Aharonov-Casher effect in quantum ring ensembles

Fateme K. Joibari,[1] Ya. M. Blanter,[1] and Gerrit E. W. Bauer[1,2]
[1]*Kavli Institute of NanoScience, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands*
[2]*Institute for Materials Research and World Premier International Research Center, Advanced Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan*

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We study the transport of electrons through a single-mode quantum ring with electric-field induced Rashba spin-orbit interaction that is subject to an in-plane magnetic field and weakly coupled to electron reservoirs. Modeling a ring array by ensemble averaging over a Gaussian distribution of energy-level positions, we predict slow conductance oscillations as a function of the Rashba interaction and electron density due to spin-orbit interaction induced beating of the spacings between the levels crossed by the Fermi energy. Our results agree with experiments by Nitta c.s. [J. Nitta, J. Takagi, F. Nagasawa, and M. Kohda, J. Phys.: Conference Series **302**, 012002 (2011) and Nagasawa *et al.* (unpublished)], thereby providing an interpretation that differs from the ordinary Aharonov-Casher effect in a single ring.

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The Aharonov-Casher (AC) effect[1] is an analog of the Aharonov-Bohm (AB) effect but is caused by the spin-orbit interaction (SOI) rather than an external magnetic field. Originally, Aharonov and Casher predicted in 1984 that a spin accumulates a phase when the electric charge is circling in an external electric field.[1] This situation is similar to a single-mode ballistic ring with Rashba spin-orbit interaction. Quantum rings in high-mobility semiconductor material have therefore attracted extensive attention, both experimentally and theoretically, as model devices to investigate fundamental quantum-mechanical phenomena.

In the AC effect, the electrons injected into a quantum ring with SOI acquire spin phases when traversing the two arms due to precession in the effective spin-orbit magnetic field. Interference of the spinor wave functions at the exit point of the ring then leads to an oscillatory conductance as a function of the spin-orbit coupling constant that in Rashba systems can be tuned by an external gate voltage. König *et al.*[2] reported the first experimental evidence of the AC effect in a single HgTe ring by measuring the phase shift of the AB-type magnetoconductance oscillations caused by tuning the Rashba SO strength. Since several sub-bands in the ring were occupied, they supported their experiments by multimode transport calculations. This study focused on the symmetry points, at which the Rashba SOI is small, and high values of the applied magnetic field.[2] Experiments on an array of rings[4] agreed well with the theory provided for a single-mode quantum ring symmetrically and strongly coupled to the leads.[3] More recently, the zero magnetic-field conductance as a function of gate field has been interpreted in terms of the modulation of (electron-density-independent) Altshuler-Aronov-Spivak (AAS) oscillations by the SOI,[5] emphasizing the importance of statistical averaging by the ring arrays.

In reality, however, the situation is not as simple as it appears. The assumed ideal link of the ring to the leads is equivalent to the strong-coupling limit in terms of a connectivity parameter.[6] The implied absence of backscattering is at odds with the interpretation of the observed oscillations in terms of AAS oscillations due to coherent backscattering.[5,7] Reference 8 addresses the effects of scattering at the contacts of a single-mode Rashba ring to the reservoirs, interpolating between the fully open and isolated ring regimes. However, the experiments[4,5] were not carried out on single rings in the one-dimensional quantum limit but on a large array of connected rings, each containing several transport channels. Some theoretical papers compute transport through an array of single-mode rings[9,10] but assuming a constant Fermi wave number, thereby disregarding the strong density changes associated with tuning the Rashba spin-orbit parameter.[7] In the present paper we offer an explanation of the robustness of the observed AC oscillations with respect to the complications summarized above.

A quantitative analysis of the multimode ring array is challenging and requires large-scale numerical simulations.[11] Here we proceed from a single single-mode quantum ring,[3] taking backscattering into account by assuming weak coupling to the electron leads and correcting for the multimode character *a posteriori* (see below). The conductance of a quantum ring can be understood as resonant tunneling through discrete eigenstates at the Fermi energy[6] that are modulated by the SOI Rashba parameter. The in-plane magnetic field[11] allows tuning of the conductance oscillations without interference of the AB oscillations (see Fig. 1). We consider a modulation of the Rashba interaction strength that is associated with an experimentally known large change in the electron density.[7] Small deviations between different rings in nanofabricated arrays can be taken into account by an ensemble averaging over slightly different single rings. We find that this procedure leads to an agreement with experiments that rivals that of previous theories.

We consider a ring with a radius of R, defined in the high-mobility two-dimensional electron gas in the xy plane. The Rashba SOI with the strength α is a known function of an external gate potential. The Hamiltonian of an electron in the ring has the form[12]

$$\hat{H}_{1D}^{(0)} = \frac{\hbar^2}{2mR^2}\left(-i\frac{\partial}{\partial\varphi}\right)^2 - \frac{\alpha}{R}(\cos\varphi\hat{\sigma}_x + \sin\varphi\hat{\sigma}_y)\left(i\frac{\partial}{\partial\varphi}\right) - i\frac{\alpha}{2R}(\cos\varphi\hat{\sigma}_y - \sin\varphi\hat{\sigma}_x), \quad (1)$$

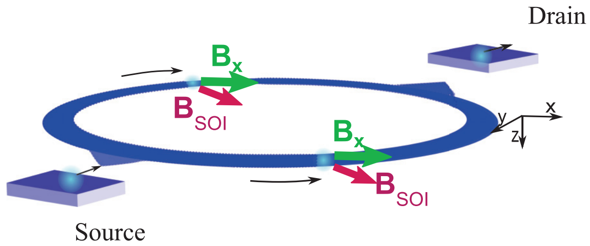

FIG. 1. (Color online) Schematic of a quantum ring weakly coupled to source and drain contacts in the presence of SOI effective field B_{SOI} and in-plane magnetic field B_x.

where m is the effective mass, φ is the azimuthal angle, and $\hat{\sigma}_i$ are the Pauli matrices in the spin space. The eigenstates are

$$E^{(0)}_{n\sigma} = E_R\left[\left(n+\frac{1}{2}\right)^2 + \frac{1}{4} + \sigma\frac{n+\frac{1}{2}}{\cos\theta}\right], \tag{2}$$

where $E_R = \hbar^2/(2mR^2)$, $\tan\theta = 2mR\alpha/\hbar^2$, the integer n is the angular momentum quantum number, and $\sigma = \pm$ denotes the spin degree of freedom.

An in-plane magnetic field B along the x direction contributes the Zeeman energy $H' = E_B\hat{\sigma}_x$, where $E_B = g\mu_B B/2$, μ_B is the Bohr magneton, and g is the effective g factor. We assume that the Zeeman energy is small compared to the (kinetic) Fermi energy and can be treated as a perturbation of the zero-field Hamiltonian, $H^{(0)}_{1D}$.

To leading order in E_B the energies $E^{(0)}_{n\sigma}$ are shifted by the in-plane field as

$$\Delta^{(2)}_{n\sigma} = \frac{E_B^2}{8E_R}\left\{\frac{\sin^2(2\theta)}{(2n\cos\theta+\sigma)\left[2(n+1)\cos\theta+\sigma\right]} + \frac{4\sin^4\frac{\theta}{2}\cos\theta}{n(\cos\theta+\sigma)} - \frac{4\cos^4\frac{\theta}{2}\cos\theta}{(n+1)(\cos\theta-\sigma)}\right\}. \tag{3}$$

The gate voltage V_g modifies the asymmetry of the electron confinement potential, thereby modulating the Rashba SOI strength α. We discuss here first the effects of varying SOI for constant Fermi energy and subsequently take the gate induced density variation into account. In the absence of a magnetic field, the energy levels move with α according to Eq. (2). The fourfold degeneracy in the absence of SOI, $E_{n,\sigma} = E_{n,-\sigma} = E_{-n-1,-\sigma} = E_{-n-1,\sigma}$, is broken when $\alpha \neq 0$ into two Kramers-degenerate doublets with $E_{n,\sigma} = E_{-n-1,-\sigma}$ [see Eq. (2)]. For $\sigma n > (<)\, 0$ the energy increases (decreases) with α as indicated in Fig. 2. The experiments by Nitta *et al.*[7] were carried out in the low-temperature regime with level spacings larger than the thermal energy,[4,5] and therefore we assume zero temperature in the following.

Resonant tunneling occurs when the energy of the highest occupied level in the quantum ring, $E_{n_F,\sigma}$, equals the chemical potential μ in the leads, i.e., $E_{n_F,\sigma}(\alpha) = \mu$, as indicated in Fig. 2. Doublets of spin-split conductance peaks merge when $\alpha = 0$, $\mu = E_{n_F,\sigma}$, and the conductance becomes twice as large. The in-plane magnetic field shifts the energy levels as $\propto B^2$. As illustrated in Fig. 3, the resonant tunneling peaks at $E_{n_F,\sigma}(\alpha,B) = \mu$ are spin split and nonparabolic. Figure 3 agrees qualitatively with the experiments[11] when

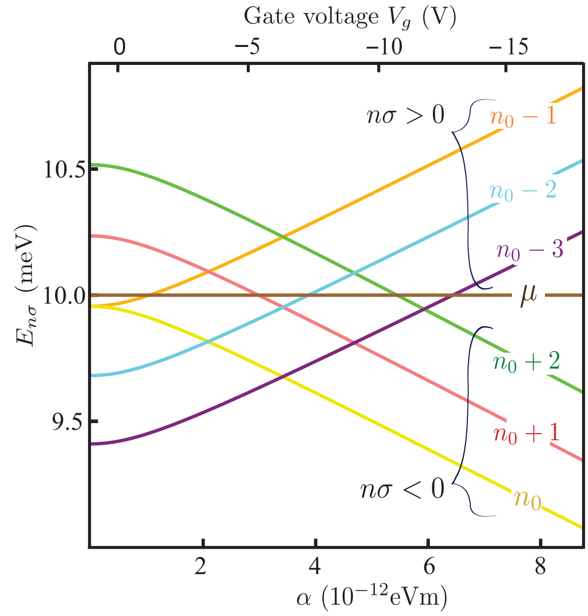

FIG. 2. (Color online) Energies of a quantum ring with the radius $R = 630$ nm close to the Fermi energy $\mu = 10$ meV as a function of the SOI strength α. Energies are labeled as $n_0 + i$ and $n_0 = 72$, for $n > 0$, whereby each level is Kramers degenerate with $-n_0 - i - 1$ and opposite spin direction. The effective mass for conduction electrons in InGaAs is $m = 0.045\, m_0$, where m_0 is the electron mass. The conductance is nonzero when μ crosses an energy level.

assuming the strong-coupling limit and justifying the apparent independence on the large μ variation with gate voltage by coherent backscattering. In the following we suggest an alternative interpretation.

According to the experiments,[4,5] α depends on the gate voltage as $\alpha(10^{-12}\text{ eV m}) = 0.424$–$0.47 \times V_G(10^{-12}\text{ V})$ and

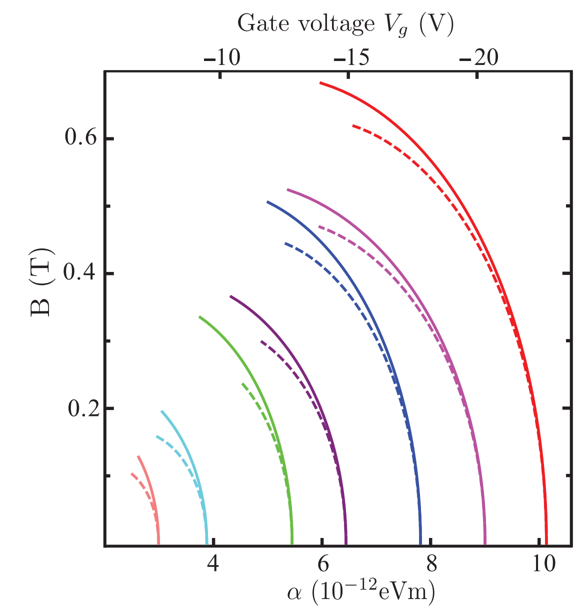

FIG. 3. (Color online) Shift of the conductance peaks, that for zero magnetic field coincide with the crossings of the Fermi energy in Fig. 2, by an in-plane magnetic field as obtained by perturbation theory. The magnetic field is seen to break Kramers spin degeneracy. The parameters are the same as in Fig. 2 and $g = -2.9$ for InGaAs.

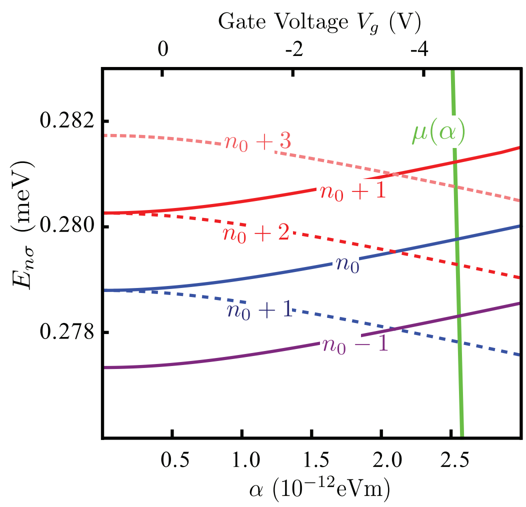

FIG. 4. (Color online) Energy levels around the Fermi energy for $\alpha \approx 2.4 \times 10^{-12}$ eV m relative to μ, that strongly depends on the gate voltage tuning α. Here $n_0 = 380$, while dashed and solid lines represent $n\sigma < 0$ and $n\sigma > 0$, respectively. Similar to Fig. 2, we label the energies for $n > 0$, keeping in mind that the levels are twofold degenerate.

on the electron density as $\alpha(10^{-12}\ \text{eV m}) = 7.81 - 3.32 \times N_s(10^{12}\ \text{cm}^{-2})$. In Fig. 4 we plot the ring energies as a function of α including the chemical potential μ, that varies much faster with α than the single-particle energies, leading to conductance peaks that as a function of gate voltage are very closely spaced. In ring arrays[4,5] we do not expect to resolve such narrow resonances due to disorder, multimode contributions, and ring size fluctuations. We can model the latter by averaging over an ensemble of rings with a Gaussian distribution of resonant energies or conductance peak positions with a phenomenological broadening parameter Γ. Figure 5 illustrates the result of the averaging procedure in the form of the normalized conductance modulations.[13] While the resonant tunneling peaks are smeared out, a slow (AC) oscillation as a function of α reappears, which represents the beating of the level spacings induced by the SOI, in qualitative agreement with experiments.

The experiments of AC oscillations in arrays with different ring radii[5] are compared in Fig. 6 with our results.

In Fig. 5, we also illustrate the effect of in-plane magnetic fields on the ensemble of Rashba rings. The magnetic field shifts the phase of the oscillation to lower values of the gate voltage or larger α and thus suppresses the amplitude of the conductance oscillations increasingly for lower values of the gate voltage. These features agree again well with those observed experimentally by Nagasawa *et al.*[11] The magnetic field splits the Kramers degeneracy, thereby leading to two sets of superimposed oscillations that might be experimentally resolved in the form of different Fourier components.

The suppression of AAS oscillations in disordered ring arrays at constant density[9] can be interpreted in favor of our model. Most previous theories[3,12] treat ideally open rings, while we consider the weak-coupling limit. Both extremes are likely not met in experiments. The intermediate regime can be modeled in terms of a connectivity parameter.[6,8] An increased coupling causes a Lorentzian smearing of the

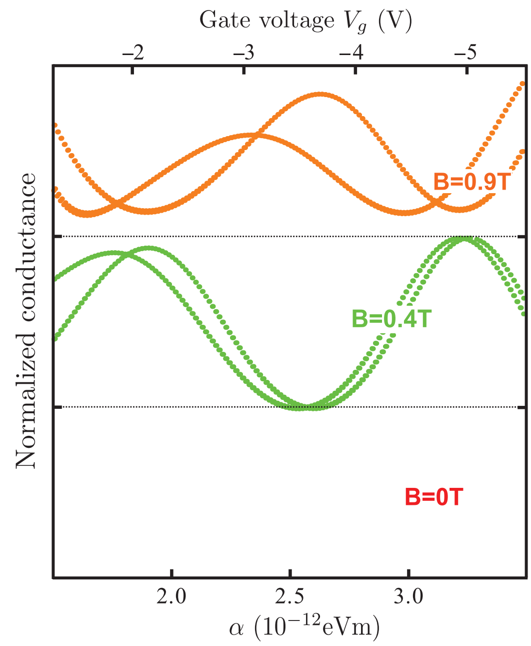

FIG. 5. (Color online) Conductance oscillations of an ensemble of rings with energy levels broadened by a Gaussian with $\Gamma = 0.003$ peV/m as a function of an in-plane magnetic field. The dashed lines are guides to the eye, to compare the oscillation amplitudes while varying the magnetic field. All amplitudes are scaled with those at $B = 0$ T that display a modulation of $(G_{\max} - G_{\min})/G_{\max} = 50\%$. The in-plane magnetic field splits Kramers degenerate spin states that evolve differently with gate voltage.

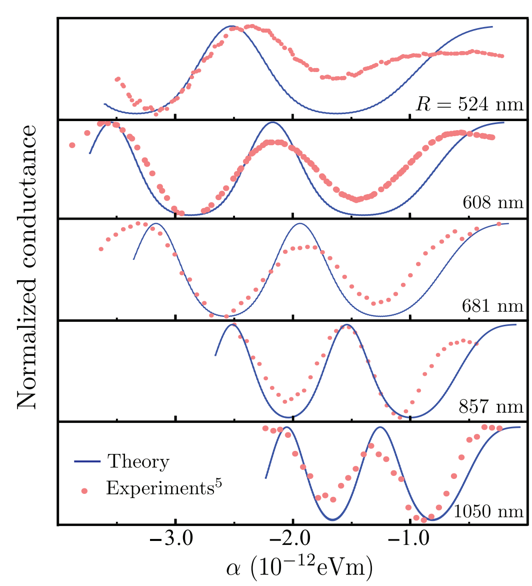

FIG. 6. (Color online) The conductance G of an array of rings modeled as an ensemble of energy levels as a function of α broadened by a Gaussian for various nominal radii R. The broadening parameters are $\Gamma = 0.005, 0.0035, 0.003, 0.002$, and 0.001 peV/m, for $R = 524, 608, 681, 857$, and 1050 nm, respectively. All amplitudes are scaled to a panel height corresponding to $(G_{\max} - G_{\min})/G_{\max} = 50\%$. We use the experimentally determined relations between the Rashba constant and electron density as before. We compare our calculations (lines) with the experimental results (points) from Ref. 5 (see also Ref. 13).

conductance peaks, which is likely to effectively enhance the phenomenological broadening of the ensemble average and cannot be resolved in the experiments. The presence of several occupied modes in the rings also contributed to the average, since each radial node can be approximated as a ring with a slightly different radius. We therefore believe that our results are robust with respect to deviations from our Hamiltonian and these deviations can be captured by the phenomenological broadening parameter Γ.

In conclusion, we investigated the conductance of single rings and an ensemble of them as a function of the Rashba spin-orbit interaction in the limit of weak coupling to the leads. We considered both constant and gate voltage-dependent density of electrons. Both situations can in principle be realized experimentally by two independent (top and bottom) gate voltages. We compare results with experiments on ring arrays in which a single gate changes both the SOI α as well as the electron density. We found that, in agreement with experiments, the ensemble averaged conductance oscillates as a function of α. The oscillations undergo a phase shift under an in-plane magnetic field, and the period varies with the ring diameter, as observed. We conclude that experiments observe SOI induced interference effects that are more complicated than the original Aharonov-Casher model but are robust with respect to the model assumptions. It should be possible to experimentally distinguish between the different models by separating effects of spatial inversion symmetry (and thereby α) and the Fermi wave-number modulation. This should be possible by employing a double gate configuration in which the electric field is varied but density is kept constant.[14]

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[1] Y. Aharonov and A. Casher, Phys. Rev. Lett. **53**, 319 (1984).

[2] M. König, A. Tschetschetkin, E. M. Hankiewicz, J. Sinova, V. Hock, V. Daumer, M. Schafer, C. R. Becker, H. Buhmann, and L. W. Molenkamp, Phys. Rev. Lett. **96**, 076804 (2006).

[3] D. Frustaglia and K. Richter, Phys. Rev. B **69**, 235310 (2004).

[4] T. Bergsten, T. Kobayashi, Y. Sekine, and J. Nitta, Phys. Rev. Lett. **97**, 196803 (2006).

[5] F. Nagasawa, J. Takagi, Y. Kunihashi, M. Kohda, and J. Nitta, Phys. Rev. Lett. **108**, 086801 (2012).

[6] M. Büttiker, Y. Imry, and M. Ya. Azbel, Phys. Rev. A **30**, 1982 (1984).

[7] J. Nitta, J. Takagi, F. Nagasawa, and M. Kohda, J. Phys.: Conference Series **302**, 012002 (2011).

[8] S. Bellucci and P. Onorato, Phys. Rev. B **78**, 235312 (2008).

[9] Zh. Zhu, Y. Wang, K. Xia, X. C. Xie, and Zh. Ma, Phys. Rev. B **76**, 125311 (2007).

[10] O. Kálmàn, P. Földi, M. G. Benedict, and F. M. Peeters, Phys. Rev. B **78**, 125306 (2008).

[11] F. Nagasawa, D. Frustaglia, H. Saarikoski, M. Kohda, K. Richter, and J. Nitta (private communication).

[12] F. E. Meijer, A. F. Morpurgo, and T. M. Klapwijk, Phys. Rev. B **66**, 033107 (2002).

[13] Experiments report resistances rather than conductances. The resistance oscillations $\delta R \ll R$ are measured on top of a large background resistance R. Thus, experiments are represented by $\delta R(\alpha \neq 0)/\delta R(\alpha = 0) = \delta G(\alpha \neq 0)/\delta G(\alpha = 0)$, which we may compare directly with our calculated normalized conductance oscillations.

[14] M. Yamaguchi, S. Nomura, K. Miyakoshi, H. Tamura, T. Akazaki, and H. Takayanagi, J. Appl. Phys. **100**, 113523 (2006).