

# Conventional superconductivity and charge-density-wave ordering in $\text{Ba}_{1-x}\text{Na}_x\text{Ti}_2\text{Sb}_2\text{O}$

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We have investigated the low-temperature physical properties of  $\text{BaTi}_2\text{Sb}_2\text{O}$  and  $\text{Ba}_{1-x}\text{Na}_x\text{Ti}_2\text{Sb}_2\text{O}$  ( $x = 0.05, 0.1, 0.15, 0.2, 0.25, 0.3$ ) by means of muon spin rotation ( $\mu\text{SR}$ ) and SQUID magnetometry. Our measurements reveal the absence of magnetic ordering below  $T_{\text{DW}} = 58$  K in the parent compound. Therefore the phase transition at this temperature observed by magnetometry is most likely due to the formation of a charge-density wave (CDW). Upon substitution of barium by sodium in  $\text{Ba}_{1-x}\text{Na}_x\text{Ti}_2\text{Sb}_2\text{O}$  we find for  $x = 0.25$  superconductivity with a maximum  $T_c = 5.1$  K in the magnetization and a bulk  $T_{c,\text{bulk}} = 4.5$  K in the  $\mu\text{SR}$  measurements. The temperature dependency of the London penetration depth  $\lambda^{-2}(T)$  of the optimally doped compound can be well explained within a conventional weak-coupling scenario in the clean limit.

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## I. INTRODUCTION

Nesting at the Fermi surface is known to be a key feature for the occurrence of either charge- (CDW) or spin-density-wave (SDW) ordering, and it is considered to be of importance for the emergence of superconductivity in some materials [e.g.,  $\text{BaFe}_{2-x}\text{Co}_x\text{As}_2$  (Ref. 1)]. The competition or coexistence of superconductivity and SDW ordering is one of the most extensively discussed topics for iron-based superconductors and for the stripe phases of cuprates.<sup>2,3</sup> The competition or coexistence between superconductivity and CDW ordering at low temperatures is less often encountered [see, e.g., in  $\text{Cu}_x\text{TiSe}_2$  (Ref. 4), in  $2\text{H-NbSe}_2$  (Ref. 5), and in  $\text{Ba}_{1-x}\text{K}_x\text{BiO}_3$  (Refs. 6 and 7)], though the development of CDW order at zero field in the normal state of superconducting  $\text{YBa}_2\text{Cu}_3\text{O}_{6.67}$  has been prominently discussed.<sup>8</sup>

The large family of stacked, layered titanium oxide pnictide compounds were long considered as potential host structures for superconductivity.<sup>9</sup> Most of these materials were identified to undergo magnetic or density wave (DW) ordering transitions at low temperatures, in the absence of superconductivity. For example,  $\text{Na}_2\text{Ti}_2\text{As}_2\text{O}$  and  $\text{Na}_2\text{Ti}_2\text{Sb}_2\text{O}$ , which crystallize in a modified anti- $\text{K}_2\text{NiF}_4$ -type structure, were found to undergo transitions to SDW-ordered states at  $T_{\text{SDW}}$  of 320 K and 115 K, respectively.<sup>10</sup>

$\text{BaTi}_2\text{Sb}_2\text{O}$  belongs to this family of compounds. Its structure consists of titanium, octahedrally surrounded by oxygen, leading to square planar  $\text{Ti}_2\text{O}$  sheets.<sup>11</sup> This compound was found to undergo a phase transition to a SDW or CDW around  $T_{\text{DW}} = 55$  K and it was proposed that below  $T_c = 1$  K it is a superconductor.<sup>12</sup> Recently, it was shown that upon substitution of barium by sodium in  $\text{Ba}_{1-x}\text{Na}_x\text{Ti}_2\text{Sb}_2\text{O}$ ,  $T_{\text{DW}}$  is lowered and eventually suppressed, while superconductivity reaches a maximum  $T_c$  of approximately 5 K.<sup>13</sup>

In this Rapid Communication we will show by a series of SQUID magnetometry and muon-spin rotation ( $\mu\text{SR}$ ) experiments that  $\text{Ba}_{1-x}\text{Na}_x\text{Ti}_2\text{Sb}_2\text{O}$  is another example for the coexistence and competition of the periodic modulations of CDW and superconductivity. Our results suggest that the

CDW ordering competes with a conventional superconducting state in these materials.

## II. EXPERIMENTAL

Standard solid-state reactions were employed to synthesize polycrystalline samples of  $\text{Ba}_{1-x}\text{Na}_x\text{Ti}_2\text{Sb}_2\text{O}$  with  $x = 0, 0.05, 0.10, 0.15, 0.20, 0.25$ , and  $0.30$ .  $\text{BaO}$  (99.99%),  $\text{BaO}_2$  (95%),  $\text{Na}_2\text{O}_2$  (95%),  $\text{Ti}$  (99.99%), and  $\text{Sb}$  (99.999%) were mixed and pressed into pellets in an argon-filled glove box. The pellets were sealed in argon-filled niobium ampoules and then sintered at  $1000^\circ\text{C}$  for 24 h. Then the samples were reground under inert atmosphere, repelletized, and sintered again for 36 h at  $1000^\circ\text{C}$ . The purity, symmetry, and cell parameters were checked by x-ray powder diffraction using a Stoe STADIP diffractometer ( $\text{Cu-K}\alpha_1$  radiation,  $\lambda = 1.54051 \text{ \AA}$ , Ge monochromator).

The magnetic properties were studied using a Quantum Design Magnetic Properties Measurement System (MPMS XL) equipped with a reciprocating sample option (RSO). Transverse-field (TF) and zero-field (ZF)  $\mu\text{SR}$  experiments were carried out at the LTF instrument at the  $\pi\text{M3}$  beamline, and at the Dolly instrument at the  $\pi\text{E1}$  beamline at the Paul Scherrer Institute (PSI), Switzerland. The superconducting pellets were cooled from above  $T_c$  in a field of  $\mu_0 H = 35$  mT for the TF experiments. The occurrence of magnetism was investigated in these samples with the ZF experiments. The  $\mu\text{SR}$  time spectra have been analyzed using the free software package MUSRFIT.<sup>14</sup>

## III. RESULTS AND DISCUSSION

In Fig. 1(a) we show magnetization  $M(T)$  data of the parent compound  $\text{BaTi}_2\text{Sb}_2\text{O}$ , in a field of  $\mu_0 H = 1.0$  T, showing a distinct kink at  $T_{\text{DW}} = 58$  K. This discontinuity was earlier attributed to either a SDW or a CDW ordering transition.<sup>12</sup>

The ZF and weak TF muon time signals for  $\text{BaTi}_2\text{Sb}_2\text{O}$  were measured above and below  $T_{\text{DW}}$  at  $T = 1.5$  K and  $T = 100$  K, as shown in Figs. 1(b) and 1(c). The measurements show neither indications of static nor fluctuating magnetism

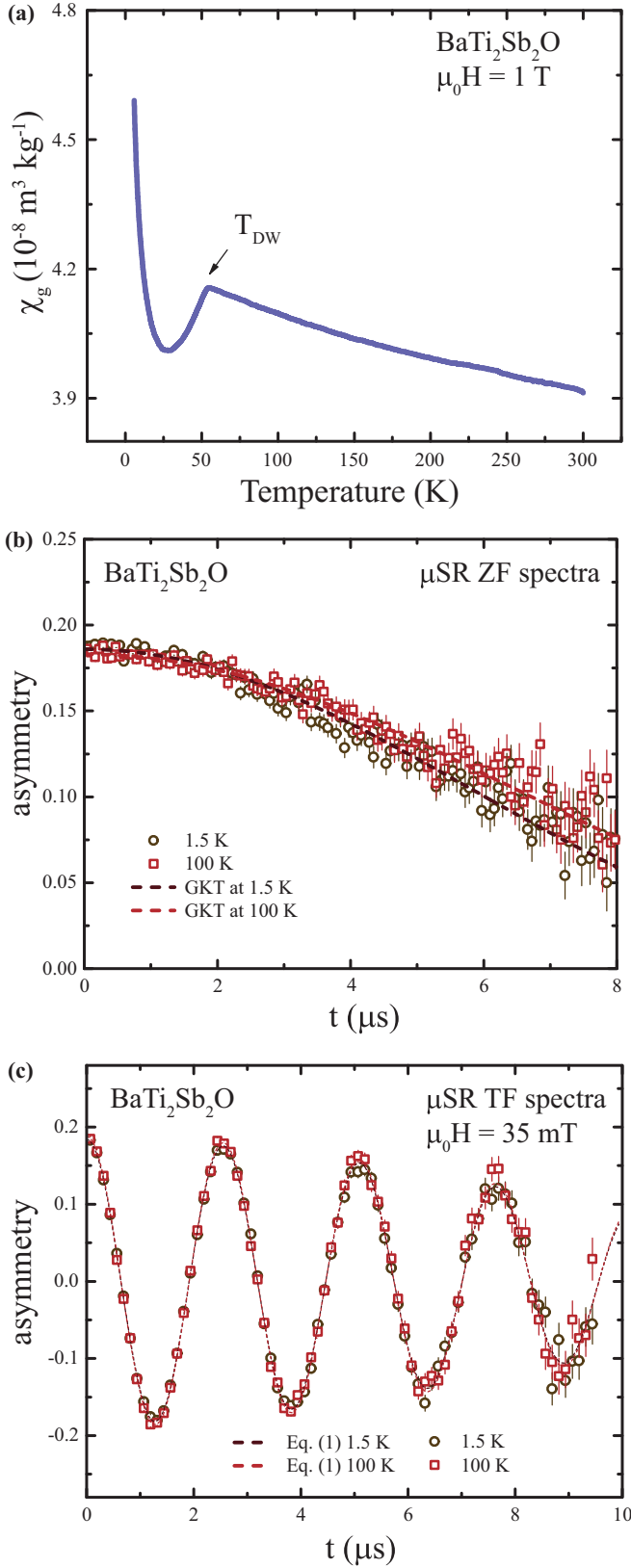


FIG. 1. (Color online) Parent compound,  $\text{BaTi}_2\text{Sb}_2\text{O}$ , characterized by (A) the temperature-dependent susceptibility in a field of  $\mu_0 H = 1$  T, (B) the ZF  $\mu\text{SR}$  spectra at 1.5 K and 100 K (the dashed lines are fits to the Gaussian Kubo-Toyabe function), and (C) TF  $\mu\text{SR}$  spectra (the dashed lines are fits to Eq. (1) at temperatures  $T = 1.5$  K and 100 K).

down to  $T = 1.5$  K. Moreover, the relaxation rates are small and show only little differences between the measurements at high temperatures and at 1.5 K. The ZF spectra are well described by a standard Gaussian Kubo-Toyabe (GKT) function,<sup>15</sup> which is typical for nuclear moments. The zero-field  $\mu\text{SR}$  spectra above and below  $T_{\text{DW}}$  do not exhibit any noticeable change in the relaxation rate, indicating the absence of a spontaneous internal field at the muon stopping site (within the sensitivity of  $\mu\text{SR}$ ). This is further supported by the weak TF measurements [Fig. 1(c)], where no reduction of the asymmetry is observed, as would be expected in case of magnetic ordering. Therefore, we can exclude that the observed transition at  $T_{\text{DW}}$  is caused by SDW ordering in the parent compound  $\text{BaTi}_2\text{Sb}_2\text{O}$ . The observed transition is therefore most likely caused by CDW ordering. These findings are in agreement with recent NMR measurements.<sup>16</sup>

Upon substitution of barium by sodium in  $\text{Ba}_{1-x}\text{Na}_x\text{Ti}_2\text{Sb}_2\text{O}$ , we find superconductivity with a maximum  $T_c = 5.1$  K in the magnetization, and a bulk  $T_{c,\text{bulk}} = 4.5$  K in the  $\mu\text{SR}$  measurements, for  $x = 0.25$ . The temperature-dependent measurements of the DC magnetic susceptibility in the vicinity to superconductivity (1.8 K to 10 K), measured in zero-field-cooled (ZFC) mode in an external field of  $\mu_0 H = 1$  mT, are shown in Fig. 2(a). The transitions to the superconducting state are depicted for six representative members of the series,  $x = 0.05, 0.1, 0.15, 0.2, 0.25,$  and  $0.3$ . In Fig. 2(b) we show the ZF muon time signals for the optimally doped sample  $x = 0.25$  at  $T = 1.5$  K and above  $T_c$  (6 K). The ZF spectra are well described by a GKT function and are overlapping for both measurements, revealing no magnetic ordering down to 1.5 K. The relaxation above  $T_c$  in the TF measurements in a field of  $\mu_0 H = 35$  mT is shown in Fig. 2(c). Similar to the ZF measurements [Fig. 2(b)], only a small relaxation arising from the randomly aligned nuclear magnetic moments is observed. The strong additional relaxation in the TF measurements below  $T_c$ , however, is solely due to the formation of the flux-line lattice (FLL) in the Shubnikov phase. As shown by Brandt, the second moment of the resulting inhomogeneous field distribution is related to the magnetic penetration depth  $\lambda$  as  $\langle \Delta B^2 \rangle \propto \sigma_{sc}^2 \propto \lambda^{-4}$ , whereas  $\sigma_{sc}$  is the Gaussian relaxation rate due to the formation of the FLL.<sup>17,18</sup> The TF  $\mu\text{SR}$  time evolutions were analyzed using the following functional form for the polarization:

$$A(t) = A(0) \exp \left[ -\frac{\sigma_{sc}^2 + \sigma_{nm}^2}{2} t^2 \right] \cos(\gamma_\mu B_{\text{int}} t + \varphi) + A_{\text{BG}}. \quad (1)$$

Here,  $A(0)$  and  $\varphi$  are the initial asymmetry and the phase of the muon ensemble, respectively,  $\sigma_{nm}$  is the damping arising from the nuclear magnetic dipole moments, which we assumed to be temperature independent and fixed to the value obtained above  $T_c$ ,  $\gamma_\mu / (2\pi) = 135.5$  MHz/T is the muon gyromagnetic ratio, and  $B_{\text{int}}$  represents the internal magnetic field at the muon stopping site. For the low-temperature measurements in the LTF instrument, part of the muon beam is stopped in the silver sample holder, resulting in a background denoted as  $A_{\text{BG}}$ .

For a weak-coupling BCS superconductor and  $B_{\text{ext}} \ll B_{c2}$ ,  $\lambda$  does not depend on external magnetic fields, whereas, e.g., in

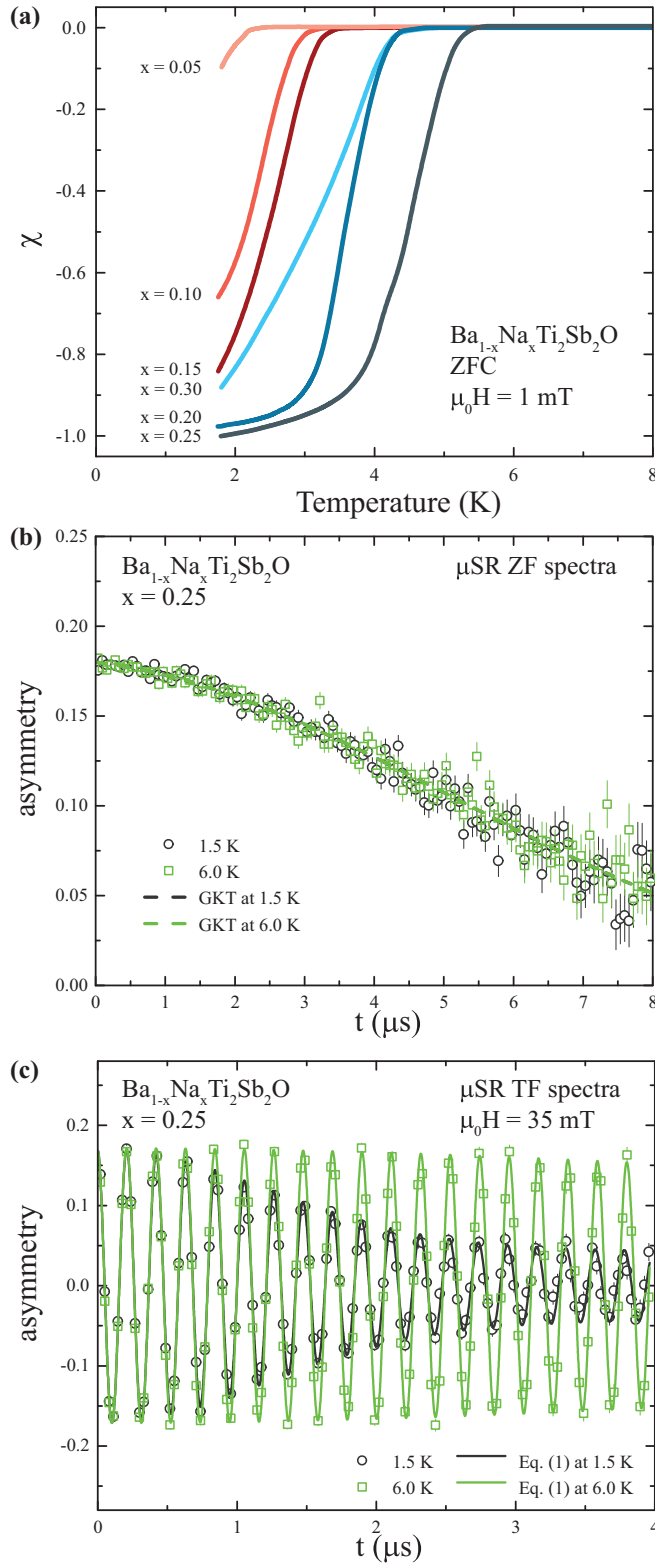


FIG. 2. (Color online) (A) The magnetic susceptibility of  $\text{Ba}_{1-x}\text{Na}_x\text{Ti}_2\text{Sb}_2\text{O}$  for  $x = 0.05, 0.1, 0.15, 0.2, 0.25, 0.3$  measured in a field of  $\mu_0 H = 1$  mT. (B) The ZF  $\mu\text{SR}$  spectra for  $x = 0.25$  above ( $6$  K) and below ( $1.5$  K)  $T_c$  (the dashed lines are the fits to the Gaussian Kubo-Toyabe function). (C) The TF  $\mu\text{SR}$  spectra for  $x = 0.25$  above ( $6$  K) and below ( $1.5$  K)  $T_c$  (the solid lines are fits to Eq. (1)). The strong relaxation of the signal at  $1.5$  K can be ascribed to the presence of the flux-line lattice.

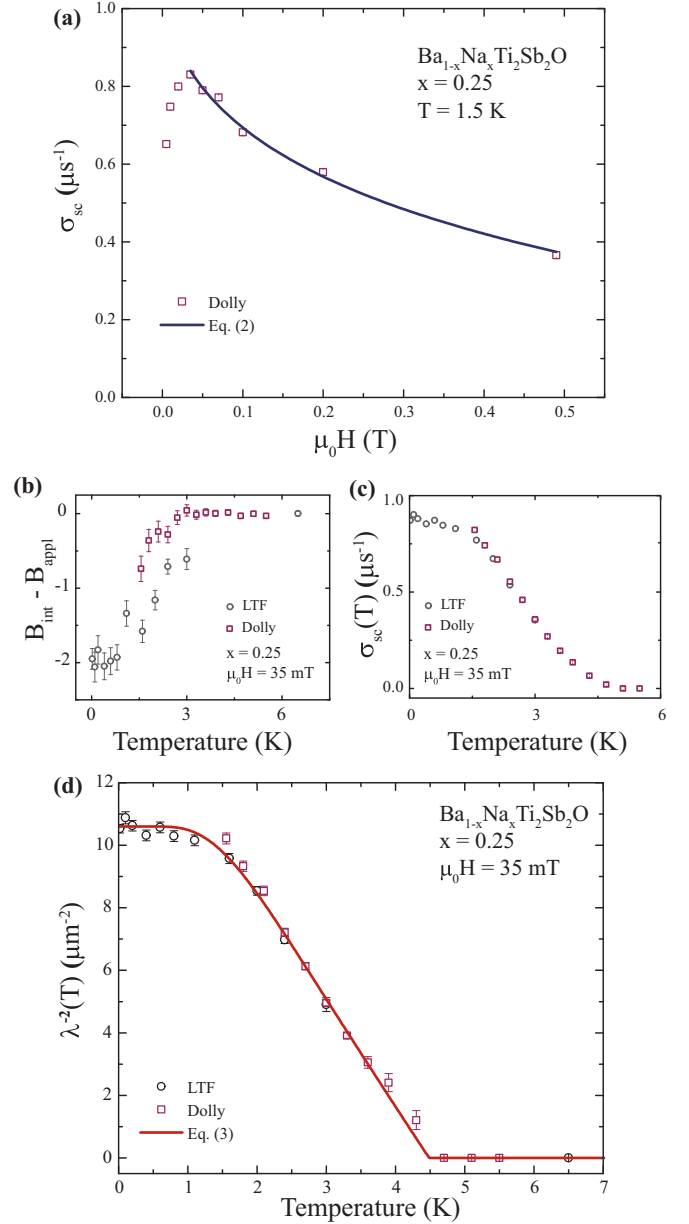


FIG. 3. (Color online) (A) Field dependence of the muon depolarization rate  $\sigma_{sc}$  at  $T \approx 1.5$  K for the optimal doping  $x = 0.25$ . The solid line corresponds to a fit of the experimental data to Eq. (2). The insets show the corresponding field dependence of  $\lambda^{-2}$  (in the Shubnikov phase). (B) The diamagnetic field shift in the superconducting state with respect to above  $T_c$  ( $B_{\text{int}} - B_{\text{appl}}$ ) for  $x = 0.25$ . (C) The temperature dependence of the muon polarization rate  $\sigma_{sc}(T)$  measured in  $\mu_0 H = 35$  mT. (D) The temperature dependence of  $\lambda^{-2}$  for  $x = 0.25$  as reconstructed from  $\sigma_{sc}(T)$  (shown in B), measured in  $\mu_0 H = 35$  mT. The solid line corresponds to a fit to Eq. (3) with  $2\Delta/(k_B T_c) = 2.9$ . Squares: Dolly instrument, circles: LTF instrument.

a multiple gap or nodal superconductor  $\lambda$  can be significantly field dependent.<sup>19–22</sup> In the case of an ideal vortex lattice of an isotropic  $s$ -wave superconductor within the Ginzburg-Landau theory, the relaxation rate  $\sigma^2$  in the superconducting state

should follow the expression<sup>18</sup>

$$\sigma_{sc} = a \left(1 - \frac{B}{B_{c2}}\right) \left[1 + 1.21 \left(1 - \sqrt{\frac{B}{B_{c2}}}\right)^3\right] \lambda^{-2}. \quad (2)$$

Here,  $a$  is a coefficient given by the symmetry of the vortex lattice (with  $a = 4.83 \times 10^4 \text{ nm}^2/\mu\text{sec}$  for triangular and  $a = 5.07 \times 10^4 \text{ nm}^2/\mu\text{sec}$  for a rectangular vortex lattice geometry<sup>18,23</sup>),  $B$  is the magnetic induction, for which we may assume  $B \simeq B_{\text{ext}}$  in the region  $\mu_0 H_{c1} \ll B_{\text{ext}} \ll \mu_0 H_{c2}$  (with  $H_{c1}$  the lower and  $H_{c2}$  the upper critical field, respectively). Equation (2) in general accounts for the reduction of  $\sigma_{sc}$  due to the stronger overlap of the vortices with increasing field. A fit of the measured  $\sigma_{sc}$  according to Eq. (2) describes the data reasonably well and yields  $B_{c2} = 1.5(1) \text{ T}$  and  $\lambda^{-2} = 10^{-5}(0.01) \text{ nm}^{-2}$  at  $T = 1.5 \text{ K}$  for  $x = 0.25$  [see Fig. 3(a)]. The value of  $B_{c2}$ , for the optimal doping  $x = 0.25$ , is in excellent agreement with previous measurements.<sup>24</sup> The parameter  $a$  was fitted to  $4.87(5) \times 10^4 \text{ nm}^2/\mu\text{sec}$  indicating that the vortex lattice in  $\text{Ba}_{1-x}\text{Na}_x\text{Ti}_2\text{Sb}_2\text{O}$  has triangular shape. To obtain maximum field contrast, we chose the magnetic field  $\mu_0 H = 35 \text{ mT}$  to study the temperature dependence of  $\lambda(T)$  for the optimally doped sample  $x = 0.25$ . Measurements down to  $T = 0.02 \text{ K}$  and  $T = 1.5 \text{ K}$  were performed in the LTF and Dolly instruments, respectively. A diamagnetic shift of the internal magnetic field  $B_{\text{int}}$  is observed below  $T_c$  [Fig. 3(b)]. The resulting temperature dependence of  $\sigma_{sc}$  is shown in Fig. 3(c). In Fig. 3(d) we show the temperature dependence of  $\lambda^{-2}(T)$  as reconstructed from  $\sigma_{sc}(T)$ , using Eq. (2). The temperature dependence of  $B_{c2}(T)$ , used in the corresponding calculation according to Eq. (2), was assumed to follow the theoretical Werthamer-Helfand-Hohenberg relation.<sup>24,25</sup>

These measurements suggest that  $\lambda^{-2}$  is virtually temperature independent below  $T \simeq 1 \text{ K}$  for the optimally doped sample. The obtained experimental temperature dependence of  $\lambda^{-2}(T)$  was tentatively analyzed within the clean limit approach for a London superconductor with an  $s$ -wave gap<sup>26</sup>

$$\frac{\lambda^{-2}(T)}{\lambda^{-2}(0)} = 1 + 2 \int_{\Delta(T)}^{\infty} \left(\frac{\partial f}{\partial E}\right) \frac{E}{\sqrt{E^2 - \Delta^2(T)}} dE. \quad (3)$$

Here  $\lambda(0)$  is the zero-temperature value of the magnetic penetration depth,  $f = [1 + \exp(E/k_B T)]^{-1}$  is the Fermi

function (with  $k_B$  the Boltzmann constant), and  $\Delta(T) = \Delta(0)\tilde{\Delta}(T/T_c)$  represents the temperature dependence of the energy gap, which can be approximated to sufficient precision as  $\tilde{\Delta}(T/T_c) = \tanh\{1.82[1.018(T_c/T - 1)^{0.51}]\}$ .<sup>27</sup> The results of this fit are  $T_c = 4.49(6) \text{ K}$  and  $\Delta(0) = 0.56(1) \text{ meV}$  with a zero-temperature magnetic penetration depth  $\lambda = 307(10) \text{ nm}$ . This corresponds to a ratio  $2\Delta/(k_B T_c) = 2.9$ , which is quite close to the value of a weak-coupling BCS superconductor  $2\Delta/(k_B T_c) = 3.5$ . There are no signs of multigap superconductivity in these data (compare Refs. 19–22), and the presented low-temperature  $\lambda^{-2}(T)$  data taken in a low magnetic field seem to be incompatible with a possible  $d$ -wave scenario.<sup>28</sup>

#### IV. CONCLUSION

We have presented magnetization and  $\mu\text{SR}$  results on the density wave (DW) ordering transition in  $\text{BaTi}_2\text{Sb}_2\text{O}$ , and on the transition to superconductivity in  $\text{Ba}_{1-x}\text{Na}_x\text{Ti}_2\text{Sb}_2\text{O}$ . The observed absence of a magnetic contribution to the  $\mu\text{SR}$  data related to the phase transition at  $T_{\text{DW}} = 58 \text{ K}$  of the parent compound  $\text{BaTi}_2\text{Sb}_2\text{O}$  is strong evidence against the SDW ordering transition theoretically proposed in Ref. 29. Therefore the observed phase transition is most likely due to CDW ordering that competes with a superconducting state. Upon substitution of barium by sodium in  $\text{Ba}_{1-x}\text{Na}_x\text{Ti}_2\text{Sb}_2\text{O}$  we find superconductivity with a maximum  $T_c = 5.1 \text{ K}$  in the magnetization, and a bulk  $T_{c,\text{bulk}} = 4.5 \text{ K}$  in the  $\mu\text{SR}$  measurements, for  $x = 0.25$ . In the TF  $\mu\text{SR}$  spectra for  $x = 0.25$  below  $T_c$  a strong relaxation of the signal is observed, which is due to the formation of the flux-line lattice. This is strong evidence for the bulk nature of the superconductivity in this material. The obtained experimental temperature dependence of  $\lambda^{-2}(T)$  can be reasonably well explained within the clean limit approach for a conventional London superconductor, which is consistent with recently published NMR and specific-heat results, as well as theoretical calculations.<sup>16,24,30</sup>

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