# <span id="page-0-0"></span>**Rashba fields in a two-dimensional electron gas at electromagnetic spin resonance**

W. Ungier<sup>1[,\\*](#page-4-0)</sup> and W. Jantsch<sup>2</sup>

<sup>1</sup>*Institute of Physics, Polish Academy of Sciences, Al. Lotnikow 32/46, 02-668 Warsaw, Poland* <sup>2</sup>*Institut fur Halbleiter- und Festk ¨ orperphysik, Johannes Kepler Universit ¨ at, A-4040 Linz, Austria ¨* (Received 26 June 2013; published 3 September 2013)

We present an analysis of the Rashba effects in a two-dimensional electron gas induced by the microwave electric and magnetic fields. We show that in the frame of the Drude model the Rashba interaction can be described by magnetic and electric corrections affecting the spin and the velocity of the electrons. We describe the rf currents making use of the conductivity tensor formalism. The electromagnetic power absorption is obtained as a function of the external electric and magnetic rf fields up to the second order of the Rashba parameter. The channels of energy transfer due to Rashba field corrections are analyzed.

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#### **I. INTRODUCTION**

The spin of an electron moving in a two-dimensional (2D) system feels an additional magnetic force arising from the Lorentz transformation of an electric field perpendicular to the sample plane in the presence of an asymmetric electric charge distribution. This spin-orbit (SO) interaction, first described by Rashba, $\frac{1}{2}$  $\frac{1}{2}$  $\frac{1}{2}$  allows us to explain many phenomena such as the Dyakonov-Perel spin relaxation,  $2^{-4}$  spin dephasing, [3](#page-4-0) a characteristic anisotropy of the  $g$  factor,<sup>[3](#page-4-0)</sup> and others.<sup>4</sup> One of the most direct exemplifications of the Rashba field is the shift of the electron spin resonance (ESR) field when an electric current is applied.<sup>[5](#page-4-0)</sup> Spin precession in the Rashba field has been proposed as the main control mechanism in the seminal concept of the Datta-Das transistor. $6$  A particular SO effect appears when an rf current with a frequency corresponding to the Larmor frequency is applied.<sup>[7](#page-4-0)</sup> In an ESR-like experiment, spin precession can be excited by a resonant microwave current as predicted by Rashba and Efros. $8$  Excitation via this current-induced ESR can be by orders of magnitude more efficient than the usual excitation of magnetic dipole transitions by a microwave magnetic field.<sup>5,8</sup> This may be of practical importance in the context of spin manipulation. $8,9$ The dynamic spin-Hall effect is another effect resulting from the current-induced spin precession $10$  and closely related to the results of this paper. Recently we showed that apart from the Rashba magnetic field, that affects the electron spin, there appears a correction of the electric force which influences the spatial dynamics of the electron motion.<sup>11</sup> This force turns out to be proportional to the time derivative of the electron's magnetic moment, i.e., to the frequency of the electron spin precession in case of rf perturbation fields.<sup>11</sup> In this paper, extending our previous work,  $\frac{11}{1}$  we consider electron spin resonance induced by not only the electric, but also by the combination of rf electric and magnetic fields, a situation which is typical for a standard ESR experiment.

#### **II. ELECTRON OSCILLATORY MOTION**

We consider a 2D system, placed in the microwave cavity of a standard ESR setup. The configuration is shown in Fig. [1.](#page-1-0) The Hamiltonian describing the dynamics of one electron between collisions with lattice imperfections is assumed in the form

$$
\hat{H} = \frac{1}{2m^*} \left( \mathbf{p} - \frac{e}{c} \mathbf{A} \right)^2 + \frac{\alpha_R}{\hbar \mu_B} \left( \mathbf{p} - \frac{e}{c} \mathbf{A} \right)
$$

$$
\times \mathbf{n} \cdot \boldsymbol{\mu} - \frac{g}{2} \mathbf{B} \cdot \boldsymbol{\mu}.
$$
 (1)

Here **p** is canonical momentum, and *m*<sup>∗</sup> is the effective electron mass. The parameter  $\alpha_R$  in the second term is a materialand sample-dependent parameter describing the magnitude of the SO Rashba coupling where  $\mu_B = e\hbar/2mc$ , the Bohr magneton, and  $\mu = \mu_B \sigma$ , the electron magnetic moment.

The last term with the *g* factor stands for the Zeeman energy in an external magnetic field,  $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1(t)$ , where  $\mathbf{B}_0$  is constant and  $\mathbf{B}_1 \|\hat{\mathbf{y}}\|$  is the rf magnetic field. We assume the sample to be placed in between the nodes of the microwave magnetic  $\mathbf{B}_1$  and electric fields  $\mathbf{E}_1(t)$  ( $\mathbf{E}_1 \parallel \hat{\mathbf{x}}$ ). Thus the vector potential **A** can be assumed in the form  $\mathbf{A} = \frac{1}{2}\mathbf{B}_{0\perp} \times \mathbf{r}$  −  $c \int_0^t \mathbf{E}_1(t') dt'$ , where  $\mathbf{B}_{0\perp}$  is the part of the field that is perpendicular to the plane of the system,  $\mathbf{B}_{0\perp} = \mathbf{n}(\mathbf{n} \mathbf{B}_0)$ . The Rashba term which arises in systems without mirror symmetry can be attributed to a built-in electric field, $\frac{1}{1}$  $\frac{1}{1}$  $\frac{1}{1}$  directed along the unit vector **n**. The Rashba SO coupling is assumed to be much stronger than the relativistic SO coupling in vacuum (for  $E_1 = 1 \text{ V/cm we have } (e\hbar/4m^2c^2)E_1 \approx (10^{-9} - 10^{-8})\alpha_R/\hbar$ , where  $\alpha_R/\hbar = 4$  m/s in Si/SiGe). Therefore we neglect the relativistic SO term due to the  $\mathbf{E}_1$ .<sup>[12–14](#page-4-0)</sup>

Some direct consequences of the Rashba SO coupling are the following. The velocity of the electron,

$$
\mathbf{v} = \frac{\partial \hat{H}}{\partial \mathbf{p}} = \mathbf{v}^{(p)} + \mathbf{v}^{(R)},
$$
 (2)

can be decomposed into a momentum velocity,  $\mathbf{v}^{(p)}$  =  $(1/m^*)[\mathbf{p} - (e/c)\mathbf{A}]$ , and a spin-dependent component  $\mathbf{v}^{(R)} =$  $(\alpha_R/\hbar \mu_B)$ **n** ×  $\mu$ .

The equation of the electron motion in the frame of the Drude model in a 2D system has the form $11$ 

$$
\frac{d\mathbf{v}}{dt} = \frac{e}{m^*} \bigg( \mathbf{E}_1 + \mathbf{E}_R + \frac{1}{c} \mathbf{v} \times \mathbf{B}_{0\perp} \bigg) - \frac{\mathbf{v}}{\tau},
$$
(3)

where the momentum relaxation time  $\tau$  is assumed to be independent of spin. The additional, spin-dependent electric field  $\mathbf{E}_R$  arises only for the electrons with unpaired spin and it

<span id="page-1-0"></span>

FIG. 1. Considered geometry: magnetic field  $\mathbf{B}_0$  and the microwave electric field are parallel and tilted by *θ* with respect to the surface normal **n**. This gives an in-plane field of  $\mathbf{E}_1(t) = -\mathbf{E}(t) \sin \theta$ . The microwave magnetic field  $\mathbf{B}_1(t)$  is directed along the (in-plane) *y* axis.

is given by the equation $11$ 

$$
\mathbf{E}_R = \frac{m^*}{e} \frac{d\mathbf{v}^{(R)}}{dt} = \frac{m^* \alpha_R}{e\hbar \mu_B} \mathbf{n} \times \frac{d\mu}{dt}.
$$
 (4)

Another consequence of the Rashba coupling is that the electron spin is affected by the SO Rashba field as well. The latter is proportional to the electron momentum velocity:

$$
\mathbf{B}_R = \frac{\alpha_R m^*}{\hbar \mu_B} \mathbf{n} \times \mathbf{v}^{(p)}.
$$
 (5)

Taking into account that the momentum velocity  $\mathbf{v}^{(p)}$  varies in time, the field which drives electron spin resonance is the sum  $\mathbf{B}_1 + \mathbf{B}_R$ . Because both  $\mathbf{B}_1$  and  $\mathbf{B}_R$  are in-plane vectors they do not directly influence the electron motion. However, they have an indirect influence on this motion through  $E_R$  defined in Eq. (4), as they drive the spin of the electron.

In the presence of the external rf electric field  $\mathbf{E}_1(t) =$ **E**<sub>1*ω*</sub> exp( $-i\omega t$ ) and similarly the magnetic field **B**<sub>1</sub>(*t*) = **B**<sub>1*ω*</sub> exp( $-i\omega t$ ) we assume **v** = **v**<sub>0</sub> + **v**<sub> $\omega$ </sub> exp( $-i\omega t$ ) and  $\mu = \mu_0 + \mu_\omega \exp(-i\omega t)$  (keeping in mind that only real parts have a physical meaning). Then, following Lax *et al.*[15](#page-4-0) we obtain from Eq. [\(3\)](#page-0-0) the equation for the Fourier amplitude of the oscillatory part of the electron velocity,

$$
\mathbf{v}_{\omega} = \frac{1}{ne} \hat{\sigma}(\omega)(\mathbf{E}_{1\omega} + \mathbf{E}_{R\omega}), \tag{6}
$$

where *n* is the surface density of the free electron gas and  $\hat{\sigma}(\omega)$ is the usual conductivity tensor, determined by the sample geometry and by the direction of  $\mathbf{B}_0$  ( $\mathbf{B}_1$  and  $\mathbf{B}_R$  are in-plane vectors). For the oscillatory term, **E***Rω*, we have

$$
\mathbf{E}_{R\omega} = -i\omega \frac{m^* \alpha_R}{e\hbar \mu_B} \mathbf{n} \times \mu_\omega.
$$
 (7)

On the other hand the equation of motion for the electron magnetic moment results from

$$
\frac{d\mu}{dt} = \gamma \left[ \frac{g}{2} (\mathbf{B}_0 + \mathbf{B}_1) + \mathbf{B}_R \right] \times \mu, \tag{8}
$$

where  $\gamma = -2\mu_B/\hbar$ . In linear approximation the solution of Eq. (8), the Fourier amplitude of the electron magnetic moment, is

$$
\boldsymbol{\mu}_{\omega} = \frac{1}{\delta n} \hat{\chi}(\omega) \bigg( \frac{g}{2} \mathbf{B}_{1\omega} + \mathbf{B}_{R\omega} \bigg), \tag{9}
$$

where *δn* [A1 in Appendix] denotes the number of electrons that occupy spin-unpaired states. The form of the tensor  $\hat{\chi}(\omega)$ [A2 in Appendix] assures that only the oscillatory part of Eq. (5),  $\mathbf{B}_{R\omega\perp}$ , which is perpendicular to  $\mathbf{B}_0$ , is essential in Eq. (8).

From Eqs. (5)–(9) one can see that the dependence of  $\mathbf{v}^{(p)}$  on the rf fields,  $\mathbf{E}_1$  and  $\mathbf{B}_1$ , is important. An approximate solution of this problem can be obtained by subtracting  $\mathbf{v}_{\omega}^{(R)}$  from both sides of Eq. (6) resulting in

$$
\mathbf{v}_{\omega}^{(p)} = \frac{1}{ne} \hat{\sigma}(\omega) \big[ \mathbf{E}_{1\omega} - ne \hat{\rho}(0) \mathbf{v}_{\omega}^{(R)} \big],\tag{10}
$$

where  $\hat{\rho}(\omega) = \hat{\sigma}^{-1}(\omega)$  is the resistivity tensor [A3 in Appendix]. Expressing the spin-dependent component of the velocity in Eq.  $(9)$  by the magnetic moment Eq.  $(8)$ , and then using an iteration method we get for the electrons that occupy spin-unpaired states

$$
\mathbf{v}_{\omega}^{(p)} \approx \frac{1}{ne} \hat{\sigma}(\omega) \left\{ [\hat{I} - \hat{\Delta}_{E}(\omega)] \mathbf{E}_{1\omega} - \hat{\Delta}(\omega) [\hat{I} - \hat{\Delta}_{B}(\omega)] \frac{g}{2} \mathbf{B}_{1\omega} \right\},
$$
(11)

where the dimensionless operators are  $\hat{\Delta}_E(\omega) = \alpha_R^2 m^* (\delta n)^{-1} (\hbar \mu_B)^{-2} \hat{\zeta}(\omega) \hat{n} \hat{\sigma}(\omega)$ , and  $\hat{\Delta}_B(\omega) =$  $\alpha_R^2 m^* (\delta n)^{-1} (\hbar \mu_B)^{-2} \hat{\zeta}(\omega) \hat{n} \hat{\sigma}(\omega)$ , and  $\hat{\Delta}_B(\omega) =$  $\alpha_R^2 m^* (\delta n)^{-1} (\hbar \mu_B)^{-2} \hat{n} \hat{\sigma}(\omega) \hat{\zeta}(\omega)$ , and  $\hat{\Lambda}(\omega) =$  $n e \alpha_R(\delta n)^{-1} (\hbar \mu_B)^{-1} \hat{\zeta}(\omega)$  with  $\hat{\zeta}(\omega) = \hat{\rho}(0) \hat{n} \hat{\chi}(\omega)$ . Here we use  $\hat{n} = -i\sigma_y$ , proportional to the Pauli spin matrix, instead of the vector product operator " $\mathbf{n} \times$ " (keeping in mind, that for arbitrary orientation of the space *n*ˆ should change sign under the space inversion).

For Si/SiGe the transverse spin relaxation time  $T_2$  is of the order of  $10^{-7}$ – $10^{-6}$  s, and the corrections  $\hat{\Delta}_E$  and  $\hat{\Delta}_B$  at the resonant frequency  $\omega = \omega_L$  are of the order of  $m^* \alpha_R^2 T_2 / \hbar^3 \approx$  $3 \times (10^{-3} - 10^{-2})$ . The factor  $\hat{\Lambda}$  is then of the order of  $1.5 \times (10^{-5} - 10^{-4})$ . Thus the influence of the magnetic field  $B_1$  on the electron momentum velocity can be negligibly small when compared to the electric field  $E_1$ . However, even if the electric field is absent, the in-plane oscillatory magnetic field causes a momentum current  $\mathbf{j}_{\omega}^{(p)} = \delta n e \mathbf{v}_{\omega}^{(p)} \approx$  $-(\delta n/n)\hat{\sigma}(\omega)\hat{\Lambda}(\omega)[\hat{I} - \hat{\Delta}_B(\omega)](g/2)B_{1\omega}$ . This current must be distinguished from eddy currents as the latter are caused by a magnetic field perpendicular to the sample plane. The current derived here appears only if some of the electrons occupy spin-unpaired states. In the opposite case, we obviously have  $\hat{\Delta}_E = \hat{\Delta}_B = \hat{\Lambda} = 0$  and  $\mathbf{j}_{\omega}^{(p)} = ne\mathbf{v}_{\omega}^{(p)} = \hat{\sigma}(\omega)\mathbf{E}_{1\omega}$ .

## **III. ELECTROMAGNETIC ABSORPTION**

The power dissipation due to the microwave field per electron can be obtained from the time derivative of the Hamiltonian (1):

$$
P(t) = \frac{\partial \hat{H}}{\partial t} = e\mathbf{E}_1 \mathbf{v} - \frac{g}{2} (d\mathbf{B}_1/dt)\boldsymbol{\mu}.
$$
 (12)

<span id="page-2-0"></span>The time averaged value of *P*,  $\lim_{T \to \infty} T^{-1} \iint_0^T P(t) dt$  can be expressed by its Fourier components as

$$
P_{\omega} = P_{\omega}^{(E)} + P_{\omega}^{(M)} = \frac{e}{2} \text{Re}(\mathbf{E}_{1\omega}^{*} \mathbf{v}_{\omega}) + \frac{\omega}{2} \frac{g}{2} \text{Im}(\mathbf{B}_{1\omega}^{*} \mu_{\omega}).
$$
 (13)

For the electrons occupying spin-paired states the velocity component  $\mathbf{v}_\omega$  in the first term of Eq. (13) is defined by Eq. [\(6\)](#page-1-0) without the spin-dependent correction  $\mathbf{E}_R$ , while the second term does not exist. Taking this into consideration, we get for the total electromagnetic power absorption (i.e., by unit area of the sample) the following equation:

$$
P_{\omega}^{(\text{tot})} = \frac{1}{2} \text{Re}[\mathbf{E}_{1\omega}^* \cdot \hat{\sigma}(\omega) \mathbf{E}_{1\omega}] + \frac{1}{2} \frac{\delta n}{n} \text{Re}[\mathbf{E}_{1\omega}^* \cdot \hat{\sigma}(\omega) \mathbf{E}_{R\omega}] + \frac{\omega}{2} \left(\frac{g}{2}\right)^2 \text{Im}[\mathbf{B}_{1\omega}^* \cdot \hat{\chi}(\omega) \mathbf{B}_{1\omega}] + \frac{\omega}{2} \frac{g}{2} \text{Im}[\mathbf{B}_{1\omega}^* \cdot \hat{\chi}(\omega) \mathbf{B}_{R\omega}], \tag{14}
$$

where the first two terms correspond to the Joule heat accompanying the spin-independent electric current  $\mathbf{j}_{\omega} = \hat{\sigma}(\omega)\mathbf{E}_{1\omega}$ , and the Rashba current  $\mathbf{j}_{R\omega} = \hat{\sigma}(\omega) \mathbf{E}_{R\omega}$ , respectively. (Notice that  $\mathbf{j}_{\omega} \neq ne\mathbf{v}_{\omega}^{(p)}$  and  $\mathbf{j}_{R\omega} \neq ne\mathbf{v}_{\omega}^{(R)}$  as  $\mathbf{v}_{\omega}^{(p)}$ , in general, is spin dependent.) The last two terms describe magnetic absorption. The first of them coincides with the classical expression, while the second one is a contribution due to the Rashba magnetic correction. The second and the fourth terms in Eq. (14) approximated up to the second order of the Rashba parameter  $\alpha_R$  are the sum of a "magnetoelectric" term:<sup>[16](#page-4-0)</sup>

$$
\delta P_{\omega}^{(1)} = \frac{m^*}{ne} \frac{\alpha_R}{\hbar \mu_B} \frac{\omega}{2} \frac{g}{2} Im[\mathbf{E}_{1\omega}^* \cdot \hat{\sigma}(\omega) \hat{n} \hat{\chi}(\omega) \mathbf{B}_{1\omega} + \mathbf{B}_{1\omega}^* \cdot \hat{\chi}(\omega) \hat{n} \hat{\sigma}(\omega) \mathbf{E}_{1\omega}],
$$
(15)

and

$$
\delta P_{\omega}^{(2)} = \left(\frac{m^*}{ne}\right)^2 \left(\frac{\alpha_R}{\hbar \mu_B}\right)^2 \frac{\omega}{2} \text{Im}\left[\mathbf{E}_{1\omega}^* \cdot \hat{\sigma}(\omega)\hat{n}\hat{\chi}(\omega)\hat{n}\hat{\sigma}(\omega)\mathbf{E}_{1\omega}\right] - \frac{ne}{\delta nm^*} \left(\frac{g}{2}\right)^2 \mathbf{B}_{1\omega}^* \cdot \hat{\chi}(\omega)\hat{n}\hat{\sigma}(\omega)\hat{\rho}(0)\hat{n}\hat{\chi}(\omega)\mathbf{B}_{1\omega}\right],
$$
(16)

where the magnetic term [second one in Eq.  $(16)$ ] in the case of the same value (in eV*/*cm) of **E**1*<sup>ω</sup>* and **B**1*<sup>ω</sup>* is about seven orders smaller  $[0.05\hbar T_2/\tau^2mc^2 \approx 6 \times (10^{-8} - 10^{-7})]$  than the electric term [first in Eq. (16)] at the resonance condition [for Si/SiGe the momentum relaxation time  $\tau \leq 10^{-11}$  s is much shorter than  $T_2 \approx (10^{-7} - 10^{-6})$  s]. The electric term is about two hundred times bigger ( $e\tau \alpha_R/\hbar \mu_B \approx 2.1 \times 10^2$ ) than the magnetoelectric one. The first term of the Eq. (16) has been recently explored as a resonant SO correction to Joule heat.<sup>[11](#page-4-0)</sup>

We note that if the sample is placed at the node of the electric or magnetic field then the magnetoelectric signal cannot be observed  $(\delta P_{\omega}^{(1)} = 0)$  for any orientation of the sample. But then there appears the possibility of separate observation of magnetic or electric signals corresponding to the appropriate term of  $\delta P_{\varphi}^{(2)}$ . In Figs. 2[–4](#page-3-0) the characteristic features of the signals  $\delta P_{\omega}^{(1)}$  and  $\delta P_{\omega}^{(2)}$  for the resonant frequency  $\omega = \omega_L$ and for different mobilities are shown. In our calculations we have assumed a real Fourier amplitude of the electric field  $\mathbf{E}_{1\omega} = [\mathcal{E} \cdot \sin \theta, 0]$  and a pure imaginary amplitude (due to the



FIG. 2. (Color online) Angular dependence of magnetoelectric *δ* $P_{\omega}^{(1)}$  signal normalized by *E B* sin *θ*.

phase shift) of the magnetic field,  $\mathbf{B}_{1\omega} = i[0,\beta]$ . The tensors  $\hat{\chi}(\omega)$  and  $\hat{\sigma}(\omega)$  are defined in the Appendix and  $\hat{n} = -i\sigma_{\nu}$ .

From Eqs. [\(5\),](#page-1-0) [\(10\)](#page-1-0) it follows that the electric field induces a Rashba field, so we can expect that some part of the absorbed electric energy due to the first term in (13) is transferred by the Zeeman channel. In fact, using the oscillatory part of Eq. [\(3\)](#page-0-0) in the two cases, spin dependent for *δn* and, respectively, spin independent for  $n - \delta n$  electrons (per cm<sup>2</sup>), and performing the summation of  $P_{\omega}^{(E)}$  in Eq. (13) over the occupied electron states due to these two cases,

$$
\sum P_{\omega}^{(E)} = \frac{e}{2} \sum \text{Re}(\mathbf{E}_{1\omega}^{*} \mathbf{v}_{\omega})
$$
  
=  $\frac{e}{2} \sum \text{Re}[\mathbf{E}_{1\omega}^{*}(ne)^{-1} \hat{\sigma}(\omega)(\mathbf{E}_{1\omega} + \mathbf{E}_{R\omega})]$   
=  $\frac{e}{2} \sum \text{Re}[(\mathbf{E}_{1\omega}^{*} + \mathbf{E}_{R\omega}^{*})(ne)^{-1} \hat{\sigma}(\omega)(\mathbf{E}_{1\omega} + \mathbf{E}_{R\omega})]$   
+  $\frac{\omega}{2} \sum \text{Im}(\mathbf{B}_{R\omega}^{*} \boldsymbol{\mu}_{\omega}),$  (17)

(the first sum on the right-hand side for the case of  $\mathbf{E}_{R\omega} = 0$ over *n* − *δn* electron states and for nonzero **E***Rω* over *δn* states, while the second sum only over *δn* states) we get

$$
\sum P_{\omega}^{(E)} = \frac{n - \delta n}{2n} \text{Re}[\mathbf{E}_{1\omega}^{*} \hat{\sigma}(\omega) \mathbf{E}_{1\omega}] \n+ \frac{\delta n}{2n} \text{Re}[(\mathbf{E}_{1\omega}^{*} + \mathbf{E}_{R\omega}^{*}) \hat{\sigma}(\omega) (\mathbf{E}_{1\omega} + \mathbf{E}_{R\omega})] \n+ \delta n \frac{\omega}{2} \text{Im}(\mathbf{B}_{R\omega}^{*} \boldsymbol{\mu}_{\omega}),
$$
\n(18)

where we have used the relation  $-(e/2)Re(E_{R\omega}^* \mathbf{v}_{\omega}^{(p)}) =$  $(\omega/2)$ Im( $\mathbf{B}_{R\omega}^*$  $\boldsymbol{\mu}_{\omega}$ ).

In analogy to Eq.  $(20)$  we obtain for the magnetic absorption [the second term of Eq.  $(13)$ ],

$$
\sum P_{\omega}^{(M)} = \frac{\omega}{2} \frac{g}{2} \sum \text{Im}(\mathbf{B}_{1\omega}^{*} \boldsymbol{\mu}_{\omega})
$$
  
\n
$$
= \delta n \frac{\omega}{2} \frac{g}{2} \text{Im} \left[ \mathbf{B}_{1\omega}^{*} (\delta n)^{-1} \hat{\chi}(\omega) \left( \frac{g}{2} \mathbf{B}_{1\omega} + \mathbf{B}_{R\omega} \right) \right]
$$
  
\n
$$
= \frac{\omega}{2} \text{Im} \left[ \left( \frac{g}{2} \mathbf{B}_{1\omega}^{*} + \mathbf{B}_{R\omega}^{*} \right) \hat{\chi}(\omega) \left( \frac{g}{2} \mathbf{B}_{1\omega} + \mathbf{B}_{R\omega} \right) \right]
$$
  
\n
$$
+ \delta n \frac{e}{2} \text{Re}(\mathbf{E}_{R\omega}^{*} \mathbf{v}_{\omega}). \tag{19}
$$

<span id="page-3-0"></span>

FIG. 3. (Color online) Angular dependence of electric-dipole ESR  $\delta P_{\omega}^{(2)}$  signal normalized by  $\mathcal{E}^2 \sin^2 \theta$ .

So, apart from the pure magnetic absorption we can separate some energy transfer by Joule heating due to the Rashba electric field correction **E***Rω* influencing spin-unpaired electrons. Thus we assume the current  $\mathbf{j}_{\omega} = \frac{\delta n}{n} \hat{\sigma}(\omega) \mathbf{E}_{R\omega}$  in the case of the absence of external rf electric field  $\mathbf{E}_{1\omega}$ . This current is induced by the in-plane oscillating magnetic field  $\mathbf{B}_1$  so it is not an eddy current.

An interesting result can be obtained from Eq. [\(17\).](#page-2-0) Combining this equation with the oscillatory part of Eq. [\(3\)](#page-0-0) we get

$$
\sum P_{\omega}^{(E)} = n \frac{m^*}{2} \langle |\mathbf{v}_{\omega}|^2 \rangle \frac{1}{\tau} + \delta n \frac{\omega}{2} \text{Im}(\mathbf{B}_{R\omega}^* \boldsymbol{\mu}_{\omega}), \qquad (20)
$$

where  $(X)$  means the average value of  $X$  over all occupied electron states.

So, the electric power absorption is equal to the oscillatory kinetic energy of the electron gas (there is a misprint in Ref. [5](#page-4-0) in the bracket sequence) transferred to the environment in the time *τ* and partially, due to the Rashba rf field, via the Zeeman channel.<sup>17</sup> As was discussed in our previous work<sup>5</sup> the Joule heat (kinetic energy of the electron gas) must be received from



FIG. 4. (Color online) Angular dependence of the magnetic  $\delta P_{\omega}^{(2)}$ signal normalized by  $\mathcal{B}^2$ .

the system to be dissipated by Rashba magnetic resonance. The minimum at  $\theta \approx 78^\circ$  in Fig. 3 corresponds to the coincidence of the cyclotron frequency,  $\omega_c \cos \theta$ , and the Larmor frequency  $\omega_L$  (maxima of the conductivity tensor elements) and thus it corresponds to the maxima of the received Joule heat. Looking at Fig. 4 we suppose that there exists an analogous transfer of energy from the Zeeman reservoir to the Joule heat due to the Rashba field **E***Rω* as in Eq. [\(19\).](#page-2-0) Unfortunately we cannot derive the equation analogous to Eq.  $(20)$ , so we cannot say anything about the role of kinetic energy in this transfer.

## **IV. CONCLUSIONS**

We analyze the Rashba effects in the frame of the Drude model which is extended by the electron spin dependencies. We have previously shown<sup>11</sup> that at electron spin resonance induced by a pure electric rf field some part of the Joule heating can be identified with magnetic absorption due to Rashba magnetic field correction. Here we demonstrate (in analogy), that in the case of pure magnetic rf field a part of the magnetic absorption may be identified with Joule heating due to Rashba electric field correction. We show, however, that an in-plane pure magnetic field can induce an in-plane electric current. This effect must be distinguished from the Faraday induction which requires perpendicular magnetic field inducing eddy currents.

## **APPENDIX**

*Number of electrons in spin-unpaired states (A1).* The surface density of electrons in spin-unpaired states is  $\delta n_e \approx$  $D(E)\mu_B B_0$ , where  $D(E) = m^*/\pi \hbar^2$  is the density of states for 2D electron gas. The magnetization of the electron gas is  $\mathbf{M}_\omega = \delta n \cdot \boldsymbol{\mu}_\omega$ .

*The susceptibility tensor (A2).* This 2D tensor connects the in-plane coordinates of the Rashba field with the in-plane coordinates of the magnetization **M***ω*.

$$
\hat{\chi} = \frac{1}{2} \begin{bmatrix} (\chi_+ + \chi_-) \cos^2 \theta & -i(\chi_+ - \chi_-) \cos \theta \\ i(\chi_+ - \chi_-) \cos \theta & \chi_+ + \chi_- \end{bmatrix},
$$

In the rotating coordinate system  $(\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})/\sqrt{2}$  (with  $\hat{\mathbf{z}}$ parallel to **B**<sub>0</sub>)  $\hat{\chi}$  is diagonal with the components  $\chi_{\pm}$  =  $\mp \gamma M_0 / (\omega \mp \omega_L + i/T_2)$  and Eq. [\(8\)](#page-1-0) has the simple form  $\mu_{\omega\pm} = (\delta n)^{-1} \chi_{\pm}(\omega) (\frac{g}{2} B_{1\omega\pm} + B_{R\omega\pm}),$  where for the vector  $\mu_{\omega}$  (and likewise for **B**<sub>1*ω*</sub> and **B**<sub>*Rω*</sub>)  $\mu_{\omega\pm} = (\mu_{\omega x} \mp i\mu_{\omega y})/\sqrt{2}$ . The static magnetization  $M_0 = \chi_0 B_0$  is defined by the Pauli paramagnetic susceptibility constant which for a 2D electron gas is equal to  $\chi_0 = D(E) \cdot \mu_B^2$ . The Larmor frequency is  $\omega_L =$ *γgB*0*/*2.

*The resistivity tensor (A3).* The 2D conductivity tensor for the configuration shown in Fig. [1](#page-1-0) is  $\hat{\sigma}(\omega)$  =  $\sigma_0[\hat{I}(1 - i\omega\tau) - i\hat{\sigma}_y\omega_c\tau\cos\theta]/[(1 - i\omega\tau)^2 + \omega_c^2\tau^2\cos^2\theta],$ where  $\sigma_0 = ne^2\tau/m^*$  is the Drude conductivity,  $\omega_c = -eB_0/m^*c > 0$  is the cyclotron frequency, and  $\hat{\sigma}_v$ is the Pauli spin matrix. The resistivity tensor  $\hat{\rho}(\omega)$  is then  $\hat{\rho}(\omega) = \sigma_0^{-1} [\hat{I}(1 - i\omega\tau) + i\hat{\sigma}_y \omega_c \tau \cos \theta].$ 

<span id="page-4-0"></span>\* Corresponding author: ungier@ifpan.edu.pl

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