Equilibration of quantum Hall edge states by an Ohmic contact

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Ohmic contacts are crucial elements of electron optics that have not received a clear theoretical description yet. We propose a model of an Ohmic contact as a piece of metal of the finite capacitance C attached to a quantum Hall edge. It is shown that charged quantum Hall edge states may have weak coupling to neutral excitations in an Ohmic contact. Consequently, despite being a reservoir of neutral excitations, an Ohmic contact is not able to efficiently equilibrate edge states if its temperature is smaller than $\hbar\Omega_c$, where Ω_c is the inverse RC time of the contact. This energy scale for a floating contact may become as large as the single-electron charging energy e^2/C.

The study of quantum Hall (QH) edge states has great importance both for theoretical understanding of a strongly correlated matter and for the development of quantum devices for electron optics. These chiral one-dimensional (1D) states are quantum analogs of the skipping orbits which appear at the edge of a two-dimensional electron gas (2DEG) in a strong magnetic field. The fact that the QH edge states behave in many ways similarly to optical beams has triggered several quantum-optics-type experiments with electrons.[1–4] One of the most important elements used to manipulate the edge states in such experiments are the Ohmic contacts. They serve as incoherent sources and detectors of the "electron beams" in experiments on controllable dephasing[2,3] and controllable energy equilibration of the edge states.[4]

Ohmic contacts are created by placing a piece of metal on top of a highly doped region in a semiconductor containing 2DEG. Strong tunneling between the edge states and the states in this doped region provides a low-resistance contact with external circuits. Ohmic contacts are very complicated electron systems from a theoretical point of view, and they are still the least understood elements of electron optics in spite of their widespread usage. A simplification arises in the regime of the integer QH effect, where edge states are commonly described using the free fermion picture.[5] Based on this picture, an idealized concept of a so-called "voltage probe"[6] has been proposed to describe floating Ohmic contacts in this regime. A voltage probe is a reservoir of electrons which absorbs all incoming electron excitations and emits new electron states with the equilibrium Fermi distribution and the electrochemical potential that takes into account the current conservation law. However, it has been shown recently,[7–11] in the context of the experiments,[2,4] that even at integer filling factors the free-fermion description of the edge states is not always correct, and that the effective theory[12] considering the edge states as collective boson excitations is a more appropriate approach. This observation calls us to reconsider the applicability of the concept of a voltage probe.

It may appear that this concept finds a theoretical justification even at the effective theory level. Indeed, it has been argued that strong tunnel coupling at a floating Ohmic contact leads to equilibration[13] and an effective elongation[14] of the QH edge channel. Therefore it seems to be natural to consider the edge channel being effectively cut into two separate parts (see Fig. 1) carrying orthogonal fermions, i.e., with zero overlap. However, edge states carry the electric charge which an Ohmic contact may have a limited ability to accommodate. To be more specific, considering an Ohmic contact as a three-dimensional piece of metal of the size L, the level spacing of neutral excitations scales as $1/L^3$. It is typically small enough to consider an Ohmic contact to be a reservoir of such excitations. On the other hand, the characteristic frequency of the charge response of an Ohmic contact scales as $1/C$, where its capacitance C is of the order of L. Therefore it scales down with the size L much slower than the level spacing and may compare to characteristic energy scales of modern mesoscopic experiments with QH edge states, which makes it impossible to fully equilibrate them. In this article, we propose a simple model of an Ohmic contact, generalizing earlier models for systems with nonchiral Luttinger liquids,[15] which is capable to correctly account its finite charge response frequency.

Floating contact and boson scattering theory. We model an Ohmic contact connecting incoming and outgoing QH edge states at filling factor $\nu = 1$ as shown in Fig. 2 and explained in the figure caption. The low-energy physics of the QH edge states[12] is described by a set of scalar boson fields $\phi_\sigma(x,t)$, where $\sigma = \pm$. The charge density operator and the current operator for incoming, $\sigma = -$, and outgoing, $\sigma = +$, states have the form $\rho_\sigma = (e/2\pi)\partial_x\phi_\sigma$ and $j_\sigma = -(e/2\pi)\partial_t\phi_\sigma$. These boson fields satisfy the following canonical commutation relations:

$$[\partial_x\phi_\sigma(x,t),\phi_{\sigma'}(y,t)] = 2\pi i\sigma\,\delta_{\sigma\sigma'}\delta(x - y). \qquad (1)$$

The system can be described by the Hamiltonian containing two parts. The first part generates the dynamics of the incoming and outgoing edge channels,[16] while the second term describes the charging energy of the Ohmic contact of a finite size:

$$\mathcal{H} = \frac{\hbar v_F}{4\pi}\sum_\sigma\int_{-\infty}^{\infty} dx(\partial_x\phi_\sigma)^2 + \frac{Q^2}{2C}, \qquad (2a)$$

$$Q = \int_{-\infty}^{0} dx\,e^{\epsilon x/v_F}[\rho_+(x) + \rho_-(x)]. \qquad (2b)$$
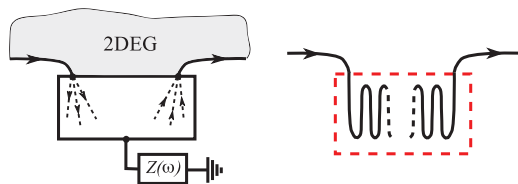
FIG. 1. (Color online) An Ohmic contact, shown on the left, is a piece of metal (white rectangle) which is placed in close proximity to the 2DEG (shown by gray shadow) and connected to an electrical circuit via the impedance $Z(\omega)$. It absorbs incident electron states, formed at the edge of the 2DEG in the QH effect regime (shown by thick black lines), and turns them to neutral electron-hole excitations (dotted lines). Then it emits equilibrium neutral excitations, and at the exit, turns them to the edge states. Motivated by earlier findings (Refs. [13] and [14]), we model this process by extending an edge state inside the piece of metal (red rectangle shown on the right) and splitting it in two uncorrelated channels. Edge states inside the metal provide their charge Q to the Ohmic contact and become neutral modes. The Ohmic contact, in turn, equilibrates these neutral modes.

Here Q is an operator of the total charge accumulated at the Ohmic contact and ϵ is a small regularization parameter, roughly given by the decay rate of neutral excitations inside the contact.[17]

Using commutation relations (1) and the Hamiltonian (2), we obtain the equations of motion for the incoming and outgoing state:

$$\sigma \, \partial_t \phi_\sigma(x,t) + v_F \partial_x \phi_\sigma(x,t) = -\frac{e}{\hbar C} Q(t) e^{\epsilon x/v_F} \theta(-x). \quad (3)$$

These equations have to be accompanied with the boundary conditions

$$\partial_t \phi_+(-\infty,t) = -(2\pi/e) j_c(t), \quad (4a)$$

$$\partial_t \phi_-(0,t) = (2\pi/e) j_{in}(t), \quad (4b)$$

where j_{in} is the current flowing into the Ohmic contact, while j_c describes *equilibrium* fluctuations of the neutral mode with the reservoir temperature T_c, originating from the Ohmic contact. Solving Eq. (3) with the boundary conditions (4), one can relate the outgoing current $j_{out}(t) = -(e/2\pi)\partial_t \phi_+(0,t)$ to the incoming current j_{in}, as illustrated in Fig. 2.
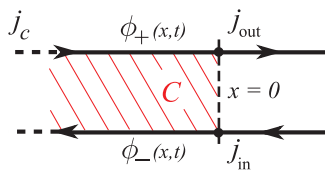

FIG. 2. (Color online) An equivalent representation of the floating Ohmic contact at filling factor $\nu = 1$ shown in Fig. 1. For convenience, we fold edge states so that they could be described by two boson fields ϕ_+ and ϕ_- of opposite chiralities. The dynamics of these fields is generated by the Hamiltonian (2), and the boundary conditions are given by Eqs. (4). The region inside the Ohmic contact, where the capacitive interaction is assumed, is shown by the red color. Note that the charge response frequency of the contact is finite, while the level spacing of neutral modes vanishes. In order to take this fact into account, we extend edge states inside the interaction region to infinity and introduce a small parameter ϵ to regularize corresponding integrals.

In order to solve Eq. (3), we apply the Fourier transform $\phi_\sigma(x,\omega) \equiv \int dt \, e^{i\omega t} \phi_\sigma(x,t)$ and rewrite them as ordinary first-order differential equations. The general solution for $x \leqslant 0$ then reads

$$\phi_\sigma(x,\omega) = \phi_\sigma(\omega) e^{i\sigma \omega x/v_F} + \frac{\sigma}{R_q C} \frac{\sum_{\sigma'} \phi_{\sigma'}(\omega)}{(i\omega - \sigma\epsilon)} e^{\epsilon x/v_F}, \quad (5)$$

where $\phi_\sigma(\omega)$ are constants of integration, and $R_q = 2\pi\hbar/e^2$ is the resistance quantum. Using the boundary conditions (4), we find the fields $\phi_\sigma(x,t)$, and then, the outgoing current

$$j_{out}(\omega) = \frac{i\omega R_q C}{i\omega R_q C - 1} j_c(\omega) - \frac{1}{i\omega R_q C - 1} j_{in}(\omega), \quad (6)$$

where we have omitted a trivial phase factor in $j_c(\omega)$ and set $\epsilon = 0$.[19] In the context of the boson scattering theory,[7,20] the boundary conditions (4) can be viewed as the incident waves, while j_{out} is the outgoing wave. Then the coefficients in front of the currents in Eq. (6) are the boson scattering amplitudes.[21]

Langevin equations. The equations of motion for the currents and the charge may be written in a yet different form:

$$\frac{dQ(t)}{dt} = j_{in}(t) - j_{out}(t), \quad (7a)$$

$$j_{out}(t) = Q(t)/R_q C + j_c(t), \quad (7b)$$

where the first equation expresses the conservation of charge. The second one is the Langevin equation, which has the following simple physical meaning. The outgoing current acquires two contributions: $Q(t)/R_q C$ is the current induced by the time-dependent potential $Q(t)/C$, and the second one, j_c, is the Langevin current source. It is easy to check that by solving these equations, one arrives at the result (6).

The advantage of this formulation is that the equations (7) can be easily generalized to account for the effects of dissipation in an Ohmic contact connected to the electric circuit. This amounts to adding a current j'_{out} and an impedance Z to the equivalent electric circuit, as shown in Fig. 3. It is convenient to present corresponding equations of motion in the frequency domain:

$$i\omega Q(\omega) = j_{out}(\omega) + j'_{out}(\omega) - j_{in}(\omega), \quad (8a)$$

$$j_{out}(\omega) = Q(\omega)/R_q C + j_c(\omega), \quad (8b)$$

$$j'_{out}(\omega) = Q(\omega)/Z(\omega)C + j_Z(\omega). \quad (8c)$$

After straightforward calculations, we present the current $j_{out}(\omega)$ in the following form:

$$j_{out}(\omega) = \sum_{p=in,c,Z} \mathcal{T}_p(\omega) j_p(\omega), \quad (9a)$$

$$\mathcal{T}_Z = -\mathcal{T}_{in} = \mathcal{T}_c - 1 = [i\omega R_q C - R_q/Z(\omega) - 1]^{-1}. \quad (9b)$$
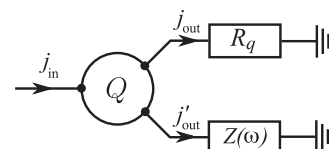

FIG. 3. Equivalent circuit representation of the Langevin equations (8). The charge conservation in the system is described by Eq. (8a), while Eqs. (8b) and (8c) for the outgoing currents are the Langevin equations with the current sources j_c and j_Z, respectively.

We note that the Langevin equation approach used here is appropriate in the case of linear quantum circuits that may be described entirely in terms of collective plasmon modes. This approach is fully consistent[22] with the Caldeira-Leggett model.[23]

Spectral functions and effective temperature. One can characterize the statistics of the current fluctuations $\delta j_{\text{out}} \equiv j_{\text{out}} - \langle j_{\text{out}} \rangle$ by the spectral density function $S(\omega)$, defined via the relation

$$\langle \delta j_{\text{out}}(\omega) \delta j_{\text{out}}(\omega') \rangle = 2\pi \delta(\omega + \omega') S(\omega). \tag{10}$$

The solution (9) allows us to express it in terms of the noise spectral densities of the currents $j_{\text{in}}(\omega)$, $j_c(\omega)$, and $j_Z(\omega)$:

$$S(\omega) = \sum_p |\mathcal{T}_p(\omega)|^2 S_p(\omega). \tag{11}$$

Here $S_p(\omega)$ are defined similarly, $\langle \delta j_p(\omega) \delta j_p(\omega') \rangle = 2\pi \delta(\omega + \omega') S_p(\omega)$, and the average is evaluated with the equilibrium state in the corresponding channel, which implies[24]

$$S_p(\omega) = \frac{2\hbar \omega G_p}{1 - e^{-\hbar\omega/T_p}}, \tag{12}$$

with $G_{\text{in}} = G_c = 1/2R_q$ and $G_Z = \text{Re}[1/Z(\omega)]$. It is easy to check that the following identity holds:

$$\sum_p |\mathcal{T}_p(\omega)|^2 G_p(\omega) = 1/2R_q. \tag{13}$$

Therefore the statistics of the current originating at the Ohmic contact is equilibrium if all the temperatures T_p are equal, and it is *nonequilibrium* otherwise.

Sufficiently far from the Ohmic contact, the outgoing state reaches the equilibrium. Therefore since the heat flux J_{out} carried by the edge state is conserved, it is convenient to define the effective temperature T_{out} of the outgoing state by using the equilibrium relation $J_{\text{out}} = \pi T_{\text{out}}^2/12\hbar$ for a 1D chiral channel,[25] which can easily be derived as follows.[26] In a chiral system, the energy flux operator is equal to the energy density operator multiplied by the velocity $J = v_F \cdot (\hbar v_F/4\pi)[\partial_x \phi(x,t)]^2 = (\pi\hbar/e^2) j^2(x,t)$. Then the heat flux can be obtained by subtracting the vacuum energy contribution: $J_{\text{out}} = \langle J \rangle - J_{\text{vac}}$. Expressing the heat flux in terms of the current noise spectral function and comparing it to the one for the equilibrium noise, we arrive at the following result:

$$T_{\text{out}}^2 = \frac{3R_q\hbar}{\pi^2} \int_{-\infty}^{\infty} d\omega [S(\omega) - \hbar\omega\theta(\omega)/R_q]. \tag{14}$$

One can see that the summation rule (13) guarantees the convergence of the integral in Eq. (14) at high frequencies.

We evaluate this integral using Eqs. (9b), (11), and (12) in the simple case where the circuit is a resistor, $Z(\omega) = R$, and with the natural assumption $T_Z = T_c$. The result reads

$$T_{\text{out}}^2 = T_c^2 + \frac{6(\hbar\Omega_c)^2}{(\pi\gamma)^2} \left[I\left(\frac{\hbar\Omega_c}{T_{\text{in}}}\right) - I\left(\frac{\hbar\Omega_c}{T_c}\right) \right], \tag{15}$$

where $\Omega_c = (R_q + R)/R_q RC$ is the inverse RC time of the Ohmic contact, and $\gamma = 1 + R_q/R$ is the circuit coupling parameter. The dimensionless function I in this equation has
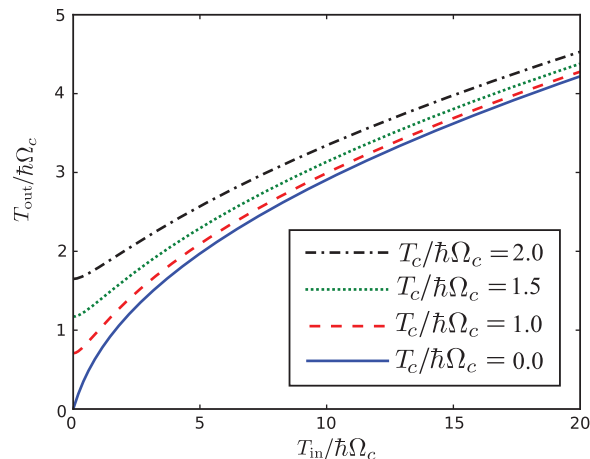

FIG. 4. (Color online) The effective temperature T_{out} of the QH edge state at $\nu = 1$ originating at a floating Ohmic contact is plotted as a function of temperature T_{in} of the incoming edge state for different values of the temperature of the Ohmic contact T_c. Note that at $T_{\text{in}} = 0$ the effective temperature of the outgoing channel saturates at a constant value, which is smaller than the contact's temperature T_c. At large values of T_{in}, it behaves as $T_{\text{out}} \propto \sqrt{T_{\text{in}}}$.

the following form:

$$I(a) = \int_0^\infty \frac{z\,dz}{z^2 + a^2} \frac{1}{e^z - 1} = \frac{1}{2}\left[\ln\left(\frac{a}{2\pi}\right) - \frac{\pi}{a} - \psi\left(\frac{a}{2\pi}\right) \right],$$

where $\psi(z)$ is the logarithmic derivative of the gamma function. Figure 4 shows T_{out} for the floating contact as a function of T_{in} for different values of T_c.

In the case of a cold Ohmic contact, $T_c = 0$, we find

$$\frac{T_{\text{out}}}{T_{\text{in}}} = \begin{cases} 1/\gamma, & \text{if } T_{\text{in}} \ll \hbar\Omega_c, \\ \sqrt{3\hbar\Omega_c/\pi\gamma^2 T_{\text{in}}}, & \text{if } T_{\text{in}} \gg \hbar\Omega_c. \end{cases} \tag{16}$$

Note that for $\gamma = 1 + R_q/R > 1$, additional cooling is provided by the dissipation in the circuit. In the case of a cold incoming state, $T_{\text{in}} = 0$, and finite T_c, we have

$$\frac{T_{\text{out}}}{T_c} = \begin{cases} \sqrt{1 - 1/\gamma^2}, & \text{if } T_c \ll \hbar\Omega_c, \\ 1 - 3\hbar\Omega_c/2\pi\gamma^2 T_c, & \text{if } T_c \gg \hbar\Omega_c. \end{cases} \tag{17}$$

Thus the ability of the Ohmic contact to equilibrate the edge state depends on the energy scale $\hbar\Omega_c$, which, for a floating contact ($\gamma = 1$), becomes comparable to the single-electron charging energy: $\hbar\Omega_c = e^2/2\pi C$. To efficiently equilibrate edge states with temperatures, e.g., in the range $T_{\text{in}} \sim 10 - 100$ mK, one needs an Ohmic contact of the size of $L \sim 10 - 100$ μm or larger.

So far, the contact's temperature T_c has been considered an independent parameter, taking (ideally) the value of the base temperature T_b. The ability of phonons to cool a contact to the base temperature can be estimated by comparing the incoming heat flux of electrons $\pi T_{\text{in}}^2/12\hbar$ to the outgoing flux $\Sigma V(T_c^5 - T_b^5)$ to phonons, where $\Sigma \simeq 0.2$ nW μm^3 K^5 is the electron-phonon coupling constant in metals,[27] and $V \simeq L^3$ is the volume of the contact. Assuming that cooling by phonons is efficient, we find the relative correction $(T_c - T_b)/T_c \simeq \pi/60\hbar\Sigma(T_c L)^3$, where we took $T_{\text{in}} = T_c$ for the estimate. On the other hand, $\hbar\Omega = e^2/2\pi C \simeq e^2/2\pi\varepsilon\varepsilon_0 L$, where

$\varepsilon \simeq 12$ for GaAs. Eliminating L, we find that $(T_c - T_b)/T_b \simeq (0.03\hbar\Omega_c/T_c)^3$, i.e., in the regime $T_c \simeq \hbar\Omega_c$ considered here, heating of the Ohmic contact is indeed weak.

Multichannel case. The generalization of our model to QH systems with the integer filling factors $\nu > 1$ is straightforward. In this case, the interactions at the edge split the spectrum of the collective modes in one charged mode and $\nu - 1$ neutral modes. For a floating contact, one may generalize the scattering theory. However, the easiest way to proceed is by noting that one should simply replace the resistance R_q in the Langevin equation (8b) with R_q/ν. Next, all the outgoing neutral modes are, obviously, at equilibrium with the Ohmic contact, because only the charged mode is coupled to its charge Q. Finally, even a screened Coulomb interaction at the edge is typically strong enough to equally distribute the heat flux over the ν electron channels[26] that are accessible experimentally. All this leads to the modification of the circuit parameter $\gamma = 1 + R_q/\nu R$ to a similar change in the charge response frequency $\Omega_c = (R_q + \nu R)/R_q RC$, and to the overall suppression of the heat flux per channel.

Expressed in terms of the new parameters, the effective temperature of an electron channel is given by $T_{out}^2 = T_c^2 + (6/\nu)(\hbar\Omega_c/\pi\gamma)^2[I(\hbar\Omega_c/T_{in}) - I(\hbar\Omega_c/T_c)]$. In particular, in this case even at small temperatures $T_{in}, T_c \ll \hbar\Omega_c$, a floating Ohmic contact is able to heat the edge states ($T_{out}/T_c = \sqrt{1 - 1/\nu}$ at $T_{in} = 0$) or cool them down ($T_{out}/T_{in} = 1/\sqrt{\nu}$ at $T_c = 0$) by redistributing the energy uniformly over the electron channels.

To conclude, we have shown that a floating Ohmic contact attached to the edge of a QH system can serve as a voltage probe only if it has a sufficiently large capacitance, so that the energy scale $\hbar\Omega_c$, where Ω_c is the inverse RC time of the contact, is much smaller than the temperature of the QH edge excitations. Such Ohmic contacts can be used to cool an edge channel with temperature larger than $\hbar\Omega_c$, and the efficiency of cooling can be increased by connecting the contact to a circuit with small resistance $R < R_q$. Finally, the equilibration by an Ohmic contact becomes more efficient at large filling factors.

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