Finite-frequency current fluctuations and self-consistent perturbation theory for electron transport through a quantum dot

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We formulate the problem of electron transport through an interacting quantum dot system in the framework of the self-consistent perturbation theory of the nonequilibrium Green's function and show that the current conservation through the central region is automatically guaranteed owing to the gauge-invariant properties of the resulting Green's functions and the generalized Ward identity. By using a generating functional for the statistics of the nonequilibrium system, we obtain general formulas for calculating the current and the current fluctuations in the presence of arbitrary time-dependent potentials. As an illustration of application, we study the interaction effects on the finite-frequency noise for electron resonant tunneling through an Anderson impurity and obtain an analytical expression for the finite-frequency current noise within the Hartree approximation, which is an extension of the results obtained by Hershfield on zero-frequency shot noise.

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I. INTRODUCTION

The transport properties of mesoscopic conductors have attracted wide research interest and are of great importance for future nanoscale electronic applications. One usually probes the dynamics of an electron in the conductors by measuring the dc and ac conductances in the linear-response regime or by driving the systems to the nonlinear out-of-equilibrium case. The coherent electron dynamical transport in a mesoscopic system can exhibit novel properties, for example, a universal charge relaxation resistance for mesoscopic capacitors was predicted theoretically[1] and was confirmed in a seminal experiment by Gabelli *et al.*[2] in which the ac conductance of a mesoscopic RC circuit at the gigahertz frequency was measured, and the violation of Kirchhoff's law of impedance addition was shown. Recent experimental advances will enable probing the electronic processes in these systems in the high-frequency region reaching the intrinsic time scales of the electron dynamics, hence, new interesting physical properties are expected to be observed.[3]

Some important progress in this field is based on the wisdom that more information about electron dynamics can be obtained by measuring the current fluctuations or the higher current moments in these systems.[4,5] The theory of full counting statistics (FCS) of the electron current, which describes the probability distribution of the transmitted charge during a fixed time interval through a mesoscopic conductor, was developed within the scattering formulation,[6–8] and the Hamiltonian formulation for the FCS was constructed based on the Anderson impurity model.[9–11] It was also shown that the real part of the ac conductance is related to the asymmetric parts of the frequency-dependent current noise by a nonequilibrium fluctuation-dissipation theorem.[12,13] The non-Markovian effects of finite-frequency quantum noise in quantum dots was addressed based on quantum master equations.[14] Therefore, at present, there are intensive research activities on the study of FCS problems both in quantum dot systems[15–17] and in diffusive conductors.[18,19]

For the study of the time-dependent and finite-frequency transport properties of mesoscopic systems, Blanter and Büttiker[5] emphasized the importance of considering electron interaction effects and pointed out that the current measured in the frequency-dependent transport experiment is a sum of the particle current and displacement current, whereas, the displacement current is zero in the static case but is essential for finite-frequency transport. A theoretical approach that can address the finite-frequency electron transport and can take into account the various electron correlation effects as well as the current conservation condition explicitly is highly worthy of investigation.[20,21]

A prototype mesoscopic system of nonequilibrium electron transport with strong Coulomb interaction is a single quantum dot coupled to left and right leads, which can be described by the Anderson impurity model. The current through the quantum dot[22] has been calculated by performing a second-order perturbation theory of the Coulomb interaction strength U. However, the authors[22] observed that the current is not conserved in this approximation when the dot level is tuned away from the particle-hole symmetry point and pointed out the necessity of treating interaction terms by a current conservation approximation. Subsequently, Hershfield[23] studied the current fluctuations in the Anderson impurity model and obtained a formula for the interaction effect on zero-frequency shot noise in the Hartree approximation. During the last decade, the shot noise and the FCS of the current of the Anderson impurity model in the low-temperature Kondo regime have attracted a great deal of research effort and interest, but we will restrict our consideration only to the resonant tunneling regime in this paper. For a noninteracting resonant tunneling model, an analytical formula of the finite-frequency noise spectrum had been obtained.[24,25] The nonsymmetrized noise spectrum of the resonant tunneling model has recently been studied within the scattering formulation[26,27] where some step and dip structures at finite frequencies were shown and the Hartree-Fock theory was applied to the multilevel quantum dot system.[28] Very recently, perturbation theory in terms of the Coulomb interaction strength was applied to study the noise spectrum of the Anderson impurity model.[29]

In the present paper, we formulate the theory of electron transport through a quantum dot based on the nonequilibrium

self-consistent perturbation method,[30] which can guarantee the gauge invariance and current conservation condition for the arbitrary time-dependent external potential. We study the effect of the potential fluctuations in the quantum dot on the finite-frequency noise within the Hartree approximation and obtain an analytical expression for the interaction effect on the current noise, which is an extension of Hershfield's result[23] to the finite-frequency case. We will show that various correlation functions of the nonequilibrium system, including the vertex correction terms, can be calculated in a systematic way by using functional derivations defined on the closed time contour and the external counting field method. Actually, the functional approach has been applied to a wide range of problems in the mesoscopic system in the literatures, e.g., current and noise characteristics for single-electron transistors in the Coulomb blockade regime[31,32] or the Kondo regime,[33] photoassisted shot noise in a mesoscopic conductor,[3] charge transport to a chaotic cavity,[34] the time evolution of the nonequilibrium quantum dot system,[35] etc.

This paper is organized as follows: In Sec. II, we discuss the current conservation condition and the generalized Ward identity for a quantum dot based on the Anderson impurity model. In Sec. III, the formula for the finite-frequency noise spectrum, including the interaction correction term in the Hartree approximation, is obtained. Section IV is devoted to the numerical calculations. In Sec. V, we summarize the results of this paper.

II. SELF-CONSISTENT PERTURBATION THEORY AND THE GENERALIZED WARD IDENTITY

We consider the electron's transport through a single level quantum dot in the presence of external ac fields, and the system will be described by the following Anderson impurity model:[36,37]

$$H = \sum_{k\eta\sigma} \epsilon_{k\eta}(t) c^{\dagger}_{k\eta\sigma} c_{k\eta\sigma} + \sum_{\sigma} \epsilon_d(t) d^{\dagger}_{\sigma} d_{\sigma} + U n_{d\uparrow} n_{d\downarrow} + \sum_{k\eta\sigma} [t_{\eta} e^{i\lambda_{\eta}(t)} c^{\dagger}_{k\eta\sigma} d_{\sigma} + \text{H.c.}], \quad (1)$$

where $\eta = L,R$ denotes the left and right leads and $\epsilon_{k\eta}(t) = \epsilon_{k\eta} + v_{\eta}(t)$ and $\epsilon_d(t) = \epsilon_d + v_0(t)$ with $v_{\eta}(t)$ and $v_0(t)$ being the ac potentials in the leads and in the dot, respectively. $\lambda_{\eta}(t)$ is the gauge potential coupled to the tunneling current from lead η to the dot. The strong Coulomb interaction term in the Hamiltonian prevents this model from obtaining an exact solution. But we can approach this problem by performing perturbation theory on the Schwinger-Keldysh closed time path contour. Using the nonequilibrium Dyson equation, we write the equation of motion for the nonequilibrium Green's function (GF) of the quantum dot as follows:

$$\left[i\frac{\partial}{\partial t} - \epsilon_d(t) \right] G_{d\sigma}(t,t') = \delta(t,t') + \int dt_1 \Sigma(t,t_1) G_{d\sigma}(t_1,t'). \quad (2)$$

It should be emphasized that the time variables t and t' can be either on the forward or on the backward branch of the time contour and the integration over t_1 is also defined on the closed time path contour. The self-energy $\Sigma(t,t')$ is obtained in the framework of self-consistent perturbation theory and can be divided into two terms,

$$\Sigma(t,t') = \Sigma^{(0)}(t,t') + \Sigma_U(t,t'), \quad (3)$$

where $\Sigma^{(0)}(t,t')$ is the dot level self-energy contributed from the tunneling between the leads and the quantum dot,

$$\Sigma^{(0)}(t,t') = \sum_{k\eta} |t_{\eta}|^2 \bar{g}_{k\eta}(t,t') \times \exp\left\{ -i \left[\lambda_{\eta}(t) - \lambda_{\eta}(t') + \int_{t'}^{t} dt_1 v_{\eta}(t_1) \right] \right\}, \quad (4)$$

with $\bar{g}_{k\eta}(t,t')$ being the bare Green's function of the lead without an external ac potential field. $\Sigma_U(t,t')$ is the self-energy due to Coulomb interaction. In the self-consistent perturbation theory, it is a functional of the full Green's functions of the quantum dot. In order to illustrate the method of calculation, we consider only the first-order approximation, and the interaction self-energy is given by the Hartree term,

$$\Sigma_U(t,t') = U \langle n_{d\bar{\sigma}}(t) \rangle \delta(t,t'). \quad (5)$$

We first study the gauge transformation properties and current conservation condition of the Green's functions of the quantum dot. By making a transformation[36] $G_{d\sigma}(t,t') = \bar{G}_{d\sigma}(t,t') e^{-i\int_{t}^{t'} dt_1 v_0(t_1)}$, the equation of motion for $\bar{G}_{d\sigma}(t,t')$ will be given by

$$\left[i\frac{\partial}{\partial t} - \epsilon_d \right] \bar{G}_{d\sigma}(t,t') = \delta(t,t') + \int dt_1 \bar{\Sigma}(t,t_1) \bar{G}_{d\sigma}(t_1,t'), \quad (6)$$

where the self-energy $\bar{\Sigma}(t,t_1) = \bar{\Sigma}^{(0)}(t,t') + \bar{\Sigma}_U(t,t')$ with

$$\bar{\Sigma}^{(0)}(t,t') = \sum_{k\eta} |t_{\eta}|^2 \bar{g}_{k\eta}(t,t') e^{-i\phi_{\eta}(t,t')}, \quad (7)$$

and in the Hartree approximation,

$$\bar{\Sigma}_U(t,t') = U \langle \bar{n}_{d\bar{\sigma}}(t) \rangle \delta(t,t'), \quad (8)$$

where the phase factor $\phi_{\eta}(t,t') = \lambda_{\eta}(t) - \lambda_{\eta}(t') + \int_{t'}^{t} dt_1 [v_{\eta}(t_1) - v_0(t_1)]$ and $\bar{G}_{d\sigma}(t,t')$ is a gauge transformation invariant quantity. If one considers a gauge transformation: $v_0(t) \to v_0(t) + \partial_t \tilde{\Lambda}(t)$ and $\lambda_{\eta}(t) \to \lambda_{\eta}(t) + \tilde{\Lambda}(t)$, then it is easy to see that the phase factor $\phi_{\eta}(t,t')$, the self-energy $\bar{\Sigma}(t,t')$, and $\bar{G}_{d\sigma}(t,t')$ are all gauge transformation invariant. Therefore, the Green's function $G_{d\sigma}(t,t')$ transforms as

$$G_{d\sigma}(t,t';\tilde{\Lambda}) = e^{-i\tilde{\Lambda}(t)} G_{d\sigma}(t,t') e^{i\tilde{\Lambda}(t')}. \quad (9)$$

The above gauge transformation is directly related to the current conservation condition in the quantum dot. Since under this gauge transformation, the change in the Hamiltonian to the first order of $\tilde{\Lambda}$ is given by

$$\delta H(t) = n_d(t) \partial_t \tilde{\Lambda}(t) + \sum_{\eta} j_{\eta}(t) \tilde{\Lambda}(t), \quad (10)$$

where $n_d(t)$ and $j_{\eta}(t)$ are the operators of the charge number in the dot and the tunneling current from lead η to the dot, respectively. The gauge transformation invariance of the action

leads to the continuity equation,

$$\partial_t \langle n_d(t)\rangle - \sum_\eta \langle j_\eta(t)\rangle = 0. \tag{11}$$

In the out-of-equilibrium steady state, the occupation number $\langle n_d(t)\rangle$ is time independent, and the current conservation condition $\sum_\eta \langle j_\eta\rangle = 0$ is satisfied.

Next, we will follow the procedure as given in Ref. 38 to give a derivation of the generalized Ward identity[39] for this quantum dot system, which is closely related to the current conservation condition. In the spin degenerate case, the spin index σ will be omitted. We consider the changes in the Green's function induced by the gauge transformation. From Eq. (9), the first-order change in G is

$$\delta G(t,t') = -i[\tilde{\Lambda}(t) - \tilde{\Lambda}(t')]G(t,t'), \tag{12}$$

and it leads to the equation,

$$\int dt_1 \left[\frac{\delta G(t,t')}{\delta v_0(t_1)}\partial_{t_1}\tilde{\Lambda}(t_1) + \sum_\eta \frac{\delta G(t,t')}{\delta \lambda_\eta(t_1)}\tilde{\Lambda}(t_1)\right] = -i[\tilde{\Lambda}(t) - \tilde{\Lambda}(t')]G(t,t'), \tag{13}$$

where the functional derivatives of G can be denoted as Λ_0 and Λ_η and they correspond to the time-ordered operator products as follows:

$$\Lambda_0(t,t';t_1) = \frac{\delta G(t,t')}{\delta v_0(t_1)} = -\langle T_C[d_\sigma(t)d_\sigma^\dagger(t')n_d(t_1)]\rangle, \tag{14}$$

$$\Lambda_\eta(t,t';t_1) = \frac{\delta G(t,t')}{\delta \lambda_\eta(t_1)} = -\langle T_C[d_\sigma(t)d_\sigma^\dagger(t')j_\eta(t_1)]\rangle. \tag{15}$$

An integration by parts in Eq. (13) and demanding that the equation is satisfied for arbitrary $\tilde{\Lambda}$ straightforwardly leads to the well-known generalized Ward identity,

$$\partial_{t_1}\Lambda_0(t,t';t_1) - \sum_\eta \Lambda_\eta(t,t';t_1) = i[\delta(t,t_1) - \delta(t',t_1)]G(t,t'). \tag{16}$$

This identity leads to a relation between the vertex functions and the self-energy, which can be demonstrated explicitly by introducing the following vertex functions:

$$\Gamma_0(t,t';t_1) = -\frac{\delta G^{-1}(t,t')}{\delta v_0(t_1)}, \tag{17}$$

$$\Gamma_\eta(t,t';t_1) = -\frac{\delta G^{-1}(t,t')}{\delta \lambda_\eta(t_1)}. \tag{18}$$

One can see that these vertex functions are related to the time-ordered operators,

$$\Lambda_0(t,t';t_1) = \int dt_2 dt_3 G(t,t_2)\Gamma_0(t_2,t_3;t_1)G(t_3,t'), \tag{19}$$

$$\Lambda_\eta(t,t';t_1) = \int dt_2 dt_3 G(t,t_2)\Gamma_\eta(t_2,t_3;t_1)G(t_3,t'). \tag{20}$$

Thereby, the generalized Ward identity Eq. (16) can be rewritten in terms of the vertex functions,

$$\partial_{t_1}\Gamma_0(t,t';t_1) - \sum_\eta \Gamma_\eta(t,t';t_1) = i[\delta(t',t_1) - \delta(t,t_1)]G^{-1}(t,t'). \tag{21}$$

This equation relates the vertex functions to the self-energy since the inverse of the Green's function $G^{-1} = G_0^{-1} - \Sigma$ is given explicitly as

$$G^{-1}(t,t') = [i\partial_t - \epsilon_d - v_0(t)]\delta(t,t') - \Sigma^{(0)}(t,t') - \Sigma_U(t,t'). \tag{22}$$

The generalized Ward identity implies the gauge invariance and current conservation condition in this problem. Therefore, it should be satisfied when the current fluctuations or time-dependent electron transport properties in this system are investigated by performing an approximation calculation of the self-energy or the vertex functions.

III. CURRENT FLUCTUATIONS AND THE HARTREE APPROXIMATION

In this section, we study the current fluctuation and statistical problems in this quantum dot system. It is well known that the central quantity in FCS calculations[6,7] is the cumulant generating function $\chi(\lambda) = \sum_{\mathbf{Q}} e^{i\mathbf{Q}\lambda}P(\mathbf{Q})$, where $\lambda = (\lambda_1,\ldots,\lambda_N)$ are the counting fields and $P(\mathbf{Q})$ is the probability for the charge $\mathbf{Q} = (Q_1,\ldots,Q_N)$ to be transferred through the respective channel[10] during the measuring time T. For the noninteracting electron system, the generating function is given by the Levitov-Lesovik formula[6] within the scattering matrix approach. It is observed that the cumulant generating function can be generalized to the system with time-dependent counting fields and it can be written in terms of the nonequilibrium Green's function defined on the closed time path contour as follows:

$$\ln\,\chi(\lambda) = \mathrm{Tr}\,\ln\,G^{-1} - \mathrm{Tr}\,\Sigma_U G + \Phi(G), \tag{23}$$

where G^{-1} is the inverse of the full Green's function of the quantum dot given explicitly by Eq. (22). The trace (Tr) implies the sum over spin index and the integration along the closed time path contour. Φ is a functional potential constructed by summing over irreducible self-energy diagrams closed with an additional Green's function line.[30] The interaction self-energy Σ_U can be obtained from the functional Φ by

$$\Sigma_U(t,t') = \frac{\delta\Phi}{\delta G(t',t)}. \tag{24}$$

One can verify that, when the counting fields λ are assumed to be time independent and the system is in the noninteracting case, the above generating functional arrives at the Levitov-Lesovik formula.

The electron current tunneling from lead η to the quantum dot can be obtained by a functional derivative of $\chi(\lambda)$ with respect to $\lambda_\eta(t)$,

$$\langle I_\eta(t)\rangle = i\frac{e}{\hbar}\frac{\delta\,\ln\,\chi(\lambda)}{\delta\lambda_\eta(t)} = -i\frac{e}{\hbar}\sum_\sigma \int dt_1 dt_2 G(t_1,t_2)\Gamma_\eta^{(0)}(t_2,t_1;t). \tag{25}$$

Here, the bare current vertex function $\Gamma_\eta^{(0)}(t_2,t_1;t)$ is given by

$$\Gamma_\eta^{(0)}(t_2,t_1;t) = \frac{\delta\Sigma^{(0)}(t_2,t_1)}{\delta\lambda_\eta(t)} = i[\delta(t_1,t)-\delta(t_2,t)]\Sigma_\eta^{(0)}(t_2,t_1). \quad (26)$$

Therefore, the current through the quantum dot is given as[32,34]

$$\langle I_\eta(t)\rangle = \frac{e}{\hbar}\sum_\sigma\int dt_1\big[G(t,t_1)\Sigma_\eta^{(0)}(t_1,t) - \Sigma_\eta^{(0)}(t,t_1)G(t_1,t)\big]. \quad (27)$$

By using the operational rules given by Langreth for contour integration,[40] it is not difficult to prove that this formula is exactly equivalent to the current formula obtained by Jauho *et al.*[41] for the time-dependent electron transport through an interacting quantum dot.

We can introduce the interaction-induced current vertex function, which is related to the self-energy of Coulomb interaction,

$$\Gamma_\eta^U(t_2,t_1;t) = \frac{\delta\Sigma_U(t_2,t_1)}{\delta\lambda_\eta(t)}. \quad (28)$$

Then, the vertex function defined in Eq. (18) is given by

$$\Gamma_\eta(t_2,t_1;t) = \Gamma_\eta^{(0)}(t_2,t_1;t) + \Gamma_\eta^U(t_2,t_1;t). \quad (29)$$

The current formula Eq. (25) indicates that the current is contributed solely from the bare current vertex. There is no contribution of the interaction vertex correction to the current in this self-consistent perturbation approach. However, we will show in the following that the interaction vertex, indeed, influences the current fluctuations.

Next, we calculate the current-current correlation functions on the time contour and find that they can be represented as the sum of two terms,

$$D_{\eta\eta'}(t,t') \equiv \langle T_C\delta I_\eta(t)\delta I_{\eta'}(t')\rangle = -\frac{e^2}{\hbar}\frac{\delta^2\ln\chi(\lambda)}{\delta\lambda_\eta(t)\delta\lambda_\eta(t')} = D_{\eta\eta'}^{(0)}(t,t') + D_{\eta\eta'}^{(c)}(t,t'), \quad (30)$$

where the bare term is

$$D_{\eta\eta'}^{(0)}(t,t') = \frac{e^2}{\hbar}\sum_\sigma\delta_{\eta\eta'}\big[G(t,t')\Sigma_\eta^{(0)}(t',t) + \Sigma_\eta^{(0)}(t,t')G(t',t)\big] + \frac{e^2}{\hbar}\sum_\sigma\int dt_1dt_2dt_3dt_4\big[G(t_1,t_2)\Gamma_{\eta'}^{(0)}(t_2,t_3;t') \times G(t_3,t_4)\Gamma_\eta^{(0)}(t_4,t_1;t)\big], \quad (31)$$

and the interaction-induced vertex correction term to the current correlation is given by

$$D_{\eta\eta'}^{(c)}(t,t') = \frac{e^2}{\hbar}\sum_\sigma\int dt_1dt_2dt_3dt_4\big[G(t_1,t_2)\Gamma_{\eta'}^U(t_2,t_3;t') \times G(t_3,t_4)\Gamma_\eta^{(0)}(t_4,t_1;t)\big]. \quad (32)$$

It should be noted that the above interaction vertex term has contributions from the current correlations of the equal spin as well as that of the opposite spin.

Among the various current correlation functions, the correlation function for current noise is of particular interest since the frequency-dependent noise spectrum of the current contains the intrinsic dynamics information of this quantum dot system. In a steady state without an external time-dependent potential, the symmetrized noise spectrum $S_{\eta\eta'}(\omega)$ is given by the Fourier transform of the correlation function of current operators $S_{\eta\eta'}(t,t') = \langle\delta I_\eta(t)\delta I_{\eta'}(t')\rangle + \langle\delta I_{\eta'}(t')\delta I_\eta(t)\rangle$. It is noted that the correlation function for current noise can be written as

$$S_{\eta\eta'}(t,t') = D_{\eta\eta'}^{>}(t,t') + D_{\eta\eta'}^{<}(t,t') = S_{\eta\eta'}^{(0)}(t,t') + S_{\eta\eta'}^{(c)}(t,t'), \quad (33)$$

where $S_{\eta\eta'}^{(0)}(t,t')$ and $S_{\eta\eta'}^{(c)}(t,t')$ are contributed from the bare term and the interaction-induced vertex correction term, respectively.

The bare term $S_{\eta\eta'}^{(0)}(t,t')$ is obtained straightforwardly by using Langreth's analytical continuation rules.[40] In the absence of an external ac potential, we can transform it to the frequency space and can express it in terms of the Green's functions of the quantum dot explicitly as[42,43]

$$\begin{aligned}S_{\eta\eta'}^{(0)}(\omega) = \frac{e^2}{\hbar}\sum_\sigma\int\frac{d\omega_1}{2\pi}&[\delta_{\eta\eta'}i\Gamma_\eta\{n_\eta(\omega_1)G^>(\omega_1+\omega) - [1-n_\eta(\omega_1+\omega)]G^<(\omega_1)\}\\ &- \Gamma_\eta\Gamma_{\eta'}(n_\eta(\omega_1)[1-n_{\eta'}(\omega_1+\omega)]G^r(\omega_1)G^r(\omega_1+\omega) + n_{\eta'}(\omega_1)[1-n_\eta(\omega_1+\omega)]G^a(\omega_1)G^a(\omega_1+\omega)\\ &+ [n_{\eta'}(\omega_1)G^a(\omega_1) - n_\eta(\omega_1)G^r(\omega_1)]G^>(\omega_1+\omega) + G^<(\omega_1)\{[1-n_{\eta'}(\omega_1+\omega)]G^r(\omega_1+\omega)\\ &- [1-n_\eta(\omega_1+\omega)]G^a(\omega_1+\omega)\} - G^<(\omega_1)G^>(\omega_1+\omega))] + \{\omega\to-\omega\}, \end{aligned} \quad (34)$$

where $n_\eta(\omega_1)$ is the Fermi distribution function in lead η. In order to obtain the interaction effect on the noise spectra, we have to calculate the vertex function by functional derivation of the interaction self-energy with respect to the counting field: $\Gamma_\eta^U(t_1,t_2;t) = \frac{\delta\Sigma_U(t_1,t_2)}{\delta\lambda_\eta(t)}$ where the interaction self energy $\Sigma_U(t_1,t_2)$ is given by Eq. (5) in the Hartree approximation. The technical details of our calculation are presented in Appendix B. After calculating the vertex function and transforming it to the frequency space, we can obtain the interaction correction to the finite-frequency current correlation function $S_{\eta\eta'}^{(c)}(\omega)$ as follows:

$$S_{\eta\eta'}^{(c)}(\omega) = \frac{e^2}{\hbar}\sum_\sigma\left[\chi_{j_\eta n}^{r,(0)}(\omega)\frac{U}{1-U\chi_{nn}^{r,(0)}(\omega)}S_{nj_{\eta'}}^{(0)}(\omega) + S_{j_\eta n}^{(0)}(\omega)\frac{U}{1-U\chi_{nn}^{a,(0)}(\omega)}\chi_{nj_{\eta'}}^{a,(0)}(\omega) + \chi_{j_\eta n}^{r,(0)}(\omega)\frac{U}{1-U\chi_{nn}^{r,(0)}(\omega)}S_{nn}^{(0)}(\omega)\frac{U}{1-U\chi_{nn}^{a,(0)}(\omega)}\chi_{nj_{\eta'}}^{a,(0)}(\omega)\right], \quad (35)$$

where the various correlation and response functions are given in Appendix A. This equation is the central result of our paper. It is a generalization of the zero-frequency noise result, obtained by Hershfield,[23] to the finite-frequency case and can be interpreted as the current noise contributed from the coupling of the density fluctuations in the quantum dot to the current fluctuations. The first term in Eq. (35) indicates that the correlation between the density and the current $S^{(0)}_{nj'_\eta}(\omega)$ can propagate forward in time via $\chi^{r,(0)}_{nn}(\omega)$ and $\chi^{r,(0)}_{j_\eta n}(\omega)$ to produce current fluctuations. The second term represents that the correlation $S^{(0)}_{j_\eta n}(\omega)$ propagates backward in time on the backward branch to produce current fluctuations. In the last term, the correlation between the densities at the quantum dot $S^{(0)}_{nn}(\omega)$ propagates forward and backward on the closed time contour simultaneously and gives rise to current fluctuations.

Thus, the noise spectrum of the current correlation between leads η and η' is given as $S_{\eta\eta'}(\omega) = S^{(0)}_{\eta\eta'}(\omega) + S^{(c)}_{\eta\eta'}(\omega)$. Since the operator of the averaged current through the quantum dot is $I(t) = [I_L(t) - I_R(t)]/2$, the noise spectrum associated with the averaged current will be given by

$$S(\omega) = \tfrac{1}{4}[S_{LL}(\omega) + S_{RR}(\omega) - S_{LR}(\omega) - S_{RL}(\omega)], \quad (36)$$

here $S(\omega)$ is real because of $S_{RL}(\omega) = S^*_{LR}(\omega)$. It should be noted that the averaged current $I(t)$ may be different from the currents measured in the leads when the capacitances of the left and right tunnel junctions are considered. Equation (36) for the noise spectrum of the average current only corresponds to the case of the quantum dot with symmetric tunnel-junction capacitances.[44] For a thorough consideration of the effects of the capacitances and the displacement currents in the quantum dot or the single-electron transistor, one might need more realistic microscopic modeling of the system than the Anderson impurity model.

IV. NUMERICAL RESULTS

To get a better understanding of the interaction effect on the current noise spectrum for the quantum dot system, in this section, we will present some numerical calculations of the current noise at zero temperature. In our calculation, we take the coupling strength Γ between the leads and the quantum dot as the units of energy and the bandwidth $D = 100$.

In order to estimate the quality of the Hartree approximation in the study of the transport properties of this system, we first make some comparisons between the results of the Hartree approximation and that of the numerical renormalization group (NRG) method[45] in the linear-response regime. The linear conductance of this system can be calculated by using the Friedel sum rule: $dI/dV = \sum_\sigma \frac{e^2}{h} \sin^2(\delta_\sigma/2)$ where the phase

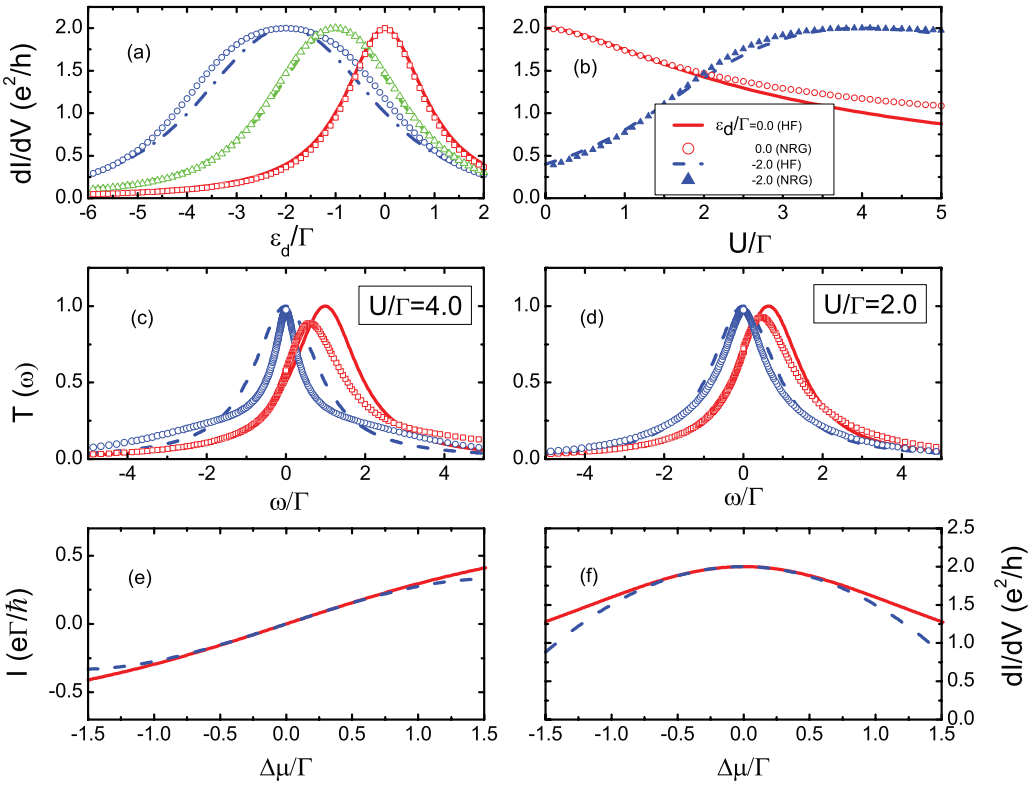

FIG. 1. (Color online) (a) The linear conductance dI/dV vs the dot level ϵ_d for the quantum dot system in the symmetric coupling case ($\Gamma_L = \Gamma_R = \Gamma$). We take the parameters in the Anderson impurity model in units of the coupling strength $\Gamma = 1.0$. The Coulomb interaction strength is $U = 0.0$ (solid line), 2.0 (dashed line), and 4.0 (dashed-dotted line), respectively. The symbols correspond to results obtained by the NRG method; (b) the linear conductance dI/dV vs the Coulomb interaction strength U for different dot levels ϵ_d; (c) and (d) show the transmission probability $T(\omega)$ vs incident electron energy ω. The parameters in panel (c): $U = 4.0$ and $\epsilon_d = 0.0$ (solid line) and -2.0 (dashed line); in panel (d), $U = 2.0$ and $\epsilon_d = 0.0$ (solid line) and -1.0 (dashed line). Panels (e) and (f) show the current and the differential conductance vs the bias voltage between the leads. The particle-hole symmetric case with $\epsilon_d = -1.0$ and $U = 2.0$ is considered. The solid lines are the results of the Hartree approximation, and the dashed lines are obtained from the analytical formulas in Ref. 47.

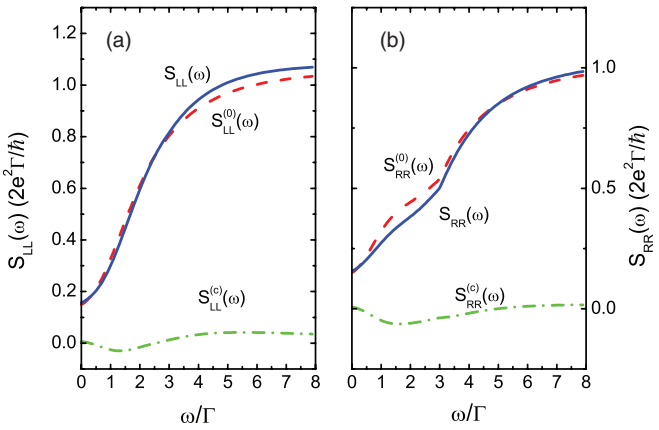

FIG. 2. (Color online) The noise spectra for quantum dot system in the symmetric coupling case ($\Gamma_L = \Gamma_R$). (a) We plot the bare noise spectrum $S^{(0)}_{LL}(\omega)$ (dashed line), the interaction correction term $S^{(c)}_{LL}(\omega)$ (dashed-dotted line), and the noise spectrum after correction $S_{LL}(\omega)$ (solid line) for the left lead; (b) the same for the right lead. The parameters used are $\epsilon_d = 0.0$, $U = 2.0$, and $\Gamma_L = \Gamma_R = 1.0$ for the symmetric case. The biased voltage between the left and the right leads is $\Delta\mu = 3.0$.

shift is $\delta_\sigma = 2\pi\langle n_{d\sigma}\rangle$.[46] The linear conductances dI/dV vs the dot level ϵ_d and the Coulomb interaction strength U are plotted in Figs. 1(a) and 1(b), respectively. Figure 1(a) shows that the conductance given by the Hartree approximation is in good agreement with the NRG results when the interaction strength U/Γ is less than 2 and is qualitatively correct when $U/\Gamma = 4.0$. In Fig. 1(b), it is somewhat of a surprise to see that the conductance, as a function of U obtained by the Hartree approximation, agrees with the NRG result even when the dot level is well below the Fermi energy ($\epsilon_d/\Gamma = -2.0$). As is well known that the Hartree approximation cannot account for the Kondo effect in the Kondo regime, so we have examined the transmission probability $T(\omega)$ as a function of incident electron energy ω for different values of U in Figs. 1(c) and 1(d). The transmission probability is obtained from the imaginary part of the dot GF $T(\omega) = -\Gamma\,\mathrm{Im}\,G^r(\omega)$. In Fig. 1(c), one can see that the narrow Kondo resonance peak in the line shape of the transmission probability $T(\omega)$ cannot be described by the Hartree approximation in the particle-hole symmetric case ($\epsilon_d = -U/2$). But, for the dot level ϵ_d being located in the resonant tunneling regime ($\epsilon_d = 0$) or in the weak Coulomb interaction case shown in Fig. 1(d), the line shapes of $T(\omega)$ given by Hartree approximation are in better agreement with the NRG results. For the system with finite bias voltage between the leads, the current I and the differential conductance dI/dV are plotted in Figs. 1(e) and 1(f), respectively, where the particle-hole symmetric case with weak Coulomb interaction $U = 2.0$ is considered. The results of the Hartree approximation are compared with that of the Fermi-liquid theory[47] in the out-of-equilibrium case. In the small bias voltage regime with $\Delta\mu/\Gamma < 0.5$, the results obtained by both methods are in good agreement. For the system with large bias voltage or away from particle-symmetric case, no general analytical expressions for the current and conductance are available in the literature, and further work might be needed to understand the non-linear-response properties of the Anderson impurity model. In the following, we will investigate the Anderson impurity model in the weak Coulomb interaction strength U case within the Hartree approximation.

To study the current noise spectra, we fix the Coulomb interaction strength at $U/\Gamma = 2.0$, the dot level at $\epsilon_d = 0.0$, and assume a finite bias voltage of $\Delta\mu = 3.0$ ($\mu_L = -\mu_R = \Delta\mu/2$) applied between the leads. For the system with symmetric coupling strength $\Gamma_L = \Gamma_R = \Gamma$, we plot the current noise spectra for the left and the right leads in Figs. 2(a) and 2(b), respectively. Figure 2(a) shows that the bare noise spectrum $S^{(0)}_{LL}(\omega)$ is a positive and increasing function of the frequency. It approaches the value of $2e^2\Gamma/\hbar$ at high frequency, which agrees with the height of noise spectrum steps found for the multilevel quantum dot system[28] (where steps of height $e^2\Gamma/\hbar$ were shown for spinless electrons). The interaction correction term $S^{(c)}_{LL}(\omega)$ shows significant dependence on the frequency. It is found that the interaction corrections for the shot noise at zero frequency has a small positive value, which agrees with the result obtained previously,[23] but the interaction correction becomes negative and more significant when the frequency increases, and it goes to a positive value again in the larger frequency region. The maximum influence of the interaction correction is achieved at the frequency which is largely determined by the energy difference between the renormalized dot level $\tilde{\epsilon}_d = \epsilon_d + U\langle n_{d\bar{\sigma}}\rangle$ and the Fermi levels of the leads. The sum of $S^{(0)}_{LL}(\omega)$ and $S^{(c)}_{LL}(\omega)$ gives the noise spectrum after the interaction correction, which is a monotonously increasing function of the frequency. Figure 2(b) shows the noise spectra for the right lead (the drain side of this system). These noise spectra exhibit more prominent features than that of the left lead. One can find a significant dip for the total noise spectrum $S_{RR}(\omega)$ at the frequency equal to the applied bias voltage ($\omega = \Delta\mu = 3.0$). At high frequency, the noise spectrum S_{RR} also approaches the step value $2e^2\Gamma/\hbar$.

The various bare correlation and response functions utilized in the calculation of the interaction correction of the noise spectrum are plotted in Fig. 3. As shown in Fig. 3(a), the density fluctuation spectrum $S^{(0)}_{nn}(\omega)$ of the quantum dot always has real positive values at finite frequencies. The real part of the density response function $\chi^{r,(0)}_{nn}(\omega)$ is negative at a low frequency, which reveals the screening effect of an electrons decreasing the Coulomb interaction strength U on the quantum dot. The imaginary part of $\chi^{r,(0)}_{nn}(\omega)$ remains negative for all frequencies due to the analytical properties of the density-density response function. The correlation and response functions between the density operator on the dot and the current operator for the left and right leads are plotted in Figs. 3(b) and 3(c), respectively.

Figure 4 shows the current noise spectra for the asymmetrically coupled quantum dot system. Since the coupling strength is $\Gamma_L \gg \Gamma_R$, we find that the magnitude of the current fluctuations in the right lead, plotted in Fig. 4(b), is much less than that of the left lead as shown in Fig. 4(a) because of the tunneling rate between the right lead and the quantum dot being much less than that of the left lead. The interaction correction terms also have negative value regions at finite frequencies both for the left and the right leads. The noise spectrum in the right lead (drain lead) shows an evident dip structure at the frequency equal to the bias voltage. It should be noted that,

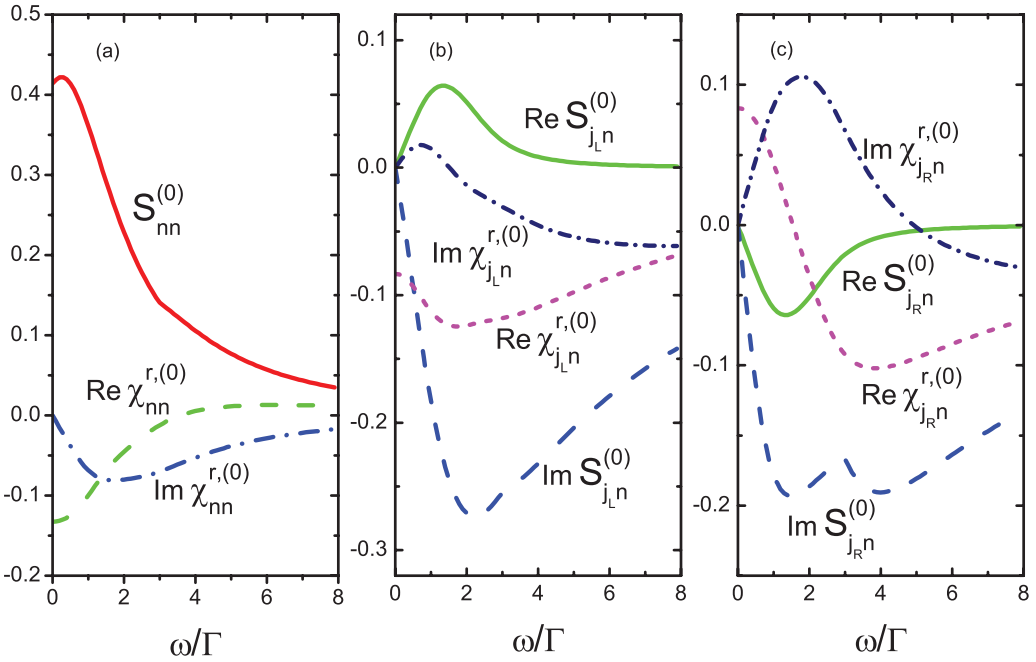

FIG. 3. (Color online) The spectra of various correlation and response functions involved in the calculation of the interaction effect on noise spectra in the symmetric coupling case (in units of $1/\Gamma$). (a) The density correlation function $S^{(0)}_{nn}(\omega)$ and the real and imaginary parts of the density response function $\chi^{(0)}_{nn}(\omega)$ in the quantum dot. (b) The real and imaginary parts of the correlation function $S^{(0)}_{j_Ln}(\omega)$ and response function $\chi^{r,(0)}_{j_Ln}(\omega)$ for the left lead. (c) The correlation function and response function of the right lead. The parameters used in the calculation are the same as in Fig. 2.

here, the dip structure of the noise spectrum is due to finite bias voltage, which is different from the dip structure found for the multilevel quantum dot system where it is caused by the energy-level spacing in the quantum dot.[28] One can expect that this kind of prominent feature of the noise spectrum can be detected in experiments.

Next, we consider the noise spectrum of the averaged current defined by Eq. (36). For the symmetric coupling case

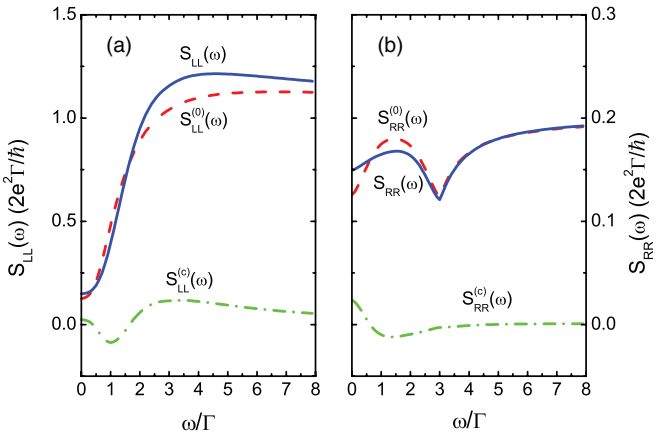

FIG. 4. (Color online) The noise spectra for the quantum dot system in the asymmetric coupling case ($\Gamma_L \gg \Gamma_R$). (a) For the left lead, we plot the bare noise spectrum $S^{(0)}_{LL}(\omega)$ (dashed line), the interaction correction term $S^{(c)}_{LL}(\omega)$ (dashed-dotted line), and the noise spectrum after correction $S_{LL}(\omega)$ (solid line); (b) the same for the right lead. We take the parameters as $\epsilon_d = 0.0$ and $U = 2.0$ and assume $\Gamma_L = 1.0$ and $\Gamma_R = 0.2$.

with different values of the dot energy level ϵ_d and a fixed biased voltage $\Delta\mu = 3.0$, the averaged current noise spectra $S(\omega)$, after taking into account the interaction correction and the corresponding interaction correction terms $S^{(c)}(\omega)$, are plotted in Figs. 5(a) and 5(b), respectively. One observes that both the noise spectrum $S(\omega)$ and the interaction correction term $S^{(c)}(\omega)$ are strongly dependent on the dot level ϵ_d. In the particle-hole symmetric point with $\epsilon_d = -1.0$, the interaction correction term $S^{(c)}(\omega)$ remains zero for all the frequencies. For the dot level $\epsilon_d > 0$, one sees that, around the frequency $\omega \approx \Delta\mu$, the noise spectra have suppression effects, and the corresponding interaction correction terms exhibit peak structures. In the high-frequency region, all of the noise spectra go to the same value which is somewhat below half the noise step height ($e^2\Gamma/\hbar$) as a result of the current cross correlation. The interaction correction terms go to zero in the high-frequency limit. In Figs. 5(c) and 5(d), we show the noise spectra and the interaction corrections for different biased voltages when the dot level is fixed at $\epsilon_d = 0$. In the zero-bias voltage case, the noise power is zero in the zero-frequency limit, which is due to the absence of thermal noise and energy dissipation at zero temperature in the equilibrium situation. It is interesting to notice that the interaction correction term also remains zero at finite frequencies at equilibrium since the quantum dot is symmetrically coupled to the left and right leads. By increasing the biased voltage, the noise power in the zero-frequency region is greatly enhanced, and the interaction corrections on the noise spectra become more significant. Recently, the interaction correction to the noise spectrum of the Anderson impurity model was studied by a second-order perturbation theory and the diagrammatic

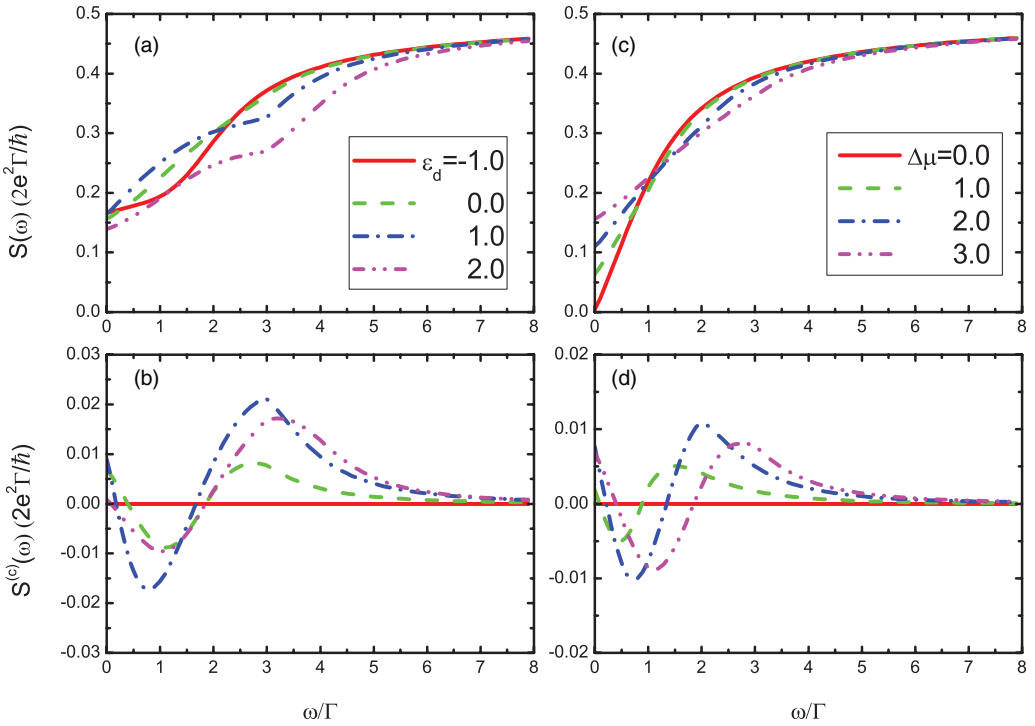

FIG. 5. (Color online) The noise spectra of the averaged current for the quantum dot system in the symmetric coupling case ($\Gamma_L = \Gamma_R = 1.0$). (a) and (c) correspond to the noise spectra after interaction correction $S(\omega)$; (c) and (d) are the interaction correction term $S^{(c)}(\omega)$. For (a) and (b), the bias voltage is fixed at $\mu_L = -\mu_R = \Delta\mu/2 = 1.5$, and the quantum dot level is tuned by gate voltage with $\epsilon_d = -1.0$ (solid line), 0.0 (dashed line), 1.0 (dash-dotted line), and 2.0 (dashed-dot-dotted line). For (c) and (d), the dot level is fixed at $\epsilon_d = 0.0$, and the bias voltage changes.

resummation technique.[29] We find the noise corrections at finite bias voltages plotted in Figs. 5(b) and 5(d) have similar peak structures to that obtained in Ref. 29 where the particle-hole symmetric regime is considered. In this self-consistent Hartree approximation, one cannot expect to take into account the spin fluctuation effects correctly, thereby, we will not

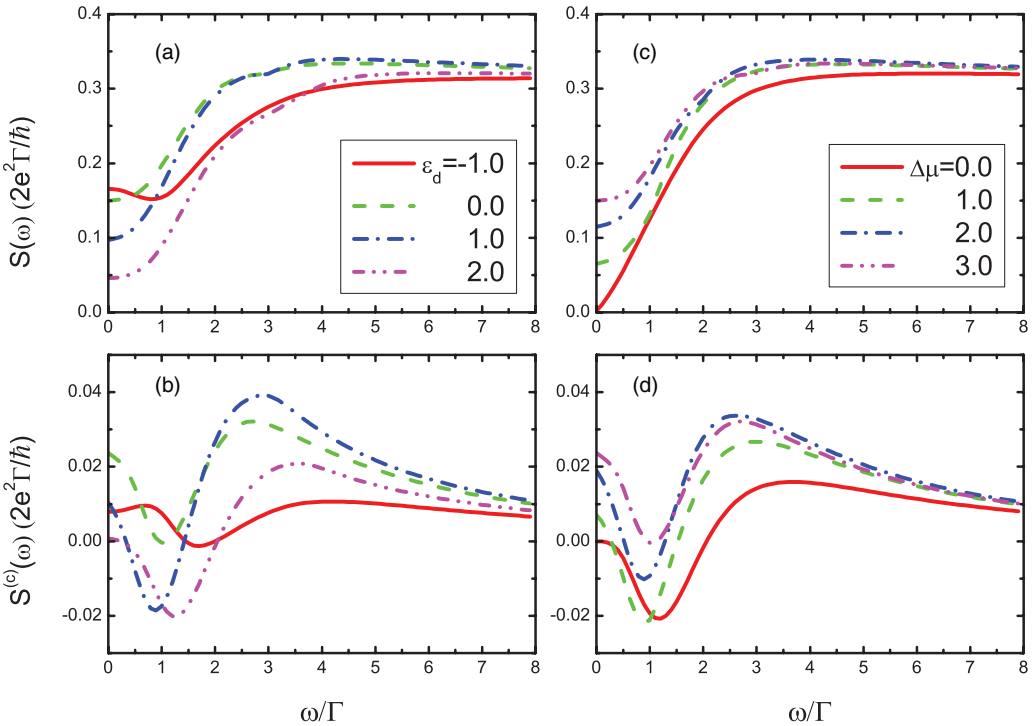

FIG. 6. (Color online) The noise spectra of the averaged current for the quantum dot system in the asymmetric coupling case ($\Gamma_L = 1.0$, $\Gamma_R = 0.2$). The others are the same as in Fig. 5.

address the Kondo effect on the noise spectrum as investigated in Ref. 29.

Figure 6 depicts the noise spectra and interaction correction terms of the averaged current in the asymmetric coupling case. It is shown that the interaction effect is more significant than that of the symmetric coupling case, and the influence of the noise spectra by tuning the dot level ϵ_d becomes larger. One of the noteworthy features of the interaction correction term is that it is nonzero at finite frequencies even in the zero-bias voltage limit as shown in Fig. 6(d). This is quite different from the symmetric coupling case. The noise spectra also show large deviations from the half-step height value ($e^2\Gamma/\hbar$) in the high-frequency limit.

V. CONCLUSIONS

In this paper, we have investigated the problem of electron transport through a quantum dot in the framework of nonequilibrium self-consistent perturbation theory and examined the current conservation condition. Based on the Anderson impurity model, we have given the current and current fluctuation formulas by using a nonequilibrium generating functional and the functional derivation method, which are valid in the presence of arbitrary external time-dependent potentials. We have calculated the interaction effect on the finite-frequency noise spectrum of the Anderson impurity model by taking into account the interaction vertex correction term within the Hartree approximation and have obtained an analytical expression for the noise correction term at finite frequencies, which corresponds to a generalization of the previous result on zero-frequency shot noise. Even though we have focused our present attention on the symmetrized noise spectrum, one can expect that the nonsymmetrized noise spectrum and the ac conductance can also be studied within the formulation presented in this paper. We believe that the self-consistent perturbation theory on the Schwinger-Keldysh contour can lead to a unified approach to many interesting problems in nonequilibrium electron transport through mesoscopic systems and will give us a deep understanding of the current fluctuation and energy dissipation phenomena.[48] The functional method provides a convenient way to study the statistics of current fluctuations. It should be noted that the Hartree approximation can only account for some interaction effects on the electron current noise in the resonant tunneling regime. Namely, only when the dot level is near the Fermi energy ($\epsilon_d \approx 0$) where no Kondo effect is involved at low temperatures and the Anderson impurity model is not in the local moment regime ($U/\Gamma < 2.0$), can the Hartree approximation with vertex corrections give reasonable results on the transport properties of the Anderson model. Therefore, it is expected that future research can treat the interacting effect beyond the Hartree approximation and can give us more information about the interaction effect on the out-of-equilibrium dynamics of electrons in the Coulomb blockade regime as well as the low-temperature Kondo regime.

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APPENDIX A

The response and correlation functions of the density operator for the one spin component are given by

$$\chi^r_{nn}(t,t') = -i\theta(t,t')\langle[n_{d\sigma}(t),n_{d\sigma}(t')]\rangle, \tag{A1}$$

and

$$S_{nn}(t,t') = \langle\{\delta n_{d\sigma}(t),\delta n_{d\sigma}(t')\}\rangle, \tag{A2}$$

respectively. The current-density response function and the correlation function are

$$\chi^r_{j_\eta n}(t,t') = -i\theta(t,t')\langle[j_{\eta\sigma}(t),n_{d\sigma}(t')]\rangle, \tag{A3}$$

and

$$S_{j_\eta n}(t,t') = \langle\{\delta j_{\eta\sigma}(t),\delta n_{d\sigma}(t')\}\rangle. \tag{A4}$$

By neglecting the vertex correction term induced by interaction effect, we can explicitly write the above functions in the frequency space as follows:

$$\chi^{r,(0)}_{nn}(\omega) = \int \frac{d\omega_1}{2\pi}(-i)[G^a(\omega_1)G^<(\omega_1+\omega) + G^<(\omega_1)G^r(\omega_1+\omega)], \tag{A5}$$

$$S^{(0)}_{nn}(\omega) = \int \frac{d\omega_1}{2\pi}[G^<(\omega_1)G^>(\omega_1+\omega) + G^>(\omega_1)G^<(\omega_1+\omega)], \tag{A6}$$

$$\chi^{r,(0)}_{j_\eta n}(\omega) = \int \frac{d\omega_1}{2\pi}\{i\Gamma_\eta n_\eta(\omega_1)[G^r(\omega_1)G^r(\omega_1+\omega) - G^a(\omega_1)G^a(\omega_1-\omega)] + i\Gamma_\eta G^<(\omega_1) \times [G^r(\omega_1+\omega) + G^a(\omega_1-\omega)]\}, \tag{A7}$$

$$S^{(0)}_{j_\eta n}(\omega) = \int \frac{d\omega_1}{2\pi}\Gamma_\eta\{[1-n_\eta(\omega_1)]G^r(\omega_1)G^<(\omega_1+\omega) - n_\eta(\omega_1)G^r(\omega_1)G^>(\omega_1+\omega) + n_\eta(\omega_1+\omega) \times G^>(\omega_1)G^a(\omega_1+\omega) - [1-n_\eta(\omega_1+\omega)] \times G^<(\omega_1)G^a(\omega_1+\omega) - G^<(\omega_1)G^>(\omega_1+\omega) - G^>(\omega_1)G^<(\omega_1+\omega)\}. \tag{A8}$$

In the Hartree approximation, the retarded/advanced Green's function is $G^{r/a}(\omega_1) = 1/[\omega_1 - \epsilon_d - U\langle n_{d\bar\sigma}\rangle \pm i\sum_\eta \Gamma_\eta/2]$. The other Green's functions are $G^<(\omega_1) = G^r(\omega_1)[i\sum_\eta \Gamma_\eta n_\eta(\omega_1)]G^a(\omega_1)$ and $G^>(\omega_1) = G^r(\omega_1)\{-i\sum_\eta \Gamma_\eta[1-n_\eta(\omega_1)]\}G^a(\omega_1)$, where $n_\eta(\omega)$ is the Fermi distribution function of the lead.

APPENDIX B

The interaction vertex function is $\Gamma^U_\eta(t_1,t_2;t) = \frac{\delta\Sigma_U(t_1,t_2)}{\delta\lambda_\eta(t)}$, and the self-energy in the Hartree approximation is

given by

$$\Sigma_U(t_1,t_2) = U\langle n_{d\bar{\sigma}}(t_1)\rangle\delta(t_1,t_2) = -iUG_{\bar{\sigma}}(t_1,t_1^+)\delta(t_1,t_2). \tag{B1}$$

In the spin degenerate case, we will omit the spin index $\bar{\sigma}$. By using the following identity:

$$\begin{aligned}\frac{\delta G(t_1,t_1^+)}{\delta\lambda_\eta(t)} &= -\int dt_2dt_3G(t_1,t_2)\frac{\delta G^{-1}(t_2,t_3)}{\delta\lambda_\eta(t)}G(t_3,t_1^+)\\ &= \int dt_2dt_3G(t_1,t_2)\big[\Gamma_\eta^{(0)}(t_2,t_3,t)\\ &\quad+\Gamma_\eta^U(t_2,t_3,t)\big]G(t_3,t_1^+),\end{aligned} \tag{B2}$$

we will derive the integral equation satisfied by the interaction vertex function. If we rewrite the vertex function $\Gamma_\eta^U(t_1,t_2;t) = \tilde{\Gamma}_\eta^U(t_1,t)\delta(t_1,t_2)$, then the integral equation for $\tilde{\Gamma}_\eta^U(t_1,t)$ is given by

$$\begin{aligned}&\tilde{\Gamma}_\eta^U(t_1,t) + iU\int dt_2G(t_1,t_2)G(t_2,t_1^+)\tilde{\Gamma}_\eta^U(t_2,t)\\ &\quad= -iU\Lambda_\eta^{(0)}(t_1,t),\end{aligned} \tag{B3}$$

where

$$\Lambda_\eta^{(0)}(t_1,t) = \int dt_2dt_3G(t_1,t_2)\Gamma_\eta^{(0)}(t_2,t_3,t)G(t_3,t_1^+). \tag{B4}$$

By introducing a function $M(t_2,t)$, which satisfies the following equation:

$$\int dt_2[\delta(t_1,t_2) + iUG(t_1,t_2)G(t_2,t_1^+)]M(t_2,t) = \delta(t_1,t), \tag{B5}$$

we then obtain the solution for the vertex function as

$$\tilde{\Gamma}_\eta^U(t_1,t) = -iU\int dt_2M(t_1,t_2)\Lambda_\eta^{(0)}(t_2,t). \tag{B6}$$

It is noticed that $\Lambda_\eta^{(0)}(t_2,t)$ is equal to the bare correlation function of the density operator on the quantum dot $\Lambda_\eta^{(0)}(t_2,t) = \langle T_C\delta j_\eta(t)\delta n_{d\bar{\sigma}}(t_2)\rangle$ without the interaction vertex correction as defined in Appendix A. The function $M(t_1,t_2)$ can be obtained by solving Eq. (B5). In the frequency space, it can be written in the matrix form of

$$\begin{aligned}&\begin{pmatrix}M^{--}(\omega) & M^{-+}(\omega)\\ M^{+-}(\omega) & M^{++}(\omega)\end{pmatrix}\\ &\quad= \frac{1}{\Delta(\omega)}\begin{pmatrix}1+U\chi_{nn}^{++,(0)}(\omega) & U\chi_{nn}^{-+,(0)}(\omega)\\ U\chi_{nn}^{+-,(0)}(\omega) & -1+U\chi_{nn}^{--,(0)}(\omega)\end{pmatrix},\end{aligned} \tag{B7}$$

with $\Delta(\omega) = [1-\chi_{nn}^{r,(0)}(\omega)][1-\chi_{nn}^{a,(0)}(\omega)]$. Therefore, an explicit formula, Eq. (B6), for the vertex function $\tilde{\Gamma}_\eta^U(t_1,t)$ is obtained, which can be substituted for Eq. (32) to get the current noise interaction correction term.

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