Magnetic relaxation due to spin pumping in thick ferromagnetic films in contact with normal metals

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Spin pumping is the most important magnetic relaxation channel in ultrathin ferromagnetic layers in contact with normal metals (NMs). Recent experiments indicate that in thick films of insulating yttrium iron garnet (YIG) there is a large broadening of the ferromagnetic resonance (FMR) lines with deposition of a thin Pt layer which cannot be explained by the known damping processes. Here we present a detailed study of the magnetic relaxation due to spin pumping in bilayers made of a ferromagnetic material (FM) and a NM. Two alternative approaches are used to calculate the transverse and longitudinal relaxation rates used in the Bloch-Bloembergen formulation of damping. In one we consider that the dynamic exchange coupling at the interface transfers magnetic relaxation from the heavily damped conduction electron spins in the NM layer to the magnetization of the FM layer while the other utilizes spin currents and the concept of the spin-mixing conductance at the interface. While in thin FM films, the relaxation rates vary with the inverse of the FM layer thickness; in thick films, they become independent of the thickness because in the FM/NM structure the FMR excitation has a surface mode character. Regardless of the thickness range the longitudinal relaxation rate is twice the transverse rate resulting in damping of the magnetization with constant amplitude characterizing a Gilbert process. The enhanced spin-pumping damping explains the experimental observations in YIG/Pt bilayers.

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I. INTRODUCTION

The manner by which the magnetization relaxes toward equilibrium is governed by the spin interactions and the detailed structure of a magnetic system, and its understanding is important from the point of view of basic physics and for technological applications. The relaxation rate determines the damping of the excitations in magnetic materials, the most important of which are the spin waves, or magnons, and also the minimum switching time for magnetization revearsal in bits of magnetic recording media. For several decades the magnetic relaxation has been investigated experimentally and theoretically in bulk and thin film materials.[1–4] In bulk magnetic insulators the relaxation occurs through intrinsic mechanisms involving magnon-magnon and magnon-phonon processes as well as extrinsic mechanisms such as scattering by impurities or sample shape irregularities.[1–4] In bulk metallic materials the relaxation is dominated by processes involving the conduction electrons.[5,6] In very thin films and multilayers new physical relaxation processes have been discovered in the last decade, the most important ones being two-magnon scattering off the irregularities at the surfaces or interfaces[7,8] and the spin pumping mechanism.[9–12]

In recent years, structures made of bilayers of ferromagnet (FM)/normal metal (NM) films have been attracting considerable interest due to the discoveries of the spin Hall effect (SHE)[13,14] and the inverse spin Hall effect (ISHE).[15,16] In a FM/NM bilayer undergoing ferromagnetic resonance (FMR) driven by microwave radiation, it has been found[15–18] that the precessing magnetization in the FM injects spins into the adjacent NM layer, creating a spin current that is converted into a charge current by means of the ISHE. This allows the conversion of spin currents into a spin-pumping dc voltage V_{SP} opening immense possibilities in the field of spintronics.[19] A very important development in this field was the demonstration that the ferrimagnetic insulator yttrium iron garnet ($Y_3Fe_5O_{12}$-YIG) can be used in FM/NM structures to study spin-charge current conversion.[20] Bilayers made with YIG and NMs with strong spin-orbit coupling, such as Pt, have drawn great attention due to their ability in converting magnetic signals into electric signals and vice versa and in transporting spin information over large distances.[20–42] Since in YIG the relaxation mechanisms involving conduction electrons are not effective, its FMR linewidth is two orders of magnitude smaller than in FMs such as permalloy (Py). As a result, the FMR and the V_{SP} spectra exhibit [20–23] many peaks corresponding to the spin-wave magnetostatic modes.[43,44] Recently it has been observed that the deposition of a Pt layer with thickness of a few nm produces an unusually large broadening of the microwave absorption lines in YIG films with thickness of several μm.[21,23,42] This cannot be explained by the known models for magnetic relaxation and poses new challenges in the field of spintronics.

This paper addresses the question of the magnetic relaxation in bilayers of FM materials and nonmagnetic metals. Central to the problem is the spin-pumping damping mechanism originating in the flow of angular momentum out of the FM layer into the NM layer. This problem can be tackled with two different formulations. One of them, more appropriate for insulating FM, consists of considering that at the FM/NM interface the spins of the FM layer interact with the NM conduction electron spins through the exchange interaction. The coupled motion of the FM magnetization with the NM spin accumulation transfers to the FM magnetization an additional relaxation from the overdamped motion of the conduction electron spins. Alternatively one can use only spin currents, but as we show here, it is necessary to consider the effect of the transverse components of the spin current in addition to the component with polarization parallel to the static magnetic field used in the conventional model.[9,10] Both treatments yield the same results. We show that the known expression for the

spin-pumping damping applies only to FM/NM bilayers that have FM-layer thickness below a critical thickness. As the FM thickness increases, the spin-pumping damping crosses over to another regime, which accounts for the observed broadening of the FMR lines in thick YIG films with deposition of a very thin Pt layer. This becomes clear when one treats the relaxation of both longitudinal and transverse components of the magnetization.

The paper is organized as follows. In Sec. II, we review the phenomenological forms for introducing the damping in the Landau-Lifshitz (LL) equations of motion of the magnetization with the main objective of laying the ground to show the importance of considering the transverse relaxation. Section III is devoted to the two most important extrinsic relaxation mechanisms in very thin FM films and bilayers: two-magnon scattering and spin pumping. We show that the spin pumping produced by the spin current polarized in the direction of the applied field accounts only for the longitudinal relaxation. In Sec. IV, we present the two alternative calculations of the transverse relaxation rate in FM/NM bilayers, one based on the exchange interaction at the interface and the other based on spin currents. In Sec. V, we show that if the FM film thickness is larger than a critical value, both of the longitudinal and transverse relaxation rates become independent of the FM film thickness, and the results explain the experimental observations in YIG/Pt. Section VI, summarizes the main results.

II. RELAXATION OF THE MAGNETIZATION IN FERROMAGNETIC MATERIALS

The relaxation of the magnetization in ferromagnetic materials manifests in several forms, the most important ways being the damping of spin waves and the time it takes to switch the magnetization in a bit of magnetic-recording media. The most widely used technique to measure the magnetic relaxation rate is FMR, in which a radio requency (rf) magnetic field is used to excite one or more spin-wave modes with small wave numbers.[45] The FMR absorption linewidth is determined by the damping experienced by the magnetization as it precesses about the equilibrium direction driven by the rf field, usually with frequency in the microwave range. In a ferromagnetic material, the atomic spins are coupled together by exchange and dipolar interactions. If the system is driven by a microwave field with frequency ω, the spins within a very small volume remain tightly parallel to each other by virtue of the exchange interaction so that the system may be described very well by the behavior of its magnetization. Even in a magnetically saturated sample, the magnetization vector varies in space due to the presence of thermal or driven spin waves so that it has to be described by $\vec{M}(\vec{r},t)$. If relaxation is neglected, the motion of the magnetization vector in space and time is governed by the LL equation,[46] $d\vec{M}(\vec{r},t)/dt = \gamma\vec{M}(\vec{r},t) \times \vec{H}_{\rm eff}(\vec{r},t)$, where $\gamma = g\mu_B/\hbar$ is the gyromagnetic ratio, μ_B is the Bohr magneton, g is the spectroscopic factor (approximately two for electron spins), and $\vec{H}_{\rm eff}(\vec{r},t)$ is an effective magnetic field to which the spins respond. There are several contributions to the effective field such as the applied external static and rf magnetic fields, magnetic dipolar, exchange, and anisotropy. Initially we assume that there is only a static magnetic field H and consider a coordinate system with the z axis in the field direction so that $H_z = H - N_z 4\pi M$, where N_z is the demagnetizing factor along the z direction and $4\pi M$ is the saturation magnetization. Of course, in the case of a film magnetized in the plane, $H_z = H$. A microwave-driving field applied perpendicularly to the static field drives the precession of the magnetization around the z direction. Relaxation can be introduced phenomenologically in the LL equation of motion in several forms. The most widely used was introduced by Gilbert[47] and consists of representing the relaxation mechanisms by a torque that pulls the magnetization toward the equilibrium direction, leading to the Landau-Lifshitz-Gilbert (LLG) equation,[1,2]

$$\frac{d\vec{M}(\vec{r},t)}{dt} = \gamma\vec{M}(\vec{r},t) \times \vec{H}_{\rm eff}(\vec{r},t) + \frac{\alpha}{M}\vec{M}(\vec{r},t) \times \frac{d\vec{M}(\vec{r},t)}{dt}, \tag{1}$$

where α is an adimensional quantity called the Gilbert damping parameter. Equation (1) is easily solved in the linear approximation by writing the magnetization as $\vec{M}(\vec{r},t) = \hat{z}M_z + \hat{x}m_x(\vec{r},t) + \hat{y}m_y(\vec{r},t)$, where $m_x, m_y \ll M_z$. Assuming initially for simplicity a uniform magnetization and no external driving or microwave dipolar field, and considering solutions for the small-signal time-varying components of the magnetization m_x and m_y in the form $\exp(i\omega t)$, Eq. (1) leads to the following linear equations:

$$i\omega m_x = (\gamma H_z + i\alpha\omega)m_y, \quad i\omega m_y = -(\gamma H_z + i\alpha\omega)m_x, \tag{2}$$

from which we find $\omega = \omega_r + i\alpha\omega$, where $\omega_r = \gamma H_z$. Equation (2) shows that if the magnetization is deviated from the equilibrium direction, its transverse components vary with $\exp(i\omega_r t)\exp(-\alpha\omega t)$, meaning that the magnetization vector precesses about the z direction with frequency $\omega_r = \gamma H_z$ and with amplitude that decays exponentially in time with a relaxation rate $\eta = \alpha\omega$. In a FMR experiment, the sample is placed in a microwave cavity with fixed driving frequency ω and the static magnetic field H is swept. At the field value for which $\omega_r \approx \omega$, an absorption line is observed with a full linewidth $2\Delta H$, where the half-width of the FMR absorption line is related to the relaxation rate by $\Delta H = \eta/\gamma$. Thus, one can express the FMR (half) linewidth in terms of the Gilbert damping parameter,

$$\Delta H_{\rm FMR} = \alpha\omega_r/\gamma. \tag{3}$$

Equation (3) is a central result of the LLG phenomenology; the FMR linewidth scales linearly with the FMR frequency. Note that in order to increase the signal-to-noise ratio, the FMR experiments are done with a low-frequency modulation in the external field and use lock-in amplification, so that one measures the field derivative of the absorption line. For a Lorentzian lineshape the peak-to-peak linewidth of the field-derivative is $\Delta H_{pp} = 2\alpha\omega_r/(\gamma\sqrt{3})$. Note also that in low-loss materials, such as YIG, the presence of spin waves is noticeable so that in FMR experiments one observes several absorption lines corresponding the spin waves with small wave numbers, the so-called magnetostatic modes.[20–24,43,44] The main mode is the one in which the magnetization precession is nearly uniform throughout the sample.

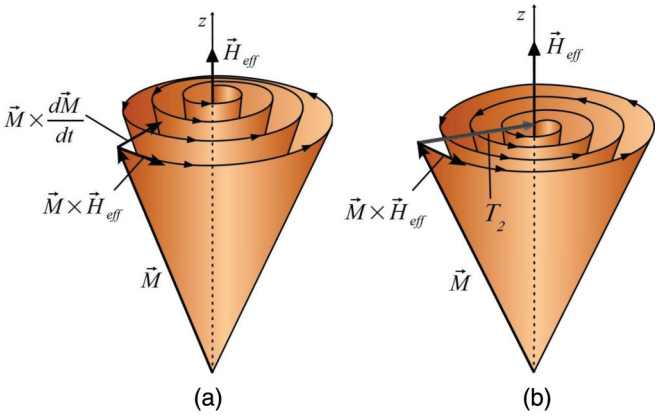

FIG. 1. (Color online) Illustration of relaxation processes of the magnetization precessing about an equilibrium direction. (a) The magnetization relaxes with constant magnitude, which is characteristic of the Gilbert damping expressed by the LLG Eq. (1). (b) Illustrates a process that occurs in many insulating ferro- or ferrimanetic materials. The transverse components of the magnetization relaxes rapidly to zero while the z component stays constant, as described by the B-B relaxation terms in Eqs. (4) and (5) for $T_1 \gg T_2$.

Although the LLG equation is the most widely used form to describe the damped motion of the magnetization, it does not apply to some important relaxation processes. The dot products of both terms on the right-hand side of Eq. (1) with $\vec{M}(\vec{r},t)$ vanish, indicating that in the Gilbert damping the magnetization vector describes a spiraling motion toward the equilibrium direction with constant magnitude,[1,2,48,49] as represented in Fig. 1(a). This is the relaxation process that prevails in bulk FM metals because the spins of the magnetic d-electrons interact strongly with the spins of the conduction s-electrons, which are in close contact with the lattice thermal bath. However, in insulating ferro- and ferrimagnetic materials with weak spin-orbit coupling, the relaxation processes take place in a quite different manner. They are better described by a relaxation form introduced by Bloch[50] for nuclear magnetic resonance[51] and adapted by Bloembergen[52] to paramagnetic[53] or ferromagnetic relaxation.[1,2] The Bloch-Bloembergen (B-B) phenomenology considers that the longitudinal and transverse components of the magnetization have different relaxation rates so that B-B equations of motion for the magnetization are writen as[1,2,48–54]

$$\frac{dm_{x,y}}{dt} = \gamma(\vec{H}_{\rm eff}\times\vec{M})_{x,y} - \frac{m_{x,y}}{T_2}, \tag{4}$$

$$\frac{dM_z}{dt} = \gamma(\vec{H}_{\rm eff}\times\vec{M})_z - \frac{M_z - M}{T_1}, \tag{5}$$

where we have omitted the time and spatial dependences for simplicity, T_1 and T_2 are, respectively, the longitudinal and transverse relaxation times, and $1/T_1$ and $1/T_2$ are the corresponding relaxation rates. In insulating FM materials, the longitudinal and transverse relaxations are governed by different physical processes so that T_1 and T_2 can be quite different. Usually T_2 is determined by the spin interactions that redistribute the energy in the precessing magnetization in the magnetic system, whereas T_1 is determined by processes that thermalize with the lattice. In many materials $T_1 \gg T_2$ so that the relaxation process occurs essentially in two steps: one in which the z component of the magnetization remains constant while the tip of the magnetization vector spirals towards the z axis with characteristic time T_2, as illustrated in Fig. 1(b), followed by another step in which the length of magnetization increases to the saturation value in a time T_1. This is essentially what happens in YIG where T_2 caused by magnon-magnon processes that conserve M_z is one order of magnitude smaller than T_1, which is long due to the weak spin-orbit coupling. Of course, the physical processes that are responsible for T_1 and T_2 occur simultaneously so that the two steps cannot actually be separated. In general, several mechanisms contribute to the relaxation acting as independent channels through which the energy flows out of the excited magnetic state so that the total relaxation rate is the sum of the individual contributions and the total half-linewidth is

$$\Delta H = \sum_{\lambda} 1/(2^{\upsilon}\gamma T_{\lambda}), \tag{6}$$

where T_λ is the relaxation time of mechanism λ and υ is an exponent that is 1 for a mechanism contributing only to the longitudinal relaxation and 0 for transverse relaxation. In closing this section, we note that many authors call the source of the FMR linewidth as Gilbert damping, regardless of its origin. As we have shown, this is not strictly correct, as some relaxation processes can be adequately described by the LLG equations but others require the B-B formulation. Only if a relaxation process has $T_2 = T_1/2$ does the magnitude of $\vec{M}$ remain constant during the relaxation, and then the B-B and the LLG forms are essentially equivalent.

III. DAMPING MECHANISMS IN THIN FILMS AND BILAYERS

It has been known for several decades that in bulk magnetic insulators the magnetic relaxation occurs through intrinsic mechanisms involving magnon-magnon and magnon-phonon processes as well as extrinsic mechanisms such as scattering by impurities or surface imperfections.[1,2] In bulk metallic materials, the relaxation is dominated by processes in which the energy of the magnetic moments is transferred directly to the spins of the conduction electrons.[4,5] In the late 1990s, it was realized that in ultrathin magnetic films the sample quality and surface roughness played an important role in the spin relaxation and that the FMR linewidth increased substantially as the film thickness decreased below certain values.[6–8,55–57] It was also realized that the linewidth increased further if certain nonmagnetic metals (NM) were deposited on FM films forming FM/NM bilayers.[11,12] The first observation was explained by Arias and Mills[7] (AM) based on a mechanism involving two-magnon scattering while the second was explained by Tserkovnyak, Bratass, and Bauer[9,10] with a new mechanism that was called spin pumping. In this section, we briefly review those two mechanisms because they are necessary for the interpretation of the experiments and for understanding the formulation of the spin-pumping damping proposed here for thick FM films in contact with NMs.

We recall that spin waves are the excitations of the magnetization in a FM material, and their quanta are called magnons. The semiclassical view of a spin wave with frequency ω_k and wave vector $\vec{k}$ is a spatially varying magnetization precessing

about the equilibrium direction with frequency ω_k and with phase $\vec{k}\cdot\vec{r}$. The variation of ω_k with $\vec{k}$, called dispersion relation, depends on the applied static field, magnetization, shape, and spin-interaction parameters.[1,2] Since the spin wave is associated with deviations of the spins from the equilibrium direction, the creation or destruction of magnons change the small-signal transverse magnetization components m_x and m_y. One magnon corresponds to one spin deviation shared by all spins. Since the saturation magnetization is $M = g\mu_B NS/V$, where N is the number of spin S in volume V, the z component of the magnetization is directly related to the total number of magnons in the sample[1,2]

$$M_z = M - (g\mu_B/V)\sum_k n_k. \tag{7}$$

The spin-spin interactions give rise to processes involving three- or four-magnon scattering that conserve both energy and momentum and constitute the main intrinsic source of spin-wave damping.[1] Processes in which an externally driven spin-wave mode relaxes into degenerate modes without momentum conservation require the presence of irregularities in the sample shape and thus are called extrinsic. Two-magnon scattering had been a well-known extrinsic relaxation mechanism in both insulating[1,58] and metallic[59,60] bulk samples when AM[7] showed that in ultrathin ferromagnetic films the two-magnon scattering due to defects and imperfections on the surfaces and interfaces were an important source of spin-wave damping.

According to AM, the low-wave number spin waves, such as the $k \approx 0$ FMR mode, have a large number of degenerate $k > 0$ modes because the dispersion relation has a broad minimum at $k \approx 10^4 - 10^5$ cm^{-1}. This is because in films magnetized in the plane, the $k = 0$ frequency is $\omega_0 = \gamma[H(H + 4\pi M - H_S)]^{1/2}$, where $H_S = 2K_S/Mt_{\rm FM}$ is the surface anisotropy field. As k increases, the frequency initially decreases due to the dipolar interaction and then increases because of exchange. The decay of the $k \approx 0$ modes into degenerate mode requires the existence of momentum nonconserving two-magnon interactions that are provided by the variation in the surface anisotropy at the defects and imperfections on the film surfaces and interfaces, which become increasingly important as the film thickness decreases. For a static magnetic field H applied in the plane of the film with thickness $t_{\rm FM}$ and magnetization M, for $H \ll 4\pi M_{\rm eff}$, the half-width at half-maximum (HWHM) can be written approximately as[7,61]

$$\Delta H_{2M} = \frac{16}{\pi}\frac{s(2K_S/M)^2}{D}\sin^{-1}\left(\frac{H}{H + 4\pi M_{\rm eff}}\right)^{1/2}\frac{1}{t_{\rm FM}^2}, \tag{8}$$

where D is the exchange stiffness constant, s is a geometrical factor characteristic of the surface roughness, $4\pi M_{\rm eff} = 4\pi M - H_S$ is the effective magnetization, and $H_S = 2K_S/(Mt_{\rm FM})$ is the surface anisotropy field. For films magnetized in the plane, the FMR frequency is[1]

$$\omega = \gamma[H(H + 4\pi M_{\rm eff})]^{1/2}, \tag{9}$$

where $\gamma = g\mu_B/\hbar \approx 2.8$ GHz/kOe is the gyromagnetic ratio. Using Eq. (8), one can transform the field dependence of the linewidth into a frequency dependence,[61]

$$\Delta H_{2M} = \frac{16}{\pi}\frac{s(2K_S/M)^2}{D} \times \sin^{-1}\left[\frac{\left(\omega^2 + \omega_M^2/4\right)^{1/2} - \omega_M/2}{\left(\omega^2 + \omega_M^2/4\right)^{1/2} + \omega_M/2}\right]^{1/2}\frac{1}{t_{\rm FM}^2}, \tag{10}$$

where $\omega_M = \gamma 4\pi M_{\rm eff}$. Equation (10) shows that the two-magnon contribution to the linewidth varies with film thickness as $1/t_{\rm FM}^2$. Regarding the frequency dependence, the linewidth vanishes at $\omega = 0$, initially increases linearly with frequency, but then it saturates at large ω. It has been shown that when the FM film is in contact with an antiferromagnetic material, the two-magnon damping is dominated by processes involving fluctuations in the exchange coupling between the two layers, resulting in very large FMR and spin-wave Brillouin light-scattering linewidths.[56,62–64] It has also been shown that the two-magnon damping has a characteristic variation with the applied field angle with the plane that allows a clear-cut determination of its contribution to the FMR linewidth in films and multilayers.[65,66]

Note that in two-magnon scattering processes, one magnon with certain wave number k is created while another with a different k is destroyed so that the total number of magnons is unchanged. According to Eq. (7), the z component of the magnetization is unchanged so that the two-magnon scattering does not contribute to the T_1 relaxation; rather, it is a T_2 process. This means that it is not correct to classify the two-magnon process as a Gilbert damping mechanism.[61] Actually the linear dependence of the Gilbert damping linewidth on the frequency, as in Eq. (3), is different from the frequency dependence of the two-magnon linewidth,[7,61] a fact that provides a convenient way to separate the two contributions in experimental data, as long as one can vary the microwave frequency over quite a large range.[49]

The concept of spin-pumping damping proposed in Refs. 9 and 10 to explain the increase in the FMR linewidth of FM films when in contact with a NM layer was based on a new physical picture: the precessing magnetization in the FM layer injects a pure spin current into the adjacent NM layer. If the FM layer is metallic, the spin transfer is governed by the reflection and transmission coefficients at the interface.[9,10] For an insulating FM, the mechanism relies on the interface exchange coupling between the spins of the d-state electrons in the FM side with the spins of the s electrons on the NM side.[20] As shown in Refs. 9 and 10 for a metallic bilayer, the spin-current density at the FM/NM interface ($y = 0$) is given by[9,10]

$$\vec{J}_S(0) = \frac{\hbar}{4\pi M^2} g_{\rm eff}^{\uparrow\downarrow}\vec{M}(y = 0,t) \times \frac{d\vec{M}(y = 0,t)}{dt}, \tag{11}$$

where $\vec{M}(y = 0,t)$ denotes the time and spatially varying magnetization at the interface plane $y = 0$, and $g_{\rm eff}^{\uparrow\downarrow}$ is the real part of the effective spin-mixing conductance including the effect of the back-flow current. In order to calculate the damping, one uses the fact that Eq. (11) has the same form of the Gilbert damping term in Eq. (1). Comparison of Eq. (11) with the damping term in Eq. (1) leads to a spin-pumping damping Gilbert parameter $\alpha_{\rm SP} = \gamma\hbar g^{\uparrow\downarrow}/(4\pi Mt_{\rm FM})$, so that

the additional half-linewidth due to spin pumping becomes[9,10]

$$\Delta H_{\rm SP} = \frac{\hbar g_{\rm eff}^{\uparrow\downarrow}\omega}{4\pi M t_{\rm FM}}. \tag{12}$$

Note that for a 10-nm-thick Py film in contact with a Pt layer, the additional FMR linewidth at a frequency of 10 GHz calculated with Eq. (12) using $4\pi M = 11$ kG and $g^{\uparrow\downarrow} = 2\times 10^{15}$ cm^{-2} is 11 Oe, which is a sizeable value compared to a 20-Oe linewidth of a single Py film. However, for a 1-μm-thick Py film, the spin-pumping linewidth reduces to 0.1 Oe, which is very small compared to the one in the bare film. This shows that the observed broadening[42] of the FMR lines in YIG films with thickness of several micrometers after deposition of a very thin Pt layer is a challenging issue.

We notice that the magnetization that enters in Eq. (11) is the vector function calculated at the interface plane, $y = 0$, whereas the magnetization in Eq. (1) is a function of space throughout the volume of the FM layer. This means that Eq. (11) does not strictly have the same form as the damping term in Eq. (1) so that at this point it is not clear if in thick FM films the spin-pumping relaxation is described by the Gilbert damping phenomenology. An alternative picture of the nature of the spin-pumping damping is revealed by the following calculation of the relaxation rate.

Due to angular momentum conservation, the spin current flowing out of the FM layer results in the relaxation of the z component of the magnetization. We write the z component of the total spin-angular momentum in the FM layer (volume V) with the precessing magnetization as $\gamma^{-1}\int dV M_z$ and consider that it relaxes toward equilibrium with a time rate $\int dA J_S(0)$, where the surface integral is carried out through the interface area. From Eq. (11), one can show[10] that the z component of the spin current pumped by the precessing FM magnetization with frequency ω is given by

$$J_{Sy}^{z}(0) = \frac{\hbar\omega g_{\rm eff}^{\uparrow\downarrow}}{4\pi M^2}[m^+(0)m^-(0)], \tag{13}$$

where $m^{\pm} = m_x \pm i m_y$. For the uniform precession mode, the volume and surface integrals above are trivial, and one obtains an equation for the time derivative of the longitudinal component of the magnetization

$$\frac{dM_z}{dt} = \frac{\gamma\hbar\omega g_{\rm eff}^{\uparrow\downarrow}}{4\pi M^2 t_{\rm FM}}[m^+(0)m^-(0)]. \tag{14}$$

Using the linear approximation $m^+m^- = m_x^2 + m_y^2 \approx 2M(M - M_z)$, Eq. (14) leads to $dM_z/dt = -(M_z - M)/T_{1\rm SP}$, where

$$\frac{1}{T_{1\rm SP}} = \frac{\gamma\hbar\omega g_{\rm eff}^{\uparrow\downarrow}}{2\pi M t_{\rm FM}}. \tag{15}$$

This result shows that the z component of the spin current produces a spin-pumping relaxation channel only for the longitudinal component of the magnetization. Using Eq. (15) in Eq. (6), we obtain the same result for the spin-pumping linewidth given by Eq. (12). Note that the well known $1/t_{\rm FM}$ dependence of the linewidth on the FM film thickness arises from the ratio between the area of the interface and the volume of the film, so that it holds only if the FMR mode is uniform over the film volume. In order to fully characterize the damping process, it is necessary to calculate also the transverse relaxation rate.

IV. RELAXATION OF THE TRANSVERSE COMPONENTS OF THE MAGNETIZATION IN FM/NM BILAYERS

In this section, we present a theoretical model for the relaxation of the transverse components of the magnetization in bilayers of a ferro- or ferrimagnetic material with a NM. Despite the fact that the microscopic origin of the spin pumping in bilayers made of FM insulator/NM is different than in FM metal/NM structures, the physical picture is the same in both cases. The spin pumping consists of the transfer of spin-angular momentum from the precessing magnetization in the FM layer into the NM layer, where the spin current is carried by the spins of the conduction electrons.[9,10] As shown in the previous section, the angular momentum that flows out of the FM layer results in a relaxation of the z component of the magnetization. Here we consider the coupled motion of the transverse components of the magnetization. This problem can be tackled with two different formulations.[67,68] One of them consists of considering that at the interface the spins of the FM interact with the NM conduction electron spins through the exchange interaction.[20,39,40,67,68] The coupled motion of the FM magnetization with the NM spin accumulation transfers to the FM magnetization an additional relaxation from the overdamped motion of the conduction electron spins. Alternatively, one can use spin currents and represent the effect of the interface by the spin-mixing conductance,[9,10] but it is necessary to consider the longitudinal and transverse polarizations. In this section, we present the former one. In both cases, we need to characterize first the spin excitations in the FM and NM layers separately.

A. Separated FM and NM layers

In the ferromagnetic layer we use a macroscopic approach and define the magnetization in terms of the spins $\vec{S}_i$ at sites i as $\vec{M}(\vec{r}) = g\mu_B\sum_i \vec{S}_i\delta(\vec{r} - \vec{r}_i)$. The evolution of the magnetization components is described by the LL with damping introduced in the B-B phenomenology, as in Eqs. (4) and (5). We consider $\vec{H}_{\rm eff} = \hat{z}H + (D/M)\nabla^2\vec{M} + \vec{h}_{\rm dip}$, where H is the in-plane applied field, D is the intraexchange stiffness parameter, and $\vec{h}_{\rm dip}$ is the dipolar field created by the spatial variation of the magnetization. From Eq. (5), one can show that the transverse variable $m^+ = m_x + iem_y$, where e is the ellipticity of the magnetization precession due to the dipolar field, satisfies the wave equation,

$$\frac{dm^+}{dt} = i\gamma H m^+ - D\nabla^2 m^+ - \eta_{\rm FM}m^+, \tag{16}$$

where $\eta_{\rm FM} = 1/T_2$ is the transverse relaxation rate. For a time dependence $\exp(i\omega t)$, Eq. (16) has wavelike solutions $\exp(\pm iky)$, where the wave number k is related to the frequency by

$$k^2 = (\omega - \omega_{\rm FM} - i\eta_{\rm FM})/\gamma D, \tag{17}$$

where $\omega_{\rm FM} = \gamma[H(H + 4\pi M)]^{1/2}$ is the FMR frequency.

In the NM layer, we designate the magnetization of the conduction electrons by $\vec{m}_N(\vec{r},t)$ and write it as[20]

$$\vec{m}_N(y) = \vec{m}_0 a_{\rm eff}\delta(y) + \delta\vec{m}_N(y), \tag{18}$$

where $\vec{m}_0 = \hat{z}\chi_N J_{\rm ex} M$ is the equilibrium magnetization, χ_N is the paramagnetic susceptibility of the conduction electrons, $a_{\rm eff} = v_e/a_s^2$ is the effective interaction range, v_e is the volume per conduction electron, a_s is the lattice constant of the localized spins at the interface on the FM side,[20] and $\delta\vec{m}_N(y,t)$ is magnetization deviation, which is related to the spin accumulation $\vec{\mu}_S$ of Refs. 9 and 10 by

$$\delta\vec{m}_N = \mu_B {\rm N}(\varepsilon)\vec{\mu}_S, \tag{19}$$

where $\rm N(\varepsilon)$ is the density of states. It can be shown that the magnetization deviation in the NM is governed by the diffusion equation[9,10,20,67,68]

$$\frac{\partial\delta\vec{m}_N}{\partial t} = \gamma_e\delta\vec{m}_N\times\vec{H} - \eta_{sf}\delta\vec{m}_N + D_N\nabla^2\delta\vec{m}_N, \tag{20}$$

where γ_e is the electron gyromagnetic ratio, η_{sf} is the relaxation rate of the spin accumulation, which is related to the electron spin-flip time τ_{sf} by $\eta_{sf} = 1/\tau_{sf}$, and D_N is the spin-diffusion constant. Using $\delta m_N^+ = \delta m_N^x + i\delta m_N^y$ one can show that

$$\frac{\partial\delta m_N^+}{\partial t} = i\omega_H\delta m_N^+ + D_N\nabla^2\delta m_N^+ - \eta_{sf}\delta m_N^+, \tag{21}$$

where $\omega_H = \gamma_e H$ is the conduction electron spin-resonance frequency. For a time dependence $\exp(i\omega t)$, Eq. (7) has solutions $\exp(\pm\kappa y)$, where

$$\kappa^2 = [i(\omega-\omega_H) + \eta_{sf}]/D_N, \tag{22}$$

which can also be expressed in terms of the spin-diffusion length λ_N using $D_N = \lambda_N^2\eta_{sf}$.

B. FM and NM layers interacting through exchange

We now consider that the ferromagnetic film with thickness $t_{\rm FM}$ is in atomic contact with a nonmagnetic metal layer (thickness t_N), as illustrated in Fig. 2. The coordinate axes have the z direction along the field and the y direction

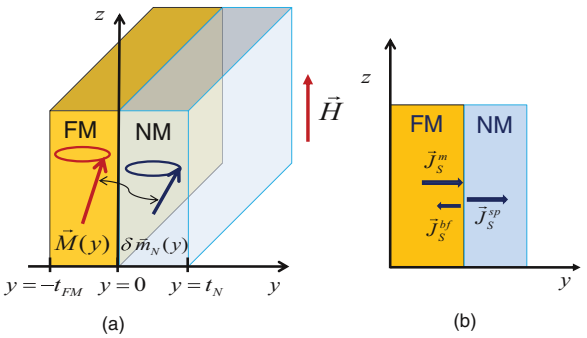

FIG. 2. (Color online) Illustration of the ferromagnet (FM)/nonmagnetic metal (NM) bilayer with the coordinate axes used to study the FM magnetization damping due to the spin-pumping process. (a) Coupling of the FM and NM magnetizations through the exchange interaction at the surface. (b) Spin currents used in an alternative calculation of the damping.

perpendicular to the interface plane at $y = 0$. At the interface sites i, the spins $\vec{s}_i$ of the conduction electrons in the NM layer interact with the spins $\vec{S}_i$ in the FM side through the s-d exchange interaction,[20] $H_{sd} = -J_{sd}\sum_i \vec{S}_i\cdot\vec{s}_i$, where J_{sd} is the exchange-coupling constant. The summation on the interface sites i can be approximated by a surface integral and the coupling between the magnetization $\vec{M}(\vec{r},t)$ in the FM side, and the magnetization $\vec{m}_N(\vec{r},t)$ of the conduction electrons in the NM side can be represented by the Hamiltonian,[20]

$$H_{sd} = -(J_{\rm ex}/A)\int dxdz\int dy\vec{M}(\vec{r},t)\,a_{\rm eff}\delta(y)\cdot\vec{m}_N(\vec{r},t), \tag{23}$$

where A is the interface area, $J_{\rm ex} = J_{sd}S/(\hbar\gamma_e M)$ is the dimensionless exchange-coupling constant, S is an effective block spin per unit cell, and M is the magnetization of the FM. In order to make the interface coupling tractable, we follow Ref. 20 and consider that the magnetizations do not vary along the interface plane and $\vec{m}_N(\vec{r},t) = \vec{m}_N(y,t)$. One obtains for the total interface exchange-coupling energy per unit area (energy density),

$$E_{\rm ex} = -J_{\rm ex}\vec{M}(y=0,t)a_{\rm eff}\cdot\vec{m}_N(y=0,t). \tag{24}$$

From Eq. (24) one can write the surface torque density on $\vec{M}(0)$ at the interface due to the exchange interaction with $\vec{m}_N(0)$,

$$\vec{\tau}_s = J_{\rm ex}a_{\rm eff}(\vec{M}\times\vec{m}_N). \tag{25}$$

Of course, the torque that $\vec{M}(0)$ exerts on $\vec{m}_N(0)$ is $-\vec{\tau}_s$. Instead of introducing the coupling by adding the torque in the equations of motion of the magnetizations, as in Ref. 20, we use for $\vec{M}$ and $\vec{m}_N$ Eqs. (16) and (20) and impose the boundary conditions on both sides of the interface. As shown long ago by Rado and Weertman,[69] the boundary condition at the interface on the FM side ($y = 0^-$) has to take into account the intraexchange interaction. It states that the total torque density vanishes at the interface

$$\vec{M}\times\frac{D}{M}\nabla_y\vec{M} + \vec{\tau}_s = 0, \tag{26}$$

where $\vec{\tau}_s$ is the surface torque density that $\vec{m}_N(0)$ exerts on $\vec{M}(0)$ at the interface due to the exchange interaction given by Eq. (25). Using a procedure similar to the one of Rado and Weertman, one can show that the magnetization deviation at the interface on the NM side ($y = 0^+$) satisfies the following boundary condition:

$$\frac{D_N}{\gamma_e}\nabla_y\delta\vec{m}_N - \vec{\tau}_s = 0, \tag{27}$$

where $-\vec{\tau}_s$ is the surface torque density acting on $\delta\vec{m}_N$.

Using the expression for the torque density [Eq. (25)] in the first boundary condition [Eq. (26)], we obtain one equation relating the transverse magnetization components at the interface,

$$M_z\frac{D}{M}\frac{\partial m^+}{\partial y}\bigg|_0 + J_{\rm ex}a_{\rm eff}\left[M_z\delta m_N^+(0) - m_N^z(0)m^+(0)\right] = 0. \tag{28}$$

Using Eq. (25) in the second boundary condition [Eq. (27)], one can find another relation between the transverse magnetization components,

$$\frac{D_N}{\gamma_e}\frac{\partial \delta m_N^+}{\partial y}\bigg|_0 - i J_{\rm ex} a_{\rm eff}[M_z \delta m_N^+(0) - m_0 m^+(0)] = 0. \quad (29)$$

In order to obtain relations between the transverse magnetizations at the interface, $m^+(0)$ and $\delta m_N^+(0)$, one needs first to find the spatial variations of $m^+(y)$ and $\delta m_N^+(y)$. For a time dependence $\exp(i\omega t)$, Eq. (16) has solutions $\exp(\pm iky)$, so that using the boundary condition $\nabla_y m^+ = 0$ at $y = -t_{\rm FM}$ one finds for the FM side,

$$m^+(y) = [\cos ky - \tan(kt_{\rm FM}) \sin ky] m^+(0). \quad (30)$$

In the NM layer, we assume for simplicity that the thickness is much larger than the diffusion length, so that for a time dependence $\exp(i\omega t)$, one obtains from Eq. (21),

$$\delta m_N^+(y) = \delta m_N^+(0) e^{-y/\lambda_N}. \quad (31)$$

Using Eq. (30) in Eq. (28) and considering that the FM layer is thin, such that one can use the linear approximation, $\tan(kt_{\rm FM}) \approx kt_{\rm FM}$, we obtain

$$[(\omega - \omega_{\rm FM} - i\eta_{\rm FM}) + \gamma\beta/t_{\rm FM}] m^+(0) = (\lambda_{\rm ex}\eta_{sf}\lambda_N/t_{\rm FM}) \delta m_N^+(0), \quad (32)$$

where $\beta = J_{\rm ex} a_{\rm eff} m_0$ and $\lambda_{\rm ex}$ is a dimensionless exchange-coupling parameter,

$$\lambda_{\rm ex} = \gamma J_{\rm ex} a_{\rm eff} M/(\eta_{sf}\lambda_N),$$

that is related to the parameter Γ in Ref. 20 by $\lambda_{\rm ex} = 1/\Gamma$. Using the Eq. (31) in Eq. (29), one finds

$$[(\omega - \omega_H - i2\eta_{sf}) + 2\lambda_{\rm ex}\eta_{sf}] \delta m_N^+(0) = (2\beta/\lambda_N) m^+(0). \quad (33)$$

Now we multiply Eqs. (32) and (33) to eliminate the magnetization variables to obtain an equation for the frequencies in the small thickness approximation $\sin kt_{\rm FM} \approx kt_{\rm FM}$,

$$[(\omega - \omega_{\rm FM} - i\eta_{\rm FM}) + \gamma\beta/t_{\rm FM}][(\omega - \omega_H - i2\eta_{sf}) + 2\lambda_{\rm ex}\eta_{sf}] = 2\beta\lambda_{\rm ex}\eta_{sf}/t_{\rm FM}. \quad (34)$$

Equation (34) leads to a second degree equation whose solutions are the complex eigen frequencies of the coupled FM-NM magnetizations,

$$\omega_1 \approx \omega_H + i\eta_{sf} \quad (35)$$

$$\omega_2 \approx \omega_{\rm FM} + i\eta_{\rm FM} + i\frac{\gamma\beta\lambda_{\rm ex}}{t_{\rm FM}}. \quad (36)$$

The real and imaginary parts of Eqs. (35) and (36) correspond, respectively, to the normal mode oscillation frequencies and relaxation rates. Clearly ω_1 is associated with the motion dominated by the conduction electron spins in the NM layer, while ω_2 is associated with the FMR of the FM layer. Since $\omega_H \sim 10^{10}$ s^{-1} and $\eta_{sf} \sim 10^{12}$ s^{-1}, the motion of the spins in Pt is heavily overdamped. The important result revealed in Eq. (36) is that the transverse relaxation rate of the FMR has, in addition to the intrinsic damping, a contribution proportional to the square of the exchange-coupling parameter $J_{\rm ex}$ and inversely proportional to the FM layer thickness. As remarked earlier, this additional damping is of the type T_2, whereas the spin-pumping relaxation in Eq. (15) is of the type T_1. In order to compare the transverse relaxation rate with the spin-pumping damping, one must relate the exchange parameter with the spin-mixing conductance. For this we use the boundary condition in Eq. (27) to relate the longitudinal component of the NM magnetization deviation with the transverse FM magnetization,

$$\delta m_N^z(0) = -\frac{\lambda_{\rm ex}}{M}{\rm Im}[\delta m_N^+(0) m^-(0)]. \quad (37)$$

Considering that $\eta_{sf} \gg \omega, \omega_H$ and $\lambda_{\rm ex} \ll 1$, we obtain from Eq. (33) the following relation, ${\rm Im}[\delta m_N^+(0)] = (\beta/\eta_{sf}\lambda_N) m^+(0)$, so that the longitudinal magnetization deviation becomes

$$\delta m_N^z(y) = -\frac{\lambda_{\rm ex}\beta}{\eta_{sf}\lambda_N M}{\rm Im}[m^+(0) m^-(0)] e^{-y/\lambda_N}. \quad (38)$$

From Eq. (38), one can obtain the spin-current density with polarization z in the NM using $J_S^z = (D_N/\gamma)\nabla_y(\delta m_N^z)$, which is, in units of angular momentum/(area.time),

$$J_S^z(0) = \frac{\beta\lambda_{\rm ex}}{M}[m^+(0) m^-(0)]. \quad (39)$$

Comparison of Eq. (39) with the spin-pumping current in Eq. (13) leads to a convenient relation between the exchange-coupling parameter and the spin-mixing conductance,

$$\beta\lambda_{\rm ex} = \frac{\hbar\omega g_{\rm eff}^{\uparrow\downarrow}}{4\pi M}. \quad (40)$$

Using Eq. (40) in (36) leads to a transverse relaxation rate

$$\eta_{\rm exch} = \frac{1}{T_2} = \frac{\gamma\hbar\omega g_{\rm eff}^{\uparrow\downarrow}}{4\pi M t_{\rm FM}}. \quad (41)$$

Comparison of Equations (15) and (41) shows that the transverse relaxation rate due to the exchange coupling between the oscillators through the interface is 1/2 the longitudinal relaxation rate due to spin pumping. This means that the exchange-coupling relaxation is simply the transverse part of the spin-pumping damping in a process that conserves the amplitude of the magnetization and characterizes the Gilbert phenomenology. This result becomes even more clear if the coupled FM/NM is treated with spin currents only, as we show in the next section.

C. Treatment with spin currents

The usual treatment of the spin-pumping damping in FM/NM bilayers is done entirely in terms of spin currents.[9,10] It turns out that, as shown in Sec. II, the calculation of the damping based on the z component of the spin currents gives information only on the longitudinal relaxation rate. Here we calculate the transverse relaxation rate due to the spin-pumping process using the transverse components of the spin current, which are also employed in the model for the ac SHE.[70,71] As done in the previous section, the interaction between the FM and NM is introduced through the boundary conditions expressed by Eqs. (26) and (27). Here the surface torque density $\vec{\tau}_s$ results from the discontinuity in the spin-current

density in the y direction at the interface,[67,68,72]

$$\vec{\tau}_s = -\left(\vec{J}_S^{sp} + \vec{J}_S^{bf}\right) \cdot \hat{y}, \tag{42}$$

where $\vec{J}_S^{sp}$, $\vec{J}_S^{bf}$ are, respectively, the spin-pumped and back-flow tensorial spin-current densities illustrated in Fig. 2(b). In an insulating ferromagnet, the spin current is carried by spin waves, and the magnon spin current in the y direction associated with the FM magnetization is given by[20,67,73]

$$\vec{J}_{Sy}^{m} = (D/M)(\vec{M} \times \nabla_y \vec{M}). \tag{43}$$

Notice that the magnon spin current [Eq. (43)] does not enter in Eq. (42) for the interface torque because it is already taken into account in the Rado-Weertman boundary condition [Eq. (26)]. Using Eqs. (42) and (43) in Eq. (26), we find that the boundary condition at the interface is $\vec{J}_{Sy}^{m} = \vec{J}_{Sy}^{sp} + \vec{J}_{Sy}^{bf}$, which means continuity of spin currents, in agreement with Refs. 39 and 40. Neglecting the ellipticity of the FM magnetization precession, one can write for the $+$ transverse component of the magnon spin current, $J_{Sy}^{m+} = J_{Sy}^{mx} + iJ_{Sy}^{my} = -iD\nabla_y m^+$. The $+$ transverse component of the pumped spin current can be calculated with $J_S^{sp+} = (D_N/\gamma)\nabla_y(\delta m_N^+)$. Using the solution of the diffusion [Eq. (21)] and the relation between $\delta m_N^+(0)$ and $m^+(0)$ given by Eq. (33) in the boundary condition [Eq. (27)], one can show that the sum of the spin-pumped and back-flow spin-current densities at the interface is given by an equation similar to the one for the longitudinal spin current,[10]

$$J_{Sy}^{sp+} + J_{Sy}^{bf+} = -\frac{\hbar\omega g_{\text{eff}}^{\uparrow\downarrow}}{4\pi M} m^+(0). \tag{44}$$

Using the expressions for the spin currents in Eq. (42), the Rado-Weertman boundary condition [Eq. (26)] at $y = 0$ leads to $iD\nabla_y m^+ - (\hbar\omega g_{\text{eff}}^{\uparrow\downarrow}/4\pi M)m^+ = 0$. Introducing the y dependence of the transverse magnetization given by Eq. (30) in this expression, one obtains

$$Dk\tan(kt_{\text{FM}}) - i\frac{\hbar\omega g_{\text{eff}}^{\uparrow\downarrow}}{4\pi M} = 0, \tag{45}$$

where k is related to the frequency ω through Eq. (17). Equation (45) is easily solved in the small thickness regime, $kt_{\text{FM}} \ll 1$, giving the complex eigenfrequency for the excitation of the transverse FM magnetization, $\omega = \omega_{\text{FM}} + i(\eta_0 + \eta_{SP})$, where η_0 represents the intrinsic damping and

$$\eta_{SP} = \frac{1}{T_{2SP}} = \frac{\gamma\hbar\omega g_{\text{eff}}^{\uparrow\downarrow}}{4\pi M t_{\text{FM}}} \tag{46}$$

is the transverse relaxation rate due to the spin-pumping process. This result coincides with Eq. (41) and confirms that the spin-pumping process gives rise to a transverse relaxation rate, which is half the longitudinal one. As shown in Sec. II, a relaxation process with $1/T_2 = 1/2T_1$ conserves the magnitude of the magnetization and characterizes a Gilbert damping process. From Eqs. (15) and (46), one finds the Gilbert damping parameter $\alpha_{SP} = \gamma\hbar g_{\text{eff}}^{\uparrow\downarrow}/(4\pi M t_{\text{FM}})$, in agreement with the well-known result.[9,10]

V. SPIN-PUMPING DAMPING IN THICK FM FILMS AND COMPARISON WITH EXPERIMENTAL DATA

The transverse relaxation rates obtained in Secs. IV B and IV C that scale with $1/t_{\text{FM}}$ are valid only in thin FM films for which one can use the linear approximation $\tan(kt_{\text{FM}}) \approx kt_{\text{FM}}$. As the FM film thickness increases, this approximation is no longer valid, and the transverse relaxation rate deviates from the relation $1/T_2 = 1/T_1$. In order to find the thickness scale of validity of the previous calculation, we use the resonance condition $\omega = \omega_{\text{FM}}$ in Eq. (17) to obtain the real part of the wave number $k_r = \sqrt{\Delta H/(2D)}$, where ΔH is the FMR HWHM. Considering that $\tan(kt_{\text{FM}}) \approx kt_{\text{FM}}$ is valid for $kt_{\text{FM}} < 0.5$, one can establish a critical FM thickness below which the relation $1/T_2 = 1/2T_1$ is satisfied,

$$t_c \approx \sqrt{D/2\Delta H}. \tag{47}$$

For YIG/Pt,[42] with $D = 5 \times 10^{-9}$ Oe cm^2 and $\Delta H \approx 3$ Oe, one obtains $t_c \approx 280$ nm, whereas for Py,[18] with $D = 2 \times 10^{-9}$ Oe cm^2 and $\Delta H \approx 20$ Oe, $t_c \approx 70$ nm. For $t_{\text{FM}} > t_c$, the linear approximation breaks down, and the transverse relaxation rate does not decrease with $1/t_{\text{FM}}$ with increasing FM film thickness. There is another more compelling reason for the change in the thickness dependence of the transverse relaxation. In thick films, only those spins that are close to the FM/NM interface within a coherence length contribute to the spin pumping.[74] Here the coherence length can be defined by $t_{\text{coh}} = v_g/\eta$, where v_g is the spin-wave group velocity and $\eta = 2\gamma\Delta H$ is the magnon number relaxation rate assumed to be dominated by the spin-pumping damping. Thus for $t_{\text{FM}} > t_{\text{coh}}$, the boundary condition $\nabla_y m^+ = 0$ must be applied at $y = -t_{\text{coh}}$. Using $v_g = 2\gamma Dk$ we find for the coherence length

$$t_{\text{coh}} = \sqrt{D/\Delta H}, \tag{48}$$

so that from Eq. (46) one can obtain an approximate result for transverse relaxation rate for FM film thickness $t_{\text{FM}} > t_{\text{coh}}$ given by

$$\eta_{SP} \approx \frac{\gamma\hbar\omega g_{\text{eff}}^{\uparrow\downarrow}}{4\pi M t_{\text{coh}}}. \tag{49}$$

Equation (49) reveals that for a FM film thickness $t_{\text{FM}} > t_{\text{coh}}$, the transverse relaxation rate is independent of the thickness. Likewise, since in thick films the FMR excitation acquires a surface-mode character with an effective volume with thickness t_{coh}, Eq. (15), for the longitudinal relaxation rate, has to be replaced by

$$\frac{1}{T_{1SP}} \approx \frac{\gamma\hbar\omega g_{\text{eff}}^{\uparrow\downarrow}}{2\pi M t_{\text{coh}}}. \tag{50}$$

Equations (49) and (50) show that for $t_{\text{FM}} > t_{\text{coh}}$ the relation $1/T_2 = 1/2T_1$ is maintained so that the spin-pumping damping in thick FM films covered with a NM layer is indeed described by the Gilbert phenomenology.

The argument of the concentration of the spin excitation within a coherence length from the interface deserves further discussion. First we note that assuming that the FMR linewidth is dominated by the spin-pumping relaxation, using $\Delta H = \eta_{SP}/\gamma$ in Equations (48) and (49), one can obtain an expression

for the coherence length in terms of material parameters,

$$t_{\rm coh} \approx \frac{4\pi M D}{\hbar\omega g_{\rm eff}^{\uparrow\downarrow}}. \tag{51}$$

Using for YIG the values $4\pi M = 1.76 \times 10^3$ G, $D = 5 \times 10^{-9}$ Oe cm, $\omega = 2\pi \times 10^{10}$ s^{-1}, and $g_{\rm eff}^{\uparrow\downarrow} = 2 \times 10^{15}$ cm^{-2}, we obtain $t_{\rm coh} \approx 600$ nm. It is well known that in a bare YIG film, the low wave-number spin waves are dominated by the dipolar interaction and are completely described by the electromagnetic boundary conditions. The exchange interaction plays a negligible role, and the spin excitations have either volume or surface character.[43,44] The FMR excitation is a volume mode with rf magnetization uniform across the thickness, and, in this case, one would not expect any concentration of spin excitation near one of the surfaces. However, the presence of the NM layer in contact with the YIG film completely changes the picture. Since the spin current vanishes at the interface with the substrate and is nonzero at the FM/NM interface, the rf transverse magnetization has to vary across the thickness so that the intraexchange interaction becomes important. For thick FM films, the variation of the rf transverse magnetization along the y direction is given by Eq. (30), and one cannot use the linear approximation $\tan(kt_{\rm FM}) \approx kt_{\rm FM}$.

In order to calculate the variation of m^+ with y we consider Eq. (17) at the resonance condition, $k^2 = -i\eta_{SP}/(\gamma D)$. Using the real and imaginary parts of k in the trigonometric functions in Eq. (30), one obtains lengthy expressions for the real and imaginary parts of $m^+(y)$, which can be easily calculated numerically. Figure 3 shows plots of the relative amplitude $|m^+(y)|^2/|m^+(0)|^2$ across the thickness of a ferromagnetic film in contact with a nonmagnetic metal layer under FMR calculated with Eq. (30) using the parameters for YIG and the following conditions. For $t_{\rm FM} \leqslant t_{\rm coh}$, the relaxation rate η_{SP} is given by Eq. (46); for $t_{\rm FM} \geqslant t_{\rm coh}$, η_{SP} is given by Eq. (49). The plots in Fig. 3 reveal that in the YIG film with $t_{\rm FM} = 0.1$ μm, which is much less than the coherence length, the magnetization amplitude is uniform across the thickness. For $t_{\rm FM} = 0.5$ μm, which is comparable but still smaller than $t_{\rm coh}$, the magnetization is notably nonuniform. For thicker films, the concentration of the rf magnetization near the FM/NM interface rapidly increases with increasing thickness. For $t_{\rm FM} = 2.0$ μm, the magnetization profile resembles that of a surface mode, in support with our argument that in thick films only those spins that are close to the FM/NM interface within a coherence length contribute to the spin pumping.

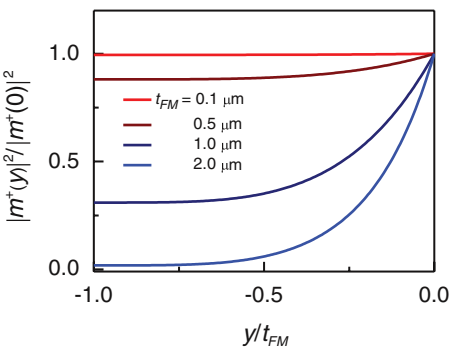

FIG. 3. (Color online) Variation of the relative rf magnetization amplitude across the thickness of a ferromagnetic film in contact with a nonmagnetic metal layer under FMR. The calculation was done with Eq. (30) using parameters appropriate for YIG for four values of the FM film thickness $t_{\rm FM}$ from top to bottom respectively: 0.1, 0.5, 1.0, and 2.0 μm.

Regarding the interpretation of the large broadening of the FMR lines observed in thick YIG films with Pt deposition, we note first that it cannot be attributed to interface two-magnon scattering relaxation. The linewidths measured[42] in YIG/Pt with the field normal and parallel to the film plane are nearly the same. However, with the field normal to the plane, the FMR frequency is at the bottom of the spin-wave manifold. As a result, there are relatively few degenerate states into which the FMR ($k = 0$) mode can decay, so the two-magnon relaxation rate is small when the field is normal, as predicted theoretically[65] and observed experimentally.[66]

Comparison of the results for the spin-pumping relaxation with the experimental FMR linewidth data in YIG/Pt recently reported[42] is not straightforward because of the uncertainties in the material parameters. The value of the spin-mixing conductance $g_{\rm eff}^{\uparrow\downarrow}$ depends on knowledge of the parameters for Pt, which have reported results that disagree by factors larger than 20.[75] We assume here the most favorable numbers and use the data for the spin-pumping/ISHE voltage generated by FMR reported in Ref. 2, $V_{SP} \propto g_{\rm eff}^{\uparrow\downarrow}\lambda_{SD}\theta_{SH}$, to estimate $g_{\rm eff}^{\uparrow\downarrow}$. With a spin-diffusion length for Pt of $\lambda_N = 1.7$ nm, from Ref. 75, and a spin Hall angle for Pt of $\theta_{SH} = 0.0037$, from Ref. 20, we obtain $g_{\rm eff}^{\uparrow\downarrow} = 2.2 \times 10^{15}$ cm^{-2}. Considering $t_{\rm coh} = 600$ nm for YIG and the measured 2.3 Oe increase in the linewidth at 10 GHz due to the Pt layer deposition,[42] we find from Eq. (49) a value for the effective spin-mixing conductance in YIG/Pt of $g_{\rm eff}^{\uparrow\downarrow} = 3.7 \times 10^{15}$ cm^{-2}. This is in order of magnitude agreement with the value determined from data for the spin-pumping/ISHE voltage, showing that the enhanced spin-pumping damping may account for the measured FMR linewidth in thick YIG films covered with a thin Pt layer.

VI. SUMMARY

In summary, we have shown that a full characterization of the spin-pumping damping in thick ferromagnetic films in contact with a nonmagnetic metal layer requires the calculation of the transverse relaxation rate in addition to the longitudinal relaxation rate that emerges from the usual treatment with only the longitudinal component of the spin current. Two alternative approaches were used to calculate the transverse relaxation rate. In one, we considered the coupled motion of the FM magnetization with the NM spin accumulation through the exchange interaction at the interface and found that the overdamped motion of the conduction electron spins transfers to the FM magnetization an additional relaxation. The other treatment involves only spin currents, but one has to consider the transverse component of the spin current in addition to the longitudinal one employed in the usual treatment.[9,10] Both treatments yield the same results. For FM layer thickness $t_{\rm FM}$ smaller than a coherence length $t_{\rm coh}$, the transverse and

longitudinal relaxation rates vary with $1/t_{FM}$ and for $t_{FM} > t_{coh}$ both relaxation rates become independent of the thickness due to the surface character of the FMR mode. In both cases, the longitudinal relaxation rate is twice the transverse rate so that the damping of the magnetization occurs with constant amplitude characterizing a Gilbert process. The enhanced spin-pumping damping explains the large broadening observed in the FMR lines of thick films of YIG with deposition of a very thin Pt layer.

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