Distributed-element circuit model of edge magnetoplasmon transport

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We report experimental and theoretical studies of edge magnetoplasmon (EMP) transport in quantum Hall (QH) devices. We develop a model that allows us to calculate the transport coefficients of EMPs in QH devices with various geometries. In our model, a QH system is described as a chiral distributed-element (CDE) circuit, where the effects of Coulomb interaction are represented by an electrochemical capacitance distributed along unidirectional transmission lines. We measure the EMP transport coefficients through single- and coupled-edge channels, a quantum point contact, and single- and double-cavity structures. These measured transmission spectra can be reproduced well by simulations using the corresponding CDE circuits. By fitting the experimental results with the simulations, we deduce the circuit parameters that characterize the electrostatic environment around the edge channels in a realistic QH system. The observed gate-voltage dependences of the EMP transport properties in gate-defined structures are explained in terms of the gate tuning of the circuit parameters in CDE circuits.

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I. INTRODUCTION

The application of radio-frequency (rf) electronic signals to a one-dimensional (1D) quantum Hall (QH) edge state excites charge density waves, called edge magnetoplasmons (EMPs),^{1–13} that travel chirally along the edge channels. Because EMPs travel coherently over more than a few millimeters, it is feasible to design coherent plasmonic circuits,¹⁴ cavities,³⁻⁷ and crystals^{14,15} for EMPs. Such plasmonic devices can be functionalized by exploiting various electrostatic tuning techniques using surface metal gates. As in typical dc transport experiments, one can switch the paths of EMP transport by selectively activating metal gates to deplete the two-dimensional electron gas (2DEG) underneath. More importantly, gate-defined structures provide additional functionality to plasmonic devices via the sensitivity of EMPs to their electrostatic environment. When the channel is covered with a metal, the group velocity of EMPs (v_{EMP}) along an etched QH boundary (bare edge channel) is significantly reduced (by more than one order of magnitude) because of the screening of the electric field.^{7–9,13} As compared to bare edge channels, those defined with gates (gate-defined edge channels) have the advantage that $v_{\rm EMP}$ is tunable, because the screening effect can be tuned by changing the distance between the channel and the metal electrode via a gate voltage.¹² This feature can be exploited to form tunable delay lines in plasmonic circuits. In addition, a beam splitter for EMPs can be constructed from a quantum point contact (QPC), whose transmission and reflection coefficients can be changed electrostatically.¹⁶⁻¹⁹ Frequency multipliers and mixers based on the nonlinearity of the transmission characteristics of a QPC have been demonstrated.²⁰ In this way, we can expect to establish integrated plasmonic circuits by combining such EMP devices with electrostatic tuning techniques.

To design and prepare an appropriate electrostatic environment for EMP transport, it is essential to correctly evaluate the effect of Coulomb interaction arising from the nonuniform charge density modulation of EMPs. However, the long-range nature of the Coulomb interaction makes the calculation cumbersome, particularly when multiple plasmonic devices are integrated on a chip. A tractable model with a certain accuracy is necessary to deal with EMP transport in actual devices. In this paper, we propose a simple method that allows us to describe the effect of Coulomb interaction and dissipation on EMP transport in terms of a distributedcircuit model. In this model, the wave equation for EMPs traveling along a single edge channel is expressed in terms of the distributed electrochemical capacitance between the channel and the ground (c_{ch} : channel capacitance). In addition, electrostatic coupling between two different channels can be modeled with distributed elements, which is expressed by the interedge capacitance (c_X : interedge capacitance).^{19,21} Once the transport characteristics of each individual device are obtained with the relevant distributed capacitance, one can simulate EMP transport in an integrated system by using the scattering matrix formalism. Since the above modeling method is developed for chiral edge channels, we call it "the chiral distributed-element (CDE) circuit model."

We apply this CDE model to EMP resonance in single and double plasmonic cavities and compare the results of the calculations with those of experiments. Our device is designed such that a single or double cavity can be selectively activated. As we shall see, the full set of cavity parameters can be controlled electrostatically, including the resonant frequency, coupling between the cavities, and coupling to the outside channels. This allows us to study several types of devices with different characteristics in one sample. We start from the simplest device, a QPC beam splitter, and then investigate single and coupled cavities. The measured characteristics of all these devices show reasonable agreement with the calculations for the corresponding CDE circuits. In particular, our experiment successfully identifies the in-phase and antiphase coupling modes in a coupled cavity predicted by the calculation. These findings ensure the validity and usefulness of the CDE model for developing future plasmonic circuits.

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The rest of this paper is organized as follows. In Sec. II, a detailed account of the CDE model is given. We start by developing a model for a single edge channel, and then extend it to other structures: coupled edge channels, a QPC, and single and coupled cavities. In Sec. III, we present experimental results and compare them with the calculations based on the CDE model. In Sec. IV, we discuss the scope of the model's application and present a few prospects and comments regarding improving the model and designing future plasmonic circuits. Section V is devoted to the conclusion.

II. CHIRAL DISTRIBUTED-ELEMENT CIRCUIT

The transport properties of an EMP, including its dispersion relation, can in principle be derived by solving the equation of motion and the continuity equation of charge at the edge of the QH system, together with the Poisson equation, which relates the excess charge of EMPs to the electric field.^{1,2} Such a hydrodynamic approach, however, is not readily applicable to real devices, where the Poisson equation must be solved in three dimensions. In this section, we give a detailed account of the CDE model, which enables us to simulate EMP transport in actual QH devices. All the essential ingredients of EMP transport, such as the wave equations along the edge channels and the scattering matrices, are naturally derived from its circuit representation.

We consider a 2DEG with electron density n_e placed in a high magnetic field *B* perpendicular to it. Its dc transport characteristics are described by the longitudinal conductivity σ_{xx} and Hall conductivity $\sigma_{xy} \approx en_e B$. We restrict our discussion to the situation $|\sigma_{xx}| \ll |\sigma_{xy}|$, i.e., the case where EMP transport is dominant and bulk plasmons are negligible. For simplicity, in the following argument we discuss only the fundamental EMP mode and neglect the acoustic mode.²

A. Unidirectional transmission line

In our circuit representation, an edge channel is modeled as a unidirectional transmission line consisting of a channel and a ground, as shown schematically in Fig. 1(a). We take the x axis to be parallel to the transmission line. We consider the relation between the voltage V(x, t) and current I(x, t) flowing in the channel at position x and time t. V(x, t) is related to the excess charge distribution $\rho(x, t)$ as $\rho = c_{ch}V$ through the channel capacitance c_{ch} , which represents the effective electrochemical capacitance (per unit length) between the channel and the ground. On the other hand, I(x, t) is related to $\rho(x, t)$ as $\partial I/\partial x = -\partial \rho/\partial t$ according to the continuity equation along the 1D channel. Here, we assume that I(x, t)and V(x, t) satisfy the relation $I(x,t) = \sigma_{xy} V(x,t)$.²² In terms of circuitry, this implies that we can define the characteristic impedance of the transmission line as $Z_{+} = V/I = 1/\sigma_{xy}$. From these three relations, we obtain the wave equation for EMPs as

$$\frac{\partial I(x,t)}{\partial t} = -\frac{\sigma_{xy}}{c_{\rm ch}} \frac{\partial I(x,t)}{\partial x}.$$
 (1)

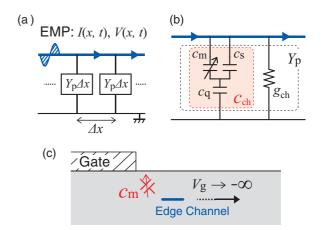


FIG. 1. (Color online) (a) Circuit representation of an EMP transmission line. (b) Components of the admittance Y_p between the channel and the environmental ground. (c) Schematic of the screening capacitance c_m of a gate-defined channel. The distance between the channel and the metal, and hence c_m , can be tuned with the gate voltage V_g .

The general solution of this equation takes the form $I(x - v_{\text{EMP}}t)$ with velocity

$$v_{\rm EMP} = \frac{\sigma_{xy}}{c_{\rm ch}},\tag{2}$$

whereas the form $I(x + v_{\text{EMP}}t)$ cannot fulfill Eq. (1). Note that the direction of propagation is determined by the sign of σ_{xy} , that is, by the sign of *B*. This unidirectional nature stems from the fact that Eq. (1) is expressed by the first derivative with respect to both space and time, and corresponds to the chiral transport nature of EMPs. In the circuit representation of the transmission line of Fig. 1(a), c_{ch} is expressed as the distributed admittance $Y_p = i\omega c_{ch}$ for generality. The chirality of the channel is marked by arrows.

Let us consider the channel capacitance c_{ch} in the circuit representation. In mesoscopic systems, the electrochemical capacitance is described as a series of the electrostatic geometrical capacitance and quantum capacitance $c_q \equiv e^2 D(E)$, where D(E) is the electron density of states.^{16,23–25} In a 1D system, $c_q = e^2 D(E) = e^2 / hv_F$, where v_F is the Fermi velocity. Further, $v_{\rm F}$ corresponds to the mean drift velocity $v_{\rm D} = (dU/dy)/B$ due to the Lorentz drift in an electric field dU/dy, which is determined by the gradient of the edge confinement potential U at the boundary of the 2DEG.²⁴ First, we consider the strong-screening limit, where Coulomb interaction between electrons can be ignored because of the screening of the electric field provided by the surrounding metals. In this limit, the electrostatic geometrical capacitance is much larger than c_q . Hence, c_{ch} is governed by c_q in the series connection; i.e., $c_{ch} \cong c_q$. This leads to $v_{EMP} \cong v_D$ from Eq. (2); electrons travel as noninteracting Fermions with the drift velocity $v_{\rm D}$ rather than as plasmons in this strong-screening limit. For example, for a smooth confinement potential U, one finds $c_q = e/(2\pi l_m^2 |dU/dy|) \cong 5 \text{ nF/m}$ and $v_D \cong 10^4 \text{ m/s}$ for a typical GaAs heterostructure at B = 5 T, where l_m is the magnetic length.²⁶

In actual QH conductors, however, the screening effect is not strong enough for this limit to be realized. Usually, v_{EMP} is

about two orders of magnitude higher than $v_{\rm D}$ ($v_{\rm EMP} \cong 10^6$ m/s in a filling factor $\nu = 1$ QH state 6-8,12,13). This is a manifestation of the strong Coulomb interaction in a bare edge channel. We include the effect of this Coulomb interaction by using the electrostatic capacitance. In principle, the long-range nature of the Coulomb interaction makes the problem essentially nonlocal, which implies that V(x) should depend on $\rho(x')$ at any position x' in the channel. For an infinitely long straight edge channel, however, because of the translational symmetry of the system, the simple relation $\rho(x) = c_{ch}(k)V(x)$ holds for a given wave number k. The effect of nonlocal $\rho(x')$ is incorporated as the k dependence of $c_{ch}(k)$ in this relation. Indeed, the calculation of $v_{\rm EMP}$ in such a scheme shows a similar k dependence of $v_{\text{EMP}} = (\sigma_{xy}/\varepsilon) \ln(\gamma/|ka|)$ for an abrupt electron density profile¹ and a soft profile² at the QH boundary. Here, ε is the dielectric constant, a is the effective width of the edge channel, and γ is a numerical constant. Comparing this expression for $v_{\rm EMP}$ with Eq. (2), one can define a k-dependent effective self-capacitance $c_{\rm s}(k) = \varepsilon / \ln(\gamma / |ka|)$. Its weak logarithmic dependence can be neglected for a certain wavelength region of interest, where $c_{\rm s}$ is expressed by considering $c_{\rm s} = \varepsilon / \gamma^*$; $\gamma^* = \ln(\gamma / |k_{\rm typ}a|)$ is a constant, and k_{typ} is the typical wave number. Because $c_{\rm s}$ is much smaller than $c_{\rm q}$ in a typical GaAs heterostructure, $v_{\text{EMP}} \cong \sigma_{xy}/c_s$ is generally much higher than $v_{\text{EMP}} \cong v_D$.

In a realistic QH device, in which such exact translational symmetry does not exist, the charge distribution $\rho(x')$ along the entire channel must be taken into account. When the device is covered with a metal electrode, however, partial screening of the long-range Coulomb interaction allows us to use a local approximation that considers the Coulomb interaction only near the channel. This is done by replacing c_s with the electrostatic screening capacitance $c_{\rm m} = \varepsilon a/d$ between the channel and the metal, where d is the distance between the channel and the covering metal. In this case, v_{EMP} is given by $v_{\text{EMP}} \cong \sigma_{xy} / c_m \cong \sigma_{xy} d / \varepsilon a$ for $d \ll a$.^{8,9,13} Generally, by considering all the contributions of c_q , c_s , and c_m , the channel capacitance c_{ch} is conceptually understood using an equivalent circuit, as shown in Fig. 1(b). The electrostatic screening capacitances c_s and c_m are connected to c_q in series. Because $c_{\rm m}$ for a metal-covered channel is generally much larger than $c_{\rm s}$ and much smaller than $c_{\rm q}$, the channel capacitance $c_{\rm ch}$ is approximated as $c_{ch} \cong c_m$.

This capacitance approximation also adequately explains the behavior of $v_{\rm EMP}$ along a gate-defined QH boundary. The capacitance $c_{\rm m}$, and hence $v_{\rm EMP}$, can be tuned with a negative gate voltage $V_{\rm g}$ because the distance between the channel and the metal varies with $V_{\rm g}$ [Fig. 1(c)].¹² In this way, the variation of $v_{\rm EMP}$ along gate-defined channels is represented as the variation of a circuit parameter $c_{\rm m}$. The velocity $v_{\rm EMP}$ can be estimated by numerically calculating $c_{\rm m}$ for a realistic electron density profile at the gate-defined QH boundary.

The longitudinal conductance σ_{xx} , which was ignored in the above discussion, causes current leakage from the channel into the bulk region and the resultant dissipation of EMPs.¹ This source of EMP dissipation can be represented by introducing a finite conductance g_{ch} between the channel and the ground in the CDE model as shown in Fig. 1(b). The distributed admittance Y_p depicted in Fig. 1(a) is then described by

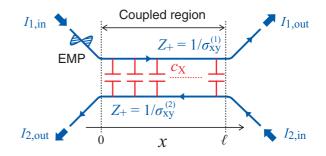


FIG. 2. (Color online) Schematic of coupled transmission lines. Two unidirectional transmission lines are coupled for length *l* through interedge capacitance c_X .

 $Y_{\rm p} = i\omega c_{\rm ch} + g_{\rm ch}$. More detailed discussion on EMP damping is given in Sec. IV.

B. Coupled transmission lines

In this subsection, we formulate EMP transport in a pair of parallel edge channels in order to extend the CDE model to cases where one must consider Coulomb interaction between edge channels. The interedge Coulomb interaction is a key factor in understanding the electron dynamics in QH edge channels, and is believed to dictate the energy relaxation,^{27–29} decoherence,^{30,31} and charge fractionalization.^{32,33} Since our CDE model allows us to parametrize and evaluate the interedge Coulomb interaction, it will be helpful for studying the nontrivial electron dynamics that emerges in coupled 1D systems.

Figure 2 shows our model for a coupled system in which two counterpropagating edge channels interact over a finite length *l* through interedge capacitance c_X . We take the *x* axis to be parallel to the edge channels in the interacting region. We assume that the two unidirectional transmission lines *i* (*i*=1 for the upper channel and 2 for the lower channel) have a characteristic impedance $1/\sigma_{xy}^{(i)}$, where $\sigma_{xy}^{(1)} = -\sigma_{xy}^{(2)} = \sigma_{xy}$, and identical channel capacitances $c_{ch}^{(1)} = c_{ch}^{(2)} = c_{ch}$. In the coupling region (0 < x < l), the charge density $\rho_i(x)$ and potential $V_i(x)$ (*i*=1, 2) of the two channels are interrelated as in the following matrix form:

$$\begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} = \begin{pmatrix} c_{ch} + c_X & -c_X \\ -c_X & c_{ch} + c_X \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}.$$
 (3)

On the other hand, the relation between the potential V_i and current I_i for each channel *i* is $I_i = \sigma_{xy}^{(i)} V_i$. From these relations, along with the continuity equation $\partial I_i / \partial x = - \partial \rho_i / \partial t$, the wave equation for the coupled system is

$$\frac{\partial}{\partial t} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = - \begin{pmatrix} U_0 & U_{12} \\ -U_{12} & -U_0 \end{pmatrix} \frac{\partial}{\partial x} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}, \quad (4)$$

where U_0 and U_{12} are the intra- and interchannel couplings, respectively:

$$U_0 = \frac{\sigma_{xy}}{c_{\rm ch}} \frac{c_{\rm ch} + c_{\rm X}}{c_{\rm ch} + 2c_{\rm X}},$$
$$U_{12} = \frac{\sigma_{xy}}{c_{\rm ch}} \frac{c_{\rm X}}{c_{\rm ch} + 2c_{\rm X}}.$$

The above wave equation has two solutions, one that fulfills $I_1 = -rI_2$ and the other that satisfies $I_2 = -rI_1$, with $r = (U_0 - \sqrt{U_0^2 - U_{12}^2})/U_{12}$. These solutions represent the coupled modes of EMPs;^{32,33} the charge ρ_1 propagating in the positive direction in the upper channel (Fig. 2) drags small amounts of negative charge $-r\rho_2$, and vice versa. Note that the interedge interaction makes the velocity of the coupled modes $v_{ct} \equiv |v_j| = \sqrt{U_0^2 - U_{12}^2}$ [j>0 (j<0) for the modes moving to the right (left)] slower than that of the uncoupled modes in the noninteracting regions (x < 0 or x > l). As an example, let us suppose that a pulse of excess charge is injected from the noninteracting region into the coupled region $(I_{1,in})$: upper left in Fig. 2). This excites the right-moving mode (ρ_1 , $(-r\rho_2)$ in the coupled region. The remainder of the charge $+r\rho_1$ reflected back into the lower left channel generates the output current I2,out. This process is called "charge fractionalization." These EMP transport characteristics are consistent with the calculations based on the Tomonaga-Luttinger liquid theory.³²

It is convenient to define the EMP scattering matrix S_{ct} of the coupled transmission lines, which relates the outgoing waves ($I_{1,out}$, $I_{2,out}$) to the incoming waves ($I_{1,in}$, $I_{2,in}$); see Fig. 2. In terms of the scattering amplitude *r* at the left (x = 0) and right (x = l) junctions and the phase evolution $\theta = \omega l/v_{ct}$ in the coupled region, S_{ct} is expressed as

$$\mathbf{S}_{\rm ct} = \frac{1}{r^2 - e^{2i\theta}} \begin{bmatrix} (-1 + r^2)e^{i\theta} & r(e^{2i\theta} - 1) \\ r(e^{2i\theta} - 1) & (-1 + r^2)e^{i\theta} \end{bmatrix}.$$
 (5)

The matrix elements of \mathbf{S}_{ct} are periodic functions of θ and hence ω .¹⁹ This periodicity can be regarded as the consequence of EMP interference propagating through the distributed capacitors. When *l* is sufficiently short, such that $\theta \ll 1$, interchannel coupling can be well approximated by a lumped capacitance $C_X = c_X l$. In this case, the expression for \mathbf{S}_{ct} reduces to

$$\mathbf{S}_{\rm ct} = \frac{1}{\sigma_{xy} + i\omega C_{\rm X}} \begin{pmatrix} \sigma_{xy} & i\omega C_{\rm X} \\ i\omega C_{\rm X} & \sigma_{xy} \end{pmatrix}.$$
 (6)

When $\omega C_X \ll \sigma_{xy}$, we find $S_{21} \cong i\omega C_X/\sigma_{xy}$. This reproduces the well-known expression for the two-probe admittance of a mesoscopic capacitor, $Y(\omega) = i\omega C_X$.^{16–19,24,25}

C. Quantum point contact

The EMP transport characteristics through a QPC involve Coulomb coupling between the input and output channels. In this subsection, we derive the scattering amplitudes of a QPC. Figure 3 depicts our model, in which two edge channels are running antiparallel along each of the split gate metals defining the QPC. We assume that the interedge Coulomb interaction exists over a length l along the split gates. As we have shown in Sec. II B, the effect of the interedge Coulomb interaction can be evaluated as the capacitance c_X .

The EMP scattering amplitude of the system is calculated by combining the contributions of the electron transport and capacitive transport. When l is much smaller than the wavelength λ_{EMP} of the EMPs, the distributed capacitance can be replaced with the lumped capacitance $C_X = c_x l$. In this case, the output currents $I_{1,\text{out}}$ and $I_{2,\text{out}}$ are given by

$$I_{1,\text{out}} = \sigma_{xy} V_{1,\text{out}}$$

= $i\omega C_X (V_{2,\text{in}} - V_{1,\text{out}}) + T_{\text{dc}} \sigma_{xy} V_{2,\text{in}} + R_{\text{dc}} \sigma_{xy} V_{1,\text{in}},$
 $I_{2,\text{out}} = \sigma_{xy} V_{2,\text{out}}$

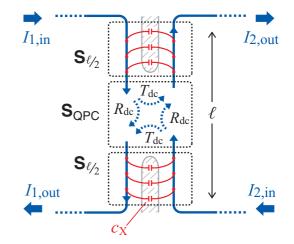


FIG. 3. (Color online) Schematic of edge channels near a QPC. Left and right channels are connected via the QPC. Interedge capacitance c_X is distributed over a length l around the QPC.

$$= i\omega C_{\rm X}(V_{1,\rm in} - V_{2,\rm out}) + T_{\rm dc}\sigma_{xy}V_{1,\rm in} + R_{\rm dc}\sigma_{xy}V_{2,\rm in}.$$

Here, T_{dc} and R_{dc} are the dc transmission and reflection probabilities for electron transport, respectively. From these equations, the scattering matrix S_{QPC-X} of the QPC is represented as

$$\mathbf{S}_{\text{QPC}-X} = \frac{1}{\sigma_{xy} + i\omega C_{\text{X}}} \begin{pmatrix} R_{\text{dc}}\sigma_{xy} & T_{\text{dc}}\sigma_{xy} + i\omega C_{\text{X}} \\ T_{\text{dc}}\sigma_{xy} + i\omega C_{\text{X}} & R_{\text{dc}}\sigma_{xy} \end{pmatrix}.$$
(7)

These scattering amplitudes are controlled by tuning T_{dc} . When the QPC is fully open (i.e., $T_{dc} = 1$), the off-diagonal element of \mathbf{S}_{QPC-X} , which we denote by t_{QPC} , is unity. In contrast, when the QPC is pinched off (i.e., $T_{dc} = 0$), EMPs are transferred only through C_X , yielding the transmission amplitude $t_{QPC} = i\omega C_X/(\sigma_{xy} + i\omega C_X)$. Note that, if we set the contribution of Coulomb coupling to zero (i.e., $C_X = 0$) in Eq. (7), the diagonal (r_{QPC}) and off-diagonal (t_{QPC}) elements of \mathbf{S}_{QPC-X} reduce to R_{dc} and T_{dc} , respectively.

When *l* is not sufficiently small to ignore the phase evolution along the interacting region, the lumped-capacitance approximation breaks down, and the distribution of c_X must be taken into account. To calculate S_{QPC-X} for the entire system, we define the **S** matrices for each section in the system; S_{QPC} is the **S** matrix for electron scattering at the QPC, and $S_{l/2}$ is that for Coulomb interaction in coupled channels with length l/2 (see Fig. 3). To combine these **S** matrices, we convert them to transmission matrices (**T** matrices: T_{QPC} and $T_{l/2}$) and calculate the **T** matrix of the entire system (T_{QPC-X}) as³⁴

$$\mathbf{T}_{\text{QPC}-\text{X}} = \mathbf{T}_{l/2} \mathbf{T}_{\text{QPC}} \mathbf{T}_{l/2}.$$
 (8)

 S_{QPC-X} is obtained by reconverting T_{QPC-X} to S_{QPC-X} .

To evaluate t_{QPC} and r_{QPC} in a device having a more complicated edge channel geometry in which these channels interact everywhere, the CDE model can be improved as much as necessary by taking the interedge capacitances elsewhere into account.

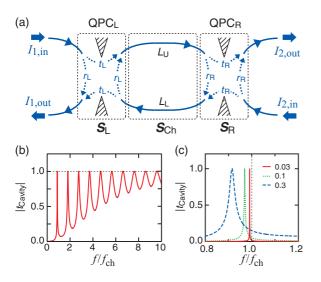


FIG. 4. (Color online) (a) Schematic of plasmonic cavity confined with QPC_L and QPC_R. (b), (c) Calculated $|t_{Cavity}|$ from Eq. (10) as a function of normalized frequency f/f_{ch} . (b) $|t_{Cavity}|$ in wide frequency band up to $f/f_{ch} = 10$ for $\alpha = 0.03$ and (c) those around the fundamental resonant frequency for $\alpha = 0.03$, 0.1, and 0.3.

D. Plasmonic cavity resonators

A plasmonic cavity is a typical application of EMPs.^{3–7} In this subsection, we derive the transmission spectra of single and double plasmonic cavities using the CDE model. We consider closed loops of edge channels having a pair of QPCs at the entrance and exit of the cavity structures. The **S** matrices for individual sections (QPCs and edge channels composing the cavities) are combined to calculate the **S** matrix of the entire system. Here, we consider the lumped capacitance across QPCs (C_X) rather than the distributed capacitance (c_x) in order to simplify the calculation. For the moment, we ignore EMP dissipation during propagation, assuming $g_{ch} = 0$.

Single plasmonic cavity. Figure 4(a) shows the schematic of a single-cavity structure. The cavity consists of two QPCs (QPC_p: p = L or R) at the entrance and exit. The scattering matrix S_p of QPC_p is given by Eq. (7). On the other hand, the scattering matrix of the looped channel in the cavity, S_{Ch} , can be described by

$$\mathbf{S}_{\mathrm{Ch}} = \begin{bmatrix} 0 & \exp\left(-ik_{\mathrm{L}}L_{\mathrm{L}}\right) \\ \exp\left(-ik_{\mathrm{U}}L_{\mathrm{U}}\right) & 0 \end{bmatrix}.$$

 S_{Ch} represents the phase evolution of the EMPs during propagation. Here, k_U and k_L represent the wave numbers of the EMPs, and L_U and L_L are the lengths of the upper and lower channels in the cavity, respectively. The S matrix of the entire cavity (S_{Cavity}) is obtained from the T matrix (T_{Cavity}) of the entire system, which is given by

$$\mathbf{T}_{\mathrm{Cavity}} = \mathbf{T}_{\mathrm{L}} \mathbf{T}_{\mathrm{Ch}} \mathbf{T}_{\mathrm{R}},\tag{9}$$

where \mathbf{T}_p is the transmission matrix of QPC_p and \mathbf{T}_{Ch} is that of the channel in the cavity; these variables are converted from \mathbf{S}_p and \mathbf{S}_{Ch} , respectively. The reflection (r_{Cavity}) and transmission (t_{Cavity}) amplitudes are the diagonal and off-diagonal terms of \mathbf{S}_{Cavity} , respectively.

When the QPCs are pinched off (electron transmission probabilities $T_{dc,L} = T_{dc,R} = 0$), the cavity couples to the

input and output channels only through C_X . In this case, the transmission spectrum is calculated as

$$|t_{\text{Cavity}}| = \left| \frac{-\alpha^2 e^{-if_{\text{ch}}\omega/2}}{(1+i\alpha)^2 - e^{-if_{\text{ch}}\omega}} \right|,\tag{10}$$

where we assume $L_{\rm U} = L_{\rm L}$. In Eq. (10), $f_{\rm ch}$ is the characteristic frequency $f_{\rm ch} = L/v_{\rm ch}$, where $L = 2L_{\rm U} = 2L_{\rm L}$ is the perimeter of the cavity, and $v_{\rm ch} = \sigma_{xy}/c_{\rm ch}$ is the characteristic velocity of the EMPs. Further, $\alpha = 2\pi f_{\rm ch}C_{\rm X}/\sigma_{xy}$ is the interedge coupling between the cavity and the leads at the QPCs.

Figures 4(b) and 4(c) show the calculated transmission spectra as a function of frequency f. As shown in Fig. 4(b), $|t_{Cavity}|$ oscillates as a function of the frequency because of the resonance of the EMPs. With increasing frequency, the resonant peaks broaden (i.e., the quality factor decreases) and, consequently, the contrast between resonance and off resonance weakens. This is because the capacitive coupling across the QPCs increases with frequency and so does the leakage of EMPs. The fundamental resonance shows a Lorentzian shape in the transmission spectrum, as displayed in Fig. 4(c). With increasing α , namely, increasing C_X , the resonant peak broadens because the leakage of EMPs increases with C_X . In addition, the increase in α reduces the resonant frequency owing to the reduced v_{EMP} in the coupled region near the QPCs.

Double plasmonic cavity. The transmission amplitude t_{Double} of a double-cavity device is derived from the combination of the **S** matrices for different sections. Figure 5(a) shows a schematic of the successive sections of a double-cavity device. The transmission matrix $\mathbf{T}_{\text{Double}}$ is obtained from those of the QPCs (\mathbf{T}_p : p = L, M, or R) and the channels inside the left and right cavities (\mathbf{T}_{Ch1} and \mathbf{T}_{Ch2}) as

$$\mathbf{T}_{\text{Double}} = \mathbf{T}_{\text{L}} \mathbf{T}_{\text{Ch}1} \mathbf{T}_{\text{M}} \mathbf{T}_{\text{Ch}2} \mathbf{T}_{\text{R}}.$$
 (11)

The scattering matrix $\mathbf{S}_{\text{Double}}$ of the double cavity is converted from $\mathbf{T}_{\text{Double}}$. The reflection (r_{Double}) and transmission (t_{Double}) amplitudes are the diagonal and off-diagonal terms of $\mathbf{S}_{\text{Double}}$, respectively. Here, we assume the perimeters of the cavities to be $L_{1\text{U}} = L_{1\text{L}} = L_1/2$ and $L_{2\text{U}} = L_{2\text{L}} = L_2/2$, where L_1 and L_2 are the perimeters of the two cavities. As for the single plasmonic cavity, we define the characteristic frequencies $f_{\text{ch1}} = v_{\text{ch1}}/L_1$ and $f_{\text{ch2}} = v_{\text{ch2}}/L_2$, where v_{ch1} and v_{ch2} denote the characteristic velocities of EMPs in these two cavities.

As we discussed in Sec. II C, the transmission amplitude of a QPC involves real (conductive) and imaginary (capacitive) parts. These two types of transmission amplitude produce very different resonance characteristics in a double-cavity device. To elucidate this difference, we consider two cavities coupled via QPC_M without input and output ports, as shown in Fig. 5(b). We describe the currents incoming to and outgoing from QPC_M as $I_{1M,in}$, $I_{2M,in}$ and $I_{1M,out}$, $I_{2M,out}$, respectively. These currents meet the following conditions, where t_M and r_M are the transmission and reflection amplitudes of QPC_M.

$$I_{1M,out} = r_M I_{1M,in} + t_M I_{2M,in} = (1 - t_M) I_{1M,in} + t_M I_{2M,in},$$

$$I_{2M,out} = t_M I_{1M,in} + r_M I_{2M,in} = t_M I_{1M,in} + (1 - t_M) I_{2M,in}.$$
(12)

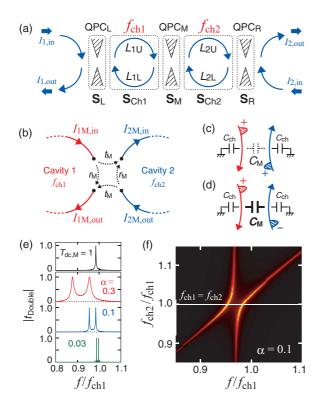


FIG. 5. (Color online) (a) Schematic of successive sections of a double-cavity structure. (b) Schematic of edge channels near QPC_M. (c), (d) Interedge coupling with closing QPC_M for dc current. (c) In in-phase coupling, the voltage drop across QPC_M is zero, so $C_{\rm M}$ can be neglected. (d) In antiphase coupling, $C_{\rm M}$ is emphasized. (e) Calculated $|t_{\rm Double}|$ of the coupled cavities at $f_{\rm ch1} = f_{\rm ch2}$ at $T_{\rm dc,M} = 1$ and $\alpha = 0.1$ (upper panel) and at $T_{\rm dc,M} = 0$ and $\alpha = 0.3$, 0.1, and 0.03 (lower three panels). (f) Calculated plot of $|t_{\rm Double}|$ as a function of normalized input frequency $f/f_{\rm ch1}$ (x axis) and $f_{\rm ch2}/f_{\rm ch1}$ (y axis) at $T_{\rm dc,M} = 0$ and $\alpha = 0.1$.

When the resonant frequency of the coupled system is $f_{\rm C}$, the conditions

$$I_{1M,in} = e^{-2\pi i f_C/f_{ch1}} I_{1M,out},$$

$$I_{2M,in} = e^{-2\pi i f_C/f_{ch2}} I_{2M,out},$$
(13)

are also satisfied. Let us consider the case of $f_{ch1} = f_{ch2}$. One solution of Eqs. (12) and (13) is always obtained at $f_C = f_{ch1} = f_{ch2}$ with $I_{1M,out} = I_{2M,out}$. This mode is the in-phase resonance of EMPs in the coupled cavities. In addition, when the coupling t_M is purely capacitive (t_M is an imaginary number), another solution is obtained at $f_C \cong (1 - t_M/i\pi)f_{ch1} = (1 - t_M/i\pi)f_{ch2}$ with $I_{1M,out} = -I_{2M,out}$. This mode is the antiphase resonance. This antiphase mode does not appear in conductive coupling, because EMPs with opposite signs cancel each other out at QPC_M.

As we discussed in Sec. II B, the interedge Coulomb interaction decreases the velocity of EMPs. For the in-phase resonance, the voltage drop across QPC_M is zero [Fig. 5(c)], so the effect of interedge coupling on v_{EMP} disappears. On the other hand, for the antiphase mode [Fig. 5(d)], the voltage drop across the QPC is doubled, emphasizing the effect of the interedge interaction. Consequently, the resonant frequency of the antiphase mode is always smaller than that

of the in-phase mode. In this way, in capacitive coupling, the resonant frequency splits into in-phase and antiphase modes, whereas only the in-phase resonance is obtained in the case of conductive coupling.

These coupling modes can be seen in simulations using Eq. (11). Figure 5(e) displays several simulated transmission spectra of a coupled cavity with $f_{ch1} = f_{ch2}$. Here, we define the coupling parameter $\alpha = 2\pi f_{ch1}C_X/\sigma_{xy}$. When QPC_M is conductive ($T_{dc,M} = 1$, uppermost panel), $|t_{Double}|$ exhibits a single resonant peak near $f/f_{ch1} = 1$ (this corresponds to the second-harmonic resonance of the combined cavity). On the other hand, when these cavities are coupled only through $C_{\rm X}$ $(T_{dc,M} = 0)$, the resonant peak splits into two (lower three panels). The splitting Δf develops with increasing α , namely increasing coupling capacitance C_X . This demonstrates that the coupling strength is parametrized with C_X in this double-cavity device. Figure 5(f) shows an image plot of the calculated $|t_{\text{Double}}|$ at $T_{\text{dc,M}} = 0$ for $\alpha = 0.1$ as a function of the normalized measurement frequency f/f_{ch1} (x axis) and detuning parameter f_{ch2}/f_{ch1} (y axis). A clear anticrossing of the two resonant modes appears at $f_{ch2}/f_{ch1} = 1$. Thus, the resonance of the EMPs exhibits dramatically different characteristics depending on whether the coupling is conductive or capacitive. Note that the mode splitting in capacitively coupled systems is a key factor in designing plasmonic band structures.^{14,15}

III. EXPERIMENTS

In this section, we present the results of EMP transport measurements. First, we compare the measured transmission amplitude t_{OPC} of a QPC with the calculation using Eqs. (5) and (6), in which interedge Coulomb interaction across the QPC is evaluated as the interedge capacitance. Second, we study the EMP resonance in a gate-defined single-cavity structure. We extract the velocity $v_{\rm EMP}$ and dissipation $g_{\rm ch}$ of EMPs [Fig. 1(b)] in the cavity from the center frequency and Q factor of the resonance. We also show that the resonant frequency, which is a function of $v_{\rm EMP}$, can be tuned with gate voltages applied to a delay gate in the cavity. Third, coupled resonance of EMPs in a double-cavity structure is reported. When the intercavity coupling is conductive, we observe a single resonant peak in the transmission spectra, while a double-peak structure appears for purely capacitive coupling, indicative of mode splitting into in-phase and antiphase resonance.

A. Device and measurement setup

Figure 6(a) shows a colored optical micrograph of the device and the measurement setup. The EMP devices were prepared in a 2DEG in a GaAs/Al_{0.3}Ga_{0.7}As heterostructure with electron density $n_e = 1.3 \times 10^{11}$ cm⁻² and mobility $\mu = 2.1 \times 10^6$ cm² V⁻¹ s⁻¹. They have three split gates [(Up, Lp); p = A, B, or C] to form QPCs and four screening gates (Sq: q = 1, 2, 3, and 4) to tune the degree of screening for each section of the upper and lower edge channels. The detailed geometry of the gate metals near the QPCs is illustrated schematically in Fig. 6(b). Plasmonic cavities are formed by activating more than one QPC. For example, a large cavity is formed with QPC_A and QPC_C [Fig. 6(c)], while a double cavity is formed with all the QPCs [Fig. 6(d)].

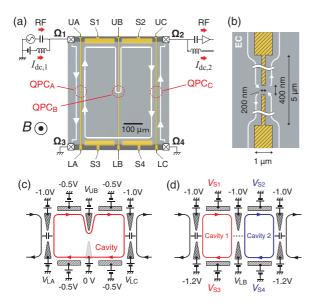


FIG. 6. (Color online) (a) Colored optical micrograph of device and measurement setup. White lines with arrows show the geometry of the edge channels forming a plasmonic cavity. (b) Schematic of gate metals of QPC_p and edge channels (ECs). (c) Schematic of large single cavity formed with QPC_A and QPC_C . (d) Schematic of double cavities formed with all QPCs.

The measurements were performed at 280 mK in a ³He cryostat with a high magnetic field (5.3 T) applied perpendicular to the 2DEG (bulk filling factor $\nu = 1$). The chirality of the edge channels was set to clockwise, as shown by the white arrows in Fig. 6(a). In this setup, we applied both dc and rf excitation to the Ohmic contact Ω_1 and simultaneously measured the dc and rf output signals from Ω_2 with the other Ohmic contacts (Ω_3 and Ω_4) grounded. The dc measurements were performed using a standard lock-in technique at 31 Hz.

The electron transmission probability T_{dc} of a gate-defined structure was measured as the collection efficiency $T_{dc} = I_{dc,2}/I_{dc,1}$ of the dc current at Ω_2 for a dc current $I_{dc,1}$ injected from Ω_1 . In the $\nu = 1$ QH state, $T_{dc} = 1$ corresponds to $G_{21} = e^2/h$, where G_{21} is the conductance from Ω_1 to Ω_2 . We confirmed $T_{dc} > 0.97$ when all the gate voltages were set to zero. This ensures that edge transport is dominant in the present device, and transport through the bulk QH state can be neglected (i.e., $\sigma_{xx} \cong 0$).

The EMP transport properties were investigated by measuring the scattering matrix element $S_{21,\text{meas}}$ for rf signals between Ω_1 and Ω_2 using a vector network analyzer. The excitation voltage applied to Ω_1 was about 1.4 mV rms.³⁵ We estimated the transmission amplitude *t* of the device as $t = S_{21,\text{meas}}/S_{21,\text{meas}}^{(0)}$, where $S_{21,\text{meas}}^{(0)}$ is the scattering matrix element for full transmission at $V_{\text{LA}} = V_{\text{LB}} = V_{\text{LC}} = 0$ V. With this calibration, the frequency dependence of the measurement system, e.g., the frequency dependence of the insertion loss of the coaxial cables and/or gain of the amplifiers, can be removed.

B. Quantum point contact

First, we investigated the electron transmission probability $T_{dc,p}$ through QPC_p (p = A, B, or C). We measured $T_{dc,p}$ as

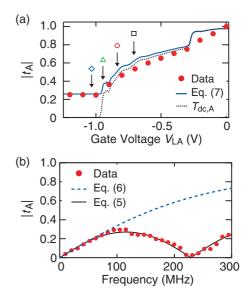


FIG. 7. (Color online) (a) Transmission amplitude $|t_A|$ of QPC_A as a function of V_{LA} measured at 80 MHz (red circles). Black dotted line is measured $T_{dc,A}$ and blue solid line is simulated $|t_A|$ from Eq. (6) assuming $C_X = 22$ fF. Open symbols show the gate voltages, where $|t_{Cavity}|$ was measured as shown in Fig. 10(a). (b) Frequency dependence of the measured $|t_A|$ with QPC_A closed ($T_{dc,A} = 0$ at $V_{LA} = -1.2$ V). Blue dashed line indicates simulation using Eq. (6) with $C_X = 22$ fF. Black solid line indicates simulation using Eq. (5) with $c_{ch} = 130$ pF/m, $c_X = 47$ pF/m, and $l = 420 \ \mu$ m.

a function of the lower gate voltage (V_{Lp}) while applying $V_{Up} = -1.0$ V to the upper gate. The measured $T_{dc,A}$ is shown in Fig. 7(a) by a black dotted line. QPC_A is formed at $V_{LA} \cong -0.25$ V, where $T_{dc,A}$ steeply decreases: $T_{dc,A} < 1$ at this QPC_A definition voltage is a result of the narrow gap of our split gates [400 nm; see Fig. 6(b)], which causes backscattering between the counterpropagating edge channels immediately after the QPC formation. Applying a more negative V_{LA} decreases $T_{dc,A}$ until it reaches $T_{dc,A} = 0$ at around $V_{LA} = -0.95$ V. The small features observed in the range $-0.6 \text{ V} < V_{LA} < -0.95 \text{ V}$ originate from scattering at impurities around the QPC. We observed a similar V_{Lp} dependence of $T_{dc,p}$ for the other two QPCs (not shown).

The absolute transmission amplitude $|t_A|$ of QPC_A measured at 80 MHz is also plotted in Fig. 7(a). Although $T_{dc,A} = 0$ below the pinch-off voltage $V_{LA} \cong -0.95$ V, $|t_A|$ remains finite even after the QPC is fully pinched off because of the Coulomb coupling across the QPC. We modeled these interactions as the interedge capacitance across QPC_A, as shown in Fig. 3. When QPC_A is closed for dc current, the system can be regarded as coupled 1D transmission lines (Fig. 2). In this case, $|t_A|$ is expected to oscillate as a function of frequency, as discussed in Sec. IIB. Figure 7(b) shows the frequency dependence of the measured $|t_A|$ at $V_{LA} = -1.2$ V (where $T_{dc,A} = 0$). The experimental data are fitted well by Eq. (5). From this fitting, the coupling parameters of the present system were found to be $c_{\rm ch} = 130 \, {\rm pF/m}, c_{\rm X} = 47 \, {\rm pF/m}, \text{ and } l = 420 \, \mu {\rm m}.$ These values of c_X and l are comparable to those obtained in our previous study.¹⁹

At low frequencies, the interedge Coulomb interaction can be simply described by the lumped capacitance C_X across the QPC. As seen in Fig. 7(b), below 100 MHz, $|t_A|$ monotonously increases with frequency, reflecting the relation $t_A \cong i \omega C_X / \sigma_{xy}$ ($\omega C_X \ll \sigma_{xy}$). Indeed, the data below 100 MHz can be well fitted with Eq. (6) with $C_X = 22$ fF. With this parameter, the measured V_{LA} dependence of $|t_A|$ shown in Fig. 7(a) is well reproduced using Eq. (7) over the entire range of V_{LA} .

C. Single plasmonic cavity

In this subsection, we present the results of transmission measurements for a single plasmonic cavity. As sketched in Fig. 6(c), our cavity is defined by using nine gate electrodes. QPC_A and QPC_C are formed by applying a constant gate voltage $V_{\text{UA}} = V_{\text{UC}} = -1.0$ V to the upper gates UA and UC and varying voltages V_{LA} and V_{LC} to the lower gates LA and LC so that the QPC openings can be tuned. The upper and lower channels in the cavity are defined by the four screening gates, which are biased at $V_{\text{S}\beta} = -0.5$ V ($\beta = 1, 2, 3, \text{ and } 4$). Additionally, gate UB was exploited to tune the perimeter L of the cavity. EMPs injected from Ω_1 enter the cavity via QPC_A and exit from QPC_C to be collected at Ω_2 .

A typical frequency dependence of the transmission amplitude $|t_{Cavity}|$ of the cavity is shown in Fig. 8. The spectrum was obtained with both QPCs pinched off ($T_{dc,A} =$ $T_{\rm dc,C} = 0$ at $V_{\rm LA} = V_{\rm LC} = -1.2$ V). A sharp peak appears near 90 MHz, corresponding to the fundamental resonance. We simulated $|t_{Cavity}|$ using Eq. (9), with the interedge coupling across the QPCs modeled as a distributed capacitance with $c_{\rm ch} = 130$ pF/m, $c_{\rm X} = 47$ pF/m, and l = 420 µm. Unlike the simulations shown in Figs. 4(b) and 4(c), the height of the resonant peaks does not reach $|t_{Cavity}| = 1$ due to EMP dissipation during propagation along the channel in the cavity. We assumed $g_{ch} = 1/260 \ \Omega^{-1} \ m^{-1}$ to include the effect of this dissipation; the mean free path of EMPs estimated from this g_{ch} is about 10 mm. The simulation (black solid line in Fig. 8) reproduces the experimental data exceedingly well, including the second-harmonic resonance around 180 MHz, indicating the validity of our CDE model. Both the experiment and simulation show that the peak height of the second-harmonic resonance is much smaller than that of the fundamental resonance. This is because, when the QPCs are closed for dc currents, EMP transmission amplitude across the QPCs is strongly suppressed near 200 MHz due to EMP interference as seen in Fig. 7(b).

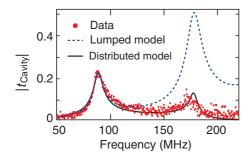


FIG. 8. (Color online) Typical $|t_{\text{Cavity}}|$ of a single plasmonic cavity measured at $V_{\text{LA}} = V_{\text{LC}} = -1.2$ V and $V_{\text{UB}} = -0.36$ V. Black solid line is simulation assuming a distributed capacitance across the QPCs. Blue dashed line shows the fitting assuming lumped capacitance in the frequency band from 50 to 120 MHz.

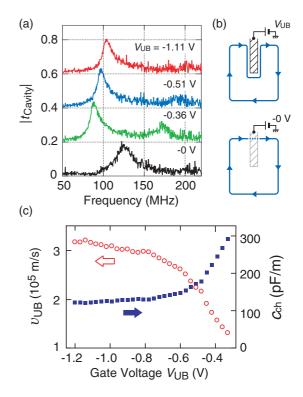


FIG. 9. (Color online) (a) Transmission spectra $|t_{Cavity}|$ of the single cavity shown in Fig. 6(c) measured at several V_{UB} . (b) Geometry of edge channel in the cavity at $V_{UB} = 0$ V (upper) and at $V_{UB} < -0.3$ V (lower). (c) Group velocity v_{UB} (open circles) and channel capacitance c_{ch} (filled squares) along the gate UB derived from the observed resonant frequencies.

At low frequencies, the calculation can be further simplified by introducing a lumped capacitance C_X across the QPCs. As shown by the dashed line in Fig. 8, the data below 120 MHz can be well fitted by the lumped-capacitance model using the parameters $C_X = 18$ fF, $v_{ch} = 2.17 \times 10^5$ m/s, and $g_{ch} = 1/260 \ \Omega^{-1} \text{ m}^{-1}$. The C_X value is close to that obtained for a single QPC in Sec. III B ($C_X = 22$ fF), and the above v_{ch} value is consistent with those previously reported for $v = 1.^{6-8,12,13}$ In the remainder of this section and the following subsection, we focus on the fundamental resonance and use the lumped-capacitance model to simplify the calculation.

Figure 9(a) shows the transmission spectra measured at several values of $V_{\rm UB}$ with both QPCs pinched off ($T_{\rm dc,A}$ = $T_{\rm dc,C} = 0$). When $V_{\rm UB}$ is changed from 0 V to -0.36 V, the resonant frequency f_{RES} shifts from 125 to 85 MHz. At $V_{\rm UB} = 0$ V, EMPs take a shortcut under the gate UB, which makes a short cavity perimeter of $L \cong 1750 \ \mu \text{m}$, whereas at $V_{\rm UB} < -0.3$ V, EMPs make a detour, and L increases to $L \cong 2250 \ \mu \text{m}$ [Fig. 9(b)]. As V_{UB} decreases further to more negative values, f_{RES} gradually increases and saturates at $f_{\text{RES}} \cong 105$ MHz at $V_{\text{UC}} < -0.8$ V. This is a consequence of the gate-voltage dependence of the velocity $v_{\rm UB}$ along gate UB.¹² We extracted the velocity $v_{\rm UB}$ by comparing the resonant frequencies for the cases with and without a detour around UB. The obtained $v_{\rm UB}$ values are shown in Fig. 9(c) as a function of V_{UB} . As V_{UB} decreases from -0.3 to -1.2 V, v_{UB} increases by a factor of 3. This suggests that, by appropriately designing the cavity structure, it is possible to vary f_{RES} electrostatically

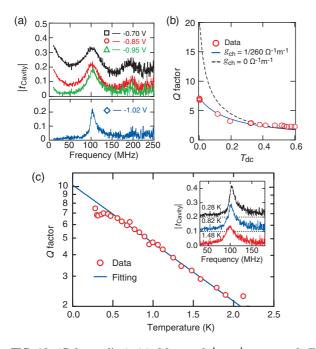


FIG. 10. (Color online) (a) Measured $|t_{\text{Cavity}}|$ at several T_{dc} $[V_{\text{L}} = -0.7, -0.85, -0.95, \text{and} -1.02 \text{ V}$; marked by open symbols in Fig. 7(a)] at $V_{\text{UB}} = -1.2 \text{ V}$. (b) Quality factor of the measured resonant peaks (circles) as a function of T_{dc} with those calculated from Eq. (9) for $g_{\text{ch}} = 1/260 \ \Omega^{-1} \text{ m}^{-1}$ (solid blue line) and for $g_{\text{ch}} = 0$ (dashed black line). (c) (Main panel) Temperature dependence of Q factor of the fundamental resonance in the single cavity. (Inset) Transmission spectra of the resonance obtained at various temperatures.

over a wide range by up to a factor of 3. In Fig. 9(c), we also plotted the channel capacitance c_{ch} along UB deduced from the v_{UB} values using Eq. (2). As V_{UB} decreases, c_{ch} decreases, reflecting the growing distance between the channel and the gate, which decreases c_m between them. The extracted value of c_{ch} (about a few hundred pF/m) is close to the value of c_m estimated from a finite element calculation (a few hundred pF/m, assuming a distance of a few micrometers between the channel and the gate metal and a channel width of a few hundred nanometers.³⁶)

Figure 10(a) shows the effect of QPC openings on $|t_{Cavity}|$. The four spectra shown were taken at different gate voltages $V_{\rm L} \equiv V_{\rm LA} = V_{\rm LC}$ ranging from -1.02 to -0.70 V [marked by open symbols in Fig. 7(a)] corresponding to different values of T_{dc} ($\equiv T_{dc,A} = T_{dc,C}$). The data show that the resonant peak becomes sharper with decreasing $V_{\rm L}$ and hence decreasing $T_{\rm dc}$. At $V_{\rm L} = -1.02$ V, where both QPCs are pinched off and the coupling is purely capacitive, the peak height becomes $|t_{\text{Cavity}}| \cong 0.2$ with $Q = f_{\text{RES}} / \Delta f_{\text{FWHM}} \cong 7$, where Δf_{FWHM} is the full width at half maximum (FWHM) of the resonant peak. As shown in Fig. 10(b), the measured Q factor increases with decreasing T_{dc} . This is consistent with the intuitive expectation that the resonance is sharper when the leakage of EMPs through the QPCs is smallest. Calculation of the Q factor using Eq. (9) reveals that $g_{ch} = 1/260 \ \Omega^{-1} \text{ m}^{-1}$ provides a good account of the obtained T_{dc} dependence of Q [solid line in Fig. 10(b)]. If we set $g_{ch} = 0$ in Eq. (9), the resultant Q factor deviates from the data at low T_{dc} , reaching Q > 20 at $T_{dc} = 0$ [dashed line in Fig. 10(b)]. This means that when the QPCs are pinched off, Q is limited not by the leakage at the QPCs, but by the dissipation of EMPs during propagation, namely, by g_{ch} . From these results, we can expect that a higher Q factor is obtained in a smaller cavity.

To explore the origin of EMP dissipation, we measured the temperature dependence of the Q factor. The inset of Fig. 10(c) shows transmission spectra obtained at several temperatures. The resonant peak is most pronounced at the lowest temperature, and the Q factor monotonically decreases with increasing temperature [Fig. 10 (main panel)]. The temperature dependence of Q is fitted well by an exponential function, $Q_0 \exp(-T_s/T_{res})$, where Q_0 is the Q factor at a temperature of 0 K, and T_s is the temperature of the system. T_{res} represents the characteristic temperature of the resonance broadening. We found $T_{res} = 1.4$ K from the fitting, which is comparable to the Zeeman gap ($\cong 1.2$ K, assuming the gfactor in the GaAs/Al_{0.3}Ga_{0.7}As heterostructure to be $|g| \cong 0.4$) at B = 5.3 T. We will discuss possible mechanisms of this temperature dependence in Sec. IV.

D. Coupled plasmonic cavities

In this subsection, we examine the coupling of the resonance in a double-cavity structure. As shown in Fig. 6(d), the double cavity is defined by activating gate LB as well as UB, and thus by forming QPC_B, which separates the entire cavity into left cavity (cavity 1) and right cavity (cavity 2) components. We applied fixed voltages of -1.0 V to the upper gates (UA, UB, and UC) and -1.2 V to gates LA and LC, but varied V_{LB} of gate LB to tune the transmission amplitude $t_{\rm B}$ of QPC_B. The resonant frequencies of cavity 1 ($f_{\rm RES1}$) and 2 ($f_{\rm RES2}$) can be independently controlled by tuning $V_{\rm SG1} \equiv V_{\rm S1} = V_{\rm S3}$ and $V_{\rm SG2} \equiv V_{\rm S2} = V_{\rm S4}$, respectively. These cavities have the same perimeters of $L_1 = L_2 = 1350 \ \mu m$. We measured the absolute transmission amplitude $|t_{\rm Double}|$ of the double-cavity device while changing $V_{\rm SG1}$ and fixing $V_{\rm SG2} = -0.3$ V.

When QPC_B is open $(T_{dc,B} \cong 0.8)$ at $V_{LB} = -0.3$ V, EMP transport through QPC_B is almost conductive $(|t_B| \cong T_{dc,B})$, as shown in Fig. 7(a). Figure 11(a) shows typical resonant spectra of the double cavity in this conductive-coupling regime, taken at several V_{SG1} values. Figure 11(c) displays the image plot of $|t_{\text{Double}}|$ as a function of V_{SG1} (x axis) and the measurement frequency (y axis). To highlight the effect of intercavity coupling, the intrinsic resonant frequencies f_{RES1} of cavity 1 and f_{RES2} of cavity 2 are indicated by the dashed and dotted lines, respectively, superposed on the image plot: f_{RES1} is measured in the absence of cavity 2 with $V_{\rm LC} = 0$ V, and vice versa. As expected, f_{RES1} varies with V_{SG1} in a manner similar to that seen in Fig. 9(c), whereas f_{RES2} is independent of V_{SG1} . Consequently, f_{RES1} and f_{RES2} cross each other as a function of V_{SG1} . The spectra shown in Fig. 11(a) were taken near the crossing point, which is indicated by the arrows in Fig. 11(c). As seen in Figs. 11(a) and 11(c), in the conductive-coupling regime, only a single resonant peak is observed, whose resonant frequency roughly lies between f_{RES1} and f_{RES2} .

On the other hand, when QPC_B is pinched off $(T_{dc,B} = 0)$ at $V_{LB} = -1.2$ V, the coupling between the two cavities is purely capacitive and is mediated by interedge Coulomb

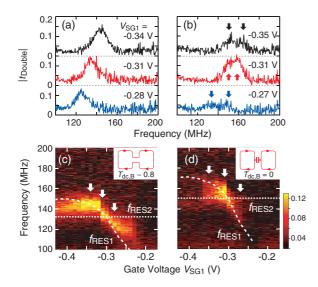


FIG. 11. (Color online) Measured transmission spectra of the coupled cavities obtained at various V_{SG1} with V_{SG2} fixed at (a) $T_{dc,B} \cong 0.8$ and (b) $T_{dc,B} = 0$. Arrows in (b) indicate the spectral peaks. Image plots of $|t_{Double}|$ measured at (c) $T_{dc,B} \cong 0.8$ and (d) $T_{dc,B} = 0$. Insets in (c) and (d) show the schematics of the edge channels in the double-cavity structure.

interaction. Figures 11(b) and 11(d) show similar measurements in this capacitive-coupling regime. The spectra taken near the crossing of f_{RES1} and f_{RES2} , shown in Fig. 11(b), reveal a double-peak structure, which is pronounced when $f_{\text{RES1}} \neq f_{\text{RES2}}$ [highlighted by the black and blue arrows in Fig. 11(b)]. Even at $f_{\text{RES1}} \cong f_{\text{RES2}}$, an additional shoulder peak structure is visible (shown by the red arrows), suggesting a splitting of the resonant frequency [Figs. 5(e) and 5(f)].

We simulated the resonance characteristics of the double cavities using Eq. (11) and compared these with the experimental results. Figure 12 shows the results of this calculation, where we assumed the lumped interedge capacitances C_p across QPC_p to be $C_A = C_C = 18$ fF, $C_B = 10$ fF, and the EMP dissipation to be $g_{ch} = 260 \ \Omega^{-1} \ m^{-1}$. These calculations match the measured results well; at $T_{dc,B} \cong 0.8$, a single resonant peak appears between f_{RES1} and f_{RES2} , whereas at $T_{dc,B} = 0$, double-peak structures are seen in the spectra. At $f_{RES1} = f_{RES2} (V_{SG1} = -0.31 \text{ V})$ and $T_{dc,B} = 0$, a shoulder peak is also reproduced in the simulation.

This excellent agreement between the experiment and the simulation demonstrates the validity of our CDE model. It also provides evidence for the crossover from conductive to capacitive intercavity coupling controlled by V_{LB} . At $f_{\text{RES1}} = f_{\text{RES2}}$, the single resonant peak observed for the conductive coupling corresponds to the in-phase mode, while the double peak (or shoulder peak) observed for the capacitive coupling indicates both the in-phase and antiphase modes. Although the size of the mode splitting in the capacitively coupled cavities cannot be quantified in the present device, clear anticrossing of two resonant frequencies [Fig. 5(f)] will be observed by optimizing the cavity structures. Furthermore, we can expect to realize plasmonic crystals^{14,15} using capacitively coupled multiple cavities. Our CDE model would assist in the design of such an integrated plasmonic device.

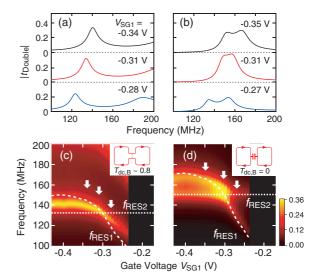


FIG. 12. (Color online) (a), (b) Calculated transmission spectra and (c), (d) image plots of $|t_{\text{Double}}|$ of the coupled cavities under the same conditions as in Fig. 11.

IV. DISCUSSION

In our CDE model, Coulomb interactions are represented as distributed electrochemical capacitances. As discussed in detail in Sec. II, the transport characteristics of plasmonic devices can be calculated by considering capacitive couplings between the edge channel and the other conductors present in the surroundings. This approximation allows us to compute the EMP transport conveniently even in integrated circuits comprising multiple plasmonic devices. However, the screening effect in low-dimensional electronic systems is not sufficient to fully screen the long-range Coulomb interactions. Therefore, in real devices, essentially all parts of the QH device can be coupled to each other.¹⁹ In an integrated system, the complicated networks of edge channels may make it unfeasible to evaluate the S matrices for individual sections. Partial screening of the edge channels by appropriately designed metal structures would solve this problem. The striking sensitivity of the EMP velocity to the presence of nearby metals¹² indicates that such screening of edge channels is indeed active; this suggests the possibility of limiting the spatial range of Coulomb coupling by using metals.

To design integrated plasmonic circuits, coherence of EMPs is a key requirement. In our CDE circuit model, loss of EMPs is parametrized with the conductance g_{ch} between the channel and the ground. With the damping included, the wave equation (1) is modified as

$$\frac{\partial I(x,t)}{\partial t} = -\frac{\sigma_{xy}}{c_{\rm ch}} \frac{\partial I(x,t)}{\partial x} - \frac{g_{\rm ch}}{c_{\rm ch}} I(x,t)$$

From this equation, the dispersion relation of EMPs is derived as

$$\omega(k) = \omega'(k) + i\omega''(k) = v_{\text{EMP}}k - i\frac{g_{\text{ch}}}{c_{\text{ch}}}$$

Thus, EMP damping $\omega''(k) = \text{Im}\omega(k)$ is represented by the circuit parameters g_{ch} and c_{ch} in our CDE model.

The advantage of the CDE model for investigating EMP damping is that we can evaluate $\omega''(k)$ from the measured resonant spectrum separately from the broadening of resonant peaks due to the loss of EMPs at the entrance and exit of a cavity. In this paper, we simulated EMP transport assuming constant g_{ch} and found good agreement between the simulation and the experimental data. However, it is known that EMP damping is not always frequency independent. It was experimentally demonstrated that the frequency dependence of EMP damping varies depending on the electrostatic environment around the sample.⁷ In the CDE model, such frequency dependences can be included into the simulation by assuming frequency dependence of g_{ch} . Once g_{ch} is obtained as a function of frequency from the comparison between the simulation and the experimental data, we can calculate EMP transport even in integrated systems including the frequency dependence, similarly to the case of constant g_{ch} .

To understand the mechanism of the damping, we need to investigate the detailed structure of edge channels.⁷ The shape of compressible and incompressible strips, electron dynamics in the edge channel, and the resultant EMP damping are sensitive to environmental parameters (such as magnetic field, electrostatic situation, and temperature). The structure of edge channels has been theoretically investigated as a function of these parameters. $^{36-38}$ On the other hand, the damping of EMPs has not yet been fully understood. For example, while it was experimentally demonstrated that the damping of EMPs is enhanced by the presence of a grounded metal near the edge channel,^{7,39} the mechanism of this metal-induced damping still remains an open question. Not only dissipation of rf signals inside the metal due to its internal resistance but also the screening and the resultant change of the shape of edge channels may enhance EMP damping. The damping of EMPs along gate-defined edge channels should be an important issue for future plasmonic circuits.

Another issue for EMP damping is its temperature dependence. We consider two possible origins for the observed temperature dependence [Fig. 10(c)]. One is the thermal smearing of the compressible strip structure^{37,38} in $\nu = 1$ QH state and the resultant enhancement of the damping in edge channels. A second possibility is the presence of thermally excited down-spin electrons in the system, for the energy scale corresponding to the characteristic temperature $T_{\rm res}$ of EMP resonance broadening is close to the Zeeman gap. Although we have not yet determined the mechanism of EMP damping, further experiments, such as space- and/or spin-resolved EMP transport measurements, would provide more insight into EMP damping.

V. CONCLUSION

A CDE circuit model has been developed to describe the EMP transport properties in a QH device. A characteristic impedance and distributed electrochemical capacitances were introduced as fundamental physical quantities that characterize the edge channels constituting plasmonic circuits. The EMP transport characteristics of various gate-defined structures, such as a QPC and single and coupled cavities were calculated using this model. Good agreement was found between the calculations and the experimental results.

The advantages of our CDE model can be summarized as follows. First, it allows the effect of Coulomb interactions to be expressed as capacitances, i.e., as circuit parameters. This provides a simple and powerful means of evaluating the impact of Coulomb interactions in electronic systems, which is essential to understanding electron dynamics in QH edge channels. Because the capacitance between two conductors can be measured experimentally or simulated using, e.g., finite element methods, the EMP transport properties can be computed using realistic parameters obtained separately. Second, the circuit representation allows us to analyze EMP transport in multiple plasmonic devices using the scattering matrix formalism. This opens a way to establish future integrated plasmonic circuits.

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