Engineering Weyl nodes in Dirac semimetals by a magnetic field

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We study the phase diagram of a Dirac semimetal in a magnetic field at a nonzero charge density. It is shown that there exists a critical value of the chemical potential at which a first-order phase transition takes place. At subcritical values of the chemical potential the ground state is a gapped state with a dynamically generated Dirac mass and a broken chiral symmetry. The supercritical phase is the normal (gapless) phase with a nontrivial chiral structure: it is a Weyl semimetal with a pair of Weyl nodes for each of the original Dirac points. The nodes are separated by a dynamically induced chiral shift. The direction of the chiral shift coincides with that of the magnetic field and its magnitude is determined by the quasiparticle charge density, the strength of the magnetic field, and the strength of the interaction. The rearrangement of the Fermi surface accompanying this phase transition is described.

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I. INTRODUCTION

During past decades, a remarkable overlap of such seemingly different areas in physics as condensed matter and relativistic physics took place (for a recent review, see Ref. 1). It was especially clearly manifested in studying graphene.[2] Such well known phenomena as the Klein paradox, the dynamics of a supercritical charge, and the dynamical generation of a Dirac mass in a magnetic field (magnetic catalysis), revealed in relativistic field theory, were first observed in studies of graphene (see Refs. 3–5). In the present paper, we will consider manifestations in condensed matter of another relativistic phenomenon: the rearrangement of the Fermi surface in three dimensional relativistic matter in a magnetic field. The original motivation for studying this phenomenon was its possible realization in magnetars and pulsars, and in heavy ion collisions.[6,7] But as we discuss below, it could also be relevant for such new materials as Dirac and Weyl semimetals.

Dirac and Weyl semimetals possess low-energy quasiparticles near the Fermi surface, which are described by the Dirac and Weyl equation, respectively.[1] As was established long ago, an example of a semimetal whose low-energy effective theory includes three-dimensional Dirac fermions is yielded by bismuth (for reviews, see Refs. 8 and 9). On the other hand, examples of the realization of the Weyl semimetals have been considered only recently.[10–12] Weyl semimetals, which are three-dimensional analogs of graphene, present a new class of materials with nontrivial topological properties.[13] Since their electronic states in the vicinity of Weyl nodes have a definite chirality, this leads to quite unique transport and electromagnetic properties of these materials.

The most interesting signatures of Dirac and Weyl semimetals discussed in the literature[11,14–19] are connected with different nondissipative transport phenomena intimately related to the axial anomaly.[20] Many of them were previously suggested in studies of heavy ion collisions (for a review, see Ref. 21).

In this paper, we will consider a different signature of Dirac semimetals: a dynamical rearrangement of their Fermi surfaces in a magnetic field. As we show below, this rearrangement is quite spectacular: a Dirac semimetal is transformed into a Weyl one. The resulting Weyl semimetal has a pair of Weyl nodes for each of the original Dirac points. Each pair of the nodes is separated by a dynamically induced (axial) vector $2\mathbf{b}$, whose direction coincides with the direction of the magnetic field. The magnitude of the vector $\mathbf{b}$ is determined by the quasiparticle charge density, the strength of the magnetic field, and the strength of the interaction. This phenomenon of the dynamical transformation of Dirac into Weyl semimetals is a condensed matter analog of the previously studied dynamical generation of the chiral shift parameter in magnetized relativistic matter in Refs. 6 and 7.

This paper is organized as follows. In Sec. II we introduce the model and set up the notations. The gap equation for the fermion propagator in the model is derived in Sec. III. We show that, at a nonzero charge density, a pair of Weyl nodes necessarily arises in the normal phase of a Dirac metal as soon as a magnetic field is turned on. In Sec. IV a perturbative solution of the gap equation describing the normal phase of the model is analyzed. A nonperturbative solution with a dynamical gap that spontaneously breaks the chiral symmetry is analyzed in Sec. V. A phase transition between the normal phase and the phase with chiral symmetry breaking is revealed and described. In Sec. VI we compare the dynamics in Dirac semimetals and graphene. A deep connection of the normal phase of a Dirac metal in a magnetic field with the quantum Hall state with the filling factor $\nu = 2$ in graphene is pointed out. The discussion of the results and conclusions is given in Sec. VII. For convenience, throughout this paper, we set $\hbar = 1$.

II. MODEL

As stated in the Introduction, the main goal of this paper is to show that a dynamical transformation of Dirac semimetals into Weyl ones can be achieved by applying an external magnetic field to the former. It is convenient, however, to start our discussion from writing down the general form of the low-energy Hamiltonian for a Weyl semimetal,

$$H^{(\mathrm{W})} = H_0^{(\mathrm{W})} + H_{\mathrm{int}}, \tag{1}$$

where

$$H_0^{(\mathrm{W})} = \int d^3r \left[v_F \psi^\dagger(\mathbf{r}) \begin{pmatrix} \boldsymbol{\sigma}\cdot(-i\boldsymbol{\nabla}-\mathbf{b}) & 0 \\ 0 & -\boldsymbol{\sigma}\cdot(-i\boldsymbol{\nabla}+\mathbf{b}) \end{pmatrix} \times \psi(\mathbf{r}) - \mu_0 \psi^\dagger(\mathbf{r})\psi(\mathbf{r}) \right] \quad (2)$$

is the Hamiltonian of the free theory, which describes two Weyl nodes of opposite (as required by the Nielsen-Ninomiya theorem[22]) chirality separated by vector $2\mathbf{b}$ in momentum space. In the rest of this paper, following the terminology of Refs. 6 and 7, we will call $\mathbf{b}$ the chiral shift parameter. There are two reasons for choosing this terminology. First, as Eq. (2) implies, vector $\mathbf{b}$ shifts the positions of Weyl nodes from the origin in the momentum space and, secondly, the shift has opposite signs for fermions of different chiralities. The other notations are as follows: v_F is the Fermi velocity, μ_0 is the chemical potential, and $\boldsymbol{\sigma} = (\sigma_x,\sigma_y,\sigma_z)$ are Pauli matrices associated with the conduction-valence band degrees of freedom in a generic low-energy model.[23] Based on the similarity of the latter to the spin matrices in the relativistic Dirac equation, we will call them pseudospin matrices.

The interaction part of the Hamiltonian describes the Coulomb interaction, i.e.,

$$H_{\mathrm{int}} = \frac{1}{2}\int d^3r\, d^3r'\, \psi^\dagger(\mathbf{r})\psi(\mathbf{r})U(\mathbf{r}-\mathbf{r}')\psi^\dagger(\mathbf{r}')\psi(\mathbf{r}'). \quad (3)$$

In order to present our results in the most transparent way, in this study we will utilize a simpler model with a contact four-fermion interaction,

$$U(\mathbf{r}) = \frac{e^2}{\kappa|\mathbf{r}|} \to g\delta^3(\mathbf{r}), \quad (4)$$

where κ is a dielectric constant and g is a dimensionful coupling constant. As we argue in Sec. VII, such a model interaction should at least be sufficient for a qualitative description of the effect of the dynamical generation of the chiral shift parameter by a magnetic field in Dirac semimetals.

Before proceeding further with the analysis, we find it very convenient to introduce the four-dimensional Dirac matrices in the chiral representation:

$$\gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \quad (5)$$

where I is the two-dimensional unit matrix, and rewrite our model Hamiltonian in a relativistic form,

$$H^{(\mathrm{W})} = \int d^3r\, \bar\psi(\mathbf{r})[-iv_F(\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}) - (\mathbf{b}\cdot\boldsymbol{\gamma})\gamma^5 - \mu_0\gamma^0] \times \psi(\mathbf{r}) + \frac{g}{2}\int d^3r\, \rho(\mathbf{r})\rho(\mathbf{r}), \quad (6)$$

where, by definition, $\bar\psi \equiv \psi^\dagger\gamma^0$ is the Dirac conjugate spinor field, $\rho(\mathbf{r}) \equiv \bar\psi(\mathbf{r})\gamma^0\psi(\mathbf{r})$ is the charge density operator, and the matrix γ^5 is

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad (7)$$

where, as is clear from the first term in the free Hamiltonian (2), the eigenvalues of γ^5 correspond to the node degrees of freedom.

The low-energy Hamiltonian of a Dirac semimetal corresponds to a special case in Eq. (6) when $\mathbf{b} = 0$, i.e.,

$$H^{(\mathrm{D})} = H^{(\mathrm{W})}|_{\mathbf{b}=0}. \quad (8)$$

Unlike Weyl semimetals, Dirac semimetals are invariant under time reversal symmetry. Note that, for the clarity of presentation, in this paper we consider only the chiral limit when the bare Dirac mass term $m_0\bar\psi(\mathbf{r})\psi(\mathbf{r})$ is absent.

In the presence of an external magnetic field, one should replace $\boldsymbol{\nabla} \to \boldsymbol{\nabla} + ie\mathbf{A}/c$, where $\mathbf{A}$ is the vector potential and c is the speed of light. Thus the Hamiltonian of the Dirac semimetal model in an external magnetic field has the following form:

$$H^{(\mathrm{D})}_{\mathrm{mag}} = \int d^3r\, \bar\psi(\mathbf{r})\{-iv_F[\boldsymbol{\gamma}\cdot(\boldsymbol{\nabla} + ie\mathbf{A}/c)] - \mu_0\gamma^0\}\psi(\mathbf{r}) + \frac{g}{2}\int d^3r\, \rho(\mathbf{r})\rho(\mathbf{r}). \quad (9)$$

Note that both this Hamiltonian and the Weyl semimetal Hamiltonian (6) are invariant under the chiral $U(1)_+ \times U(1)_-$ symmetry, where $+$ and $-$ correspond to the node states with $+1$ and -1 eigenvalues of the γ_5 matrix, respectively. The currents connected with the $U(1)_+$ and $U(1)_-$ symmetries are anomalous. However, because these symmetries are Abelian, one can introduce conserved charges for them.[24]

III. GAP EQUATION

In this section, we will derive the gap equation for the fermion propagator in the Dirac semimetal model (9) and show that, at a nonzero charge density, a nonzero $\mathbf{b}$ *necessarily* arises in the normal phase as soon as a magnetic field is turned on.

In model (9), we easily find the following free fermion propagator:

$$iS^{-1}(u,u') = [(i\partial_t + \mu_0)\gamma^0 - v_F(\boldsymbol{\pi}\cdot\boldsymbol{\gamma})]\delta^4(u-u'), \quad (10)$$

where $u = (t,\mathbf{r})$ and $\boldsymbol{\pi} \equiv -i\boldsymbol{\nabla} + e\mathbf{A}/c$ is the canonical momentum. In the rest of the paper, we will choose the vector potential in the Landau gauge, $\mathbf{A} = (0,xB,0)$, where B is a strength of the external magnetic field pointing in the z direction.

An ansatz for the full fermion propagator can be written in the following form (we will see that this ansatz is consistent with the Schwinger-Dyson equation for the fermion propagator in the mean-field approximation):

$$iG^{-1}(u,u') = [(i\partial_t + \mu)\gamma^0 - v_F(\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) + \gamma^0(\tilde{\boldsymbol{\mu}}\cdot\boldsymbol{\gamma})\gamma^5 + v_F(\mathbf{b}\cdot\boldsymbol{\gamma})\gamma^5 - m]\delta^4(u-u'). \quad (11)$$

This propagator contains dynamical parameters $\tilde{\boldsymbol{\mu}}$, $\mathbf{b}$, and m that are absent at tree level in Eq. (10). Here m plays the role of the dynamical Dirac mass and $\mathbf{b}$ is the chiral shift.[6,7] By taking into account the Dirac structure of the $\tilde{\boldsymbol{\mu}}$ term, we see that it is related to the anomalous magnetic moment μ_{an} (associated with the pseudospin) as follows: $\tilde{\boldsymbol{\mu}} \equiv \mu_{\mathrm{an}}\mathbf{B}$. It should be also emphasized that the dynamical parameter μ in the full propagator may differ from its tree-level counterpart μ_0 [see Eq. (13) below].

In order to determine the values of these dynamical parameters, we will use the Schwinger-Dyson (gap) equation for the

fermion propagator in the mean-field approximation, i.e.,

$$iG^{-1}(u,u') = iS^{-1}(u,u') - g\{\gamma^0 G(u,u)\gamma^0 - \gamma^0 \mathrm{tr}[\gamma^0 G(u,u)]\}\delta^4(u-u'). \quad (12)$$

The first term in the curly brackets describes the exchange (Fock) interaction and the last term presents the direct (Hartree) interaction.

Separating different Dirac structures in the gap equation, we arrive at the following set of equations:

$$\mu - \mu_0 = -\frac{3}{4}g\langle j^0\rangle, \quad (13)$$

$$\mathbf{b} = \frac{g}{4v_F}\langle \mathbf{j}_5\rangle, \quad (14)$$

$$m = -\frac{g}{4}\langle\bar\psi\psi\rangle, \quad (15)$$

$$\tilde{\boldsymbol{\mu}} = \frac{g}{4}\langle\boldsymbol{\Sigma}\rangle. \quad (16)$$

The fermion charge density, the axial current density, the chiral condensate, and the anomalous magnetic moment condensate on the right-hand side of the above equations are determined through the full fermion propagator as follows:

$$\langle j^0\rangle \equiv -\mathrm{tr}[\gamma^0 G(u,u)], \quad (17)$$

$$\langle \mathbf{j}_5\rangle \equiv -\mathrm{tr}[\boldsymbol{\gamma}\gamma^5 G(u,u)], \quad (18)$$

$$\langle\bar\psi\psi\rangle \equiv -\mathrm{tr}[G(u,u)], \quad (19)$$

$$\langle\boldsymbol{\Sigma}\rangle \equiv -\mathrm{tr}[\gamma^0\boldsymbol{\gamma}\gamma^5 G(u,u)]. \quad (20)$$

Note that the right-hand sides in Eqs. (19) and (20) differ from those in Eqs. (17) and (18) by the inclusion of an additional γ^0 matrix inside the trace. Since, according to Eq. (5), the γ^0 matrix mixes quasiparticle states from different Weyl nodes, we conclude that while $\langle j^0\rangle$ and $\langle \mathbf{j}_5\rangle$ describe the charge density and the axial current density, the chiral condensate $\langle\bar\psi\psi\rangle$ and the anomalous magnetic moment condensate $\langle\boldsymbol{\Sigma}\rangle$ describe internode coherent effects.

As is known,[25–27] in the presence of a fermion charge density and a magnetic field, the axial current $\langle \mathbf{j}_5\rangle$ is generated even in the free theory. Therefore, according to Eq. (14), the chiral shift $\mathbf{b}$ is induced already in the lowest order of the perturbation theory. As a result, a Dirac semimetal is *necessarily* transformed into a Weyl one, as soon as an external magnetic field is applied to the system (see also a discussion in Sec. IV).

In order to derive the propagator $G(u,u')$ in the Landau-level representation, we invert $G^{-1}(u,u')$ in Eq. (11) by using the approach described in Appendix A of Ref. 7. For our purposes here, the expression for the propagator in the coincidence limit $u' \to u$ is sufficient [cf. Eq. (A26) in Ref. 7]:

$$G(u,u) = \frac{i}{2\pi l^2}\sum_{n=0}^{\infty}\int\frac{d\omega\,dk^3}{(2\pi)^2}\frac{\mathcal{K}_n^-\mathcal{P}_- + \mathcal{K}_n^+\mathcal{P}_+\theta(n-1)}{U_n}, \quad (21)$$

where $\mathcal{P}_\pm \equiv \frac{1}{2}(1 \pm is_\perp\gamma^1\gamma^2)$ are the pseudospin projectors, $l = \sqrt{c/|eB|}$ is the magnetic length, and $s_\perp = \mathrm{sgn}(eB)$. Also, by definition, $\theta(n-1) = 1$ for $n \geqslant 1$ and $\theta(n-1) = 0$ for $n < 1$. The functions $\mathcal{K}_n^\pm$ and U_n with $n \geqslant 0$ are given by

$$\mathcal{K}_n^\pm = [(\omega+\mu \mp s_\perp v_F b)\gamma^0 \pm s_\perp\tilde\mu + m - v_F k^3\gamma^3]\big\{(\omega+\mu)^2 + \tilde\mu^2 - m^2 - (v_F b)^2 - (v_F k^3)^2 - 2nv_F^2|eB|/c \mp 2s_\perp[\tilde\mu(\omega+\mu) + v_F bm]\gamma^0 \pm 2s_\perp(\tilde\mu + v_F b\gamma^0)v_F k^3\gamma^3\big\} \quad (22)$$

and

$$U_n = \big[(\omega+\mu)^2 + \tilde\mu^2 - m^2 - (v_F b)^2 - (v_F k^3)^2 - 2nv_F^2|eB|/c\big]^2 - 4\{[\tilde\mu(\omega+\mu) + v_F bm]^2 + (v_F k^3)^2[(v_F b)^2 - \tilde\mu^2]\}, \quad (23)$$

where we took into account that the only nonvanishing components of the axial vectors $\mathbf{b}$ and $\tilde{\boldsymbol{\mu}}$ are the longitudinal projections b and $\tilde\mu$ on the direction of the magnetic field. Note that the zeros of the function U_n determine the dispersion relations of quasiparticles.

IV. PERTURBATIVE SOLUTION

In order to obtain the leading order perturbative solution to the gap equations, we can use the free propagator on the right-hand side of Eqs. (17) through (20), i.e.,

$$S(u,u) = \frac{i}{2\pi l^2}\sum_{n=0}^{\infty}\int\frac{d\omega\,dk^3}{(2\pi)^2} \times \frac{[(\omega+\mu_0)\gamma^0 - v_F k^3\gamma^3][\mathcal{P}_- + \mathcal{P}_+\theta(n-1)]}{(\omega+\mu_0)^2 - (v_F k^3)^2 - 2nv_F^2|eB|/c}. \quad (24)$$

Note that unlike the high Landau levels with $n \geqslant 1$, where both spin projectors $\mathcal{P}_+$ and $\mathcal{P}_-$ contribute, the lowest Landau level (LLL) with $n = 0$ contains only one projector $\mathcal{P}_-$. The reason for this is well known. The Atiyah-Singer theorem connects the number of the zero energy modes (which are completely pseudospin polarized) of the two-dimensional part of the Dirac operator to the total flux of the magnetic field through the corresponding plane. This theorem states that LLL is topologically protected (for a discussion of the Atiyah-Singer theorem in the context of condensed matter physics, see Ref. 28).

By making use of Eq. (24), we straightforwardly calculate the zeroth order result for the charge density,

$$\langle j^0\rangle_0 = \frac{\mu_0}{2v_F(\pi l)^2} + \frac{\mathrm{sgn}(\mu_0)}{v_F(\pi l)^2}\sum_{n=1}^{\infty}\sqrt{\mu_0^2 - 2nv_F^2|eB|/c} \times \theta\big(\mu_0^2 - 2nv_F^2|eB|/c\big), \quad (25)$$

and the axial current density

$$\langle \mathbf{j}_5\rangle_0 = -\frac{e\mathbf{B}\mu_0}{2\pi^2 v_F c}. \quad (26)$$

As to the chiral condensate and the anomalous magnetic moment condensate, they vanish, i.e., $\langle\bar\psi\psi\rangle_0 = 0$ and $\langle\boldsymbol{\Sigma}\rangle_0 = 0$. This is not surprising because both of them break the chiral $U(1)_+ \times U(1)_-$ symmetry of the Dirac semimetal Hamiltonian (9). Then, taking into account Eqs. (15) and (16),

we conclude that both the Dirac mass m and the parameter $\tilde{\boldsymbol{\mu}}$ are zero in the perturbation theory.

After taking into account the gap equations (13) and (14), the results in Eqs. (25) and (26) imply that there is a perturbative renormalization of the chemical potential and a dynamical generation of the chiral shift. Of special interest is the result for the axial current density given by Eq. (26). This is generated already in the free theory and known in the literature as the topological contribution.[26,29] Its topological origin is related to the following fact: in the free theory, only the LLL contributes to both the axial current and the axial anomaly. By combining Eqs. (14) and (26), we find

$$\mathbf{b} = -\frac{ge\mathbf{B}\mu_0}{8\pi^2 v_F^2 c}. \tag{27}$$

This is our principal result, which reflects the simple fact that at $\mu_0 \neq 0$ (i.e., nonzero charge density), the absence of the chiral shift is not protected by any additional symmetries in the normal phase of a Dirac metal in a magnetic field. Indeed, in the presence of a homogeneous magnetic field pointing in the z direction, the rotational $SO(3)$ symmetry in the model is explicitly broken down to the $SO(2)$ symmetry of rotations around the z axis. The dynamically generated chiral shift parameter $\mathbf{b}$ also points in the same direction and does not break the leftover $SO(2)$ symmetry. The same is true for the discrete symmetries: while the parity $\mathcal{P}$ is preserved, all other discrete symmetries, charge conjugation $\mathcal{C}$, time reversal $\mathcal{T}$, $\mathcal{CP}$, $\mathcal{CT}$, $\mathcal{PT}$, and $\mathcal{CPT}$ are broken. Last but not least, the chiral shift does not break the chiral $U(1)_+ \times U(1)_-$ symmetry considered in Sec. II. This implies that the dynamical chiral shift is necessarily generated in the normal phase of a Dirac semimetal in a magnetic field, and the latter is transformed into a Weyl semimetal.

In the phase with a dynamically generated chiral shift $\mathbf{b}$, the quasiparticle dispersion relations, i.e., $\omega_{n,\sigma} = -\mu + E_{n,\sigma}$, are determined by the following Landau-level energies (recall that we assume that the magnetic field points in the z direction):

$$E_{0,\sigma} = v_F(s_\perp b + \sigma k^3), \quad n = 0, \tag{28}$$

$$E_{n,\sigma} = \pm v_F\sqrt{(s_\perp b + \sigma k^3)^2 + 2n|eB|/c}, \quad n \geqslant 1, \tag{29}$$

where $\sigma = \pm$ corresponds to different Weyl nodes. The corresponding dispersion relations are shown graphically in Fig. 1. Note a qualitatively different character (compared to higher Landau levels) of the dispersion relations in the LLL given by the straight lines whose signs of slope correlate with the Weyl nodes. Of course, this correlation is due to the complete polarization of quasiparticle pseudospins in the LLL. As seen from Fig. 1, the effect of the chiral shift is not only to shift the relative position of the Weyl nodes in momentum space, but also to induce a chiral asymmetry of the Fermi surface.[6,7]

V. NONPERTURBATIVE SOLUTION: PHASE TRANSITION

The magnetic catalysis phenomenon, which takes place for planar as well as three-dimensional relativistic charged fermions because of the dimensional reduction,[30,31] implies that at vanishing μ_0, the ground state in the model at hand is characterized by a nonzero Dirac mass m that spontaneously breaks the chiral symmetry. (For the corresponding studies in graphene, see Ref. 32.)

Such a vacuum state can withstand a finite stress due to a nonzero chemical potential. However, as we discuss below, when μ_0 exceeds a certain critical value $\mu_{\rm cr}$, the chiral symmetry restoration and a new ground state are expected. The new state is characterized by a nonvanishing chiral shift parameter $\mathbf{b}$ and a nonzero axial current in the direction of the magnetic field. Since no symmetry of the theory is broken, this state is the *normal* phase of the magnetized matter that happens to have a rather rich chiral structure. This phase was considered in the previous section.

Let us describe this transition in more detail. The value of the dynamical Dirac mass m in the vacuum state can be easily calculated following the same approach as in Ref. 31. At weak coupling, in particular, we can use the following expression for the chiral condensate:

$$\langle\bar{\psi}\psi\rangle \simeq -\frac{m}{4\pi^2 v_F}\left(\Lambda^2 + \frac{1}{l^2}\ln\frac{v_F^2}{\pi m^2 l^2}\right), \tag{30}$$

obtained in the limit of a small mass (which is consistent with the weak coupling approximation), using the gauge invariant

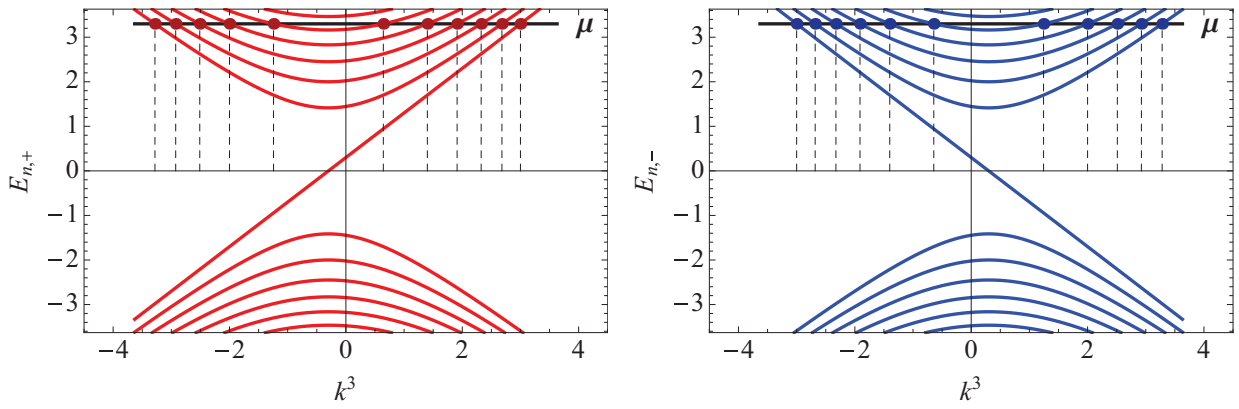

FIG. 1. (Color online) Dispersion relations of the quasiparticles from different Weyl nodes and their Fermi surfaces. The two dispersion relations are the mirror images of each other.

proper-time regularization. Here the ultraviolet momentum cutoff Λ can be related, for example, to the value of lattice spacing a as follows: $\Lambda \simeq \pi/a$. Finally, by taking into account gap equation (15), we arrive at the solution for the dynamical mass,

$$m \simeq \frac{v_F}{\sqrt{\pi} l} \exp\left(-\frac{8\pi^2 v_F l^2}{g} + \frac{(\Lambda l)^2}{2}\right). \tag{31}$$

This zero-temperature, nonperturbative solution exists for $\mu_0 < m$.

The free energies of the two types of states, i.e., the nonperturbative state with a dynamically generated Dirac mass (and no chiral shift) and the perturbative state with a nonzero chiral shift (and no Dirac mass) become equal at about $\mu_0 \simeq m/\sqrt{2}$. This is analogous to the Clogston relation in superconductivity.[33]

At the critical value $\mu_{cr} \simeq m/\sqrt{2}$, a first order phase transition takes place. Indeed, both these solutions coexist at $\mu_0 < m$, and while for $\mu_0 < \mu_{cr}$ the nonperturbative (gapped) phase with a chiral condensate is more stable, the normal (gapless) phase becomes more stable at $\mu_0 > \mu_{cr}$. Note that at $\mu_0 < m$ the chemical potential is irrelevant in the gapped phase: the charge density is absent there. On the other hand, at any nonzero chemical potential, there is a nonzero charge density in the normal (gapless) phase. Therefore, at $\mu_{cr} \simeq m/\sqrt{2}$, there is a phase transition with a jump in the charge density, which is a clear manifestation of a first order phase transition.

VI. DIRAC SEMIMETALS VS GRAPHENE IN A MAGNETIC FIELD

It is instructive to compare the states in the magnetized Dirac semimetals and graphene. First of all, we would like to point out that the chiral shift is a three-dimensional analog of the Haldane mass,[34,35] which plays an important role in the dynamics of the quantum Hall effect in graphene. Indeed, in the formalism of the four-component Dirac fields in graphene, the Haldane mass condensate is described by the same vacuum expectation value as that of the axial current in three dimensions:[36]

$$\langle \bar{\psi}\gamma^3\gamma^5\psi\rangle = -\,\mathrm{tr}[\gamma^3\gamma^5 G(u,u)] \tag{32}$$

for a magnetic field pointing in the z direction, which is orthogonal to the graphene plane; cf. Eq. (18). Moreover, similar to the solution with the chiral shift, the solution with the Haldane mass (with the same sign for both spin-up and spin-down quasiparticles) describes the normal phase: it is a singlet with respect to the $SU(4)$ symmetry, which is a graphene analog of the chiral group in Dirac and Weyl semimetals.

Also, in graphene, there is a phase transition similar to that described in the previous section. It happens when the LLL is completely filled.[36] In other words, the quantum Hall state with the filling factor $\nu = 2$ in graphene is associated with the normal phase containing the Haldane mass.

As is well known, the Haldane mass leads to the Chern-Simons term in an external electromagnetic field.[34,35] This feature reflects a topological nature of the state with the filling factor $\nu = 2$ in a graphene. As was recently shown in Ref. 37, the chiral shift term

$$\bar{\psi}(\mathbf{b}\cdot\boldsymbol{\gamma})\gamma^5\psi \tag{33}$$

leads to an induced Chern-Simons term of the form $\frac{1}{2}b_\mu\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}A_\nu$ in Weyl semimetals [here b_μ is a four-dimensional vector $(0,\mathbf{b})$]. Therefore, it should also be generated in Dirac semimetals in a magnetic field. Note, however, the following principle difference between them: while in Weyl semimetals the chiral shift $\mathbf{b}$ is present in the free Hamiltonian, it is dynamically generated in the normal phase of Dirac semimetals in a magnetic field [see Eq. (27)]. Like in graphene, the generation of the Chern-Simons term implies a topological nature of the normal state in this material.[38]

VII. DISCUSSION

In the present paper, we considered manifestations of a relativistic phenomenon, the rearrangement of the Fermi surface in three-dimensional matter in a magnetic field, in a Dirac semimetal. It was shown that its normal phase at a nonzero charge density and in a magnetic field has a nontrivial chiral structure: it is a Weyl semimetal with a pair of Weyl nodes for each of the original Dirac points. The nodes are separated by a dynamically induced chiral shift that is directed along the magnetic field. The phase transition between the normal phase and the phase with chiral symmetry breaking is revealed, and the rearrangement of the Fermi surface accompanying this phase transition is described.[39]

Although we studied a simple model with a contact four-fermion interaction, we believe that the present qualitative results apply equally well to more realistic models. The studies in the relativistic Nambu-Jona-Lasinio (NJL) model (with a contact interaction) on the one hand[6,7] and in QED on the other[40] strongly support the validity of this statement. Namely, the dynamical generation of the chiral shift in a magnetic field and at a nonzero fermion density is a universal phenomenon.

In the present study we analyzed the simplest model: it is isotropic (in the absence of the magnetic field), has no gap (i.e., no bare Dirac mass), and no Zeeman term for spin. In real materials, such as bismuth,[8,9] the presence of an anisotropy, a gap, and the Zeeman term for spin should be taken into account. This task, as well as the generalization of the study to the case of nonzero temperature, is outside the scope of the present paper and will be considered elsewhere. Here we just want to mention that the effects of a bare Dirac mass term and temperature were studied in the NJL model in Ref. 7. It was shown there that the chirality remains a good approximate quantum number even for massive fermions in the vicinity of the Fermi surface, provided the mass is sufficiently small, i.e., $m \ll \mu$. As for the temperature effects,[7] an interesting feature of the chiral shift is that it is insensitive to the temperature when $T \ll \mu$, and *increases* with temperature $T \gtrsim \mu$.

A natural extension of this work would be the study of the phase diagram of a Weyl semimetal in a magnetic field and

a nonzero charge density. Because of a built-in chiral shift in the Hamiltonian of such a material, one may expect that its phase with chiral symmetry breaking, realized at zero (or low) density, should be inhomogeneous, possibly, a Larkin-Ovchinnikov-Fulde-Ferrel (LOFF) –like one (cf. Refs. 41–43). We expect that a first order phase transition between the normal phase and the phase with chiral symmetry breaking will take place in that case too.

Note added in proof. Recently, an experimental observation of the transition from a Dirac semimetal to a Weyl semimetal in a magnetic field was reported.[44]

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