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Kinetics of excitations on the Fermi arcs in underdoped cuprates at low temperatures

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The Fermi-liquid-like (FL) resistivity recently observed in clean Hg1201 below the pseudogap temperature was related to carriers at the nodal points on the Fermi surface (FS) (N. Barišić *et al.*, arXiv:1207.1504, doi:10.1073/pnas.13019891109). We show that this has important implications for the electronic spectrum of underdoped (UD) cuprates as a whole. Photoemission experiments (angle-resolved photoemission spectroscopy) in other cuprates picture the spectrum as "metallic arcs" separated from each other by regions with large energy gaps. We rigorously solve the kinetic equation in such a model. The Fermi arcs' carriers contribute to the FL resistivity, if scattering between the opposite nodal points admits the umklapp processes. The Hall coefficient defines the effective number of carriers on the arcs and at weak magnetic fields it has a positive sign. All parameters that determine the arcs' widths are measurable experimentally. We conclude that the T^2 resistivity gives support to the Fermi arcs' concept and argue that the idea of a reconstructed FS in UD cuprates is not consistent with the latter.

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High temperature superconductivity (HTSC) in cuprates has challenged the condensed matter physics community for more than 25 years. Besides the fact that the pairing mechanisms are unknown, even in the paramagnetic phase cuprates demonstrate a strange non-Fermi-liquid energy spectrum. Thus, for underdoped (UD) cuprates photoemission experiments find the coherent excitations only within narrow "arcs" at the nodal points of the "bare" Fermi surface (FS) (for a review, see Ref. 1). While the general consensus is that strong electron-electron correlations in the vicinity of the Mott insulating antiferromagnetic phase are responsible for non-Fermi-liquid behavior, a unifying theory is still missing.

Recent developments concerning the discovery of quantum oscillations (QOs) in the vortex state have revealed small Fermi-liquid (FL) pockets in UD *ortho II* $YBa_2Cu_3O_{7-x}$ (YBCO).[2] In the literature the interpretation of these findings varies from a scenario of the *two* pockets formed on the "bare" FS reconstructed in a hypothetical spin- or density-wave phase transition at higher temperatures[2] and the idea of a *single* pocket inherent in the details of the band structure.[3]

Most recently one more feature was reported on for Hg1201.[4] Besides T_c, the temperature of the superconducting transition and T^*, the pseudogap temperature, there exists a new scale, T^{**}, on the phase diagram of Hg1201 below which resistivity manifests a distinct T^2 dependence that is typical for FLs.[4] An interval of the order of T^{**} itself separates T^{**} and T^* from each other. Above T^* resistivity is proportional to temperature, as is usual in cuprates.

It was emphasized in Ref. 4 that the new feature is not unique for Hg1201 and is seen in the resistivity data available for a few other UD cuprates [YBCO and $La_{2-x}Sr_xCuO_4$ (LSCO)]. Consequently, the question arises whether the emergent T^2-resistivity regime[4] in Hg1201 has something in common with small pockets first deduced in Ref. 2 from low frequency quantum oscillations in *ortho II* YBCO in high magnetic fields.

Photoemission [angle-resolved photoemission spectroscopy (ARPES)] and the magnetotransport studies are two independent experimental techniques. In what follows, we demonstrate that the T^2-FL resistivity[4] can be readily explained in terms of a simple model of normal carriers on the Fermi arcs. So far the photoemission experiments were mostly performed on two-layer $Bi_2Sr_2CaCu_2O_8$ (Bi2212), single-layer Bi2201, and LSCO. Thereby, the analysis below gives strong support to the supposition that the existence of the Fermi arcs is an indispensable feature of the energy spectrum in all UD cuprates. Recall that ARPES finds *single-sided arcs* at each of the four nodal points in the Brillouin zone (BZ), as opposed to *two small pockets*, at $(0, \pm\pi)$,$(\pm\pi,0)$ and $(\pm\pi/2, \pm\pi/2)$ points expected at the FS reconstruction via a spin- or a charge-density-wave (CDW) transition hypothesized first in Ref. 2 (see the recent review[5]).

In addition, it is self-evident that the Cooper instability must be related to pairing of excitations on the opposite arcs. Then, should electrons on the "arcs," in point of fact, form closed pockets, the onset of superconductivity would build on the latter the d-wave superconducting gap below T_c, contrary to the linear in T (Ref. 3) electronic specific heat at low temperature in *zero fields*. In turn, since it is outside the nodal points, the pocket[3] is too small in size to admit the umklapp processes and cannot contribute to finite conductivity. Thereby, carriers at the Fermi arcs and at the small pocket belong to two different subsystems.

That the new regime[4] bears a nontrivial character can be seen already from the analysis of the Drude formula for resistivity $\rho(T) = [m^*/ne^2\tau(T)]$. In layered cuprates the FL T^2 contribution could be conveniently presented as

$$(s/c)(e^2/\pi\hbar)\Delta\rho(T) = \text{const}(T/\varepsilon_F)^2 \qquad (1)$$

(c is the lattice constant perpendicular to the CuO_2 plane; s is the number of conducting layers per unit cell).

In theory, the T^2 dependence signifies the FL regime as long as $T \ll \varepsilon_F$ in the whole temperature interval, i.e., the Fermi energy *must be large*. Substitution of T^{**} and use of the resistivity data, e.g., for YBCO (see Fig. 3 in Ref. 4) gives $\varepsilon_F \approx (\text{const})^{1/2}290$ K at $T^{**} = 190$ K (dimensionless

const ~ 1). The fact that the quadratic dependence describes resistivity of YBCO up to $T^{**} \approx (2/3)\varepsilon_F$ puts the straightforward FL interpretation [Eq. (1)] in question. The same estimates give close results for Hg1201, LSCO, and YBCO (see the data shown in Ref. 4, Fig. 2). Actually, such an analysis of the Drude expression was first performed for the T^2 mobility of carriers in LSCO in Ref. 6, with the same conclusions that ε_F is too small for the routine FL physics to apply.

After searching through various mechanisms, from the charge-wave order, superconducting fluctuations to the FL regime via the hypothetic pocket, such as was deduced from QO in UD YBCO, the authors[4] associated the T^2 regime with carriers at the nodal points on the Fermi surface. Although qualitative arguments[4] in SI (9) of Ref. 4 are somewhat vague, we adopted this point of view. Below we suggest the model of the Fermi arcs and solve it rigorously. The T^2-resistivity regime at low enough temperatures is obtained after taking explicitly into account the umklapp scattering between the opposite nodal points. Forestalling results of the analysis below, the physics behind it is, loosely speaking, the total number of *carriers on the Fermi arcs* that must be taken as the number n of charge carriers in the Drude formula. After that, the Fermi energy in Eq. (1) turns out to be large, $\varepsilon_F^* = p_F^2/2m^*$, of the order of the chemical potential (m^* is the renormalized band mass).

Before to proceeding further, we note in passing that discussions regarding whether the ARPES technique distinguishes between "arcs" and "pockets" were continued until recently (e.g., Ref. 7). We adopt the view that arcs are quite literary "single-sided arcs" without "hidden contours" on their backside.

In early experiments[8] the arc's length was found to be proportional to temperature at all dopings so that at extrapolation to $T = 0$ the arc would collapse onto a single point. Results[8] were revised in Refs. 9–11, and it is now well established that the length of the arcs in the pseudogap phase is finite down to the lowest temperatures, although the arc shrinks with a decrease in the dopant concentration. ARPES reveals the large energy gaps outside the arcs' end points in the normal phase *directly above* T_c.[10–12]

One must stipulate again that the surface problems continue to plague ARPES experiments in Hg1201 and especially in YBCO.[13] Yet, the general consensus is that, except for possibly minor details, the basic physics behind these ARPES results is of a general character. With that in mind, we suggest that in clean UD cuprates at low temperature, but still in the normal phase, the energy spectrum of excitations inside the Fermi arc is the ordinary $\varepsilon(\vec{p}) = v_F(p - p_F)$. (Actually, there is a structure in the energy spectrum seen, in particular, as kinks for the Fermi velocity v_F near the chemical potential; v_F changes only by an insignificant numerical factor.[14]) The angular dependence $(t_\perp/2)|\cos(k_x a) - \cos(k_y a)|$ of the large energy gap (pseudogap) that grows from the arc's ending points resembles the profile of the d-wave superconducting gap. In UD cuprates $t_\perp$ is considerable larger than Δ_0, the amplitude of the superconducting d-wave gap.[10–12]

ARPES intensity is proportional to the single-particle spectral function $A(\omega,\vec{k})$ (see e.g., Ref. 1). The latter is related in the following way to the imaginary part of the retarded Green's function $G(\vec{k};\omega) = [\omega - \varepsilon(\vec{k}) + \mu - \Sigma(\vec{k};\omega)]^{-1}$:

$$A(\omega,\vec{k}) = -\frac{1}{\pi}\frac{\mathrm{Im}\Sigma(\vec{k};\omega)}{[\omega - \varepsilon(\vec{k}) + \mu - \mathrm{Re}\Sigma(\vec{k};\omega)]^2 + [\mathrm{Im}\Sigma(\vec{k};\omega)]^2}. \tag{2}$$

In the experimental procedure (e.g., Ref. 8) the positions of the excitations on the ungapped parts of the "bare" FS, in accordance with Eq. (2), are expected to coincide with maxima of $A(\omega,\vec{k})$ at $\omega = 0$. Inside each arc the Green's function can be written in the customary form as first obtained for the electrons interacting with phonon in metals[15] and for the "Kondo lattice" problem (in the mean field approximation):[16]

$$G^{R(A)}(\vec{p};\omega) = \frac{Z}{\omega - Z[\varepsilon(\vec{p}) - \mu] \pm i/\tau(\omega)} \cong \frac{Z}{\omega - Z\xi \pm i/\tau(\omega)}. \tag{3}$$

Here $\xi = v_F(p - p_F)$ is the perpendicular to the FS component of the "bare" energy dispersion, and $\tau(\omega)$ is the relaxation time. In (3) the sign $(+)$ or $(-)$ in front of the imaginary part stands for the retarded (R) and the advanced (A) Green's functions, correspondingly. The residue is $Z = [1 - \partial\Sigma(\omega)/\partial\omega]^{-1}$; $m^* = Z^{-1}m$ is the effective mass.[15]

The temperature dependence of the imaginary part $\mathrm{Im}\Sigma(\vec{p};\omega)$ can be obtained *rigorously* by the analytical continuation of the "skeleton" self-energy diagram with three internal electronic lines.[17] While this is all that one needs to know for the spectral function $A(\omega,\vec{k})$ in Eq. (2), it is not enough for finding the conductivity. Indeed, contributions into $\mathrm{Im}\Sigma(\vec{p};\omega)$ come about from all the scattering channels, whereas the conductivity in metals is finite exclusively due to the umklapp processes in which electrons transfer the momentum to the lattice. Although a *large* enough FS admits the umklapp scattering, the imaginary part $\mathrm{Im}\Sigma(\vec{k};\omega)$ determines contributions relevant to conductivity only *by the order of magnitude*. In ordinary metals, to calculate the resistivity for an arbitrary FS, one has to resort to the approximate variational principle (maximum entropy production). In other words, measuring resistivity usually gives *no* specific information on the Fermi surfaces. Instead, for the model of small arcs, the behavior of resistivity as a function of temperature and doping concentration becomes one of the most important means for verifying the basic supposition.

To simplify equations, we calculate the conductivity using the kinetic equation. In hindsight the rigorous Green's function formalism allows to express the final results via renormalized masses and the strength of interactions. The kinetic equation reads

$$e\left(\vec{E} + \frac{1}{c}[\vec{v} \times \vec{H}]\right)\vec{\nabla}_{\vec{p}} n(\vec{p}) = \left(\frac{dn}{dt}\right)_{\mathrm{coll}}, \tag{4}$$

where $n(\vec{p})$ is the distribution function for carriers with charge e, and $\vec{E}$ and $\vec{H}$ are the electric and the magnetic fields, correspondingly. The consensus is that strong electron-electron correlations determine the fundamentals of the cuprates' physics. Correspondingly, at least, as far as phonons with large

momenta are "frozen out" at lower temperatures, the kinetics will be due to strong Coulomb (e-e) interactions as well.

The textbook integral $(dn/dt)_{\rm coll}$ balances "gains" and "losses" at the (e-e) collisions:

$$\left(\frac{dn_{\vec{p}}}{dt}\right)_{\rm coll} = 2\iint |V(\vec{p}_1\vec{p}_3;\vec{p}\,\vec{p}_2)|^2 \frac{d^2\vec{p}_1 d^2\vec{p}_2}{(2\pi)^4} \times \delta(\varepsilon_1+\varepsilon_3-\varepsilon-\varepsilon_2)\{(\text{gains})-(\text{losses})\}, \quad (5)$$

where $\varepsilon_i \equiv \varepsilon_i(\vec{p}_i)$, gains $= n(\vec{p}_1)n(\vec{p}_3)[1-n(\vec{p})][1-n(\vec{p}_2)]$, and the expression for losses follows after the obvious permutations.

Assuming that Fermi arcs in UD cuprates are located not far from the four points $(\pm\pi/2, \pm\pi/2)$ in the BZ, consider the system in a weak electric field $\vec{E}$ along the diagonal $(-\pi,-\pi)-(\pi,\pi)$. Then the contribution into the current from the pair of nodal points on the perpendicular diagonal can be neglected. Substituting $\varepsilon(\vec{p}) \to \varepsilon(\vec{p}) - \vec{p}\cdot\vec{u}$ in $n_0(\vec{p}) = (e^{\varepsilon(\vec{p})/T}+1)^{-1}$ (where $\vec{u}$ is the drift velocity) gives the Fermi distribution in the presence of a nonzero current. Expanding over small $\vec{u}$, $n(\vec{p}) \approx n_0(\vec{p}) + n_1(\vec{p})$ for $n_1(\vec{p})$, one has

$$n_1(\vec{p}) = (\vec{p}\cdot\vec{u})\left(\frac{1}{4T}\right)\cosh^{-2}\left(\frac{\varepsilon_p}{2T}\right). \quad (6)$$

The conservation law $p^{\parallel}+p_2^{\parallel} = p_1^{\parallel}+p_3^{\parallel}-K^{\parallel}$ for the momentum components along the diagonal invokes the umklapp vector $\vec{K} = (2\pi/a, 2\pi/a)$ (for the perpendicular components $p^{\perp}+p_2^{\perp} = p_1^{\perp}+p_3^{\perp}$). With the energy spectrum $\varepsilon(\vec{p})$ near the two opposite nodal points,

$$\varepsilon(\vec{p}) = \pm v_F(p^{\parallel} \mp p_F) + (p^{\perp})^2/2m, \quad (7)$$

it gives

$$p^{\parallel}+p_2^{\parallel}-p_1^{\parallel}-p_3^{\parallel}-K^{\parallel} = v_F^{-1}[\varepsilon(\vec{p})+\varepsilon(\vec{p}_2)+\varepsilon(\vec{p}_1)+\varepsilon(\vec{p}_3)-\Sigma] = 0. \quad (8)$$

In (8),

$$\Sigma \equiv \{(p^{\perp})^2+(p_2^{\perp})^2+(p_1^{\perp})^2+(p_3^{\perp})^2\}/2m^* - 4\mathrm{A}. \quad (9)$$

Here $\mathrm{A} = v_F p_F[(K/4p_F)-1] \equiv (v_F p_F)\kappa$.

It follows from (8) that the δ function $\delta(\varepsilon_1+\varepsilon_3-\varepsilon-\varepsilon_3)$ in (5) can be written in either of the two forms:

$$\delta(\varepsilon_1+\varepsilon_3-\varepsilon-\varepsilon_2) = (1/2)\delta(\varepsilon_1+\varepsilon_3-\Sigma/2) \quad \text{or} \quad \delta(\varepsilon_1+\varepsilon_3-\varepsilon-\varepsilon_2) = (1/2)\delta(\varepsilon+\varepsilon_2-\Sigma/2). \quad (10)$$

The right-hand side (rhs) in Eq. (5) which is linear in $\vec{u}$ with the help of Eq. (10) acquires the form

$$\text{rhs} = \iint |V(1;2)|^2 \frac{d^2\vec{p}_1 d^2\vec{p}_2}{T(4\pi)^4} \frac{(\vec{K}\cdot\vec{u})\delta(\varepsilon+\varepsilon_2-\Sigma/2)}{\cosh(\varepsilon/2T)\cosh[(\varepsilon-\Sigma/2)/2T]\cosh(\varepsilon_1/2T)\cosh[(\varepsilon_1-\Sigma/2)/2T]}. \quad (11)$$

[In (11), $|V(1;2)|^2 \equiv |V(\vec{p}_1\vec{p}_3;\vec{p}\,\vec{p}_2)|^2$.] Since $d^2\vec{p}_1 d^2\vec{p}_2 = dp_1^{\perp}dp_2^{\perp}dp_1^{\parallel}dp_2^{\parallel} \equiv v_F^{-2}dp_1^{\perp}dp_2^{\perp}d\varepsilon_1 d\varepsilon_2$, one can directly integrate over $d\varepsilon_1$ in (11):

$$\int \frac{d\varepsilon_1}{\cosh[\varepsilon_1/2T]\cosh[(\varepsilon_1-\Sigma/2)/2T]} = \frac{\Sigma}{\sinh(\Sigma/4T)}. \quad (12)$$

Combining (11) with the linear in the electric field term in (4), rhs $= ev_F E(dn_{p,o}/d\varepsilon)$, and integrating both sides over $\vec{p}$ ($d^2\vec{p} \equiv v_F^{-1}dp^{\perp}d\varepsilon$) gives

$$-eE\int_{\Delta p^{\perp}} dp^{\perp} = (Ku)T\iiint |V(1;2)|^2 \frac{dp^{\perp}dp_1^{\perp}dp_2^{\perp}}{4^2\pi^4 v_F^3} \times \left(\frac{\Sigma/4T}{\sinh(\Sigma/4T)}\right)^2. \quad (13)$$

The integral on the left-hand side is over the "arc width" $\delta = 2\Delta p^{\perp} = (\Delta\varphi)p_F$. The exact definition would depend on the details of the arc's structure.[14] In what follows, δ is the model parameter.

At low temperatures the main contribution into the integral Eq. (13) comes from the small Σ of Eq. (9): $\langle(p^{\perp}/p_F)^2\rangle \approx (K/4p_F-1)$. Correspondingly, for $p^{\perp}$ to lie inside the arc's length δ, the square root $\kappa^{1/2} = \sqrt{K/4p_F-1}$ has to be small enough. (For the umklapp processes to be effective at low T, A must be positive: $\mathrm{A} = v_F p_F[(K/4p_F)-1] > 0$.)

Reduction of the quadratic in momenta term in Σ [Eq. (9)] to the diagonal form simplifies the threefold integral in (13) to

$$I \equiv \iiint \frac{dp^{\perp}dp_1^{\perp}dp_2^{\perp}}{(\pi)^4}\left(\frac{\Sigma/4T}{\sin h(\Sigma/4T)}\right)^2 = \frac{2\sqrt{2}}{\pi^3}\int_0^{\infty}\sqrt{v}dv\left[\frac{\{(v/4mT)-\mathrm{A}/T\}}{\sin h\{(v/4mT)-\mathrm{A}/T\}}\right]^2. \quad (14)$$

Then

$$I = \frac{2\sqrt{2}}{\pi^3}\int_{-4m\Delta}^{\infty}\sqrt{(s+4m\mathrm{A})}ds\left[\frac{s/4mT}{\sinh(s/4mT)}\right]^2, \quad (15)$$

and at low temperatures ($T \ll \mathrm{A}$)

$$I \cong \frac{8p_F^3}{3\pi}\sqrt{\frac{\mathrm{A}}{\varepsilon_F}}\left(\frac{T}{\varepsilon_F}\right)\left(1-\frac{\pi^2 T}{40\mathrm{A}}-\cdots\right). \quad (16)$$

With matrix elements $|V(1;2)|^2 \equiv |V(\vec{p}_1\vec{p}_3;\vec{p}\,\vec{p}_2)|^2$ in the dimensionless form, $|V(1;2)|^2 = |\tilde{V}(1;2)|^2\nu_0^{-2}(\varepsilon_F)$ [here $\nu_0(\varepsilon_F) = (m/2\pi\hbar^2)$ is the "bare" two-dimensional (2D) density of states for one spin direction], one has $eE\delta = up_F(|\tilde{V}(1;2)|^2/v_F^3)T(\pi/m)^2 I$; that gives for the drift velocity

$$u = \frac{-eE\delta v_F^3}{|\tilde{V}(1;2)|^2 T p_F(\pi/m)^2 I}. \quad (17)$$

Substitution of $n_1(\vec{p})$ in its form in Eq. (6) into the expression for the current, $j_{2\rm D} = 4e\int v_F\frac{d^2\vec{p}}{(2\pi)^2}n_1(\vec{p})$, after trivial trans-

formations, and using $d^2\vec{p} \equiv v_F^{-1} dp^{\perp} d\varepsilon$ gives the in-layer resistance

$$\rho_{2D}(T) = \left(\frac{\pi\hbar}{e^2}\right) \frac{4\pi^2 |Z\tilde{V}(1;2)|^2}{3(\Delta\varphi)^2} \sqrt{\frac{A}{\varepsilon_F^*}} \left(\frac{T}{\varepsilon_F^*}\right)^2. \quad (18)$$

(Electrons on the two opposite arcs contribute equally into the current, hence, the additional factor 2 in j_{2D} above is beside the spin.)

The T^2-dependent resistivity, Eq. (18), is the central result of this Rapid Communication. Notice that renormalization of the interaction $\tilde{V}(1;2) \to Z\tilde{V}(1;2)$ and of the mass $m^* = Z^{-1}m$ is already taken into account in Eq. (18). The combination $\varepsilon_F^* = p_F^2/2m^*$ is the renormalized band Fermi energy; here and below ε_F^* is the convenient shorthand notation for the actual expansion parameter. In Eq. (18) $|Z\tilde{V}(1;2)|$ is the only parameter that characterizes the strength of interactions between carriers. In fact, all other matrix elements for scattering between the arcs at different nodal points do not involve the umklapp processes and drop out from Eqs. (5) and (11) at calculations of conductivity.

Let us do some estimates. First, taking $m^* \approx 4m$ and the lattice parameter $a \simeq 3.86 \times 10^{-8}$ cm for YBCO, one would obtain $\varepsilon_F^* \approx 3000$ K and $T^{**}/\varepsilon_F^* \approx 1/15$. [For comparison, the Fermi energy of Tl2201 estimated from data on QO (Ref. 18) is $\varepsilon_F \approx 5000$ K.] In Eq. (18), $A/\varepsilon_F^* = 2|K/p_F - 1| \equiv 2\kappa$. Substitution of the same data for YBCO ($p = 0.09$)[4] gives for the arc a reasonable estimate, $\Delta\varphi \approx 0.37\kappa^{1/4} \approx 21^{\circ}\kappa^{1/4}$ [assuming $|Z\tilde{V}(1;2)| \sim 1$]. There, only ARPES can determine positions of the nodal points in the BZ, however, even if κ were small, $\kappa^{1/4} \sim 1$.

In the theoretical formulas the deviations from the quadratic T dependence [Eq. (18)] occur at $T \approx (40/\pi^2)A \sim 8\kappa\varepsilon_F^*$ [see Eq. (16)]. Such a value of T^{**} would seem to be too high. In the estimates above it was implicitly assumed that in actual fact, T^{**} is due to switching on other mechanisms, say, scattering on the phonons or the pseudogap diminishing at $T \approx 200$ K.[4,12] With regard to the $1/p$ dependence on p, the dopants' concentration for the FL resistivity in Hg2201,[4] notice the factor $(\Delta\varphi)^2$ in the denominator of Eq. (18): If the arcs have had two sides, $(p_F\Delta\varphi)^2$ would be proportional to the "area" of the "enclosed pocket".

Let us turn to the Hall effect. In nonzero magnetic fields H the Lorentz force causes the flow of excitations perpendicular to the electric field on arcs on the second diagonal. Calculations of the transverse drift velocity are identical to the ones for conductivity. The Hall coefficient R_H is

$$R_H = \frac{\pi^2}{ec(\delta p_F)}. \quad (19)$$

In literature, the FS of UD cuprates is routinely called the holelike FS centered at (π,π). In semiconductors the concept of holes emerges when the chemical potential is close to the top of an electronic band. This point of view is not applicable to UD cuprates because the underlying "bare" FS is not small. Instead, since the system with the electronically half-filled BZ is in the insulating Mott state, I argue that, counting off from the insulating limit, the carriers on the Fermi arcs are holes, $e > 0$ and $R_H > 0$. [In Eq. (19), $\delta p_F/\pi^2 = \Delta\varphi(p_F^2/\pi^2)$ stands for the effective number of carriers on the Fermi arcs.]

All the parameters in (18) and (19) are experimentally accessible. In fact, leaving aside for a moment the matrix element in Eq. (18), ARPES directly measures the difference $(K/4p_F) - 1$ at $(\pi/2,\pi/2)$ (e.g., for LSCO see Ref. 12). From the experimental data on resistivity one then determines the dependence of the arc width $\Delta\varphi$ in Eq. (18) on the concentration of doped holes p. $\Delta\varphi$ can be independently obtained using the expression for the Hall coefficient, Eq. (19). After that, even the value of the matrix element can be found.

Recent experiments[19–21] reveal the tendency to form the competing charge ordered phase in UD cuprates. So far, however, the field-induced charge-density wave is either ordered at too low a temperature[19] or charge order manifests itself in the fluctuation regime.[20,21] It was also pointed out that the tendency to a CDW order may have something in common with the stripe ordering in the $La_{2-x}Ba_xCuO_4$ (214) compounds at $x = 1/8$.[22] On the phase diagram of Hg1201 the T^2 regime in Ref. 4 is limited by $p \approx 0.12$–0.13 (Fig. 4A in Ref. 4) and is not in conflict with the observations.[19–21]

The derivation of Eq. (19) assumes the classical regime $\omega_c\tau \ll 1$ [$\omega_c = (eH/m^*c)$]. The singularity of the situation is that by the order of magnitude τ is typically $\tau \propto \varepsilon_F/T^2$, i.e., can be very large in clean samples. For example, at $T = 100$ K the magnetic field must be much smaller than 3–5 T. Most published data for R_H in Hg1201 and YBCO in such a temperature range are only known for the considerably higher fields (see Ref. 23 and references therein). The theoretical interpretation of the Hall effect in strong fields and the energy spectrum consisting of the Fermi arcs for charged carriers is absent. [In the context of the model one can suggest another interpretation to the provision $\omega_c\tau \ll 1$. The transverse momentum component obeys the equation $\partial p^{\perp}/\partial t = (ep_F/cm^*)H$. Then the deviation $\Delta p^{\perp} \simeq \omega_c p_F \tau$ in the transverse direction for the time period τ should not exceed the arc's length δ.]

Finally, we make a few comments about what is known concerning the paramagnetic phase in UD cuprates below the pseudogap temperature T^*: $T_c < T < T^*$. In the introductory part it was argued that experiments[3] disprove the case of the Fermi surface reconstructed via a spin- or charge-wave transition in UD *ortho II* YBCO. For Hg2201 and YBCO there are experimental evidences in favor of the transition into the so-called "loop current phase "(see Ref. 24 and the experimental reference therein). By its symmetry,[25] such an order parameter would only break the tetragonal symmetry in the positions of pairs of the nodal points in the BZ, and one would expect no other changes in the above results except an anisotropy in the T^2-resistivity contribution. [An uncertainty regarding the structure of the electronic spectrum in the pseudogap phase, however, was brought about recently by observation of the *two* phase transitions in the (nearly) *optimal doped* single-layer Bi2201 at $T = T_c$ and $T = T^*$.[26]

To conclude, for temperatures above the superconducting dome the analysis of ARPES data[9–12] makes it possible to develop the model in which in the normal state carriers in the Fermi arcs occupy narrow valleys in the momentum space restricted on each side by regions with the large energy gap. Inside each valley the excitations' spectrum is close to the ordinary energy spectrum of free holes. Strong short-range

electron-electron interactions are assumed to prevail for clean UD cuprates in the pseudogap phase in relaxation between carriers on the Fermi arcs. Scattering of carriers between four arcs at the nodal points in the BZ was treated in the framework of the semiclassical kinetic equation. Renormalization of the FL parameters owing to the electron-electron correlations is taken into account in the final results. The explicit treatment of the umklapp processes gave the finite value for conductivity. At low enough temperatures the calculated resistivity of the normal phase is proportional to T^2, in excellent agreement with the experimental data.[4] It is shown that the Hall effect in clean UD cuprates is extremely sensitive to the value of the magnetic fields at these temperatures. Presumably, even the sign of the Hall coefficient may depend on the magnetic field strength. Experimental data for the Hall coefficient for low magnetic fields are missing.

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