Chiral anomaly and classical negative magnetoresistance of Weyl metals

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We consider the classical magnetoresistance of a Weyl metal in which the electron Fermi surface possesses nonzero fluxes of the Berry curvature. Such a system may exhibit large negative magnetoresistance with unusual anisotropy as a function of the angle between the electric and magnetic fields. In this case the system can support an additional type of plasma wave. These phenomena are consequences of the chiral anomaly in electron transport theory.

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Materials with nontrivial topological properties have attracted considerable interest since the discovery of topological insulators.[1] One type of such materials is the so-called Weyl semimetals, characterized by the the presence of points of band touching (Dirac points).[2–10] In this paper, we study the metallic counterparts of these materials—the Weyl metals, where Dirac points are hidden inside a Fermi surface. We show that these materials may exhibit large negative magnetoresistance with unusual anisotropy. This negative magnetoresistance is connected to the triangle anomaly and a related effect—the chiral magnetic effect, but it occurs in the classical regime, where the electron mean free path is short compared to the magnetic length. We also find an additional type of plasma wave in these systems.

At low magnetic field $\mathbf{B}$ and at relatively high temperature Landau quantization can be neglected, and electron transport in metals can be described using the semiclassical Boltzmann kinetic equation

$$\frac{\partial n_{\mathbf{p}}}{\partial t} + \dot{\mathbf{r}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{r}} + \dot{\mathbf{p}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} = I_{\mathrm{coll}}\{n_{\mathbf{p}}\}. \tag{1}$$

Here $n_{\mathbf{p}}(\mathbf{r},t)$ is the electron distribution function, $\mathbf{p}$ is the quasimomentum, $I_{\mathrm{coll}}\{n_{\mathbf{p}}\}$ is the collision integral, and

$$\dot{\mathbf{r}} = \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} + \dot{\mathbf{p}} \times \mathbf{\Omega}_{\mathbf{p}}, \tag{2a}$$

$$\dot{\mathbf{p}} = e\mathbf{E} + \frac{e}{c}\dot{\mathbf{r}} \times \mathbf{B}. \tag{2b}$$

The last, "anomalous," term in Eq. (2a), proportional to the Berry curvature

$$\mathbf{\Omega}_{\mathbf{p}} = \nabla_{\mathbf{p}} \times \mathbf{A}_{\mathbf{p}}, \quad \mathbf{A}_{\mathbf{p}} = i\langle u_{\mathbf{p}}|\nabla_{\mathbf{p}} u_{\mathbf{p}}\rangle, \tag{3}$$

was introduced in Ref. 11. (See also reviews on the subject in Refs. 12 and 13.) In systems with time-reversal symmetry $\mathbf{\Omega}_{\mathbf{p}} = \mathbf{\Omega}_{-\mathbf{p}}$, while in centrosymmetric systems $\mathbf{\Omega}_{\mathbf{p}} = -\mathbf{\Omega}_{-\mathbf{p}}$. Thus, in systems which are both time and centrosymmetric $\mathbf{\Omega}_{\mathbf{p}} = \mathbf{0}$. In this case the magnetoresistance described by Eq. (1) is positive and is governed by the parameter $(\omega_c \tau_{\mathrm{tr}})^2$.[14] Here ω_c is the cyclotron frequency and τ_{tr} is the electron transport mean free path. The Berry curvature is divergence-free except at isolated points in $\mathbf{p}$ space, which are associated with band degeneracies. As a result, in the case where the electronic spectrum has several valleys, they can be characterized by integers (see, for example, Ref. 15)

$$k^{(i)} = \frac{1}{2\pi\hbar}\oint d\mathbf{S} \cdot \mathbf{\Omega}_{\mathbf{p}}^{(i)} = 0, \pm 1, \ldots. \tag{4}$$

Here the index "i" labels the valleys, and $d\mathbf{S}$ is the elementary area vector. Nonzero values of $k^{(i)}$ are realized if, near the degeneracy points, electrons can be described by the massless Dirac Hamiltonian[16]

$$H = \pm v\boldsymbol{\sigma} \cdot \left(-\hbar\nabla - \frac{e}{c}\mathbf{A} \right). \tag{5}$$

Here $\mathbf{A}$ is the vector potential, $\boldsymbol{\sigma}$ is the operator of pseudospin, v is the quasiparticle velocity, and the signs $\pm$ correspond to the different chiralities of the Weyl fermions.

It is well known that massless Dirac fermions exhibit a chiral anomaly which can be understood in the language of level crossing in the presence of a magnetic field.[17] According to the Nielsen-Ninomiya theorem,[18] the number of valleys with opposite chiralities (positive and negative values of $k^{(i)}$) should be equal, and so $\sum_i k^{(i)} = 0$. Recently, gapless semiconductors with topologically protected Dirac points (Weyl semimetals) have attracted significant attention.[2–10] Both time-reversal-breaking[2] and noncentrosymmetric[4] versions of these systems have been proposed. In the absence of a random potential and doping, the chemical potentials in these systems are at the Dirac points.

In this article we consider the case where in equilibrium the chemical potential $\mu = \mu_i$ measured from the Dirac points is finite, and show that the semiclassical Eqs. (1) and (1) can yield a substantial anomaly-related negative magnetoresistance. The latter also exhibits unusual anisotropy as a function of the angle θ between $\mathbf{E}$ and $\mathbf{B}$. Here μ_i is the chemical potential in the ith valley, measured from the Weyl point. Using Eqs. (2) we get

$$\begin{aligned} \dot{\mathbf{r}} &= \left(1 + \frac{e}{c}\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}\right)^{-1} \left[\mathbf{v} + e\mathbf{E} \times \mathbf{\Omega}_{\mathbf{p}} + \frac{e}{c}(\mathbf{\Omega}_{\mathbf{p}} \cdot \mathbf{v})\mathbf{B}\right], \\ \dot{\mathbf{p}} &= \left(1 + \frac{e}{c}\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}\right)^{-1} \left[e\mathbf{E} + \frac{e}{c}\mathbf{v} \times \mathbf{B} + \frac{e^2}{c}(\mathbf{E} \cdot \mathbf{B})\mathbf{\Omega}_{\mathbf{p}}\right], \end{aligned} \tag{6}$$

where $\mathbf{v} = \partial\epsilon_{\mathbf{p}}/\partial\mathbf{p}$. Substituting Eqs. (6) into Eq. (1) we get the kinetic equation in the form

$$\frac{\partial n_{\mathbf{p}}^{(i)}}{\partial t} + \left(1 + \frac{e}{c}\mathbf{B}\cdot\mathbf{\Omega}_{\mathbf{p}}^{(i)}\right)^{-1}\left[\left(e\mathbf{E} + \frac{e}{c}\mathbf{v}\times\mathbf{B} + \frac{e^2}{c}(\mathbf{E}\cdot\mathbf{B})\Omega_{\mathbf{p}}^{(i)}\right)\frac{\partial n_{\mathbf{p}}^{(i)}}{\partial\mathbf{p}} + \left(\mathbf{v} + e\mathbf{E}\times\mathbf{\Omega}_{\mathbf{p}}^{(i)} + \frac{e}{c}(\mathbf{\Omega}_{\mathbf{p}}^{(i)}\cdot\mathbf{v})\mathbf{B}\right)\frac{\partial n_{\mathbf{p}}^{(i)}}{\partial\mathbf{r}}\right] = I_{\text{coll}}^{(i)}\{n_{\mathbf{p}}^{(i)}\} \tag{7}$$

(cf. Ref. 19). Let us consider the case where $\mu \gg T$ and $\hbar\omega_c = \hbar|e|v^2B/c\mu$, and assume that the conductivity of the system is determined by elastic scattering. Then the collision integral I_{coll} in Eq. (7) describes elastic intra- and intervalley scattering characterized by intravalley τ_{intr} and intervalley τ scattering times, respectively. If $\mu\tau_{\text{intr}},\mu\tau \gg 1$, then these characteristic times can be calculated using standard perturbation theory, while all interference corrections to these quantities can be neglected. We assume that $\tau_{\text{intr}} \ll \tau$, the anisotropy of the distribution function within each valley can be neglected, and the latter depends only on the energy ϵ: $n_{\mathbf{p}}^{(i)} = n^{(i)}(\epsilon)$. In this case the transport scattering time in the absence of magnetic field τ_{tr} is essentially τ_{intr}. Denoting by $\rho^{(i)}(\epsilon)$ the density of states,[12,13]

$$\rho^{(i)}(\epsilon) = \int\frac{d\mathbf{p}}{(2\pi\hbar)^3}\left(1 + \frac{e}{c}\mathbf{B}\cdot\mathbf{\Omega}_{\mathbf{p}}^{(i)}\right)\delta(\epsilon_{\mathbf{p}} - \epsilon), \tag{8}$$

in the homogeneous case we get the kinetic equation in the form

$$\frac{\partial n^{(i)}(\epsilon)}{\partial t} + \frac{k^{(i)}}{\rho^{(i)}(\epsilon)}\frac{e^2}{4\pi^2\hbar^2c}(\mathbf{E}\cdot\mathbf{B})\frac{\partial n^{(i)}(\epsilon)}{\partial\epsilon} = I_{\text{coll}}^{(i)}\{n^{(i)}(\epsilon)\}, \tag{9}$$

where the collision integral now includes only intervalley scattering. For this, we will use the relaxation time approximation

$$I_{\text{coll}}^{(i)} = -\frac{\delta n^{(i)}(\epsilon)}{\tau}, \tag{10}$$

where $\delta n^i(\epsilon)$ is the deviation of the distribution function from its equilibrium value.

The electron density and entropy density in the ith valley are

$$N^{(i)} = \int d\epsilon\,\rho^{(i)}(\epsilon)n^{(i)}(\epsilon), \tag{11}$$

$$S^{(i)} = -\int d\epsilon\,\rho^{(i)}(\epsilon)\{(n^{(i)}(\epsilon)\ln n^{(i)}(\epsilon) + [1 - n^{(i)}(\epsilon)]\ln[1 - n^{(i)}(\epsilon)]\}. \tag{12}$$

Integrating Eq. (9) over $\rho^{(i)}(\epsilon)d\epsilon$ we get the conservation law for particle number in each valley:

$$\frac{\partial N^{(i)}}{\partial t} + \nabla\cdot\mathbf{j}^{(i)} = k^{(i)}\frac{e^2}{4\pi^2\hbar^2c}(\mathbf{E}\cdot\mathbf{B}) - \frac{\delta N^{(i)}}{\tau}, \tag{13}$$

$$\mathbf{j}^{(i)} = \int\frac{d\mathbf{p}}{(2\pi\hbar)^3}\left[\mathbf{v} + e\mathbf{E}\times\mathbf{\Omega}_{\mathbf{p}}^{(i)} + \frac{e}{c}(\mathbf{\Omega}_{\mathbf{p}}^{(i)}\cdot\mathbf{v})\mathbf{B}\right]n_{\mathbf{p}}^{(i)}. \tag{14}$$

Thus, in the presence of electric and magnetic fields, the number of particles in the ith valley, $N^{(i)}$, is not conserved even if $\tau \to \infty$. This is the chiral anomaly which was originally introduced in field theory in Refs. 20, and later discussed in the context of electron band structure theory in Ref. 17, and in the theory of superfluid ^{3}He.[21,22] It is interesting that the anomaly can be understood completely in the framework of the semiclassical kinetic equation (1), characterized by $k^{(i)}$,[23] and that the term proportional to $\mathbf{E}\cdot\mathbf{B}$ in Eq. (13) is the same as obtained in Ref. 17 in the ultraquantum limit.

Equation (7) represents a low-energy effective theory. To see why the number of electrons in an individual valley is not conserved one has to take into account the spectral flow process which brings the energy levels (together with electrons occupying them) from one Dirac point to another through the bulk of the valence band, as schematically shown in Fig. 1. Such a possibility exists only in the presence of a magnetic field.

The existence of the chiral anomaly results in a rather unusual mechanism for the negative magnetoresistance. The easiest way to calculate the magnitude of the effect is to estimate the rate of entropy production in the presence of an electric field,

$$\dot{S} = \sum_i\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}\frac{\left(\delta n_{\mathbf{p}}^{(i)}\right)^2}{\tau}\frac{1}{n_{\mathbf{p}}^0\left(1 - n_{\mathbf{p}}^0\right)} = \frac{\sigma E^2}{T}. \tag{15}$$

At small $\mathbf{E}$, the stationary solution to Eq. (9) is

$$\delta n^{(i)}(\epsilon) = -\frac{k^{(i)}}{\rho^{(i)}(\epsilon)}\frac{e^2\tau}{4\pi^2\hbar^2c}(\mathbf{E}\cdot\mathbf{B})\frac{\partial n_0(\epsilon)}{\partial\epsilon}. \tag{16}$$

For simplicity let us assume there are only two valleys, with $k_{1,2} = \pm 1$, and with the same quasiparticle velocity v. Let the z axis be parallel to $\mathbf{B}$. Then, from Eqs. (15) and (16) we get an anomaly-related contribution to the component σ_{zz} of the

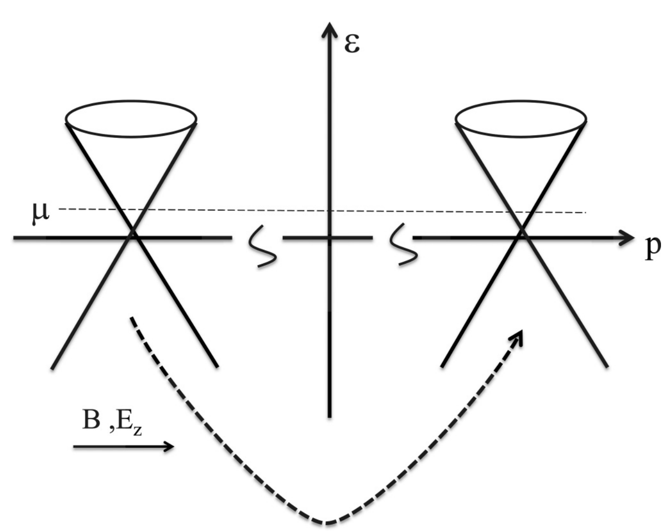

FIG. 1. Schematic three-dimensional electron spectrum in a Weyl metal. Only two valleys in the electron spectrum are shown. The dashed line indicates the direction of the electron spectral flow in the presence of parallel electric and magnetic fields.

conductivity tensor,

$$\sigma_{zz} = \frac{e^2}{4\pi^2\hbar c}\frac{v}{c}\frac{(eB)^2v^2}{\mu^2}\tau. \tag{17}$$

Note that σ_{zz} given by Eq. (17) is an increasing function of the magnetic field. All other anomaly-related components of the conductivity tensor σ_{ij} are zero. In other words, the anomaly-related current can flow only in the direction of $\mathbf{B}$. One can also understand this fact by noticing that, at $\mathbf{E} = \mathbf{0}$, and in the presence of a magnetic field, Eq. (14) gives an expression for the current density (the chiral magnetic effect)[24,25]

$$\mathbf{j} = e\sum_i \mathbf{j}_i = \frac{e^2}{4\pi^2\hbar^2 c}\mathbf{B}\sum_i k^{(i)}\mu^{(i)}. \tag{18}$$

Here we assume that the electron distribution functions in the individual valleys have equilibrium forms. In the case of a global equilibrium, all $\mu_i = \mu$, and the contributions to Eq. (18) from different valleys cancel each other. According to Eq. (13), in the presence of electric and magnetic fields, an imbalance of electron populations and, consequently, a difference between the the chemical potentials μ_i is created. As a result, there is a finite current density, which can relax only via intravalley scattering. In agreement with Eq. (17), its value is proportional to τ, its direction is parallel to $\mathbf{B}$, and it responds only to the component of the electric field parallel to $\mathbf{B}$.

There is a significant difference between the anomaly-related [Eq. (17)] and the conventional Drude contributions $\sigma_{ij}^{(D)}(\mathbf{B})$ to the $\mathbf{B}$ dependence of the conductivity tensor. For an isotropic Fermi surface and in the relaxation time approximation, all components of $\sigma_{ij}^{(D)}$, except for $\sigma_{zz}^{(D)}$, are decreasing functions of B. For an anisotropic Fermi surface, at $(\omega_c\tau_{tr})^2 \ll 1$, there is a $\mathbf{B}$ dependence of $\sigma_{zz}^{(D)}$ as well, which can be estimated as

$$\sigma_{zz}^{(D)}(0) - \sigma_{zz}^{(D)}(\mathbf{B}) \sim \sigma_{zz}^{(D)}(0)(\omega_c\tau_{tr})^2. \tag{19}$$

Here $\sigma^{(D)}(0) = e^2\nu v^2\tau_{tr}/3$ is the Drude conductivity, and $\nu \sim \mu^2/v^3$ is the density of states at the Fermi level. For small magnetic fields, both the Drude and the anomalous contributions to the resistance scale as B^2, and the anomaly-related contribution [Eq. (17)] dominates the magnetoresistance, provided that

$$\frac{\tau}{\tau_{tr}}\frac{1}{(\mu\tau_{tr})^2} > 1. \tag{20}$$

Generically, in small-gap semiconductors, the parameter $\tau/\tau_{tr} \gg 1$, because the intervalley scattering requires a large momentum transfer. If the scattering potential is smooth, this parameter becomes exponentially large. Even in the case of an anisotropic Fermi surface, depending on symmetry there could be a direction of $\mathbf{E}$ for which $\sigma_{zz}^{(D)}$ is independent of $\mathbf{B}$. At small values of $\mu \ll T$ the conductivity is determined by electron-hole scattering.[6,7] In this case one has to substitute μ for T in Eqs. (17) and (20), while the parameter $\tau/\tau_{tr} \gg 1$ is exponentially large.

For $\omega_c\tau_{tr} \gg 1$, the $\mathbf{B}$ dependence of $\sigma_{zz}^{(D)}(\mathbf{B})$ saturates and it becomes independent of $\mathbf{B}$. In contrast, the deviation of the anomaly-related contribution from the quadratic-in-$\mathbf{B}$ behavior takes place at much higher magnetic fields. Thus, σ_{zz} could be a nonmonotonic function of $\mathbf{B}$. Finally, the anomaly-related contribution to the conductivity tensor may be distinguished by its unusual frequency dependence: it is controlled by the parameter $(\omega\tau)^2$, rather than by the conventional parameter $(\omega\tau_{tr})^2$. Here ω is the frequency of the electric field.

At low values of μ the anomaly-related contribution to the conductivity can be even larger than the Drude contribution $\sigma^{(D)}$. In this case the system supports an additional type of weakly damped plasma wave with a frequency

$$\omega_p \sim \pm\sqrt{\frac{e^2}{\pi\hbar c}\frac{v}{c}\frac{eBv}{T}}, \quad \mu = 0, \tag{21}$$

provided that $\omega_p \gg \tau^{-1}$.

The approach based on the semiclassical equations of motion, Eqs. (1) and (2), is valid if $\mu \gg \hbar\omega_c$. In the opposite, ultraquantum, limit $\omega_c\tau_{tr} \gg 1$, the anomaly-related negative magnetoresistance has been previously discussed in Refs. 17 and 26. In this case the spectrum of the Dirac equation has the form

$$\epsilon_n(p_z) = \pm v\sqrt{2n\frac{\hbar e}{c}B + p_z^2}, \quad n = 1,2,\ldots. \tag{22}$$

For the $n = 0$ case $\epsilon_0 = \pm v p_z$, where $\pm$ correspond to different valleys. In other words, the $n = 0$ Landau level is chiral: the branches of the spectrum with $\epsilon_0 = \pm v p_z$ correspond to different valleys, as shown in Fig. 2. Consider the case where both the chemical potential and the temperature are small compared to the energy difference between the zeroth and the first Landau levels, i.e., $\mu, T < \hbar v/L_B$, where $L_B = \sqrt{\hbar c/eB}$ is the magnetic length. In this case only chiral branches of the spectrum are occupied by electrons. Contributions to the current from branches of the spectrum with different chiralities can relax only by intervalley scattering processes characterized by τ. If the electric field is applied in the z direction, electrons move according to the law $\dot{p}_z = eE_z - p_z/\tau$, $v_z = \pm v$, and we get the following expression for the conductivity:[17]

$$\sigma_{zz} = \frac{\tau e^2 v}{4\pi^2\hbar L_B^2}. \tag{23}$$

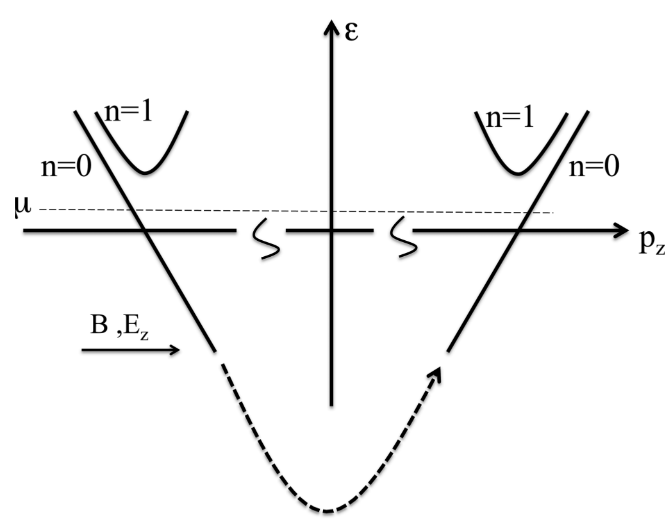

FIG. 2. Schematic electron spectrum of a Weyl metal in the ultraquantum limit. $n = 0,1$ label Landau levels. The dashed line indicates the direction of the electron spectral flow in p_z space in the presence of a z component of the electric field.

By the same token we can obtain an expression for the plasma frequency at zero wave vector,

$$\omega_{\rm p}^2 = r_s \frac{2v^2}{\pi L_B^2}, \tag{24}$$

where $r_s = e^2/\kappa\hbar v$, and κ is the dielectric constant. Equation (24) is valid if $r_s < 1$.

The fact that Eq. (24) does not have a classical limit ($\hbar \to 0$) is a particular example of a general property of collective modes in the massless Dirac plasma.[27] We note, however, that in three dimensions and at $\mathbf{B} = \mathbf{0}$ the plasma frequency is proportional to μ. In contrast, Eq. (24) is independent of the value of μ and remains finite even when $\mu = 0$.

Equations (23) and (24) are valid only for Weyl metals where time-reversal symmetry is preserved. In systems with no time-reversal symmetry contributions to σ_{zz} linear in $\mathbf{B}$ are allowed. The magnitude of these contributions is not universal and depends on the details of the mechanism of time-reversal-symmetry violation.

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