Devil's staircases and continued fractions in Josephson junctions

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Detailed numerical simulations of the *IV* characteristics of a Josephson junction under external electromagnetic radiation show the devil's staircase within different bias current intervals. We have found that the observed steps form very precisely continued fractions. Increase of the amplitude of the radiation shifts the devil's staircase to higher Shapiro steps. An algorithm for the appearance and detection of subharmonics with increasing radiation amplitude is proposed. We demonstrate that the subharmonic steps registered in the well-known experiments by Dayem and Wiegand [Phys. Rev. **155**, 419 (1967)] and Clarke [Phys. Rev. B **4**, 2963 (1971)] also form continued fractions.

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I. INTRODUCTION

Josephson junctions are regarded as excellent model systems for studying a variety of nonlinear phenomena in different fields of science[1,2] such as frequency locking, chaos, charge-density waves, transport in superconducting nanowires, interference phenomena, and others.[3–6] These phenomena, and especially properties of the Shapiro steps[7] (SSs) in Josephson junctions, are very important for technical applications.[8]

In a Josephson system driven by external microwave radiation, the so-called devil's staircase (DS) structure has been predicted as a consequence of the interplay of the Josephson plasma frequency and the applied frequency (see Refs. 9 and 10 and references therein). To stress the universality in the scenario presented, we note that the devil's staircase appears in other systems including infinite spin chains with long-range interactions,[11] frustrated quasi-two-dimensional spin-dimer systems in magnetic fields,[12] systems of strongly interacting Rydberg atoms,[13] and the fractional quantum Hall effect.[14] We note the recent paper by Hriscu and Nazarov[15] in which the synchronization of Josephson and Bloch oscillations results in the quantization of transresistance, which also leads to a devil's staircase structure for this coupled system. A series of fractional synchronization regimes (devil"s staircase) in a spin-torque nano-oscillator driven by a microwave field was experimentally demonstrated.[16] In Ref. 17 it is considered that a devil's staircase shows a high degree of self-organization.

A detailed experimental investigation of the subharmonic SSs in superconductor–normal metal–superconductor (SNS) junctions was made by Clarke.[18] He found that the application to a junction of rf electromagnetic radiation of frequency Ω induced constant-voltage current steps at voltages $(n/m)\hbar\Omega/(2e)$, where n and m are positive integers. The results were explained based on the idea that the phase difference in a Josephson junction is increasing in time in a uniform manner and the current-phase relation is nonsinusoidal. The junction generates harmonics when it is biased at some voltage, and these harmonics may synchronize with the applied radiation to produce the steps. Another well-known experiment on the behavior of thin-film superconducting bridges in a microwave field by Dayem and Wiegand[19] also demonstrates the production of constant-voltage steps in the *IV* characteristics. Some experimental results are explained by a nonsinusoidal current-phase relation.[20,21] Ben-Jacob and coauthors[10] found subharmonic steps within the resistively and capacitively shunted junction (RCSJ) model with a purely sinusoidal current-phase relation.[22,23]

In this paper we clearly show by high-precision numerical simulations that the *IV* characteristic of a Josephson junction under microwave radiation exhibit a DS structure of subharmonic Shapiro steps. To show that we have a devil's staircase, we demonstrate its self-similar structure by analyzing our results in terms of continued fractions.[24,25] We stress that the present description helps to simplify the analysis of the experimental and simulated results, as it provides a predictive power. We show that the results in the published papers are easily classified based on our continued fraction formula (3). In particular, we show that the steps observed in many previous experiments[3,18,19,26–30] and numerical simulations[4,9,10,31] form continued fractions. We analyze the data of the experiments of Clarke [see Ref. 18 and Fig. 9(a) there] and Dayem and Wiegand (see Ref. 19 and Fig. 16 there) in terms of continued fractions and show that the steps observed in these papers also form very precisely continued fractions.

II. MODEL AND METHODS

Assuming the RCSJ model, we employ the following system of equations for the phase difference φ across the junction, taking into account the external radiation with frequency ω and amplitude A:

$$\dot{V} + \sin(\varphi) + \beta\dot{\varphi} = I + A\sin(\omega t), \quad (1)$$

$$\dot{\varphi} = V. \quad (2)$$

Here the dc bias current I and ac amplitude A are normalized to the critical current I_c, the voltage V to $V_0 = \hbar\omega_p/(2e)$ (ω_p

is the plasma frequency), and time t to ω_p^{-1}. β is the dissipation parameter ($\beta = \beta_c^{-1/2}$, where β_c is McCumber's parameter). In this study we investigate an underdamped Josephson junction (JJ), with $\beta = 0.2$, which exhibits hysteresis in its *IV* characteristics. The overdot indicates a derivative with respect to the dimensionless time. In our simulations we used mostly 0.05 as the step in time, 10^4 as the time domain for averaging with 10^3 units before averaging, and 10^{-5} as the step in the bias current.

To find the *IV* characteristics of the JJ, we solve this system of nonlinear differential equations (1) and (2) using the fourth-order Runge-Kutta method. As a result, we find the temporal dependence of the voltage in the JJ at a fixed value of the bias current I. Then the current value is increased or decreased by a small amount of δI (the bias current step) to calculate the voltage at the next point of the *IV* characteristics. We use the final phase and voltage achieved at the previous point of the *IV* characteristics as the initial conditions for the next current point. The average of the voltage V is given by $V = \frac{1}{T_f - T_i}\int_{T_i}^{T_f} V(t)dt$ where T_i and T_f determine the interval for the temporal averaging. The details of the simulation procedure are described in Ref. 32.

III. DEVIL'S STAIRCASE AND CONTINUED FRACTIONS

Figure 1(a) shows the *IV* characteristics of the Josephson junction at $\omega = 0.5$ and $A = 0.8$. We see that there is no hysteresis in comparison with the case at $A = 0.1$ shown in the inset, and chaos is developed in some current intervals. There is manifestation of the second harmonic, i.e., an integer Shapiro step at $V = 2\omega = 1$, and the fifth and sixth harmonics, at $V = 2.5$ and $V = 3$, respectively.

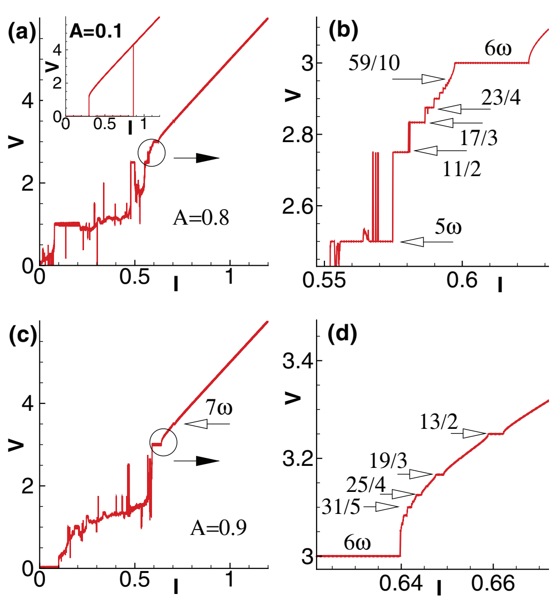

FIG. 1. (Color online) Simulated current-voltage characteristics of a Josephson junction under external electromagnetic radiation with $\omega = 0.5$ and different radiation amplitudes A. (b) and (d) show enlarged views of the encircled devil's staircases in (a), below, and (c), above, the sixth principal SS harmonic.

Let us consider carefully the part of the *IV* characteristic (IVC) marked by a circle which is enlarged in Fig. 1(b). A series of steps in the form of $(N - 1/n)\omega$, where $N = 6$ and n is a positive integer, is observed between 5ω and 6ω. We note that these steps are approaching the sixth harmonic from below. As A is increased the chaotic region is expanded and the DS structure disappears. Instead it develops above the sixth SS harmonic. Figure 1(c) shows the *IV* characteristic of the same Josephson junction at $A = 0.9$ with the DS structure which is enlarged in Fig. 1(d). The steps are approaching the 6ω harmonic from above and follow the formula $(N + 1/n)\omega$, again with $N = 6$ and n a positive integer.

The analysis of the various observed staircase structures leads us to the conclusion that in general the steps follow the formula for continued fractions given by

$$V = \left(N \pm \cfrac{1}{n \pm \cfrac{1}{m \pm \cfrac{1}{p \pm \ddots}}} \right) \omega, \qquad (3)$$

where $N,n,m,p,\ldots$ are positive integers. We will call the terms that involve only N the first-level terms or the Shapiro step harmonics. The other terms describe the subharmonics, or the fractional steps. Those involving N and n we call the second-level terms, those with N, n, and m the third-level terms, etc.

Usually mathematicians use a positive sign to express continued fractions.[24,25] We have included the minus sign for convenience only; this allows us to easily analyze the subharmonics in the chosen interval of voltage (or frequency). Another reason to use continued fractions with a negative sign is the following. The formula with positive signs puts physically equal sequences of subharmonics at different levels of the formula. Consider the sequences $3/2, 4/3, 5/4, \ldots$ and $1/2, 2/3, 3/4, \ldots$ which describe subharmonics placed at the same distance from the first Shapiro step; i.e., at ω. In all positive continued fractions they are related to the different levels described respectively by the formulas $N + 1/n$ and $(N-1) + 1/(n + 1/m)$ with $N = 1$ in the first case and $N = 1, n = 1$ in the second case. Including the negative sign allows us to use $N \pm 1/n$, with $+$ for the first and $-$ for the second sequence, and to keep $N = 1$ for both sequences.

The algorithm of continued fractions is schematically presented in Fig. 2. We show by the numbers in circles the SS harmonics (red online). The second level of continued fractions gives two groups of subharmonic steps (blue online): $(N - 1) + (1/n)$ and $N - (1/n)$. The first group is approaching the $(N - 1)$th SS, and second one is approaching the Nth SS. So, if the sequence in an interval (a,b) is building to approach the step a, we need to take the $+$ sign, and if the sequence is approaching the step b, then $-$. To find subharmonics corresponding to the third level we first determine the interval we are interested in; this entails choosing n and $n + 1$, which are then kept constant, as m is varied. Each of them leads to the appearance of two other groups, approaching the first and second terms. In Fig. 2 we show the sequences of the third level between the subharmonics with $n = 1$ and $n = 2$ also. Other sequences are formed by the same algorithm.

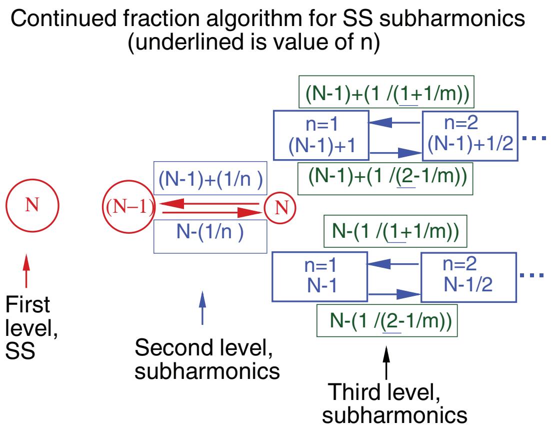

FIG. 2. (Color online) Schematic demonstration of the appearance of continued fractions in IV-characteristic of Josephson junction under external electromagnetic radiation. N is the SS number, n and m are positive integers.

In view of Eq. (3) we note that the continued fraction description is not directly related to the steady-state dynamics of the junction. For any choice of the quantities $N, n, m, \ldots$, one can of course always reduce the right-hand side of Eq. (3) to the form p/q, where p and q are some positive integers, in a ratio that reflects the phase locking between the Josephson oscillations and external radiation. However the continued fraction form of Eq. (3) provides a very convenient way of mapping the potentially infinite number of these ratios to the observable hierarchical structure in the IV characteristic. We mention that the continued fraction is used in the classification of the devil's staircase in other contexts, for one-dimensional lattice gases,[33,34] for Wigner lattices,[35] and for the Frenkel-Kontorova model.[36]

IV. SELF-SIMILARITY

We now set out to show the different levels of continued fractions of the devil's staircase. The DS in the IV characteristic of the Josephson junction at $\omega = 2$ and $A = 0.5$ is presented in Fig. 3.

The one-loop IV characteristic, which is obtained by sweeping the bias current from $I = 0$ to $I = 1.2$ and back down to $I = 0$, is shown in the inset to Fig. 3(a). Here we see that the return current is low enough to allow the $V = 2$ step to develop. The steps reflect the second level of continued fractions $(N - 1/n)\omega$ with $N = 1$. There is no half-integer step at $(1/2)\omega$ in the IVC [cf. Figs. 1(a) and 1(c)], because of the larger value of the return current at these chosen parameters.

The staircase bounded by the subharmonics 3/4 and 4/5 and marked by a rectangle in Fig. 3(a) is enlarged in Fig. 3(b). In particular, we see the sequence $4/5, 7/9, 10/13, 13/17, \ldots$, reflecting the third-level continued fraction $[N - 1/(n + 1/m)]\omega$ with $N = 1$, $n = 4$ and the sequence $3/4, 7/9, 11/14, 15/19, \ldots$, reflecting $[N - 1/(n - 1/m)]\omega$ with $N = 1$, $n = 5$. Moreover, the part between the steps $\frac{7}{9}\omega$ and $\frac{4}{5}\omega$ also marked by a rectangle in this figure is enlarged in Fig. 3(c). We found here the steps $7/9, 11/14, 15/19, 19/24$, reflecting the fourth level of

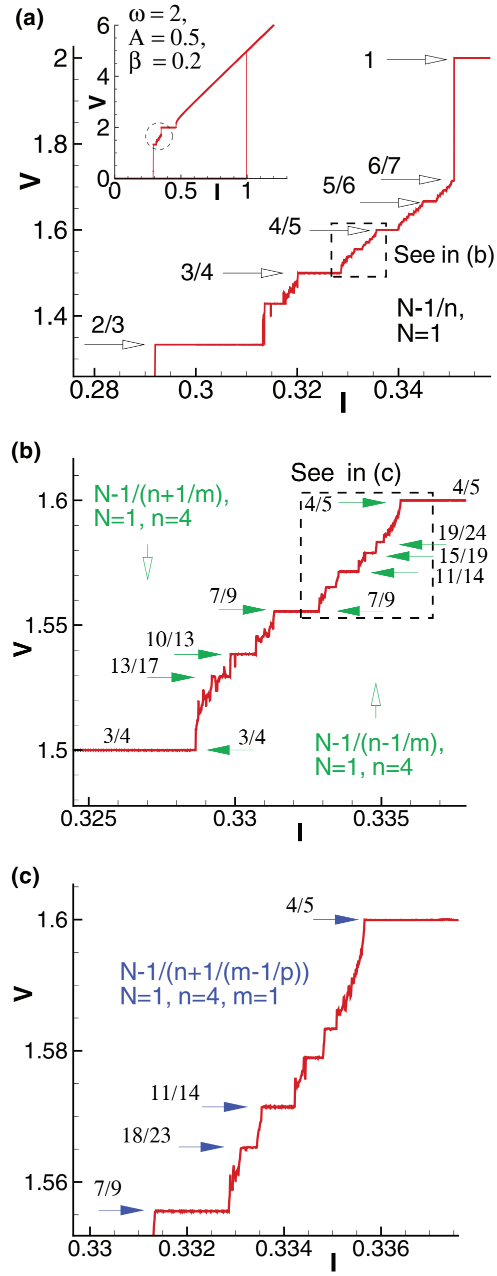

FIG. 3. (Color online) The manifestation of the continued fractions in the IV characteristic of a Josephson junction at $\omega = 2$ and $A = 0.5$. (a) The steps in the interval between the zeroth and first SS; (b) the steps between $3\omega/4$ and $4\omega/5$ marked by the rectangle in (a); (c) the steps $7\omega/9$ and $4\omega/5$ marked by the rectangle in (b).

continued fractions $\{N - 1/[n + 1/(m + 1/p)]\}\omega$ with $N = 1$, $n = 4$, and $m = 1$, and the sequence $4/5, 11/14, 18/23$, reflecting $\{N - 1/[n + 1/(m - 1/p)]\}\omega$ with $N = 1$, $n = 4$, and $m = 2$. The voltages found in our high-precision numerical simulations coincide with the corresponding values calculated by formula (3).

We should like to emphasize that we deal with a problem in the physics of synchronization. This means that stable periodic orbits exist for the junction (or pendulum) at frequencies that are rational multiples of the drive frequency.

In principle, we could have any rational multiple, and in this sense the set of stable orbits constitutes a dense set, unlike that of the harmonics. It is at this stage that the mathematical concept of the continued fraction paves the way. A continued fraction forms a representation for any irrational (but not transcendental) number. This makes for a dense, yet countable, set. The fact that the set is dense means that between any two rationals we can find a countably infinite number of rationals, and this we have pointed out in our description of *levels*. The self-similar property of the devil's staircase is reproduced by the progressive generation of the subharmonics within it.[37] The algorithm presented here for the generation of levels, giving the structure of the continued fraction, prescribes such a progressive generation as well. The Josephson system provides a unique opportunity to take this mathematical idea and show its physical application. This is yet another aspect of the precision in the frequency measurements in the Josephson system that we have been able to reproduce here with our very high-precision simulations.

V. ANALYSIS OF THE EXPERIMENTAL RESULTS

Let us finally discuss the experimental results for the subharmonic steps in the *IV* characteristic of a Josephson junction in the presence of rf radiation. Our main statement is that the set of constant-voltage steps found in previous experiments[18,19,26,27] are structured such that they are reproduced by continued fractions.

We first consider the experiments of Clarke, and in particular look at Fig. 9(a) in Ref. 18. In Fig. 4(a) we reproduce these experimental results and compare them with continued fractions in the corresponding intervals of voltage. The voltage is normalized to the value of the first Shapiro step. In the experimental paper the subharmonic $1/2$ is registered between the zeroth and first Shapiro steps, reflecting the sequence $N-1/n$ with $N=0, n=2$. In the second SS interval (1,2) a series 1,3/2,5/3 is fixed which follows $V=(N-1/n)$ with $N=2$. In the third (2,3) and fourth (3,4) SS intervals the steps at voltages $3/1, 5/2, 7/3$ and $4/1, 7/2, \ldots, 13/4$ follow the fractions $V=(N+1/n)$ with $N=2$ and $N=3$, respectively. In the last series, it was only the $10/3$ step that was not noticed in the experimental paper.

The subharmonics which were experimentally measured by Dayem and Wiegand in Ref. 19 precisely follow the continued fraction formulas also. Figure 16 of Ref. 19 shows the *IV* characteristics at different power levels, for applied microwave radiation at 4.26 GHz. In Fig. 4(b) we also reproduce these experimental results and compare them with continued fractions. Subharmonic steps in the SS intervals (0,1) and (1,2) were found. The analysis shows that the steps $0, 1/2, 2/3, 3/4$ follow $(N-1/n)$ with $N=1$ and the series $1/n$ is just $(N+1/n)$ with $N=0$. For clarity we enlarge this part of the figure in the inset. In the SS interval (1,2) the experiment shows the steps $2/1, 3/2, 4/3, 5/4$ according to $N+1/n$ with $N=1$, and $1, 3/2, 5/3$ according to $N-1/n$ with $N=2$. It seems that there is a misprint in the original paper: the step around $V=4\mu V$ denoted as $1/5$. Actually, it is the step $2/5$ and it follows the third level of continued fractions $N+1/(n+1/m)$ with $N=0, n=2$, and $m=2$. We see also in the analyzed figure the signature of the step

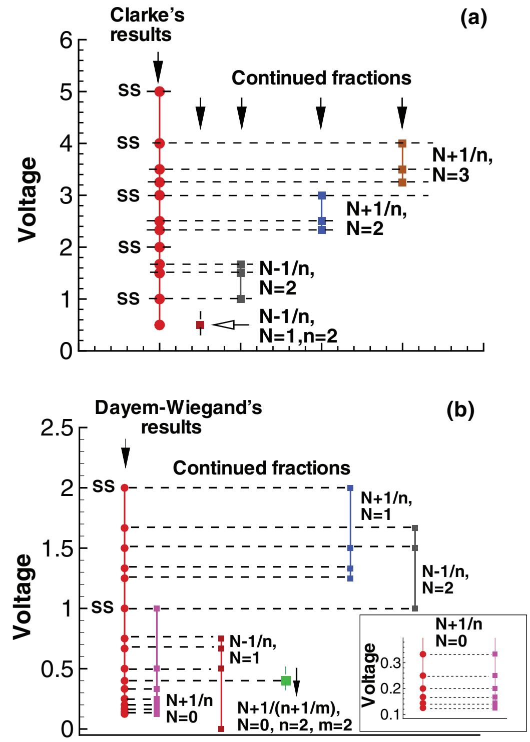

FIG. 4. (Color online) Comparison of the experimental results of (a) Clarke (Ref. 18) and (b) Dayem and Wiegand (Ref. 19) with continued fractions. Filled circles show the experimental results, and squares different continued fractions.

$3/5$ between $1/2$ and $2/3$, following $N-1/(n-1/m)$ with $N=1$, $n=3$, and $m=2$, which was not marked by the authors.

We note that in Ref. 26 the authors observed two series of subharmonic steps up to the sixth order ($n=6$) experimentally. We consider these to be special cases of Eq. (3): the first series corresponds to $V=(0+1/n)\omega$ and the second to $V=(1+1/n)\omega$.

We also note that steps with ratios p/q corresponding to higher values of p and q generally feature less prominently in the data. This is so because a given rational p/q forms a more stable orbit, the smaller q is, in the sense that it occupies a larger basin of attraction in the phase space of the system. The way the continued fraction is defined makes it clear that, given two rationals, the continued fraction at the next level introduces rationals between the two, so that for the latter level, q is increased. As mentioned above, we are numerically proving the mathematical idea of synchronization at a countably infinite number of frequencies given by p/q times the drive frequency. Yet the precision of measurements, experimental or numerical, naturally puts a limit to the observations that are to serve as proof.

One of the main purposes of the present work is to demonstrate the fact that the developed algorithm, represented by Eq. (3), simplifies the analysis of experimental results. Equation (3) accurately reproduces the observed structure in the Shapiro steps. Furthermore, its application is not only limited to the system considered here. It might also be useful for the analysis of subharmonics in other systems, in particular, for subharmonics observed in high-T_c superconductors within an applied magnetic field (Ref. 38, Fig. 1). Two groups of observed steps (1,1/2,1/3,1/5) and (0,2/3,4/5) correspond to the second level of continued fractions $(0 + 1/n)$ and $(1 - 1/n)$, respectively. The steps 2/5 and 3/5 are at the third level $[0 + 1/(3 - 1/2)]$ and $[1 - 1/(2 + 1/2)]$. The half-integral constant-voltage steps in high-T_c grain boundary junctions in Ref. 39 are related to the second level of continued fractions. The steps observed in the quantum Hall effect (see Ref. 15) are also easy to classify by Eq. (3).

Reports on measurements of dc electron transport and microwave dynamics of thin-film hybrid Nb/Au/CaSrCuO/YBaCuO planar Josephson junctions were presented in Ref. 27. The authors observed tunnel-like behavior, and oscillations synchronous with the applied radiation at integer and half-integer steps. For a junction fabricated on a *c*-oriented yttrium barium copper oxide (YBCO) film a devil's staircase structure was observed under microwave irradiation at 4.26 GHz.

VI. SUMMARY

A detailed numerical simulation of the *IV* characteristics of a Josephson junction under external electromagnetic radiation shows the presence of a devil's staircase as a function of the bias current. We have shown the particular structure of the continued fraction that reproduces this staircase. Increasing the amplitude of the radiation shifts the devil's staircase to higher Shapiro steps, and the algorithm that reveals the structure of the staircase does contain this effect.

We have used the well-known RCSJ model to make high-precision simulations of the *IV* characteristics. By analyzing the results of these simulations and comparing them to the available experimental data, we have stressed that the self-similar structure of the Shapiro step subharmonics can be understood in terms of the continued fractions formula, as given by Eq. (3). We have related this formula directly to the simulated and observable results, within the RCSJ-model approximation. We have found that the subharmonic steps registered in the experiments by Dayem and Wiegand, Clarke, and many others also form continued fractions. The continued fraction form of Eq. (3) provides a very convenient way of systematically mapping the fine details of the devil's staircase to the observable hierarchy of steps in the *IV* characteristic.

Recently, systems of coupled Josephson junctions also have attracted great interest in both the scientific and technologically minded communities. In coupled systems of junctions, in which capacitive and inductive coupling become important, one would expect a gradual destruction of the uncoupled DS structure as the coupling strength increases. In future work it would be interesting to investigate the extent to which the present model can help us explain the more complicated nonlinear phenomena that are found in systems of coupled junctions.

Another very interesting application of the methods developed in the present paper is in topological superconductivity. Topological superconductors are currently being investigated intensively.[40,41] They support Majorana fermions which are expected to be used for realization of quantum gates that are topologically protected from local sources of decoherence (see Ref. 42). The authors of this paper report the observation of the fractional ac Josephson effect in a semiconductor-superconductor nanowire junction as a signature of Majorana quasiparticles. The use of subharmonics for the detection of the Majorana fermions is a very interesting but unsolved problem. Its solution may provide additional information on Majorana physics and may warrant special consideration in a more detailed investigation.

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