Fokker-Planck approach to the theory of the magnon-driven spin Seebeck effect

L. Chotorlishvili,[1] Z. Toklikishvili,[2] V. K. Dugaev,[1,3,4] J. Barnaś,[5,6] S. Trimper,[1] and J. Berakdar[1]
[1]*Institut für Physik, Martin-Luther-Universität Halle-Wittenberg, Heinrich-Damerow-Straße 4, 06120 Halle, Germany*
[2]*Department of Physics, Tbilisi State University, Chavchavadze Avenue 3, 0128 Tbilisi, Georgia*
[3]*Department of Physics, Rzeszów University of Technology, al. Powstanców Warszawy 6, 35-959 Rzeszów, Poland*
[4]*Department of Physics and CFIF, Instituto Superior Técnico, TU Lisbon, Avenida Rovisco Pais, 1049-001 Lisbon, Portugal*
[5]*Faculty of Physics, Adam Mickiewicz University, ul. Umultowska 85, 61-614 Poznań, Poland*
[6]*Institute of Molecular Physics, Polish Academy of Sciences, ul. Smoluchowskiego 17, 60-179 Poznań, Poland*

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Following the theoretical approach by J. Xiao *et al.* [Phys. Rev. B **81**, 214418 (2010)] to the spin Seebeck effect, we calculate the mean value of the total spin current flowing through a normal metal/ferromagnet interface. The spin current emitted from the ferromagnet to the normal metal is evaluated in the framework of the Fokker-Planck approach for the stochastic Landau-Lifshitz-Gilbert equation. We show that the total spin current depends not only on the temperature difference between the electron and the magnon baths, but also on the external magnetic field and magnetic anisotropy. Apart from this, the spin current is shown to saturate with increasing magnon temperature, and the saturation temperature increases with increasing magnetic field and/or magnetic anisotropy.

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I. INTRODUCTION

If two ends of a conductor are held at different temperatures, electrons from the hot end diffuse towards the cold one.[1] This phenomenon, discovered by Seebeck, is the basis for various thermoelectric charge-transport effects and plays a key role in the development of energy-saving technologies. With the emergence of spintronics as a new area of mesoscopic physics, whose main objective is to utilize the electron spin in device operations, spin-related thermoelectricity has become of high interest.[2] Even though the generation of electromotive force by a temperature gradient has been known for a long time, the spin analog of the Seebeck effect, known as the spin Seebeck effect (SSE), was discovered only very recently.[3] In the latter experiment, a temperature gradient along a ferromagnetic slab generated a pure spin current over a long distance, much longer than in typical injection experiments, where spin current (and also spin voltage) disappears over distances longer than the spin-diffusion length.[3] The SSE was observed not only in metallic ferromagnets (such as Co_2MnSi)[4] or semiconducting ferromagnets (e.g., GaMnAs),[5] but also in magnetic insulators $LaY_2Fe_5O_{12}$[6] and (Mn,Zn)Fe_2O_4.[7] Explanation of the effect observed experimentally in insulating magnets cannot rely on conduction electrons and requires a more general approach.

The Seebeck effect is usually quantified by the Seebeck coefficient S which is defined as the ratio of the generated electric voltage ΔV to the temperature difference ΔT, $\Delta V = -S\Delta T$. The magnitude of the Seebeck coefficient S depends on the scattering rate and the density of electron states at the Fermi level, and thus it is different in different materials. In the case of SSE, the spin voltage is formally determined by $\mu_\uparrow - \mu_\downarrow$, where $\mu_{\uparrow(\downarrow)}$ are the electrochemical potentials for spin-up and spin-down electrons, respectively. Usually, the density of states and the scattering rate for spin-up and spin-down electrons are different, which results in different Seebeck constants for the two spin channels. Therefore, in a metallic magnet subjected to a temperature gradient, the electrons in different spin channels generate different driving forces, leading to a spin voltage that drives a nonzero spin current.

In this paper we study the spin current flowing through the normal metal/ferromagnet interface due to the thermal bias applied to the system. We consider the system and the model studied recently by Xiao *et al.*[8] However, we use a different approach and also consider in detail the influence of an external magnetic field and of the magnetic anisotropy. As in Ref. 8, we assume that the electron-phonon interactions in both normal-metal and ferromagnetic subsystems are predominant, as compared to the interface effects. Therefore, the phonon and electron reservoirs in both normal metal and ferromagnet thermalize internally before the thermal equilibrium between the ferromagnet and normal metal appears. In terms of the local temperature, which is based on the hierarchy of relaxation times, this means that the temperatures of the phonon ($T^p_{N(F)}$) and the electron ($T^e_{N(F)}$) baths are equal in both the normal metal (N) and the ferromagnet (F), $T^p_N = T^e_N = T_N$, $T^p_F = T^e_F = T_F$. However, there is a difference in temperatures of the subsystems, $T_N \neq T_F$, which is externally controlled. This difference drives the SSE. The interaction between the normal-metal and the ferromagnet subsystems is mediated via the magnon bath. The temperature of the magnon bath deviates from the temperature of the electron and the phonon baths, $T^m_F \neq T_N$ and $T^m_F \neq T_F$.

As shown in Ref. 8, two different spin currents contribute to the total spin current flowing through the normal metal/ferromagnet interface. One of them is the spin current emitted from the ferromagnet to the normal metal due to the thermally activated magnetization dynamics in the ferromagnet. This spin current is referred to as the spin-pump current, $\vec{I}_{sp}$. The second contribution to the total spin current has the opposite nature and flows in the opposite direction—from normal metal to ferromagnet. This contribution follows from the thermal noise in the normal metal and will be referred to as the spin-torque current, $\vec{I}_{fl}$. In order to evaluate the spin current flowing through the interface between normal metal and ferromagnet, Xiao *et al.*[8] used the linearized Landau-Lifshitz-Gilbert (LLG) equation and found that the spin current is proportional to the difference in the temperatures of the magnon system T_F^m and electron subsystem in the normal metal T_N^e. Here we address this problem using a different method, which is based on the Fokker-Planck equation for the stochastic LLG equation. We also distinguish between the influence of magnetic anisotropy and external magnetic field on the spin Seebeck effect. As in the linearized approach the role of magnetic anisotropy is similar to that of an external magnetic field and can be described by some effective field, this is not the case when fluctuations are large. We demonstrate that the spin current obtained within the framework of the linear response theory is a particular case of the result obtained using the Fokker-Planck approach, and corresponds to the low-temperature approximation for the magnon temperature. Apart from this, we show that the spin current saturates with increasing magnon temperature, and the saturation temperature increases with increasing magnetic field and/or magnetic anisotropy.

The paper is organized as follows. In Sec. II we describe the model. The Fokker-Planck equation is solved in Sec. III, where two cases are distinguished: (i) the case with dominant external field, and (ii) the case where the uniaxial magnetic anisotropy field is dominant. Our summary and final conclusions are in Sec. IV.

II. GENERAL BACKGROUND

We consider a ferromagnetic metallic layer which is in direct contact with a nonmagnetic metallic layer. Magnetization dynamics of the ferromagnet will be described by the LLG equation in the macrospin approximation.[9–11] Following Xiao *et al.*,[8] we assume that strong electron-phonon interaction assures local thermal equilibrium between electrons and phonons in both ferromagnetic and normal-metal layers, $T_F^p = T_F^e = T_F$, $T_N^p = T_N^e = T_N$. However, the magnon temperature in the ferromagnetic layer is different from the corresponding temperature of electrons, $T_F^m \neq T_F$.[8]

At finite temperatures, the thermally activated magnetization dynamics in the ferromagnet gives rise to a spin current emitted from the ferromagnet to the normal metal. This effect is known as the spin pumping.[12–14] The corresponding expression for the spin current density reads[8]

$$\vec{I}_{sp} = \frac{\hbar}{4\pi}[g_r \vec{m}(t) \times \dot{\vec{m}}(t) + g_i \dot{\vec{m}}(t)], \tag{1}$$

where g_r and g_i are the real and imaginary parts of the dimensionless spin mixing conductance of the ferromagnet/normal metal $(F|N)$ interface, while $\vec{m}(t)$ is a dimensionless unit vector along the magnetization direction.

In turn, the thermal noise in the normal-metal layer leads to the spin current flowing from the normal metal to the ferromagnet,[12]

$$\vec{I}_{fl}(t) = -\frac{M_s V}{\gamma}\gamma \vec{m}(t) \times \vec{h}'(t), \tag{2}$$

where M_s is the saturation magnetization, V is the total volume of the ferromagnet, and γ is the gyromagnetic factor. Apart from this, $\vec{h}'(t)$ is the random magnetic field with the following correlation function in the high-temperature limit, $k_B T \gg \hbar\omega_0$,

$$\langle \gamma h_i'(t)\gamma h_j'(t')\rangle = \sigma'^2 \delta_{ij}\delta(t-t') \tag{3}$$

for $i,j = x,y,z$. Here, ω_0 is the ferromagnetic resonance frequency, $\sigma'^2 = 2\alpha'\gamma k_B T_N / M_s V$, and $\alpha' = \gamma\hbar g_r / 4\pi M_s V$ is the magnetization damping constant related to the spin pumping. Using Eqs. (1) to (3), the total average spin current flowing across the interface can be written in the form

$$\langle \vec{I}_s \rangle = \frac{M_s V}{\gamma}[\alpha' \langle \vec{m} \times \dot{\vec{m}}\rangle + \gamma \langle \vec{m} \times \vec{h}'\rangle], \tag{4}$$

while the magnetization dynamics is described by the stochastic LLG equation,

$$\dot{\vec{m}} = -\gamma \vec{m} \times (H_{\rm eff}\hat{z} + \vec{h}) + \alpha \vec{m} \times \dot{\vec{m}}, \tag{5}$$

where $H_{\rm eff}$ is the effective magnetic field which consists of the external constant magnetic field H_0 oriented along the z axis and magnetic anisotropy field, $H_A m_z$, with $H_A = 2K_1/M_s$ and K_1 being the anisotropy constant. For $K_1 > 0$ the magnetic anisotropy is of easy-axis type, while for $K_1 < 0$ it is of easy-plane type. Apart from this, in the above equation $\hat{z}$ is the unit vector along the z axis, $\vec{h}$ is the total random field, while α is the total magnetic damping constant.[8] This constant includes the contributions from the bulk damping constant α_0 associated with the lattice random field h_0 and from the damping constant α' associated with the contact to the normal metal [random field $\vec{h}'(t)$].

We assume that the random contributions from the unrelated noise sources are independent and therefore the correlation function for the total random magnetic field can be factorized in the following form:

$$\langle \gamma h_i(t)\gamma h_j(t')\rangle = \sigma^2 \delta_{ij}\delta(t-t'), \tag{6}$$

where $\sigma^2 = 2\alpha\gamma k_B T_F^m / M_s V$, and $\alpha T_F^m = \alpha_0 T_F + \alpha' T_N$.

III. FOKKER-PLANCK EQUATION AND THE SPIN CURRENT

In Ref. 8, the stochastic LLG equation was linearized near the relevant equilibrium. Here, to evaluate the mean current $\langle \vec{I}_{sp}\rangle$ for the stochastic LLG, we derive the Fokker-Planck equation for the distribution function $f(\vec{m},t)$. The derivation procedure follows Ref. 15 and is outlined in the Appendix. As a result, one finds

$$\frac{\partial f}{\partial t} = \frac{1}{1+\alpha^2}\frac{\partial}{\partial \vec{m}}\left\{(\vec{m}\times\vec{\omega}_{\rm eff}) f + \alpha \vec{m}\times(\vec{m}\times\vec{\omega}_{\rm eff}) f - \frac{\sigma^2}{2(1+\alpha^2)}\vec{m}\times\left(\vec{m}\times\frac{\partial f}{\partial \vec{m}}\right)\right\}, \tag{7}$$

where $\vec{\omega}_{\rm eff} = \gamma H_{\rm eff}\hat{z} = (0,0,\omega_{\rm eff})$, with $\omega_{\rm eff} = \gamma H_{\rm eff}$.

The stationary solution of Eq. (7) for the distribution function has the form

$$f(\vec{m}) = Z^{-1}\exp\left(\beta \int \vec{\omega}_{\rm eff}\cdot d\vec{m}\right), \quad Z = \int \exp\left(\beta\int \vec{\omega}_{\rm eff}\cdot d\vec{m}\right) d^3\vec{m}, \tag{8}$$

where we introduced the following notation: $\beta = 2\alpha(1+\alpha^2)/\sigma^2 \approx 2\alpha/\sigma^2 = M_s V/\gamma k_B T_F^m$. The limits of weak and strong magnetic anisotropy are of particular interest. Therefore, in the following we will consider both situations separately, and start with the case of a weak anisotropy field.

A. Weak magnetic anisotropy

In the case of weak magnetic anisotropy, when the external magnetic field is dominant, $\omega_0 = \gamma H_0 \gg \omega_p = \gamma H_A$, the effective field in Eq. (8) is $\vec{\omega}_{\rm eff} = (0,0,\gamma H_0)$. Using Eqs. (1) and (8) we find the mean values of the magnetization components:

$$\begin{aligned} \langle m_x m_y\rangle &= \langle m_x m_z\rangle = \langle m_y m_z\rangle = 0,\\ \langle m_{x,y}\rangle &= 0, \quad \langle m_z\rangle = L(\beta\omega_0),\\ \langle m_{x,y}^2\rangle &= \frac{1}{\beta\omega_0}L(\beta\omega_0),\\ \langle m_z^2\rangle &= 1 - \frac{2}{\beta\omega_0}L(\beta\omega_0), \end{aligned} \tag{9}$$

where $L(x) = \coth x - \frac{1}{x}$ is the Langevin function, which has the asymptotics $L(x) \approx x/3, x \ll 1$. Then, taking into account Eq. (9), one finds the mean value of the spin current,

$$\langle I_{sz}\rangle = \frac{M_s V}{\gamma}\{\alpha'\omega_0(1 - \langle m_z^2\rangle) - \gamma\langle(m_x h_y' - m_y h_x')\rangle\}. \tag{10}$$

The last term in Eq. (10) can be evaluated using the linear-response theory, i.e., by linearizing the LLG equation in the vicinity of the equilibrium point, $\langle m_z\rangle = L(\beta\omega_0)$. After straightforward but laborious calculations one obtains

$$\langle m_x h_y' - m_y h_x'\rangle = \frac{\sigma'^2}{\gamma}\langle m_z\rangle. \tag{11}$$

Combining Eqs. (10) and (11) one can write the final result for the spin current density in the form

$$\langle I_{sz}\rangle = 2\alpha' k_B L\left(\frac{M_s V H_0}{k_B T_F^m}\right)(T_F^m - T_N). \tag{12}$$

We see that the average spin current depends on two physical parameters: (i) the ratio, $M_s V H_0/k_B T_F^m$ of the magnetic energy in the external field to the thermal energy corresponding to the magnon temperature, and (ii) the difference between the magnon temperature and the temperature of the electron-phonon bath in the normal metal, $(T_F^m - T_N)$. The dependence of the average spin current on the field is factorized in the Langevin function, Eq. (12). Therefore, introducing the notation $\langle I_{sz}\rangle_0$ for the mean spin current calculated in the linear response approach, $\langle I_{sz}\rangle_0 = 2\alpha' k_B(T_F^m - T_N)$,[8] one can rewrite Eq. (12) in the compact form as $\langle I_{sz}\rangle = L(\frac{M_s V H_0}{k_B T_F^m})\langle I_{sz}\rangle_0$.

In the limit of a low magnon temperature, $H_0/T_F^m > \frac{k_B}{M_s V}$, we have $\langle I_{sz}\rangle \approx \langle I_{sz}\rangle_0$. This means that the spin current calculated using the Fokker-Planck approach, without a linearization of the system, gives the same result as that obtained in the linear response. The physical reason for this is clear. Indeed, in the case of a strong magnetic field, the magnetization vector tends to be aligned along the field direction, and nonlinear effects in the magnetization dynamics related to large deviation from the equilibrium are less relevant. However, in the opposite case, corresponding to the high magnon temperature, $H_0/T_F^m < \frac{k_B}{M_s V}$, and strong magnetization fluctuations, the nonlinear effects in the magnetization dynamics are much more important. Consequently, the spin current is different from $\langle I_{sz}\rangle_0$ and reads $\langle I_{sz}\rangle = \frac{2}{3}\alpha' M_s H_0(1 - T_N/T_F^m)$. We see that the maximum value of the spin current corresponds to the hot magnon bath and saturates at $\langle I_{sz}\rangle = \frac{2}{3}\alpha' M_s H_0$. In turn, the spin current from linear response theory increases linearly with the magnon temperature.

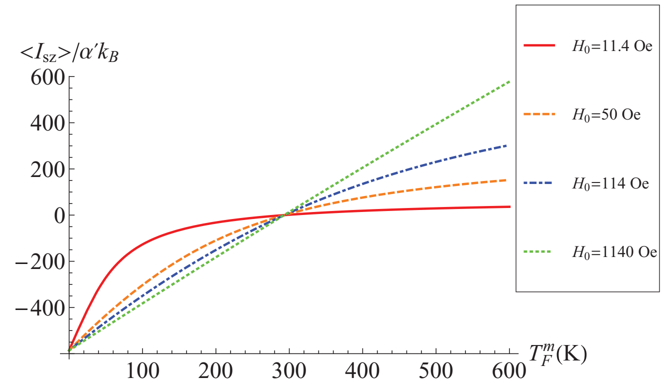

FIG. 1. (Color online) Dependence of the spin current $\langle I_{sz}\rangle$ on magnon temperature T_F^m for the following parameters (Ref. 8): $M_s = 800$ G, $V = 1.6 \times 10^{-18}$ cm^3, $T_N = 293$ K. The parameters correspond to Py = $Ni_{80}Fe_{20}$ alloy (Ref. 8). The spin current is measured in units of $\alpha' k_B$ and is shown for different amplitudes of the magnetic field. The dotted (green) line corresponds to the linear response theory.

The dependence of the spin current on the magnetic field and magnon temperature is plotted in Figs. 1 and 2. In particular, in Fig. 1 the dependence of the total spin current on the magnon temperature is plotted for different values of the external magnetic field. The dotted (green) line corresponds to the linear response theory, whereas all other curves correspond to the Fokker-Planck approach. We note that in contrast to the linear response theory, the Fokker-Planck approach leads to the spin current that saturates for high magnon temperatures. This, in turn, means that the linear response is valid only in a narrow range of the magnon temperatures. Note that all the curves cross the same point for $T_F^m = 0$ as there are then no thermal fluctuations in the magnon system, and thus the only contribution comes from thermal noise in the nonmagnetic film. Deviation from the description based on the linearized model is especially large for small magnetic fields and appears already at low magnon

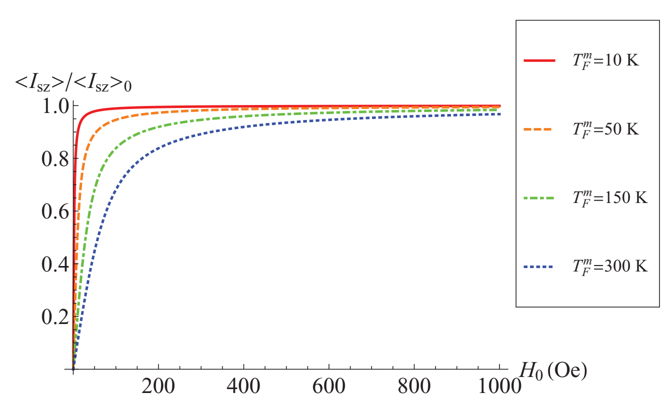

FIG. 2. (Color online) Dependence of the ratio $\langle I_{sz}\rangle/\langle I_{sz}\rangle_0$ on the magnetic field H_0 for the following parameters (Ref. 8): $M_s = 800$ G, $V = 1.6 \times 10^{-18}$ cm^3. The parameters correspond to the Py = $Ni_{80}Fe_{20}$ alloy (Ref. 8). Different curves correspond to different values of the magnon temperature.

temperatures (see the curve for $H_0 = 11.4$ Oe in Fig. 1). In this case, the saturation of the spin current also appears at low temperatures. For higher magnetic field, the deviation is smaller and appears at higher magnon temperatures. This behavior is reasonable as the fluctuations at low magnetic fields are larger, and therefore difference between linearized model and description based on the Fokker Planck equation appears at lower magnon temperatures. Obviously, the curves also cross at the point corresponding to $T_F^m = T_N$, where the spin current vanishes. This directly follows from Eq. (12). In the nonlinear regime $L(\frac{M_s V H_0}{k_B T_F^m} < 1) \approx \frac{M_s V H_0}{3k_B T_F^m}$ and for spin current we have $\langle I_{sz}\rangle = \frac{2\alpha' M_s V H_0}{3T_F^m}(T_F^m - T_N)$. Consequently the magnon temperature T_F^m for which we observe saturation of the spin current $\langle I_{sz}\rangle = \frac{2}{3}\alpha' M_s V H_0$ basically is defined by the following inequalities: $T_F^m > \frac{M_s V H_0}{k_B}, T_F^m > T_N$.

Figure 2 shows the dependence of the ratio $\langle I_{sz}\rangle/\langle I_0\rangle$ on the magnetic field for different values of the magnon temperatures T_F^m. This figure clearly shows that the saturation field increases with increasing the magnon temperature. Apart from this, it is interesting to note that $\langle I_{sz}\rangle/\langle I_0\rangle$ tends to zero in the limit of zero magnetic field. This behavior can be accounted for by noting that $\omega_0 \to 0$ for zero magnetic field, so the zero-field magnetic fluctuations are large. In this limit the description based on the linearized model gives finite spin current, while that based on the Fokker-Planck equation gives vanishing spin current.

B. Strong magnetic anisotropy

In the presence of a magnetic anisotropy, the situation is more complicated. Now, $\vec{\omega}_{\text{eff}} = \omega_{\text{eff}}(m_z)\hat{z}$, where $\omega_{\text{eff}}(m_z) = \gamma H_{\text{eff}} = \gamma(H_0 + H_A m_z)$. The stationary solution to Eq. (7) is

$$f = Z_a^{-1} \exp\left[\frac{2\alpha}{\sigma^2}\left(\omega_0 m_z + \frac{\omega_p m_z^2}{2}\right)\right], \qquad (13)$$

where $Z_a = \int \exp[\frac{2\alpha}{\sigma^2}(\omega_0 m_z + \frac{\omega_p m_z^2}{2})]d^3\vec{m}$ is a normalization factor, and $\omega_p = \gamma H_A$. Using Eq. (13) and calculating the spin current we find

$$\langle I_{sz}\rangle = \alpha' k_B\{T_F^m A(1 - \langle m_z^2\rangle) + 2T_F^m B(\langle m_z\rangle - \langle m_z^3\rangle) - 2T_N\langle m_z\rangle\}. \qquad (14)$$

Here, the mean values $\langle m_z\rangle$, $\langle m_z^2\rangle$, and $\langle m_z^3\rangle$ are given by

$$\langle m_z\rangle = \frac{e^A \sinh(A)}{\sqrt{B}G(A,B)} - \frac{A}{2B};$$
$$\langle m_z^2\rangle = -\frac{1}{2B} + \left(\frac{A}{2B}\right)^2 + \frac{e^A[2B\cosh(A) - A\sinh(A)]}{2B^{3/2}G(A,B)};$$
$$\langle m_z^3\rangle = \frac{3}{4}\frac{A}{B^2} - \left(\frac{A}{2B}\right)^3 + \frac{e^A[-2AB\cosh A + (A^2 - 4B - 4B^2)\sinh(A)]}{4B^{5/2}G(A,B)}. \qquad (15)$$

The following notation has been introduced in the above equations: $A = \frac{M_s V H_0}{k_B T_F^m}$, $B = \frac{K_1 V}{k_B T_F^m}$, and $G(A,B) = e^{2A}F(\frac{A+2B}{\sqrt{B}}) - F(\frac{A-2B}{\sqrt{B}})$, where $F(x) = \exp(-x^2)\int_0^x \exp(y^2)dy$ is the Dawson function.[16] The above formula, Eqs. (14) and (15), will be now used to calculate the influence of magnetic anisotropy on the asymptotic behavior of the average spin current.

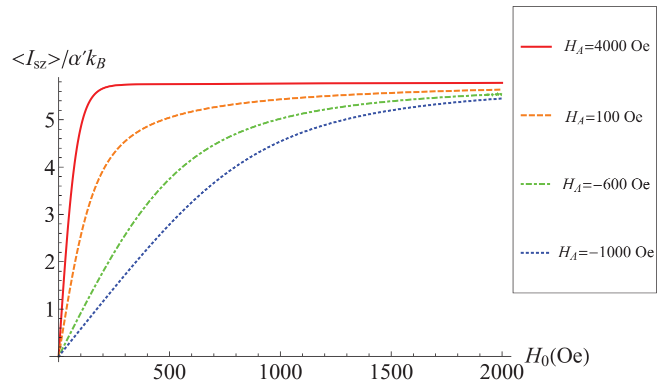

FIG. 3. (Color online) The dependence of spin current $\langle I_{sz}\rangle$ on the field H_0 for the following parameters (Ref. 8): $T_F^m = 293$ K, $T_N = 290.07$ K, $\Delta T = T_F^m - T_N = 2.93$ K, $4\pi M_s = 4000$ G, $V = 1.6 \times 10^{-18}$ cm^3. These parameters correspond to manganese spinel ferrite films ($MnFe_2O_4$; Ref. 17). The spin current is measured in units of $\alpha' k_B$. Different curves correspond to different values of H_A.

The effect of magnetic anisotropy is demonstrated in Fig. 3, where the spin current is shown as a function of the external field for the indicated values of H_A. It is evident that for the parameters assumed in Fig. 3 the magnetic anisotropy magnifies the spin current for $H_A > 0$ (easy-axis anisotropy). This behavior is qualitatively similar to that observed for external field H_0. The difference comes from the fact that the effective role of anisotropy depends on the magnetization—the anisotropy field changes sign when the m_z component is reversed. One may also say that the effect of magnetic anisotropy adds to the effect of magnetic field. In turn, the easy-plane anisotropy ($H_A < 0$) reduces the spin current, i.e., reduces the effect of external field. Generally, the spin current saturates with increasing H_0. However, in the presence of easy-axis (easy-plane) magnetic anisotropy, the saturation is reached at H_0 lower (larger) than in the absence of the magnetic anisotropy.

IV. SUMMARY AND CONCLUSIONS

We have studied the spin Seebeck effect in a system consisting of normal-metal and ferromagnetic films subjected to a temperature gradient. Using the Fokker-Planck equation we have derived analytical expressions for the averaged spin current flowing through the interface between the layers. This current consists of two parts: The first part is the spin current which occurs due to the thermally activated magnetization dynamics in the ferromagnetic layer, $\langle I_{sp}\rangle$. The second part, $\langle I_{fl}\rangle$, flows in the opposite direction and arises from thermal fluctuations in the normal metal.

We have considered two special cases—with and without magnetic anisotropy. The obtained analytical results describe the dependence of the mean spin current on the external magnetic field and the magnetic anisotropy field, and on the difference between the magnon temperature and the temperature of the electron-phonon bath in the normal metal.

The dependence of the spin current on the thermal gradient was already analyzed in Ref. 8, where the corresponding stochastic Landau-Lifshitz-Gilbert equation was linearized. However, the dependence on magnetic anisotropy was not considered there. We have shown that the magnetic field enhances the spin current, which should be observable experimentally. In the absence of the anisotropy, the dependence of spin current on the magnetic field is factorized by the Langevin function, Eq. (12). In the limit of a low magnon temperature, $H_0/T_F^m > \frac{k_B}{M_s V}$, the spin current calculated using the approach based on the Fokker-Planck equation (without linearization of the Landau-Lifshitz-Gilbert equation) gives the same result as that obtained in the framework of linear response theory. From the physical point of view this result is rather clear. In the case of a strong magnetic field, the magnetization vector tends to be aligned along the field and the nonlinear effects in the magnetization dynamics concerning large deflection from the equilibrium position are less relevant. However, in the opposite case corresponding to the high magnon temperature $H_0/T_F^m < \frac{k_B}{M_s V}$ and larger fluctuations of the magnetization vector, nonlinear effects in the magnetization dynamics are more important. Consequently, the behavior of the spin current is different from that found in the linear response description.

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APPENDIX: DERIVATION OF THE FOKKER-PLANCK EQUATION

For the derivation of the Fokker-Planck equation we follow Ref. 15 and use the functional integration method in order to average the dynamics over all possible realizations of the random noise field. First, we rewrite Eq. (5) in the following form:

$$\dot{\vec{m}} = -\frac{1}{1+\alpha^2}\vec{m}\times[\vec{\omega}_{\text{eff}} + \vec{\zeta}(t)] - \frac{\alpha}{1+\alpha^2}\vec{m}\times(\vec{m}\times\vec{\omega}_{\text{eff}}), \tag{A1}$$

where $\vec{\omega}_{\text{eff}} = \gamma H_{\text{eff}}\hat{z} = (0,0,\omega_{\text{eff}})$, and $\vec{\zeta}(t) = \gamma\vec{h}(t)$ is a random Langevin field with the following correlation relations:

$$\langle\vec{\zeta}(t)\rangle = 0,$$
$$\langle\zeta_i(t);\zeta_j(t')\rangle = \frac{2\alpha\gamma k_B T_m^F}{M_s V}\delta_{ij}\delta(t-t') \equiv \sigma^2\delta_{ij}\delta(t-t'). \tag{A2}$$

We introduce the probability distribution for the random Gaussian noise $\vec{\zeta}$:

$$F[\vec{\zeta}(t)] = \frac{1}{Z_\zeta}\exp\left[-\frac{1}{\sigma^2}\int_{-\infty}^{+\infty} d\tau\,\zeta^2(\tau)\right], \tag{A3}$$

where $Z_\zeta = \int D\vec{\zeta}\,F$ is the noise partition function; $D\vec{\zeta}$ denotes the functional integration over all realizations of $\vec{\zeta}(\tau)$. For all n we have

$$\int D\vec{\zeta}\,\frac{\delta^n F[\vec{\zeta}]}{\delta\zeta_{\alpha_1}\delta\zeta_{\alpha_2}\ldots\delta\zeta_{\alpha_n}} = 0. \tag{A4}$$

Using Eq. (A3), the average of any noise the functional $A[\vec{\zeta}(t)]$ can be written as

$$\langle A[\vec{\zeta}]\rangle_\zeta = \int D\vec{\zeta}\,A[\vec{\zeta}]F[\vec{\zeta}]. \tag{A5}$$

Using the identity $\frac{\delta\zeta_\alpha(\tau)}{\delta\zeta_\beta(t)} = \delta_{\alpha\beta}\delta(\tau - t)$ and (A4) and (A5) for $n = 1,2$, it is easy to obtain the correlation relations (A2).[15] The Fokker-Planck equation corresponding to Eq. (A1) can be written for the distribution function as

$$f(\vec{M},t) \equiv \langle\vec{\pi}(t,[\vec{\zeta}])\rangle_\zeta, \quad \vec{\pi}(t,[\vec{\zeta}]) \equiv \delta(\vec{M} - \vec{m}(t)) \tag{A6}$$

on the sphere $|\vec{M}| = 1$. Taking into account the relation[15] $\dot{\vec{\pi}} = -\frac{\partial\vec{\pi}}{\partial\vec{M}}\dot{\vec{m}}$ and the equation of motion (A1) we deduce the following Fokker-Planck equation:

$$\frac{\partial f}{\partial t} = \frac{1}{1+\alpha^2}\frac{\partial}{\partial\vec{m}}\{(\vec{m}\times\vec{\omega}_{\text{eff}})f + [\vec{m}\times(\vec{m}\times\vec{\omega}_{\text{eff}})]f + \vec{m}\times\langle\vec{\zeta}(t)\vec{\pi}(t,[\vec{\zeta}])\rangle\}. \tag{A7}$$

To calculate $\langle\vec{\zeta}(t)\vec{\pi}(t,[\vec{\zeta}])\rangle$ we use the standard procedure,[15] which yields

$$\langle\vec{\zeta}(t)\vec{\pi}(t,[\vec{\zeta}])\rangle = -\frac{\sigma^2}{2(1+\alpha^2)}\vec{m}\times\frac{\partial f}{\partial\vec{m}}. \tag{A8}$$

Inserting Eq. (A8) into Eq. (A7) we find the Fokker-Planck equation, Eq. (7).

[1] N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Saunders College, Philadelphia, 1976), pp. 253–258.
[2] M. Hatami, G. E. W. Bauer, Q. Zhang, and P. J. Kelly, Phys. Rev. B **79**, 174426 (2009).
[3] K. Uchida, S. Takahashi, K. Harii, J. Ieda, W. Koshibae, K. Ando, S. Maekawa, and E. Saitoh, Nature (London) **455**, 778 (2008).
[4] S. Bosu, Y. Sakuraba, K. Uchida, K. Saito, T. Ota, E. Saitoh, and K. Takanashi, Phys. Rev. B **83**, 224401 (2011).
[5] C. M. Jaworski, J. Yang, S. Mack, D. D. Awschalom, J. P. Heremans, and R. C. Myers, Nat. Mater. **9**, 898 (2010).
[6] K. Uchida, J. Xiao, H. Adachi, J. Ohe, S. Takahashi, J. Ieda, T. Ota, Y. Kajiwara, H. Umezawa, H. Kawai, G. Bauer, S. Maekawa, and E. Saitoh, Nat. Mater. **9**, 894 (2010).
[7] K. Uchida, T. Nonaka, T. Ota, H. Nakayama, and E. Saitoh, Appl. Phys. Lett. **97**, 262504 (2010); D. Qu, S. Y. Huang, Jun Hu, Ruqian Wu, and C. L. Chien, Phys. Rev. Lett. **110**, 067206 (2013).

[8] J. Xiao, G. E. W. Bauer, K. C. Uchida, E. Saitoh, and S. Maekawa, Phys. Rev. B **81**, 214418 (2010).
[9] K. D. Usadel, Phys. Rev. B **73**, 212405 (2006); J. L. Garcia-Palacios and D. A. Garanin, *ibid.* **70**, 064415 (2004); S. I. Denisov, T. V. Lyutyy, and P. Hänggi, Phys. Rev. Lett. **97**, 227202 (2006).
[10] R. F. L. Evans, D. Hinzke, U. Atxitia, U. Nowak, R. W. Chantrell, and O. Chubykalo-Fesenko, Phys. Rev. B **85**, 014433 (2012).
[11] Y. P. Kalmykov, W. T. Coffey, U. Atxitia, O. Chubykalo-Fesenko, P. M. Dejardin, and R. W. Chantrell, Phys. Rev. B **82**, 024412 (2010); W. T. Coffey and Y. P. Kalmykov, J. Appl. Phys. **112**, 121301 (2012).
[12] J. Foros, A. Brataas, Y. Tserkovnyak, and G. E. W. Bauer, Phys. Rev. Lett. **95**, 016601 (2005).
[13] Y. Tserkovnyak, A. Brataas, and G. E. W. Bauer, Phys. Rev. Lett. **88**, 117601 (2002).
[14] H. Adachi, K. Uchida, E. Saitoh, and S. Maekawa, Rep. Prog. Phys. **76**, 036501 (2013).
[15] H. Risken, *The Fokker-Planck Equation: Methods of Solution and Applications* (Springer, Berlin, 1989); D. A. Garanin, Phys. Rev. B **55**, 3050 (1997); D. A. Garanin, V. V. Ishchenko, and L. V. Panina, Teor. Math. Fiz. **82**, 242 (1990) [Theor. Math. Phys. **82**, 169 (1990)].
[16] *Handbook of Mathematical Functions*, edited by M. Abramovitz, National Bureau of Standards Applied Mathematics Series (Dover, New York, 1964), Vol. 55.
[17] X. Zuo, A. Yang, and S.-D. Yoon, Appl. Phys. Lett. **87**, 152505 (2005).