

**$\nu = 5/2$  fractional quantum Hall state in low-mobility electron systems: Different roles of disorder**

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We report the observation of a fully developed fractional quantum Hall state at  $\nu = 5/2$  in GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As quantum wells with mobility well below  $10^7$  cm<sup>2</sup>/Vs. This is achieved either by strong illumination or reducing the barrier Al composition without illumination. We explain both results in terms of screening of the ionized remote impurity (RI) potential by nearby neutral shallow donors. Despite the dramatic improvement in the transport features, the energy gap  $\Delta_{5/2}$  is limited to a rather small value ( $\sim 100$  mK), which indicates that once the RI potential is well screened and the  $5/2$  state emerges, the size of  $\Delta_{5/2}$  is limited by the mobility, i.e., by background impurities.

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The fractional quantum Hall (FQH) state at even-denominator filling factor  $\nu = 5/2$  in the  $N = 1$  first-excited Landau level (LL), whose origin has remained enigmatic since its discovery over 20 years ago,<sup>1</sup> is currently the focus of extensive studies. This is mainly because it constitutes a prime candidate for a non-Abelian state of matter<sup>2</sup> and is considered as a potential platform for implementing topological quantum computation.<sup>3</sup> Experimental access to the  $\nu = 5/2$  state, however, has been severely limited by the extraordinary requirements imposed on the sample quality by the small energy scale involved. Indeed, it was only after a sample with very high mobility of  $\mu = 1.7 \times 10^7$  cm<sup>2</sup>/Vs became available that a fully developed  $\nu = 5/2$  state with exact quantization was demonstrated.<sup>4</sup>

Disorder in remote-doped two-dimensional electron systems (2DESs) has two main sources: ionized remote impurities (RIs) and background impurities (BIs).<sup>5</sup> After nontrivial technological advances in molecular-beam epitaxy reducing BIs to achieve  $\mu \sim 1.7 \times 10^7$  cm<sup>2</sup>/Vs, the ultra-high-mobility regime of  $\mu \sim 3 \times 10^7$  cm<sup>2</sup>/Vs has become accessible by replacing the conventional one-side-doped single heterostructure by a quantum well (QW) doped from both sides.<sup>6</sup> This allows one to place RIs further away from the 2DES while keeping the same electron density  $n_s$ . For such structures with a typical setback distance of 100 nm, the contribution of RI scattering to  $\mu$  is minor and  $\mu$  is dominated by BI scattering.<sup>5</sup> The reduced disorder has led to the emergence of new correlated states in the  $N = 1$  LL.<sup>7-9</sup> At the same time, the  $\nu = 5/2$  energy gap  $\Delta_{5/2}$  has been observed to increase<sup>10,11</sup> and effects of disorder on  $\Delta_{5/2}$  have been discussed.<sup>10-13</sup> Although it is widely believed that  $\mu \geq 10^7$  cm<sup>2</sup>/Vs is necessary for observing a well-developed  $5/2$  state, the exact criteria dictating its emergence and the mechanism limiting  $\Delta_{5/2}$  are still unknown.

In this paper, we demonstrate that a fully developed  $\nu = 5/2$  state can be observed in GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As QWs with  $\mu < 10^7$  cm<sup>2</sup>/Vs, without the need for illumination or special doping schemes.<sup>14,15</sup> We first show that, by strong illumination, a fully developed  $\nu = 5/2$  state can be established in a sample with  $x = 0.34$  and  $\mu$  as low as  $4.8 \times 10^6$  cm<sup>2</sup>/Vs. Furthermore, we show that, instead of illumination, reducing the barrier Al composition systematically improves the FQH features, leading to a dramatic emergence of a fully developed  $5/2$  state at  $x = 0.25$ . We explain both results in terms of

screening of RI potential by nearby neutral shallow donors. Activation measurements reveal different roles of disorder in dictating the emergence of the  $5/2$  state and limiting  $\Delta_{5/2}$ .

We studied samples with a standard structure, 30-nm-wide GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As QWs modulation doped with Si from both sides at setback distances of 100 nm above and 120 nm below the QW. We employed conventional delta doping in the Al<sub>x</sub>Ga<sub>1-x</sub>As alloy. A series of samples with different  $x$  and Si sheet doping concentrations ( $N_{\text{Si}}$ ) were grown. Here,  $x$  was determined from low-temperature photoluminescence using the energy of the exciton transition from the Al<sub>x</sub>Ga<sub>1-x</sub>As barrier.<sup>16</sup> Transport experiments were performed on 4-mm square specimens with InSn (50:50) contacts diffused at each corner. A standard lock-in technique with excitation current of 20 nA and frequency ranging from 3 to 17 Hz was used. The samples were cooled in the mixing chamber of a dilution refrigerator with a base temperature well below 20 mK.

We first show effects of illumination on the FQH features in the range  $2 < \nu < 3$ . Figure 1(a) shows the longitudinal ( $R_{xx}$ ) and Hall ( $R_{xy}$ ) resistances of a sample with  $x = 0.34$  and  $N_{\text{Si}} = 2 \times 10^{12}$  cm<sup>-2</sup>, taken without illumination. As expected for the relatively low  $\mu$  of this sample ( $\mu = 4 \times 10^6$  cm<sup>2</sup>/Vs at  $n_s = 2.48 \times 10^{11}$  cm<sup>-2</sup>), only a poorly developed minimum at  $\nu = 5/2$  is observed.  $R_{xx}$  at  $\nu = 5/2$  is not thermally activated and the minimum appears only as a result of the flanks that rise at low temperatures.<sup>1,17</sup> As the sample is successively illuminated with a red LED and  $n_s$  is increased, the visibility of the FQH features continues to improve until  $n_s$  reaches  $3.60 \times 10^{11}$  cm<sup>-2</sup> with  $\mu = 4.8 \times 10^6$  cm<sup>2</sup>/Vs, where it saturates and no longer increases upon further illumination at moderate intensity. Under these conditions, clear  $R_{xx}$  minima become visible at  $\nu = 8/3$  and  $7/3$  as well as at  $\nu = 5/2$  [Fig. 1(b)]. This level of improvement achieved with moderate illumination (i.e., LED current of a few mA) is common and is believed to be a consequence of the larger Coulomb energy and improved  $\mu$ , both reflecting the higher  $n_s$ . However, the observed FQH features are not yet fully developed; the  $R_{xx}$  minima do not tend to zero and the plateaus in  $R_{xy}$  are not entirely defined even at the lowest temperature ( $T \sim 10$  mK).

A dramatic improvement is observed when the sample is further illuminated at much higher intensity (i.e., LED current of  $\sim 15$  mA) [Fig. 1(c)]. Now the FQH states at  $\nu = 5/2$ ,  $8/3$ , and  $7/3$  are fully developed, with  $R_{xx}$  going to

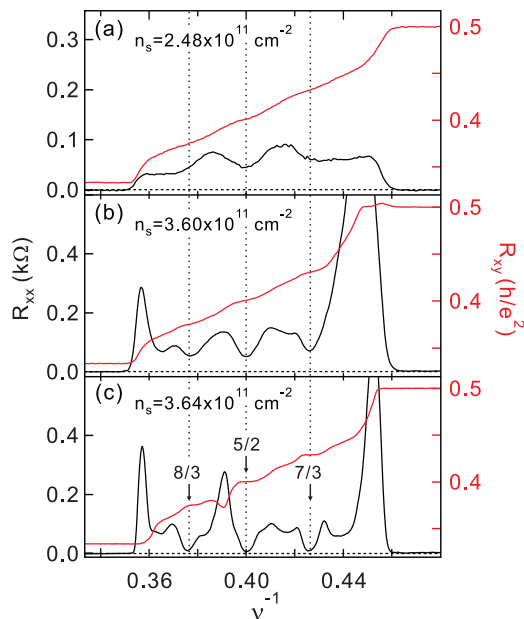


FIG. 1. (Color online)  $R_{xx}$  and  $R_{xy}$  of the  $x = 0.34$  sample. (a) Before illumination, (b) after moderate illumination, and (c) after strong illumination.  $T$  is well below 20 mK. All data were taken with the same contact configurations.

zero and  $R_{xy}$  showing well-developed plateaus. Temperature-dependent measurements demonstrate that  $R_{xx}$  at  $\nu = 5/2$  follows an activated behavior  $R_{xx} \propto \exp(-\Delta_{5/2}/2T)$ , from which an energy gap of  $\Delta_{5/2} = 93$  mK can be estimated. Note that the above scenario, commonly used to account for the improvement in the FQH features by illumination, does not apply to the present case, because  $n_s$  increased only by 1%, and accordingly,  $\mu$  barely changed with the strong illumination. These observations suggest that the type of disorder that limits the mobility and the one that governs the emergence of a fully developed  $5/2$  state are distinct. Although a rather poor correlation between  $\mu$  and the appearance of the  $5/2$  state was previously noted in the ultra-high- $\mu$  regime ( $\gtrsim 3 \times 10^7$  cm<sup>2</sup>/Vs),<sup>14</sup> the value of  $\mu$  here is far outside that range and, indeed, lower than any of those at which a fully developed  $\nu = 5/2$  state has ever been reported.

To clarify the mechanism underlying the dramatic improvement in the FQH features, we first note the fact that Si donors in  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  ( $0.22 < x < 0.4$ ) can exist in two different states: a substitutional hydrogenic shallow donor state and a deep center called a “DX center” accompanying lattice relaxation.<sup>18</sup> When cooled in the dark, almost all Si donors become DX centers for  $0.3 \lesssim x < 0.4$ , which can be transformed into shallow donors by illumination at low  $T$ . This results in an increase in  $n_s$  because shallow donors have a larger energy offset with respect to the QW state in GaAs. A second important fact is that  $N_{\text{Si}} \gg n_s$ , so that the saturation of  $n_s$  does not necessarily imply that all DX centers have been transformed into shallow donors. When placed in a slowly varying potential created by ionized donors, shallow donors ( $d^0$ ) may become polarized by displacing their electronic wave functions, thereby screening the disorder potential. The essence of our scenario is that while DX centers have almost no screening capability due to their strongly localized wave

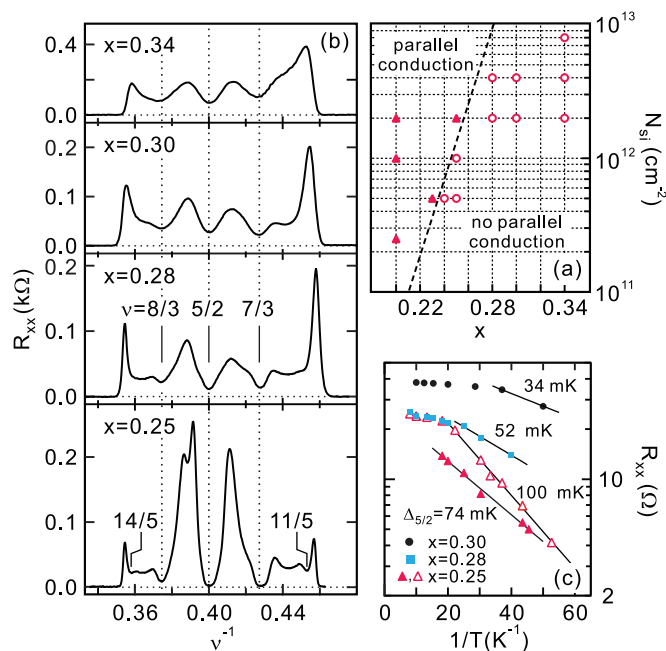


FIG. 2. (Color online) (a) Mapping of samples in the  $x$ - $N_{\text{Si}}$  plane indicating the presence or absence of parallel conduction without illumination. (b)  $R_{xx}$  taken without illumination from samples with different  $x$  ( $N_{\text{Si}} = 1$ – $2 \times 10^{12}$  cm<sup>-2</sup>). From top to bottom,  $n_s = 2.48, 2.57, 2.73$ , and  $2.65 \times 10^{11}$  cm<sup>-2</sup>.  $T$  is well below 20 mK. For  $x = 0.34$ , data taken with a different configuration than in Fig. 1(a) are shown. (c) Arrhenius plot of  $R_{xx}$  at  $\nu = 5/2$  for samples with different  $x$ . Open triangles show results for the sample shown in Fig. 3(b). Lines indicate slopes corresponding to  $\Delta_{5/2} = 100, 74, 52$ , and 34 mK.

functions, shallow donors may provide good screening due to their much more extended wave functions. Hence, the dramatic improvement in the FQH features after strong illumination could be explained by the enhanced screening provided by an increased number of  $d^0$ .

DX centers lie deep in the band gap of  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  for  $x \sim 0.4$  and gradually approach the conduction band bottom with decreasing  $x$ , being taken over by the hydrogenic donor level for  $x < 0.22$ .<sup>18</sup> Hence, by changing  $x$  in the barrier, one expects that the donor state can be varied in a more controlled manner than is possible by illumination. Figure 2(a) maps the samples we investigated for this purpose in the  $x$ - $N_{\text{Si}}$  plane. As  $x$  is decreased for a given  $N_{\text{Si}}$ , even without illumination, parallel conduction sets in, whose onset depends critically on  $x$  and  $N_{\text{Si}}$ . The onset of parallel conduction provides a measure of the wave-function overlap between neighboring donors, which reflects the average donor distance ( $\propto N_{\text{Si}}^{-1/2}$ ) and the spatial extent of individual donor wave functions. The results shown in Fig. 2(a) reveal that the donor state drastically changes with  $x$ , even in the range of  $x > 0.22$  where DX centers are believed to be mostly relevant.<sup>19</sup>

Figure 2(b) depicts  $R_{xx}$  taken without illumination from four samples with different  $x$  ranging from 0.34 to 0.25. These samples have the same  $N_{\text{Si}} = 2 \times 10^{12}$  cm<sup>-2</sup> except the one with  $x = 0.25$ , for which  $N_{\text{Si}}$  was reduced to  $1 \times 10^{12}$  cm<sup>-2</sup> ( $\equiv N_0$ ) to avoid parallel conduction [Fig. 2(a)]. The data clearly demonstrate a systematic improvement of the FQH

features with decreasing  $x$ . The most dramatic change occurs when  $x$  is decreased from 0.28 to 0.25, where the FQH states at  $\nu = 5/2, 8/3$ , and  $7/3$  become fully developed. The very low disorder in the  $x = 0.25$  sample is also seen from the emerging FQH features at  $\nu = 11/5$  and  $14/5$  and from an additional minimum between  $\nu = 5/2$  and  $8/3$ , which is a precursor to the re-entrant insulating state.<sup>7,8</sup> Note that all four samples have similar  $n_s$  of  $(2.61 \pm 0.13) \times 10^{11} \text{ cm}^{-2}$ , while  $\mu$  varies as  $4.0, 5.0, 5.5$ , and  $6.4 \times 10^6 \text{ cm}^2/\text{Vs}$  for  $x = 0.34, 0.30, 0.28$ , and  $0.25$ . This change in  $\mu$  is considered to reflect the BI concentration in the  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  barriers, which generally tends to decrease with decreasing  $x$ . Figure 2(c) shows activated behavior at  $\nu = 5/2$ , from which  $\Delta_{5/2} = 74 \text{ mK}$  is estimated for the  $x = 0.25$  sample. Samples with  $x = 0.28$  and  $0.30$  also show activated behavior, but with smaller  $\Delta_{5/2}$ .

As we already discussed,  $\mu$  is unlikely to be the reason for the dramatic change in the FQH features with  $x$ . Rather, we note that the  $x = 0.25$  sample is close to the onset of parallel conduction, which lies between  $N_{\text{Si}} = N_0$  and  $2N_0$  for  $x = 0.25$  [Fig. 2(a)]. The spatial extent of the donor wave functions at  $x = 0.25$  can be estimated from the average donor distance,  $N_{\text{Si}}^{-1/2} = 7\text{--}10 \text{ nm}$ . For  $x = 0.28$ , on the other hand, parallel conduction does not occur even when  $N_{\text{Si}}$  is increased to  $4N_0$  [Fig. 2(a)], indicating that the donor wave function is much more localized.<sup>20</sup> These results lend strong support for our scenario that the screening of RI potential by nearby

donor electrons is playing an essential role in the emergence of a fully developed  $5/2$  state.<sup>21</sup>

Further insights are provided by examining the dependence on the doping density. Figures 3(a) and 3(b) compare two samples with the same  $x = 0.25$  but with lower  $N_{\text{Si}}$  of (a)  $0.15N_0$  and (b)  $0.5N_0$ . These samples have similar  $n_s$  despite the largely different  $N_{\text{Si}}$ .<sup>22</sup> We show  $R_{xx}$  in three representative  $\nu$  ranges to compare the influence of  $N_{\text{Si}}$  on the electronic states in different LLs. For  $N_{\text{Si}} = 0.5N_0$ , the  $\nu = 5/2$  state is equally well developed as in the case for  $N_{\text{Si}} = N_0$ . This is also confirmed by an activation measurement, which shows an even larger gap of  $\Delta_{5/2} = 100 \text{ mK}$  [Fig. 2(c)]. Further reducing  $N_{\text{Si}}$  to  $0.15N_0$ , however, significantly deteriorates the visibility of the  $5/2$  state. We note that  $N_{\text{Si}} = 0.15N_0$  is close to  $n_s/2$ , which implies that nearly all donors are ionized and so the data represent the case without screening by donor electrons or correlation in the donor charge distribution.<sup>23,24</sup> Our result for  $N_{\text{Si}} = 0.5N_0$  shows that, when the donor wave function is sufficiently extended, a donor-electron density of  $N_{\text{Si}} - n_s/2 = 0.37N_0$ , which is only three times the ionized-donor density  $\sim n_s/2 = 0.13N_0$ , is sufficient to establish a fully developed  $5/2$  state.

Effects of self-screening are also clearly visible for the  $N = 2$  LL; stripe and bubble phases<sup>25,26</sup> are well developed for  $N_{\text{Si}} = 0.5N_0$  but not for  $0.15N_0$ . Although not as dramatic, effects on the  $N = 0$  LL are also discernible.

The screening of RI potential can be further enhanced by reducing  $x$  from 0.25 to 0.24 while keeping  $N_{\text{Si}} = 0.5N_0$  [Fig. 3(c)]. This is manifested by the better developed re-entrant insulating state and the stripe phase showing a larger anisotropy. A poor correlation between  $\mu$  and the  $R_{xx}$  features is also evident here. Despite these clear improvements in the  $R_{xx}$  features, activation measurements detected no significant change in  $\Delta_{5/2}$  ( $=100 \text{ mK}$ ). This presents the intriguing question as to whether there should be a precise correspondence between the quality of the  $R_{xx}$  features and the size of  $\Delta_{5/2}$ .

In Fig. 4, we compare the measured  $\Delta_{5/2}$  of our samples with those reported in the literature for 30-nm QWs.<sup>7–11</sup> The inset plots the data as a function of  $n_s$ , a measure of the Coulomb energy, showing that the sizes of  $\Delta_{5/2}$  in our samples are much smaller than previously reported for similar  $n_s$ . As shown in Refs. 10 and 12, plotting  $\Delta_{5/2}$  as

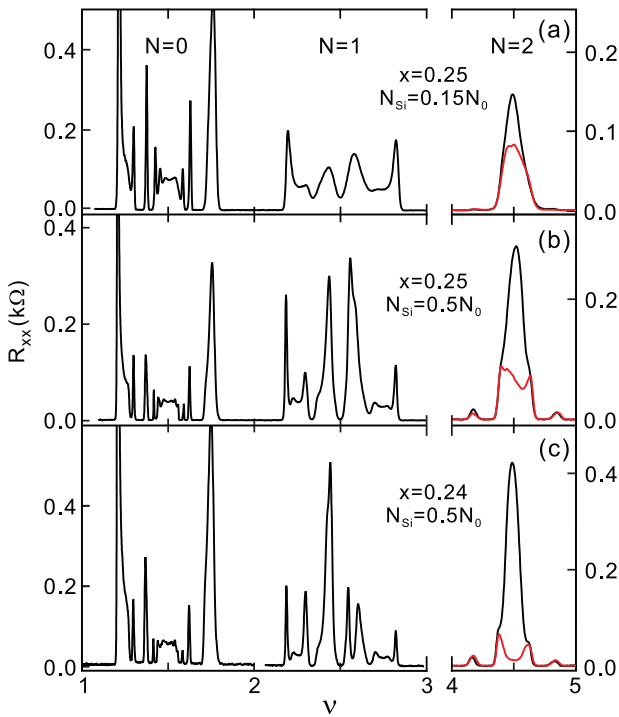


FIG. 3. (Color online)  $R_{xx}$  taken without illumination from samples with different sets of  $x$  and  $N_{\text{Si}}$ . (a) 0.25 and  $0.15N_0$ , (b) 0.25 and  $0.5N_0$ , and (c) 0.24 and  $0.5N_0$  ( $N_0 = 10^{12} \text{ cm}^{-2}$ ). From top to bottom,  $n_s = 2.43, 2.64, 2.66 \times 10^{11} \text{ cm}^{-2}$  and  $\mu = 6.4, 7.6, 7.2 \times 10^6 \text{ cm}^2/\text{Vs}$ .  $T$  is well below  $20 \text{ mK}$  for  $\nu \leq 3$ , while  $40 \text{ mK}$  for  $\nu \geq 4$ . The two curves for  $\nu \geq 4$  show  $R_{xx}$  measured in the two orthogonal current directions.

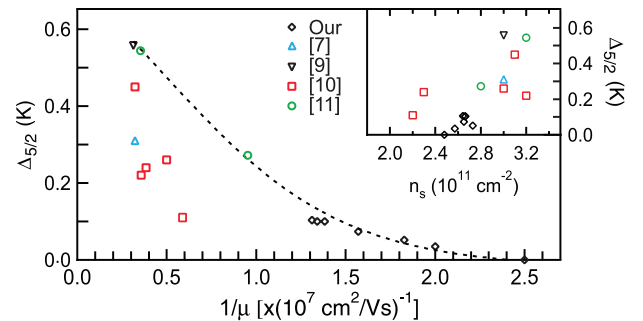


FIG. 4. (Color online) Comparison between  $\Delta_{5/2}$  of our samples and values reported in the literature, plotted versus inverse mobility  $\mu^{-1}$  (main panel) and  $n_s$  (inset). Dashed line is a guide indicating the maximum  $\Delta_{5/2}$  at each  $\mu^{-1}$ .

a function of  $\mu^{-1}$  makes it clear that  $\Delta_{5/2}$  tends to decrease with decreasing  $\mu$ , i.e., with increasing BI concentration. Here, two important observations are noteworthy. First, Fig. 4 does not include pseudogap data,<sup>17,27</sup> demonstrating that a well-developed  $5/2$  state with activation behavior can be observed in the  $\mu$  range much lower than anticipated from the previous analysis.<sup>10</sup> Our data, in turn, indicate that the  $\mu^{-1}$  dependence of  $\Delta_{5/2}$  is more gentle than the linear decrease expected from the simple lifetime broadening argument. Second, in contrast to the dramatic change in the quality of the  $R_{xx}$  features with  $x$ , the variation of  $\Delta_{5/2}$  in our samples is rather small and seems to be governed by the values of  $\mu$  instead. This can be understood by noting that in our samples a large part (more than 80%) of the intrinsic gap is taken away by disorder due to BIs, which masks the influence of RIs on  $\Delta_{5/2}$ . These observations elucidate the different roles played by the two dominant sources of disorder; while the emergence of a fully developed  $\nu = 5/2$  state is dictated by the RI potential, at  $\mu < 10^7$  cm<sup>2</sup>/Vs the size of the gap is mostly limited by the BI potential. In turn, the scattering of  $\Delta_{5/2}$  reported in the

ultra-high- $\mu$  regime, where the influence of BIs is considered to be small, may be due to the different levels of RI screening, which can arise from details of the doping schemes or illumination procedure. This implies that the total gap reduction is not given by a simple sum of the individual contributions from different sources of disorder. These insights, combined with the improvements in the mobility and sample design, will help to further enhance the stability of the  $\nu = 5/2$  state.

*Note added.* Recently, Refs. 28–30 were published. Reference 28 studies the impact of short-range disorder associated with interface roughness on the  $\nu = 5/2$  gap, while Ref. 29 reports quantitative analysis of disorder broadening based on the measured gap at  $\nu = 5/2, 7/2, 7/3$ , and  $8/3$ . Reference 30 demonstrates the effectiveness of the scheme presented in this work for back-gated structures.

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<sup>19</sup>The charge state of DX centers is an issue of long standing debate (Ref. 18), and we will not go into its detail. Nevertheless, we

believe that our results cast a different light on the donor state in Al<sub>x</sub>Ga<sub>1-x</sub>As ( $0.22 < x < 0.4$ ).

<sup>20</sup>It has been argued that the number ratio of shallow and deep centers change with  $x$ . According to Schubert and Ploog [*Phys. Rev. B* **30**, 7021 (1984)], the ratios of shallow donors at  $x = 0.25$  and  $0.28$  are 55 and 40%, respectively. Since  $N_{Si}$  is twice as high in the  $x = 0.28$  sample, the number of shallow donors alone cannot account for the dramatic improvement for  $x = 0.25$ .

<sup>21</sup>Near the onset of parallel conduction, donor wave functions may overlap to form locally conducting clusters, which do not contribute to transport but can provide screening. The fact that the  $x = 0.25$  samples with  $N_{Si} = N_0$  and  $0.5N_0$  show  $\nu = 5/2$  states of equal quality indicates that the formation of such clusters is not essential.

<sup>22</sup>In these samples, additional doping with  $N_{Si} = 0.25N_0$  was incorporated on each side at another 60-nm setback to compensate unintentional trap states and ensure the same  $n_s$ .

<sup>23</sup>E. Buks, M. Heiblum, and H. Shtrikman, *Phys. Rev. B* **49**, 14790 (1994).

<sup>24</sup>The fact that the visibility of the  $\nu = 5/2$  state in the  $x = 0.30$  sample with much higher  $N_{Si} = 2N_0$  is similar to that in the  $N_{Si} = 0.15N_0$  sample (with  $x = 0.25$ ) seems to suggest that such correlation is of secondary importance. Yet, considering the different freeze-out temperature for distinct  $x$ , we cannot entirely exclude the effect.

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