Local structure of NaNbO₃: A neutron scattering study

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We report the results of a neutron diffraction study of structural evolution in sodium niobate, NaNbO₃, which is the parent compound for lead-free ferroelectric materials, as a function of temperature from 15 to 930 K over six phases. The Rietveld analysis of the high-resolution powder neutron diffraction data shows the variation in the structure from cubic to rhombohedral ferroelectric structures. However, the refinements on local structure by the pair distribution function (PDF) method indicates that there are only three basic patterns of the local structure, and the ground states of NaNbO₃ in the low-temperature antiferroelectric and ferroelectric phases have the R3csymmetry, even though in the long range the system shows the *Pbcm* symmetry or the coexistence of two phases. The origin of the complex phase behavior and its implications on the performance as lead-free ferroelectrics are discussed.

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I. INTRODUCTION

Ferroelectric materials are widely used in many electromechanical components and electronic devices, such as actuators, sensors, field-effect transistors, and ultrasonic transducers. Due to strong piezoelectric response, Pb(Zr,Ti)O₃ (PZT) is the most widely employed ferroelectric material at present, even though lead has recently been banned from many commercial applications and materials, for example, from solder, glass, and pottery glaze, due to concerns regarding its toxicity. Therefore a concerted effort was mounted to develop lead-free piezoelectric ceramics. A large number of studies have been focused on the discovery of better ferroelectrics near the morphotropic phase boundaries (MPB) in alkaline niobatebased perovskite solid solutions.¹⁻³ In particular Saito, et al.⁴ have designed a new lead-free ferroelectric based on the MPB in the perovskite-rich region of (K,Na)NbO₃, which shows excellent piezoelectric performance comparable to those of PZT. Interestingly, their mother compound, sodium niobate, NaNbO₃, is known for its complex phase behavior, and there are a number of conflicting reports on its structure.^{5–20} Thus there is a strong need for understanding the structure and phase transformation of sodium niobate.

At high temperatures, NaNbO₃ takes the structure of simple perovskite. But as the temperature is lowered,^{5–7} pressure is applied⁸ or the sample size is reduced,⁹ NaNbO₃ undergoes a surprisingly complex sequence of structural phase transitions. Its rich phase diagram is generally described by Glazer and Megaw,¹⁰ which contains at least six phases, varying from cubic at high temperature, through tetragonal, orthorhombic, and, finally, to rhombohedral structure below room temperature. This complexity invited many scientists to study the temperature-induced phase changes in NaNbO3 and its soft mode mechanism for structural change.^{5,8,11–15} However, the results are seriously contradictory to each other, particularly at low temperatures. Above 913 K, it has a paraelectric cubic phase Pm3m, and when cooling down, it evolves through a series of antiferrodistortive phases, tetragonal (T_2) P4/mbm, orthorhombic (T_1) Cmcm, orthorhombic (S) Pbnm,

orthorhombic (R) Pbnm, orthorhombic (P) Pbcm phases, and forms a rhombohedral R3c phase at low temperatures. Mishra *et al.* replace the space group Pnmm of S and R phases with the Pbnm group according to a high-resolution powder neutron diffraction study.¹⁶ They also point out that the antiferroelectric phase Pbcm and the ferroelectric phase R3c coexist and have competing interaction at room temperature and below.¹⁷ Further in the transition mechanism, the highand room-temperature phase transitions are considered to be due to the NbO₆ octahedral tilting, while the low-temperature transitions are related to the off-centered displacements of niobium atoms and octahedral distortions of NbO₆.¹⁸⁻²⁰ In spite of extensive use of experimental techniques and theoretical studies, surprisingly large amounts of controversies are still present on the details of the structure and process of phase transitions of NaNbO₃. In this paper, we use the Rietveld and pair distribution function (PDF) analyses of high-resolution neutron scattering data to probe the local structure of each phase and details of each phase of NaNbO₃.

II. RESULT AND DISCUSSION

A. Rietveld analysis of powder neutron diffraction data

Powder sample of NaNbO₃ (99.997% purity) was purchased from Alpha Aesar Co., and the pulsed neutron diffraction measurements were performed with the NPDF spectrometer of the Lujan Center of Los Alamos National Laboratory at various temperatures. The results were first analyzed using the Rietveld refinement (EXRGUI, revision 1225). The neutron diffraction pattern at 930K shown in Fig. 1 was analyzed on the basis of an ideal cubic structure with the space group Pm3m. The cubic phase is well fitted with the structural parameters of the ideal perovskite, with Na at (0,0,0), Nb at (0.5,0.5,0.5), O at (0.5,0.5,0) and a = b = c = 3.9507 Å. All the Bragg reflections present could be indexed as main cubic perovskite reflections. When temperature is lowered to 880 K, the appearance of the superlattice peaks shown in Fig. 1 indicates the phase transition Intensity

2



190

6

1

4

Q

FIG. 1. (Color) Evolution the neutron diffraction patterns for NaNbO₃ as temperature is varied from 930 to 15 K. Each pattern is representative for one phase of NaNbO₃. Superlattice peaks are marked with arrows.

to the tetragonal *P4/mbm* space group, which is induced by M and R zone boundary condensation involving the rotation of NbO₆ octahedra. Between 820 and 770K, NaNbO₃ has an orthorhombic *Cmcm* space group phase. The appearance of additional superlattice peaks (see Fig. 1) is the sign for phase transition and multiplicity of the unit cell. When the temperature further goes down, numerous controversies are drawn on the space groups and phase transitions. As the temperature falls in the range of 700–600 K, Ahtee *et al.* ⁶ find that it belongs to the *Pnmm* space group symmetry with



FIG. 2. (Color online) Observed (solid spheres), calculated (open circles), and difference (crosses) profiles obtained after the Rietveld refinement of NaNbO₃ using the mixture of phases with rhombohedra R3c group (70%) and orthorhombic *Pbcm* group (30%) at 15 K. Peaks marked with arrows are the ones belong only to the R3c group according to the analysis of GASA software.

the cell dimensions $2 \times 2 \times 2$ with respect to the elementary perovskite cell by x-ray diffraction measurements. However, Mishra *et al.* argue that it undergoes a phase transition to the *Pbnm* space group with the $\sqrt{2} \times \sqrt{2} \times 12$ cell dimensions at 700 K and $\sqrt{2} \times \sqrt{2} \times 6$ cell dimensions at 600 K through neutron diffraction measurements. Our neutron diffraction patterns can be fitted well by the *Pbnm* space group parameters with $\sqrt{2} \times \sqrt{2} \times 6$ cell dimensions, partly supporting Mishra *et al.*, but not much difference is seen between the patterns for 700 and 600 K. Note that our model includes a small portion (4.5%) of a phase with *Cmcm* space group at 700 K.

Between room temperature and 490 K, the orthorhombic Pbcm space group seems to achieve the best fit in the Rietveld refinement. Below room temperature, as detected by Mishra et al., the orthorhombic Pbcm group and rhombohedra R3c group coexists within the entire temperature range, including the lowest temperature. When we compare every peak at low and room temperatures, it seems that most of the supperlattice peaks at low temperatures can find their original formation at room temperature, in spite of some shifts in the Q value and changes in intensity, still, some peaks are an exclusive result of the R3c space group symmetry, as shown in Fig. 2. The evolution of phase transitions in our Rietveld refinement is shown in Table I and the lattice parameters, parameters of every atom position, thermal factor, phase fraction, and the agreement factors are shown in Table II. The definition of parameters in the table can be found in Ref. 21.

B. Pair distribution function analysis above room temperature

In the antiferroelectric and ferroelectric phases, NaNbO₃ undergoes lattice distortion, with tilting and atomic off-center displacements in the short range. However, the diffraction patterns are not sensitive to detailed local displacements during the phase transition. In order to determine differences between the local and average structures, the PDF method was employed to probe the evolution of the local structure, especially for those at temperatures below 490 K.

The PDF, denoted g(r) in the following formula, describes all the interatomic distances present in the structure, and is obtained from the total structure function S(Q) through the Fourier transformation²²

$$g(r) - 1 = \frac{1}{2\pi^2 r \rho_0} \int_0^{+\infty} Q[S(Q) - 1] \sin(Qr) dQ$$

Because S(Q) includes both the Bragg and diffuse scattering intensities, the PDF can describe aperiodic as well as periodic structure. From 900 to 600 K, the PDF calculated (PDFGUI 1.0b) with the same fitting parameters as those in the Rietveld analysis shows good agreement with the experimental PDF, indicating that the crystallographic structure provides also a good description of the local atom arrangement above 600 K. Figure 4 is an example of the experimental PDF agreeing with the calculated PDF using the Rietveld model at 820K. In Fig. 3, within the basic cubic perovskite cell (around 4 Å), the two primary peaks corresponding to Nb-O and Na-O are at r = 1.9 and 2.8 Å, which indicates that Nb is at the center of an NbO₆ octahedron, and there is no obvious distortion in the unit cell.

Temperature	930 K	880 K	820–770 K	700 K	600 K	490–300 K	190–15 K					
Space group	Pm3m	P4/mbm	Стст	Cmcm + Pbnm	Pbnm	Pbcm	Pbcm + R3c					
Symmetry	Cubic	Tetragonal	Orthorhombic			Orthorhombic (AFE)	Rhombohedra (FE)					

TABLE I. Phase evolution of NaNbO3 at various temperatures.

When temperature is lowered to 490K, through the PDF analysis we find that the structure model obtained by the Rietveld refinement actually shows very poor agreement at small r, as shown in Fig. 4, which suggests that for r <5.2 Å the structure is actually not in the *Pbcm* space group. The PDF provides the data for true atomic distances, which cannot easily be determined from the average structure analysis by the Rietveld analysis. If we use the parameters for the low temperature phase R3c group to refine all the atom positions and thermal factors, we can get an excellent fit to the PDF below 5.2 Å as shown in Fig. 5. The same conclusion is obtained through the PDF fitting from 490 to 300 K. Therefore, in this range of temperature, the *Pbcm* space group describes only the long-range symmetries and atomic positions. On the other hand, within the short range up to 5.2 Å, the correct local structure is surprisingly the R3c space group. This size of the domain is slightly smaller than the rhombohedral R3cunit cell, in which it keeps the character and symmetry of the R3c group, though distorted. These small R3c domains gradually stack into the Pbcm space group through the entire structure. Depending on the models resulted from the Rietveld refinement (detailed information given in Table II), the *a* and b axes of the *Pbcm* unit cell rotate by 45° from those of the R3c unit cell; so it is most likely that the atoms form twin structures (as the Na and O atoms shown in Figure 6) in R3c domains and along its diagonal direction, the whole structure can result in Pbcm symmetry. Moreover, the R3c space group is a ferroelectric phase while the *Pbcm* space group is antiferroelectric, which means that the local domains in the R3c group have a spontaneous polarization and they align in antiparallel arrangements to revolve into the *Pbcm* group. Along the c axis, the length of the *Pbcm* unit cell (c = 15.48 Å) is about twice as the R3c unit cell (c = 7.82 Å) and about four times as the elementary cubic cell (3.95 Å), so every basic polarized cell also form similar twin structures to make the polarizations

TABLE II. Structural parameters of NaNbO₃ obtained by the Rietveld analysis from 930 to 15 K.

TEMPERATURE: 930 K					TEMPERATURE: 880 K				
Atoms	X	Y	Ζ	Uiso	Atoms	X	Y	Ζ	Uiso
NA	0	0	0	0.0531	NA	0	0.5	0.5	0.06416
NB	0.5	0.5	0.5	0.01331	NB	0	0	0	0.02759
0	0	0.5	0.5	0.04169	01	0	0	0.5	0.03643
					O2	0.2754	0.2247	0	0.03643
Lattice parameters (Å)					Lattice parameter	ers (Å)			
a = b = c = 3.9507			$\alpha = \beta = \gamma =$	= 90°	a = b = 5.5786	c = 3.9510	$\alpha = \beta = \gamma =$	90°	
Agreement facto	or				Agreement facto	or			
wRp = 0.0667			Rp = 0.0469	9	wRp = 0.0667		Rp = 0.0469		
$CHI^{2} = 8.495$			$RF^2 = 0.1596$		$CHI^2 = 7.207$				
				TEMPERA	TURE: 700 K				
Phase 1(<i>Cmcm</i>)				Phase 2(<i>Cmcm</i>)					
Atoms	X	Y	Ζ	Uiso	Atoms	X	Y	Ζ	Uiso
Nal	-0.0074	0	0.25	0.04329	Na1	0.6887	0.8774	0.25	0.02367
Na2	0.4926	0	0.25	0.04329	Na2	1.1592	0.3899	0.1155	0.02367
Nb	0.75	0.75	0	0.02401	Nb1	0.5	0	0	0.00499
01	0	-0.2844	0	0.02216	Nb2	05242	-0.0321	0	0.00812
O2	-0.2304	0	-0.0691	0.02216	01	-0.5049	-0.0079	0.25	0.08941
O3	0.7546	-0.2574	0.25	0.02216	O2	0.0332	-0.5012	0.1338	0.08941
					O3	0.7288	0.263	0.0543	0.08941
					O4	0.6805	0.312	0.3894	0.08941
					O5	0.6886	0.3073	0.7237	0.08941
Lattice parameters (Å)					Lattice parameters (Å)				
a = 7.8705 $b = 7.8805$ $c = 7.8681$					a = 5.3688	b = 5.8333	c = 23.56499		
$lpha=eta=\gamma=90^\circ$					$\alpha = \beta = \gamma = 9$	0°			
Agreement factor					Phase ratio				
wRp = 0.1059 $Rp = 0.0644$		$CHI^{2} = 20.35$		$RF^2 = 0.2990$ Phase1 = 4.5% Phase			Phase $2 = 95$.	5%	



FIG. 3. (Color online) Neutron pair distribution function G(r) analysis of NaNbO₃ at 820 K with the *Cmcm* parameters from the Rietveld analysis. This phase is well described by the *Cmcm* structure. Models obtained by the Rietveld analysis can fit the PDF from 930 to 600 K quite well. This is an example that shows the accuracy of the Rietveld model at high temperatures.

stack antiparallel through the whole structure as shown in Fig. 6.

It is traditionally considered that the spontaneous polarization in ferroelectric and antiferroelectric phases of NaNbO₃ originates from the distortion of the lattice and off-centered displacement of Nb atoms. Figure 7 shows the PDF of NaNbO₃ at a very short range, which is within the range of an elemental perovskite unit cell from 930 to 15 K. We can see that the first peak, which corresponds to the Nb-O bond, is broad and asymmetric but without a split down to 600 K (six green curves



FIG. 4. (Color online) Neutron pair distribution function G(r) analysis of NaNbO₃ at 490 K with the *Pbcm* group. Note that this structure model can describe the PDF only above 5.2 Å.



FIG. 5. (Color online) Neutron pair distribution function G(r) analysis of NaNbO₃ at 490 K with the *R3c* group. Note that this phase can describe the structure profile only below 5.2 Å.

shown in Fig. 7). As temperature falling down to 300 K (four blue curves shown in Fig. 7), the peak due to Nb-O bond is separated into two subpeaks, one around r = 1.9 Å and the other around r = 2.1 Å, and a Na-O bond peak appears at r = 2.4 Å as well. When the temperature is above 600 K, where the structure is paraelectric, the Nb-O bond length is 1.95 Å and the Na-O bond length is 2.8 Å. Therefore, in the antiferroelectric phases, the Nb atom in the octahedral NbO₆ is off-centered by 0.15 Å or 7.7%, while the Na atom in NaO₈ is off-centered by 0.4 Å or by 14%. This means Na has even a larger off-centered displacement than Nb, as shown in Fig. 8,



FIG. 6. (Color online) (a) Atomic arrangement of NaNbO₃ at 490 K. Yellow balls are Na, green balls are niobium, and purple balls are oxygen. *a* and *b* are *R3c* axes and *a'* and *b'* are *Pbcm* axes. (b) Construction of the basic cells (\sim 4 Å) and local polarizations. Every arrow corresponds to a spontaneous polarization in an elementary perovskite cell.



FIG. 7. (Color) Neutron pair distribution function G(r) analysis of NaNbO₃ from 930 to 15 K up to 4 Å. Note that the result can be divided into three groups with the evolution of temperature: 900–600 K(green lines), 490–300 K(blue lines), and 190–15 K(red lines). This indicates that only two basic structural changes happen at a short range. Peaks resulted from Na-O and Nb-O bonds are marked with arrows and obvious Nb-O bond lengths changes at very short distance from 490 to 15 K are also emphasized in the inset.

and both displacements contribute to the formation of local dielectric polarization.

C. Pair distribution function analysis at low temperatures

When NaNbO₃ is cooled down below room temperature, it exhibits ferroelectric behaviors due to the spontaneous polarization formed in its structure. A great deal of research has been focused on the structure in this range of temperature, and it is generally agreed that two phases, R3c and Pbcm, coexist. Our Rietveld analysis also shows the coexistence of the two phases. Two-phase coexistence in a compound at low



FIG. 8. (Color online) Depictions of NaO_{12} at 930 K (left top) and at 15 K (left bottom), and NbO_6 at 930 K (right top) and at 15 K (right bottom) according to the PDF analysis. Na and Nb displacements are exaggerated for clarity.



FIG. 9. (Color online) Neutron PDF analysis of NaNbO₃ at 15K with R3c and Pbcm group coexistence model from the Rietveld analysis. Note that the differences gradually vanish above 10 Å.

temperatures usually signifies that at least one of the phases is metastable. The PDF analysis explained why this occurred.

As shown in Fig. 9, if we use the model obtained from the Rietveld refinement, the PDF fitting confirms the two phase coexistence at r > 10 Å. However, it is clearly shown in Fig. 10 that below 10 Å, *R3c* is the only phase group that correctly describes the local atomic distribution. Additionally, largest off-centering displacements of Na atoms and the most significant distortions in Na-O-Na correlations are indicated (similar to that shown in Fig. 6), which is the result of the flexibility of Na atomic position. The radius of sodium ion is very small (1.02 Å),²³ whereas it is surrounded by 12 nearest



FIG. 10. (Color online) Neutron pair distribution function G(r) analysis of NaNbO₃ at 15 K with only the *R3c* group. Note that the differences are very small below 10 Å and begin to become obvious above 10 Å.

oxygen atoms (1.40 Å in radius²³) with the average Na-O distance of 2.79 Å. These oxygen atoms form a cage with very large space for an Na atom to move around. Of course, Nb atoms also have off-centered displacement, as believed conventionally. But the nominal ionic size of Nb (0.74 Å)²³ is much larger for the volume of the octahedron formed by the six oxygens surrounding Nb. Thus the picture of Nb rattling in the O₆ cage is invalid.

Below 190 K, NaNbO3 falls in the ferroelectric state, with the parallel spontaneous polarization. Again, if we compare the PDF in the very short range (three red lines in Fig. 7), namely within the size of the elemental perovskite unit cell, the splitting of the first peak confirms the Nb-O variations in the NbO₆ octahedra, one at r = 1.87 Å and the other at r =2.1 Å, which is attributed to the off-centered displacement of Nb atom. Especially, the two Nb-O peaks are not identical and the ratio of their intensities is about 1.5. In some previous studies, Nb was considered to move towards the [111] direction of the NbO₆. But in such a case the split Nb-O peaks should have the 1:1 intensity ratio. Our PDF result seems to indicate that Nb moves towards a direction between (111) and (110). Moreover, with the evolution of temperature below 190K, the peaks of Nb-O bond lengths prefer to stay the same, but the Na-O bond correlation peak shifts very significantly. During cooling, the amplitude of thermal vibration of Na becomes very small, which leaves Na to more clearly off-centered positions. At 15K, two well-defined Na-O peaks are formed at r = 2.4 and 3.1 Å, inferring that Na atoms have about 14% off-centering from the center of the oxygen cage, much larger than the off-centering of Nb atoms. Also, the ratio of the intensities of these two peaks is again about 1.5, though the intensities are relatively small due to broadening of average Na-O correlations. That means Na may have a similar direction of movement as Nb. The coincidence of Na and Nb off-center displacements contribute to the formation of large dielectric polarization.

From 490 to 300 K, the space group at short range is always R3c, which is a ferroelectric (FE) state. However, the structure over the long distance beyond 10 Å is always antiferroelectric (AFE) with the *Pbcm* group symmetry. This can only mean that nanoscale twins are formed in the R3c phase with the ferroelectric polarization changing from one twin to another. Above room temperature, this kind of average AFE structure appears through the whole lattice. Below 190 K, some of the FE domains are larger, while much of the FE domains remain in the nanoscale twins, resulting in apparent coexistence of the R3c and Pbcm phases.

The results of PDF fitting clearly indicate that the true ground state for the local structure of NaNbO₃ below 490 K is actually the *R3c* structure. It is the real phase of NaNbO₃ at low temperatures, but they tend to form nanotwins, resulting in the appearance of the *Pbcm* phase at long ranges. It should be noted that if the twins are regularly spaced they should result in superlattice diffraction, and a different structure with a large unit cell. That such superlattice diffraction peaks are not observed implies the spacings of nanotwins are irregular. As noted by Hendrick and Teller²⁴ for partially ordered layered lattices of graphite, such irregular, random arrangement of twins can produce sharp diffraction peaks with the average lattice structure, in this case *Pbcm*, rather than broad

diffraction patterns corresponding to the short structural coherence length.

The most probable cause of the formation of nanotwins is the low mobility of Na ions at low temperatures. The off-centering of Na is quite large, as much as 0.4 Å, greatly contributing to large ferroelectric polarization. But at the same time the energy barrier to move from one off-centered position to another must be significant, resulting in trapping of Na ion in one of these off-centered positions at low temperatures. This makes formation of large FE domains difficult, resulting in nanotwin states. The large barrier for motion also explains the hysteric effect naturally without invoking the idea of quantum paraelectricity.²⁵ On the other hand, as we mentioned earlier, replacing Na with other elements such as K can improve its ferroelectric and piezoresponse performance. The ionic radius of K⁺ is larger than that of Na⁺ by 0.25 Å,²³ reducing the off-centering to only 0.1 Å. The energy barrier for such a small off-centering is negligible, making K⁺ much more mobile than Na⁺, although the contribution of K⁺ to ferroelectricity is smaller. Apparently, the new lead-free ferroelectric materials⁴ were synthesized by improving the mobility of alkali ion at the expense of losing some polarization, by partially replacing Na by K.

III. CONCLUSIONS

In summary, we have discovered the local aspect of evolution of the structure and space group of sodium niobate from 930 K down to 15 K through the combination of the standard neutron scattering diffraction Rietveld analysis and the local pair distribution function (PDF) study. The longrange crystallographic phase undergoes six phase transitions, through cubic, tetragonal, orthorhombic, and rhombohedral. However, the study of the short-range structure demonstrates that in spite of such complexity there are only three main patterns in the local structures, as shown in Fig. 7. At high temperatures (above 490 K), local dielectric polarization is absent. In the intermediate temperature range (down to 300 K), Nb starts to become off-centered, but Na remains centered. Only below 300 K, both Nb and Na become off-centered. The study of the local structure brought further new information. Especially at low temperatures, where NaNbO₃ experiences antiferroelectric and ferroelectric transitions, our local PDF shows that the short-distance structure is always in the R3cspace group, which is actually the real ground state. In the long range, however, the nanotwined structure of the R3c cells brings the appearance of the Pbcm space group. Our discovery solves the long-standing controversies in numerous neutron studies of sodium niobate at low temperature and explains why NaNbO3 is a good base compound for promising new ferroelectric materials that can substitute lead-containing toxicant materials.

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