$. Only K_0 survives in the absence of backscattering $r=0$, in which case, one obtains, by comparing with Eq. (3), $C_0 = C_g$ and $R_q = h/(2e^2)$, half of the result of spinless electrons for a large dot h/e^2.[3] $\Pi_{a\eta}(\tau) = \langle \eta(\tau)\hat{a}(\tau)\eta\hat{a}\rangle$ is a polarization operator computed from the quadratic part of the action with the result

$$\Pi_{a\eta}(\omega) = -(1/2\pi v_F)[\ln(E_C/\Gamma) + \beta(\omega/\Gamma)],$$
$$\beta(x) = -(1 + 2i/x)\ln(1 - ix). \tag{5}$$

NFL behavior in the charge susceptibility is signaled by logarithmic singularities in the computation of $\Pi_{a\eta}$ cutoff by the charging energy E_C. They arise in the contraction $\langle\hat{a}\hat{a}\rangle\langle\eta\eta\rangle$ and essentially originate from the fact that the Majorana operator $\hat{a}$ has zero dimension for energies between Γ and E_C.

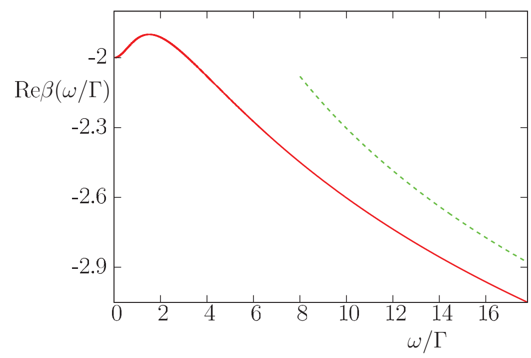

FIG. 1. (Color online) Real part of the function β as a function of the ratio ω/Γ. The dotted line, $-\ln x$, gives the NFL asymptotical behavior caused by the local Majorana fermion.

The function $\beta(\omega/\Gamma)$ describes the crossover between the FL and NFL responses for $\omega \ll \Gamma$ and $\omega \gg \Gamma$ respectively, its real part is shown in Fig. 1. For $\omega \ll \Gamma$, we use the expansion

$$\beta(x) \simeq -2 + x^2/6 + ix^3/6 - 3x^4/20 + \cdots, \quad x \ll 1 \tag{6}$$

inserted in Eq. (4) and compare with Eq. (3) to extract C_0 and R_q. At vanishing frequency, the static susceptibility, and C_0, coincide precisely with Ref. 39. Remarkably and similarly to the spinless case, we find no correction to the charge relaxation resistance $R_q = h/(2e^2)$ for $r \neq 0$ due to the absence of a linear term in Eq. (6). This result confirms FL behavior at low energy. Indeed, using the Fermi-liquid approach elaborated in Ref. 16, where lead electrons are coherently backscattered with a phase shift proportional to the static charge susceptibility,[41] one easily derives the Shiba relation and shows that $R_q = h/(2e^2)$ for an arbitrary transmission of the QPC. Note that Γ vanishes at $N_0 = 1/2$ where the system is always a NFL.

We now turn to the opposite limit of weak transmission of the QPC. The system is adequately described[3,17] by the tunnel Hamiltonian $H = H_0 + H_C + H_T$ where

$$H_T = t \sum_{k,k',s=\uparrow,\downarrow} (d^\dagger_{ks} c_{k's} + c^\dagger_{k's} d_{ks}) \tag{7}$$

transfers electrons between the (large) dot and the lead with operators c_k and d_k respectively; the index s refers, e.g., to the two spin polarizations. The free-electron part reads $H_0 = \sum_{a=c,d,s} \varepsilon_k a^\dagger_{ks} a_{ks}$ for dot and lead electrons. H_T either decreases or increases the dot charge by one unit and thus does not commute with H_C. Far from charge degeneracy, the perturbative approach of Ref. 3 can be reproduced with an additional factor 2 that accounts for spin degeneracy. One readily obtains $R_q = h/(2e^2)$, again in agreement with the Fermi-liquid picture. Perturbation theory however breaks down close to charge degeneracy $N_0 \simeq 1/2$ where NFL physics starts to play a role. In this region, the charge states other than 0 and 1 can be disregarded and a mapping to the two-channel Kondo model formulated[3,17] where the two charge states are represented by a spin 1/2 with $\hat{N} = \frac{1}{2} + S_z$. The vicinity to charge degeneracy $h_0 = E_C(1 - 2N_0)$ defines a local magnetic field coupled to S_z. Our study of charge fluctuations is then translated to a study of the local spin susceptibility in the two-channel Kondo model.

For $h_0 \ll T_K$, where T_K is the Kondo temperature, two regimes have been identified[22] in the renormalization-group (RG) analysis: NFL properties dominate for frequencies (energies) $\Gamma = h_0^2/(2T_K) < \omega < T_K$ while a FL response is obtained at smaller frequencies $\omega < \Gamma$. The crossover is investigated analytically using the SO(8) representation[27] of the two-channel Kondo model, which provides a simple description of the NFL fixed point.[47,48] The bulk fermions, with two spin species and two channels, have a nonlocal representation in terms of eight chiral Majorana fermions. With no impurity, the free Hamiltonian reads

$$H_0^{\mathrm{K}} = \frac{-i v_F}{2} \sum_{j=1}^{8} \int_{-\infty}^{+\infty} dx\, \chi_j(x) \partial_x \chi_j(x). \tag{8}$$

The Majorana fermions $\chi_{1,2,3}$ generate the spin current, $\chi_{4,5,6}$ the flavor current, and χ_7, χ_8 the charge current.

In the presence of the Kondo impurity coupled only to the spin current, the NFL infrared fixed point is simply characterized by the twisted boundary conditions $\chi_j(0^-) = -\chi_j(0^+)$ for $j = 1,2,3$. Absorbing this $\pi/2$ phase shift into a redefinition of the fields, $\chi_{1,2,3}(x) \to \mathrm{sgn}(x)\chi_{1,2,3}(x)$, one recovers the free Hamiltonian form Eq. (8) also at the infrared fixed point. A finite local magnetic field h_0 destabilizes this fixed point with the relevant perturbation[47] $H_{\mathrm{b}} = i(h_0/\sqrt{T_K/v_F})\chi_1 \hat{a}$ of scaling dimension 1/2, where $\chi_1 = \chi_1(0)$. The local Majorana fermion $\hat{a}$ describes the residual impurity spin. The Hamiltonian $H_{IR} = H_0^K + H_b$ is equivalent to the two-dimensional Ising model with a boundary magnetic field, a correspondence that has been used to calculate the one-body Green's function along the FL to NFL crossover.[48] For energies $\omega \ll \Gamma$, the relevant boundary term H_{b} restores the continuity of $\chi_1(x)$ at $x = 0$. We thus recover a Fermi liquid as the even number of twisted fields (χ_2 and χ_3) indicates.[27]

Quite generally, the impurity spin can be expanded over the different operators allowed by conformal field theory (CFT). At low energy, the leading term is

$$S_z = i\sqrt{\frac{v_F}{T_K}}\, \chi_1 \hat{a}, \tag{9}$$

in accordance with H_b. The spin susceptibility $\chi_s(\tau) = -(v_F/T_K)\langle \chi_1(\tau)\hat{a}(\tau)\chi_1\hat{a}\rangle$ is obtained by noting the equivalence between the Hamiltonian H_{IR} and the quadratic action $S_F + S_{BS}^0$, derived in the large transparency case. We identify $\eta = \chi_1$ and $2\Gamma = h_0^2/T_K$ and find

$$\chi_{s0}(\omega) = \frac{1}{2\pi T_K}\left[\ln\left(\frac{T_K}{\Gamma}\right) + \beta(\omega/\Gamma)\right], \tag{10}$$

describing the FL-NFL crossover. At large frequency $\omega \gg \Gamma$, the spin susceptibility exhibits NFL features

$$\chi_{s0}(\omega) = \frac{1}{2\pi T_K}\left[\ln\left(\frac{T_K}{\omega}\right) + \frac{i\pi}{2}\right], \tag{11}$$

in agreement with the prediction of conformal field theory[22] and Abelian bosonization.[37] The absence of a linear term in Eq. (6) requires, for the calculation of R_q, to include the leading irrelevant perturbation to H_{IR},

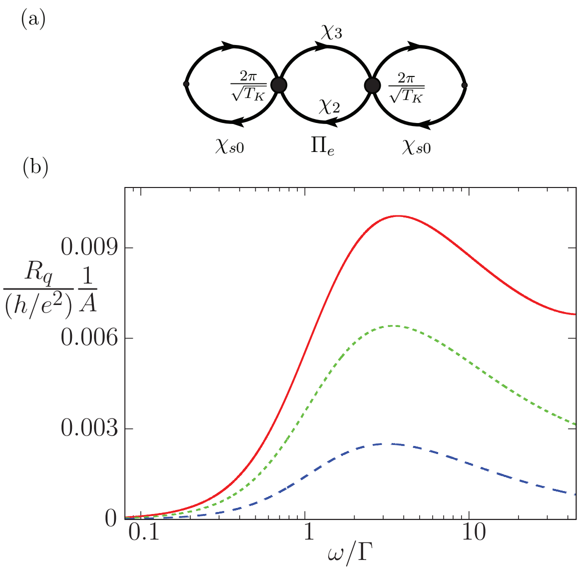

FIG. 2. (Color online) (a) Vertex correction to the spin susceptibility χ_s to second order in δH. (b) Universal charge relaxation resistance valid at all transmissions showing the increase towards the NFL regime. It is plotted here for $B = 6,7,10$ (solid, dotted, and dashed lines). At weak transmission, the maxima of R_q are respectively 4.03, 7.02, and 55.1 in units of h/e^2. We note that the quantized result $R_q = h/2e^2$, recovered at zero frequency, is not visible in this plot computed in the scaling limit $A \gg 1$.

$$\delta H = \frac{2\pi v_F^{3/2}}{\sqrt{T_K}}\, \chi_1\chi_2\chi_3\hat{a}. \tag{12}$$

The only linear in frequency correction to the spin susceptibility comes from the vertex correction, shown Fig. 2(a), $\delta\chi_s(\omega) = \chi_{s0}(\omega)\Pi_e(\omega)\chi_{s0}(\omega)$ where $\Pi_e(\omega)$ is the electron-hole pair susceptibility. At zero temperature, $\Pi_e(\omega) = i\pi\omega$, interpreted as the dissipation produced by electron-hole excitations. Expanding the spin susceptibility $\chi_s = \chi_{s0} + \delta\chi_s$ to linear order in ω, we arrive at the Shiba relation

$$\mathrm{Im}\chi_s(\omega) = \hbar\pi\omega\, \chi_s(0)^2, \tag{13}$$

equivalent to the charge relaxation resistance $R_q = h/(2e^2)$. This result confirms the validity of the Fermi-liquid picture[16] also at low transmission.

Finally, both for weak and almost perfect transparency, we examine the regime of intermediate frequencies $\omega \sim \Gamma$, where the expansion Eq. (3) is no longer relevant. Nevertheless, keeping $\omega \ll T_K, E_C$ and splitting the charge susceptibility into real and imaginary parts,

$$e^2\chi_C(\omega) = C_0(\omega) + i\omega\, C_0(\omega)^2\, R_q(\omega), \tag{14}$$

one can define frequency-dependent capacitance and charge relaxation resistance. This definition is relevant for experiments where the real and imaginary parts are measured separately.[2] At weak transmission and $\omega \ll T_K$, we extract the universal form

$$\frac{R_q(\omega)}{h/e^2} = A\,\Phi\left(\frac{\omega}{\Gamma}\right) = A\,\frac{\Gamma}{\omega}\frac{\mathrm{Im}\beta(\omega/\Gamma)}{[B + \mathrm{Re}\beta(\omega/\Gamma)]^2}, \tag{15}$$

with $A = T_K/\Gamma$. The dimensionless function $\Phi(x)$ is plotted in Fig. 2(b) for different values of $B = \ln(T_K/\Gamma)$.

Remarkably, the same scaling form involving the function Φ is obtained in the opposite limit of weak backscattering. In the scaling limit where $N_0 \to 1/2$ and $\omega \ll E_C$, one has $\alpha(\omega) \simeq 1$, $\sin(\pi N_0) \simeq 1$, $K_2 \simeq 0$, and $K_0 \ll K_1$ in Eqs. (4). As a result, one recovers Eq. (15) for the charge relaxation resistance R_q with $B = \ln(E_C/\Gamma)$ and $A = E_C/(4\gamma r^2 \Gamma)$. The universality of the FL-NFL crossover has been anticipated by Matveev[39] who argued that the two-channel Kondo model influences the phase diagram beyond weak transparency.[49] To summarize briefly, we have shown that the presence of Majoranas in a quantum RC circuit results in a subtle charge dynamics which can, in principle, be revealed using current technology.[46] The FL to NFL crossover produces a visible increase of the charge relaxation resistance which can be probed through admittance measurements. We anticipate the possibility that NFL behavior emerges also for a superconducting wire supporting Majorana fermions at his edges.[29,30,32–36] Majorana fermions can also be manipulated using a microwave cavity.[50] Other interesting directions concern the role of an asymmetry between channels which can be engineered through Zeeman effects for example.[51]

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