Rutgers relation for the analysis of superfluid density in superconductors

H. Kim,[1,2] V. G. Kogan,[1] K. Cho,[1] M. A. Tanatar,[1,2] and R. Prozorov[1,2,*]
[1]*The Ames Laboratory, Ames, Iowa 50011, USA*
[2]*Department of Physics & Astronomy, Iowa State University, Iowa 50011, USA*
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It is shown that the thermodynamic Rutgers relation for the second-order phase transitions can be used for the analysis of the superfluid density data irrespective of complexities of the Fermi surface, structure of the superconducting gap, pairing strength, or scattering. The only limitation is that critical fluctuations should be weak so that the mean-field theory of the second-order phase transitions is applicable. By using the Rutgers relation, the zero-temperature value of the London penetration depth $\lambda(0)$ is related to the specific heat jump ΔC and the slope of upper critical field dH_{c2}/dT at the transition temperature T_c, provided the data on $\Delta\lambda = \lambda(T) - \lambda(0)$ are available in a broad temperature domain. We then provide a way to determine $\lambda(0)$, the quantity difficult to determine within many techniques.

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I. INTRODUCTION

The London penetration depth λ is one of the most important characteristic length scales of superconductors. The temperature dependent $\lambda(T)$ is the subject of many studies as it provides information on the symmetry of the order parameter.[1,2] It is used to calculate the superfluid density $\rho(T) \equiv \lambda^2(0)/\lambda^2(T)$. Determination of $\lambda(0)$ is critical because the shape of $\rho(T)$, extracted from the data on $\Delta\lambda(T)$, depends sensitively on the value of $\lambda(0)$ adopted, and a wrong $\lambda(0)$ could lead to incorrect conclusions on the superconducting order parameter.

A tunnel diode resonator (TDR) technique provides, perhaps, the most precise measurements of the variation in λ with temperature $\Delta\lambda = \lambda(T) - \lambda(0)$. With additional sample manipulation by coating it with a lower-T_c superconductor, the absolute value of $\lambda(0)$ can be determined as well but with much lower precision compared to $\Delta\lambda$.[3] Other techniques which are used to measure $\lambda(0)$ include muon spin rotation (μSR),[4] infrared spectroscopy,[5] the microwave cavity perturbation technique,[6] and local probes.[7,8] However, each of these techniques has its own limitations. μSR measures averaged $\lambda(T,H)$ in the mixed state, and the extrapolated to $H = 0$ value is used to estimate $\lambda(0)$. In infrared spectroscopy, $\lambda(0)$ is deduced from the measured plasma frequency, which is not a precisely determined quantity.[5] Local probes, such as a scanning superconducting quantum interference device[7] and magnetic force microscopy[8] magnetometry, infer $\lambda(0)$ from the analysis of magnetic interactions between a relevant probe and a magnetic moment induced in a superconductor.[9]

In this paper, we show that the thermodynamic relation between the specific heat jump ΔC and the slope of thermodynamic critical field $\partial H_c/\partial T$ at the superconducting transition temperature T_c, first proposed by Rutgers,[10] can be rewritten in terms of measurable quantities—the slopes of the upper critical field and of the superfluid density in addition to specific heat jump. As a general thermodynamic relation, valid at the second-order transition (excluding critical fluctuation region), it is applicable for any superconductor irrespective of the pairing symmetry, scattering, or multiband nature of superconductivity as we verified on several well-known systems. Such general applicability of the Rutgers relation offers a method of estimating $\lambda(0)$ if ΔC and dH_{c2}/dT at T_c are known. This idea is checked on Nb and MgB_2 and is applied to several unconventional superconductors. In all cases, we use $\Delta\lambda(T)$ measured by the TDR technique and the literature data for the other two quantities, except for $YBa_2Cu_3O_{7-\delta}$ (YBCO), where its $\rho(T)$ was taken from Ref. 6. In all studied cases, the method works well, and determined values of $\lambda(0)$ are in agreement with the literature.

A. Thermodynamic Rutgers relation

The specific heat jump at T_c in materials where the critical fluctuations are weak is expressed through the free-energy difference $F_n - F_s = H_c^2/8\pi$,[10,11]

$$\Delta C = T_c \frac{\partial^2}{\partial T^2}\frac{H_c^2}{8\pi}\bigg|_{T_c} = \frac{T_c}{4\pi}\left(\frac{\partial H_c}{\partial T}\right)^2_{T_c}. \tag{1}$$

Here, C is measured in erg cm^{-3} K^{-1}, and T is measured in Kelvins. Within the mean-field Ginzburg-Landau (GL) theory, near T_c, the thermodynamic critical field $H_c = \phi_0/2\sqrt{2}\pi\xi\lambda$ with

$$(\xi,\lambda) = \frac{(\xi_{GL},\lambda_{GL})}{\sqrt{1-t}}, \quad t = \frac{T}{T_c}. \tag{2}$$

Here, ξ and λ are the coherence length and the penetration depth, and the constants ξ_{GL},λ_{GL} are of the same order but not the same as the $T = 0$ values $\xi(0)$ and $\lambda(0)$. Hence, we have

$$\Delta C = \frac{\phi_0^2}{32\pi^3\xi_{GL}^2\lambda_{GL}^2 T_c}, \tag{3}$$

where ξ_{GL} is related to the slope of $H_{c2}(T)$ at T_c,

$$T_c\frac{\partial H_{c2}}{\partial T}\bigg|_{T_c} = \frac{\partial H_{c2}}{\partial t}\bigg|_{t=1} = -\frac{\phi_0}{2\pi\xi_{GL}^2}. \tag{4}$$

It is common to introduce the dimensionless superfluid density $\rho = \lambda^2(0)/\lambda^2$ with the slope at T_c given by

$$T_c\frac{\partial \rho}{\partial T}\bigg|_{T_c} = \frac{\partial \rho}{\partial t}\bigg|_{t=1} = -\frac{\lambda^2(0)}{\lambda_{GL}^2}. \tag{5}$$

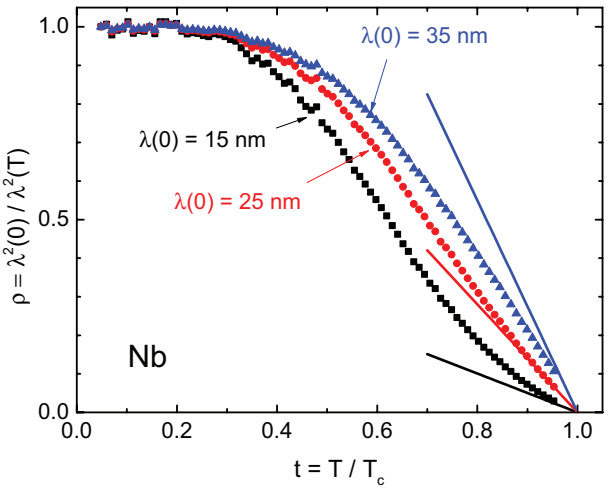

FIG. 1. (Color online) Superfluid density $\rho(t)$ calculated from Eq. (7) using the TDR data on $\Delta\lambda(T)$ and assuming $\lambda(0) = 15$, 25, and 35 nm. Straight lines have the slope ρ' estimated from Eq. (6) for each $\lambda(0)$.

We then obtain

$$\Delta C = \frac{\phi_0}{16\pi^2\lambda^2(0)T_c}(H'_{c2}\rho')_{t=1}, \tag{6}$$

where the primes denote derivatives with respect to t.

It should be stressed that being a thermodynamic relation that holds at a second-order phase transition, applicability of a Rutgers formula is restricted only by the possible presence of critical fluctuations. In particular, it can be applied for zero-field phase transitions in materials with anisotropic order parameters and Fermi surfaces, multibands, etc., which makes it a valuable tool in studying a great majority of new materials.

For anisotropic materials, Eq. (1) is, of course, valid since the condensation energy and H_c do not depend on direction. However, already in Eq. (3), the field direction should be specified. In the following, we discuss situations with $\boldsymbol{H}$ parallel to the c axis of uniaxial crystals. Hence, the quantities in Eq. (6) should have subscripts: ρ_{ab}, $\lambda_{ab}(0)$ and $H_{c2,c}$. We omit these subscripts for brevity. A general case of anisotropic material with an arbitrary field orientation requires separate analysis.

II. DETERMINATION OF λ(0)

The superfluid density needed for the analysis of the experimental data and comparison with theoretical calculations depends on the choice of $\lambda(0)$,

$$\rho(t) = \frac{\lambda^2(0)}{[\lambda(0) + \Delta\lambda(t)]^2}. \tag{7}$$

Figure 1 shows an example of this dependence of $\rho(t)$ on $\lambda(0)$ for Nb. Symbols represent $\rho(t)$ calculated from measured $\Delta\lambda(t)$ with $\lambda(0)$ chosen as 15, 25, and 35 nm. Clearly, the calculated $\rho(t)$ is sensitive to the choice of $\lambda(0)$. The straight solid lines have the slope $\rho'(1)$ calculated by using Eq. (6) for each $\lambda(0)$. We used $\Delta C = 137.2$ mJ mol^{-1} K^{-1} = 126 450 erg cm^{-3} K^{-1} (Ref. 12) since, in the formulas used here, the specific heat is per unit volume.[13] Using $H'_{c2}|_{T_c} = 440$ Oe/K (Ref. 14), we obtain $-\rho'(1) = 0.49$, 1.4, and 2.7 for 15, 25, and 35 nm, respectively. Although the choice of $\lambda(0) = 25$ nm shows reasonable agreement, for the choices of 15 and 35 nm, the slopes calculated using the data and Eq. (7) are determined by Eq. (6) under- and overestimates, respectively. Note that, with $\lambda(0) = 15$ nm, the temperature dependence of ρ is pronouncedly concave near $t = 1$ and $-\rho'(1)$ is smaller than 1. The idea of our method is to utilize the Rutgers relation (6) and to choose such a $\lambda(0)$ that would not contradict the thermodynamics near T_c.

To this end, we rewrite Eq. (6) in the form

$$\frac{\rho'(1)}{\lambda^2(0)} = \frac{16\pi^2 T_c \Delta C}{\phi_0 H'_{c2}(1)}. \tag{8}$$

Here, the right-hand side is determined from independent measurements of ΔC and H_{c2}. Thus, by taking a few test values of $\lambda(0)$, calculating $\rho(t)$ and its slope at $t = 1$, we can decide which $\lambda(0)$ and $\rho[t,\lambda(0)]$ obey the Rutgers relation.

We first apply this method to two well-studied superconductors—conventional Nb and two-band MgB_2. For Nb, we obtain $|\rho'|/\lambda^2(0) \approx 2240\ \mu\text{m}^{-2}$ using the same thermodynamic quantities as for Fig. 1.[12,14] We now take a set of values for $\lambda(0)$ shown in top left panel of Fig. 2 and plot $|\rho'|/\lambda^2(0)$ vs $\lambda(0)$. The value of $\lambda(0) = 28 \pm 2$ nm satisfying the Rutgers relation is obtained from the intersection of the calculated curve with the value expected from Eq. (8) (shown by a gray band that takes into account experimental uncertainties in determining ΔC and H'_{c2}). It is consistent with the literature values varying between 26 and 39 nm.[12,15] The final calculated superfluid density with the choice of $\lambda(0) =$ 30 nm is shown in Fig. 2(b). The solid line is determined with the calculated slope $|\rho'(1)| = 2$ as predicted for isotropic s-wave superconductors (see the Appendix).

In addition to the aforementioned uncertainties, determination of the experimental $|\rho'(1)|$ is not trivial even if the quality of measurement is excellent since $\rho(t)$ near $t = 1$ is often significantly curved due to several experimental artifacts, most importantly, due to the influence of the normal skin effect near T_c, which is more pronounced for higher-frequency measurements on highly conducting materials. The TDR technique typically uses ~10 MHz, so this effect is weak in most of the materials concerned. Analyzing the data for different superconductors, we have found that the data in the regime between $t = 0.8$ and 0.95 work well for the determination of $\rho'(1)$. The experimental $|\rho'(1)|$ in this paper is determined from the best linear fit of $\rho(t)$ data in this range.

The same procedure can be employed for a well-known multigap superconductor MgB_2 (shown in the bottom row of Fig. 2) where $|\rho'|/\lambda^2(0)$ is estimated to be $130 \pm 12\ \mu\text{m}^{-2}$ by using $\Delta C = 133$ mJ mol^{-1} K^{-1} (Ref. 16), $|H'_{c2}(1)| =$ 0.45 T/K (Ref. 17) within a ±5% error. The determined $\lambda(0) = 84 \pm 10$ nm is in good agreement with 100 nm estimated by the μSR technique.[18,19] For $\lambda(0) = 84$ nm, the calculated slope $|\rho'(1)| = 0.91$ agrees with the expected theoretical value of 0.92 (see the Appendix).

The method described has also been used for $SrPd_2Ge_2$ for which $\lambda(0)$ was not clear. By using the determined $\lambda(0)$, we have shown that $SrPd_2Ge_2$ is a single-gap s-wave superconductor.[20]

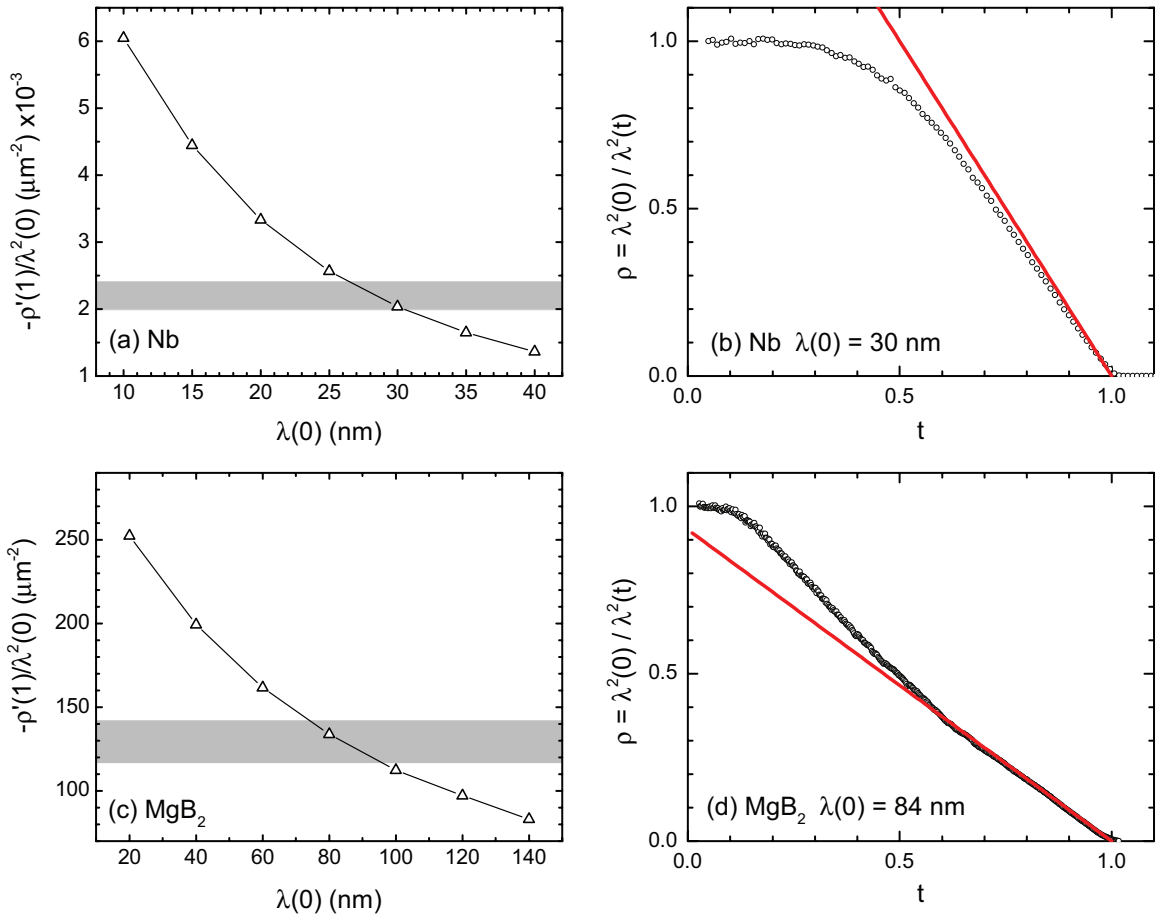

FIG. 2. (Color online) Top row: Nb, Bottom row: MgB_2. Left column: variation in $|\rho'|/\lambda^2(0)$ as a function of $\lambda(0)$ for Nb and MgB_2. Shaded horizontal bands are the estimated values of the right-hand side of Eq. (8) with literature values of ΔC and $H'_{c2}(1)$ including experimental uncertainties. Right column: superfluid density for the best value of $\lambda(0)$ that satisfies the Rutgers relation, Eq. (8).

A. Unconventional superconductors

Here, we examine the validity of our approach for some superconductors for which the necessary experimental quantities have been reported in the literature. Where possible, we use $H_{c2}(T)$ determined from the specific heat jump because resistive and magnetic measurements may actually determine the irreversibility field, which may differ substantially from the thermodynamic H_{c2}.[38]

We have selected LiFeAs, $FeTe_{0.58}Se_{0.42}$, $YBa_2Cu_3O_{7-\delta}$, and $MgCNi_3$, representing a stoichiometric pnictide, a chalcogenide, a d-wave high-T_c cuprate, and a close to magnetic instability s-wave superconductor, respectively. ΔC, dH_{c2}/dT, and $\lambda(0)$, for the selected compounds, have been measured by various techniques by different groups. The superfluid density was calculated from the penetration depth measured by using a TDR technique at Ames Laboratory, except for YBCO for which anisotropic superfluid density was determined by the

TABLE I. V_c is the volume of the unit cell. ΔC is the specific heat jump at T_c in $\mathrm{mJ\,mol^{-1}\,K^{-1}}$. dH_{c2}/dT is the slope of H_{c2} at T_c. $\rho'_{\mathrm{Rut}} = (d\rho/dt)_{\mathrm{Rut}}$ is the calculated slope using Eq. (6) where $t = T/T_c$. ρ'_{exp} is an experimental slope with given $\lambda(0)$.

Compound	V_c (Å^3)	T_c (K)	$\Delta C/T_c$ ($\mathrm{mJ\,mol^{-1}\,K^{-2}}$)	$\lvert dH_{c2}/dT\rvert_{T_c}$ (T/K)	$\lambda(0)$ (nm)	$-\rho'_{\mathrm{Rut}}$	$-\rho'_{\mathrm{exp}}$
Nb	35.937 (Ref. 21)	9.3 (Ref. 12)	14.8 (Ref. 12)	0.044 (Ref. 14)	30[a]	2.0	1.8
MgB_2	29.064 (Ref. 22)	39 (Ref. 16)	3.4 (Ref. 16)	0.45 (Ref. 17)	84[b]	0.91	0.83
LiFeAs	90.252 (Ref. 23)	15.4 (Ref. 24)	20 (Ref. 24)	3.46 (Ref. 25)	200 (Ref. 26)	1.2	1.1
$FeTe_{0.58}Se_{0.42}$	87.084 (Ref. 27)	14 (Ref. 28)	20 (Ref. 28)	13 (Ref. 29)	500[c]	1.4	1.5
$YBa_2Cu_3O_{7-\delta}$	173.57 (Ref. 32)	23 (Ref. 33)	61 (Ref. 34)	1.9 (Ref. 35)	120 (Ref. 3)	3.0	2.15–4.98 (Ref. 6)
$MgCNi_3$	54.496 (Ref. 36)	7 (Ref. 36)	129 (Ref. 36)	2.6 (Ref. 32)	232 (Ref. 37)	1.8	2.0

[a]Determined in this paper.
[b]Determined in this paper.
[c]An average value over 430–560 nm (Refs. 29–31).

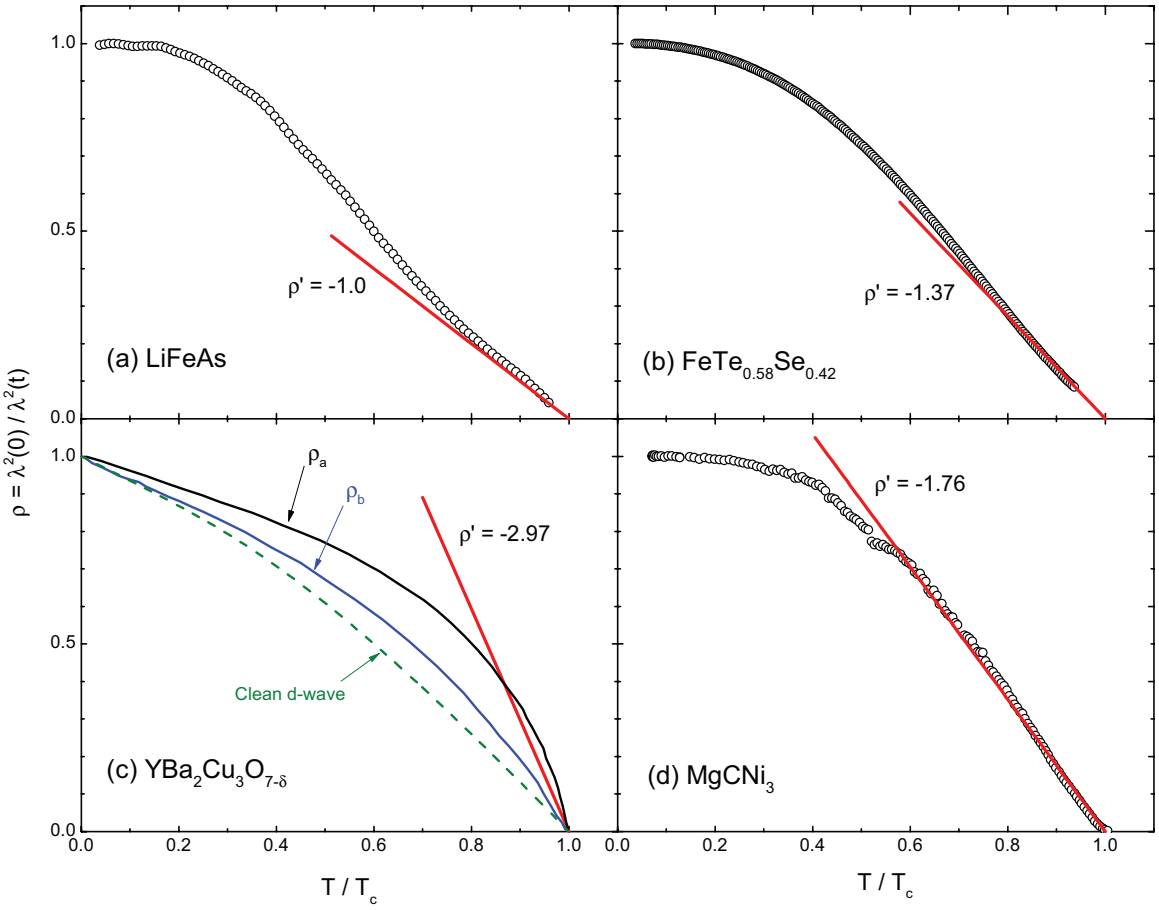

FIG. 3. (Color online) Experimental superfluid density $\rho = \lambda^2(0)/\lambda^2(T)$ in LiFeAs, $FeTe_{0.58}Se_{0.42}$, $YBa_2Cu_3O_{7-\delta}$, and $MgCNi_3$ with $\lambda(0) = 500$, 200, 120, and 232 nm, respectively. The straight lines in each panel were estimated with the Rutgers formula. Parameters used for the calculation are summarized in Table I.

microwave cavity perturbation technique.[6] Thermodynamic parameters are discussed in a number of papers.[27,39,40] In-depth discussions of the specific heat are given in Refs. 33 and 40. Table I summarizes parameters used in the calculations. Figure 3 shows experimental superfluid density in LiFeAs, $FeTe_{0.58}Se_{0.42}$, $YBa_2Cu_3O_{7-\delta}$, and $MgCNi_3$ with $\lambda(0) = 500$, 200, 120, and 232 nm, respectively. The agreement between $\rho'_{\rm Rut}$ calculated with the Rutgers relation and $\rho'_{\rm exp}$ extracted from the data on $\Delta\lambda(t)$, given possible uncertainties in the input experimental parameters, is rather remarkable.

III. SUMMARY

In conclusion, we have shown that the thermodynamic relation based on the Rutgers formula can be used for the analysis of the superfluid density. Based on this relation, a method to estimate $\lambda(0)$ is developed. As a test, it was successfully applied to reproduce known $\lambda(0)$ in Nb and MgB_2. We used this relation to verify reported literature values of $\lambda(0)$ for several unconventional superconductors of different band structures, gap anisotropies, and pairing symmetries.

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APPENDIX: THEORETICAL RESULTS RELEVANT FOR THE ANALYSIS OF THE SUPERFLUID DENSITY

1. Penetration depth in anisotropic materials

It is known[41] that, in isotropic materials,

$$\rho'(1) = \lambda^2(0)/\lambda^2_{\rm GL} = -2. \tag{A1}$$

It is easy to reproduce this result for the free-electron model of the normal state; it is shown below, however, that this value holds for any Fermi surface provided the order parameter is isotropic.

Here, we are interested in relating $\lambda(0)$ and $\lambda_{\rm GL}$, the T independent part of λ near T_c, for anisotropic Fermi surfaces and order parameters. We start with a known relation,

$$(\lambda^2)^{-1}_{ik} = \frac{16\pi^2 e^2 N(0) T}{c^2} \sum_{\omega} \left\langle \frac{\Delta^2 v_i v_k}{\beta^3} \right\rangle, \tag{A2}$$

which holds at any temperature for *clean* materials with arbitrary Fermi-surface and order parameter anisotropies.[2,42] Here, $N(0)$ is the density of states at the Fermi level per spin, $\beta^2 = \Delta^2 + \hbar^2\omega^2$ with $\hbar\omega = \pi T(2n+1)$, $\Delta(\boldsymbol{k}_F, T) = \Psi(T)\Omega(\boldsymbol{k}_F)$ is the zero-field order parameter, which, in

general, depends on the position $\boldsymbol{k}_F$ on the Fermi surface, and $\langle\cdots\rangle$ stands for averaging over the Fermi surface. The function $\Omega(\boldsymbol{k}_F)$, which describes the variation in Δ along the Fermi surface, is normalized: $\langle\Omega^2\rangle = 1$.

Equation (A2) is obtained within the model of factorizable effective coupling $V(\boldsymbol{k},\boldsymbol{k}') = V_0\,\Omega(\boldsymbol{k})\,\Omega(\boldsymbol{k}')$.[43] The self-consistency equation of the weak-coupling theory takes the form

$$\Psi(\boldsymbol{r},T) = 2\pi T N(0) V_0 \sum_{\omega>0}^{\omega_D} \langle \Omega(\boldsymbol{k}) f(\boldsymbol{k},\boldsymbol{r},\omega)\rangle, \qquad \text{(A3)}$$

where f is the Eilenberger Green's function, which, for the uniform current-free state, reads: $f = \Delta/\beta = \Psi\Omega/\beta$. The order parameter near T_c is now readily obtained

$$\Psi^2 = \frac{8\pi^2 T_c^2 (1-t)}{7\zeta(3)\langle\Omega^4\rangle}, \qquad \text{(A4)}$$

which reduces to the isotropic BCS form for $\Omega = 1$. We substitute this in Eq. (A2) to obtain, near T_c,

$$(\lambda^2)^{-1}_{ik} = \frac{16\pi e^2 N(0)\langle\Omega^2 v_i v_k\rangle}{c^2\langle\Omega^4\rangle}(1-t), \qquad \text{(A5)}$$

from which the constants $\lambda_{\rm GL}$ for any direction readily follow.

As $T \to 0$, the sum over the Matsubara frequencies in Eq. (A2) can be replaced with an integral according to $2\pi T\sum_\omega \to \int_0^\infty d(\hbar\omega)$,

$$(\lambda^2)^{-1}_{ik}(0) = \frac{8\pi e^2 N(0)}{c^2}\langle v_i v_k\rangle. \qquad \text{(A6)}$$

For free electrons, this reduces to the London value $\lambda^2 = mc^2/4\pi e^2 n$ where $n = 2mN(0)v^2/3$ is the electron density.

Hence, we get, for the slope of the in-plane superfluid density,

$$\rho'_{ab}(1) = -\frac{\lambda^2_{ab}(0)}{\lambda^2_{{\rm GL},ab}} = -2\frac{\langle\Omega^2 v_a^2\rangle}{\langle v_a^2\rangle\langle\Omega^4\rangle}. \qquad \text{(A7)}$$

Similarly, one can define $\rho'_c(1)$ for which v_a should be replaced with v_c in Eq. (A7). In particular, we have

$$\frac{\rho'_c(1)}{\rho'_{ab}(1)} = \frac{\langle v_a^2\rangle}{\langle v_c^2\rangle}\frac{\langle\Omega^2 v_c^2\rangle}{\langle\Omega^2 v_a^2\rangle} = \frac{\gamma_\lambda^2(0)}{\gamma_\lambda^2(T_c)}. \qquad \text{(A8)}$$

E.g., for MgB_2 with $\gamma_\lambda(0) \approx 1$, $\gamma_\lambda(T_c) \approx 2.6$, we estimate $\rho'_c(1) \approx 0.15\,\rho'_{ab}(1)$.

It is instructive to note that $\rho'(1)$ reduces to the isotropic value of -2 for *any* Fermi surface provided the order parameter is constant, $\Omega = 1$.

2. MgB_2

Consider a simple two-band model with the gap anisotropy given by

$$\Omega(\mathbf{k}) = \Omega_{1,2}, \quad \mathbf{k} \in F_{1,2}, \qquad \text{(A9)}$$

where F_1, F_2 are two sheets of the Fermi surface. $\Omega_{1,2}$'s are assumed to be constants, in other words, we model MgB_2 as having two different s-wave gaps. The normalization $\langle\Omega^2\rangle = 1$ then gives

$$\Omega_1^2\nu_1 + \Omega_2^2\nu_2 = 1, \quad \nu_1 + \nu_2 = 1, \qquad \text{(A10)}$$

where $\nu_{1,2} = N_{1,2}/N(0)$ are the relative densities of states.

Based on the band-structure calculations,[44,45] ν_1 and ν_2 of our model are $\approx$0.56 and 0.44. The ratio is $\Delta_2/\Delta_1 = \Omega_2/\Omega_1 \approx 3$. Then, the normalization (A10) yields $\Omega_1 = 0.47$ and $\Omega_2 = 1.41$.

Furthermore, we use the averages over separate Fermi sheets calculated in Ref. 44:

$\langle v_a^2\rangle_1 = 33.2$, $\langle v_a^2\rangle_2 = 23$ cm^2/s^2. With this input, we estimate

$$\rho'_{ab}(1) = -0.92. \qquad \text{(A11)}$$

It should be noted that this number is sensitive to a number of input parameters. The procedure described above, see Fig. 2, gives $\rho'_{ab}(1) \approx -0.91$.

Since only even powers of Ω enter Eq. (A7), the same analysis of the slope $\rho'(1)$ can, in fact, be exercised for materials modeled by two bands with the $\pm s$ symmetry of the order parameter for which Ω's have opposite signs. If the bands relative densities of state $\nu_{1,2}$ and the averages $\langle v_a^2\rangle_{1,2}$ are comparable to each other and are similar to those of MgB_2, we expect a similar $|\rho'(1)| \approx 1$ for clean crystals.

3. *d*-wave

It can be shown that $\Omega = \sqrt{2}\cos 2\phi$ for closed Fermi surfaces as rotational ellipsoids (in particular, spheres) or open ones as rotational hyperboloids (in particular, cylinders).[46] A straightforward algebra gives

$$\rho'_{ab}(1) = -4/3. \qquad \text{(A12)}$$

4. Scattering

In the limit of a strong nonmagnetic scattering for an arbitrary Fermi surface but a constant s-wave order parameter we have, see, e.g., Ref. 2,

$$(\lambda^2)^{-1}_{ik} = \frac{8\pi^2 e^2 N(0)\langle v_i v_k\rangle\tau}{c^2\hbar}\Delta\,\tanh\frac{\Delta}{2T}. \qquad \text{(A13)}$$

Here, τ is the average scattering time. It is worth noting that the dirty limit does not make much sense for anisotropic gaps because T_c is suppressed even by nonmagnetic scattering in the limit $\tau \to 0$. At $T = 0$, we have

$$(\lambda^2)^{-1}_{ik}(0) = \frac{8\pi^2 e^2 N(0)\langle v_i v_k\rangle\tau}{c^2\hbar}\Delta(0), \qquad \text{(A14)}$$

whereas, near T_c,

$$(\lambda^2)^{-1}_{ik} = \frac{8\pi^2 e^2 N(0)\langle v_i v_k\rangle\tau}{c^2\hbar}\frac{\Delta^2}{2T_c}, \qquad \text{(A15)}$$

Since for nonmagnetic scattering, T_c and $\Delta(T)$ are the same as in the clean case, in particular, $\Delta = 8\pi^2 T_c^2(1-t)/7\zeta(3)$, we obtain

$$\rho'(1) = -\frac{4\pi^2 T_c}{\Delta(0)} = -\frac{4\pi e^\gamma}{7\zeta(3)} = -2.66. \qquad \text{(A16)}$$

We, thus, conclude that scattering causes the slope $\rho'(1)$ to increase.

Evaluations of scattering effects on the slope ρ' near T_c for anisotropic gaps and Fermi surfaces are more involved because both T_c and Δ are affected even by nonmagnetic scattering. The case of a strong pair breaking is an exception: $\lambda^{-2} = \lambda_0^{-2}(1 - t^2)$ that immediately gives $\rho'(1) = -2$.

*Corresponding author: prozorov@ameslab.gov

[1] R. Prozorov and R. W. Giannetta, Supercond. Sci. Technol. **19**, R41 (2006).

[2] R. Prozorov and V. G. Kogan, Rep. Prog. Phys. **74**, 124505 (2011).

[3] R. Prozorov, R. W. Giannetta, A. Carrington, P. Fournier, R. L. Greene, P. Guptasarma, D. G. Hinks, and A. R. Banks, Appl. Phys. Lett. **77**, 4202 (2000).

[4] J. E. Sonier, Rep. Prog. Phys. **70**, 1717 (2007).

[5] D. N. Basov and T. Timusk, Rev. Mod. Phys. **77**, 721 (2005).

[6] S. Kamal, R. Liang, A. Hosseini, D. A. Bonn, and W. N. Hardy, Phys. Rev. B **58**, R8933 (1998).

[7] T. M. Lippman, B. Kalisky, H. Kim, M. A. Tanatar, S. L. Budko, P. C. Canfield, R. Prozorov, and K. A. Moler, Physica C **483**, 91 (2012).

[8] L. Luan, O. M. Auslaender, T. M. Lippman, C. W. Hicks, B. Kalisky, J.-H. Chu, J. G. Analytis, I. R. Fisher, J. R. Kirtley, and K. A. Moler, Phys. Rev. B **81**, 100501 (2010).

[9] J. R. Kirtley, Rep. Prog. Phys. **73**, 126501 (2010).

[10] A. Rutgers, Physica (Utrecht) **1**, 1055 (1934).

[11] L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii, *Electrodynamics of Continuous Media*, 2nd ed. (Butterworth-Heinemann, Oxford, 1984), Vol. 8.

[12] H. W. Weber, E. Seidl, C. Laa, E. Schachinger, M. Prohammer, A. Junod, and D. Eckert, Phys. Rev. B **44**, 7585 (1991).

[13] To convert ΔC, which is commonly reported in $\mathrm{mJ\,mol^{-1}\,K^{-1}}$ into $\mathrm{erg\,cm^{-3}\,K^{-1}}$, one needs to calculate the mass density, which requires crystallographic information. For niobium, we use parameters found in Ref. 21. The crystal structure of elemental niobium belongs to the space group Im-$3m$ (No. 229) with lattice parameters $a = b = c = 0.3303$ nm, and the corresponding volume is $V = 0.036\ \mathrm{nm}^3$. There are two molecular units per volume ($Z = 2$). Using these values, we obtain the converted $\Delta C = 137.2\ \mathrm{mJ\,mol^{-1}\,K^{-1}}$ (Ref. 12) $= 126\,450\ \mathrm{erg\,cm^{-3}\,K^{-1}}$.

[14] S. J. Williamson, Phys. Rev. B **2**, 3545 (1970).

[15] B. W. Maxfield and W. L. McLean, Phys. Rev. **139**, A1515 (1965).

[16] F. Bouquet, R. A. Fisher, N. E. Phillips, D. G. Hinks, and J. D. Jorgensen, Phys. Rev. Lett. **87**, 047001 (2001).

[17] S. L. Bud'ko, C. Petrovic, G. Lapertot, C. E. Cunningham, P. C. Canfield, M.-H. Jung, and A. H. Lacerda, Phys. Rev. B **63**, 220503 (2001).

[18] C. Niedermayer, C. Bernhard, T. Holden, R. K. Kremer, and K. Ahn, Phys. Rev. B **65**, 094512 (2002).

[19] K. Ohishi, T. Muranaka, J. Akimitsu, A. Koda, W. Higemoto, and R. Kadono, J. Phys. Soc. Jpn. **72**, 29 (2003).

[20] H. Kim, N. H. Sung, B. K. Cho, M. A. Tanatar, and R. Prozorov, Phys. Rev. B **87**, 094515 (2013).

[21] M. Ito, H. Muta, M. Uno, and S. Yamanaka, J. Alloys Compd. **425**, 164 (2006).

[22] J. Nagamatsu, N. Nakagawa, T. Muranaka, Y. Zenitani, and J. Akimitsu, Nature (London) **410**, 63 (2001).

[23] J. H. Tapp, Z. Tang, B. Lv, K. Sasmal, B. Lorenz, P. C. W. Chu, and A. M. Guloy, Phys. Rev. B **78**, 060505 (2008).

[24] F. Wei, F. Chen, K. Sasmal, B. Lv, Z. J. Tang, Y. Y. Xue, A. M. Guloy, and C. W. Chu, Phys. Rev. B **81**, 134527 (2010).

[25] K. Cho, H. Kim, M. A. Tanatar, Y. J. Song, Y. S. Kwon, W. A. Coniglio, C. C. Agosta, A. Gurevich, and R. Prozorov, Phys. Rev. B **83**, 060502 (2011).

[26] F. L. Pratt, P. J. Baker, S. J. Blundell, T. Lancaster, H. J. Lewtas, P. Adamson, M. J. Pitcher, D. R. Parker, and S. J. Clarke, Phys. Rev. B **79**, 052508 (2009).

[27] D. C. Johnston, Adv. Phys. **59**, 803 (2010).

[28] D. Braithwaite, G. Lapertot, W. Knafo, and I. Sheikin, J. Phys. Soc. Jpn. **79**, 053703 (2010).

[29] T. Klein, D. Braithwaite, A. Demuer, W. Knafo, G. Lapertot, C. Marcenat, P. Rodière, I. Sheikin, P. Strobel, A. Sulpice, and P. Toulemonde, Phys. Rev. B **82**, 184506 (2010).

[30] P. K. Biswas, G. Balakrishnan, D. M. Paul, C. V. Tomy, M. R. Lees, and A. D. Hillier, Phys. Rev. B **81**, 092510 (2010).

[31] H. Kim, C. Martin, R. T. Gordon, M. A. Tanatar, J. Hu, B. Qian, Z. Q. Mao, R. Hu, C. Petrovic, N. Salovich, R. Giannetta, and R. Prozorov, Phys. Rev. B **81**, 180503 (2010).

[32] C. P. Poole, Jr., H. A. Farach, R. J. Creswick, and R. Prozorov, in *Superconductivity*, 2nd ed., Superconductivity Series (Academic, Amsterdam, 2007), p. 201.

[33] J. S. Kim, B. D. Faeth, and G. R. Stewart, Phys. Rev. B **86**, 054509 (2012).

[34] Y. Wang, B. Revaz, A. Erb, and A. Junod, Phys. Rev. B **63**, 094508 (2001).

[35] U. Welp, W. K. Kwok, G. W. Crabtree, K. G. Vandervoort, and J. Z. Liu, Phys. Rev. Lett. **62**, 1908 (1989).

[36] L. Shan, K. Xia, Z. Y. Liu, H. H. Wen, Z. A. Ren, G. C. Che, and Z. X. Zhao, Phys. Rev. B **68**, 024523 (2003).

[37] G. MacDougall, R. Cava, S.-J. Kim, P. Russo, A. Savici, C. Wiebe, A. Winkels, Y. Uemura, and G. Luke, Physica B **374-375**, 263 (2006).

[38] A. Carrington, A. P. Mackenzie, and A. Tyler, Phys. Rev. B **54**, R3788 (1996).

[39] P. C. Canfield and S. L. Bud'ko, Annu. Rev. Condens. Matter Phys. **1**, 27 (2010).

[40] G. R. Stewart, Rev. Mod. Phys. **83**, 1589 (2011).

[41] A. Fetter and P. Hohenberg, *Superconductivity*, edited by R. Parks, Superconductivity Vol. 2 (Dekker, New York, 1969), p. 823.

[42] V. G. Kogan, Phys. Rev. B **66**, 020509 (2002).

[43] D. Markowitz and L. P. Kadanoff, Phys. Rev. **131**, 563 (1963).

[44] K. D. Belashchenko, M. van Schilfgaarde, and V. P. Antropov, Phys. Rev. B **64**, 092503 (2001).

[45] H. J. Choi, D. Roundy, H. Sun, M. L. Cohen, and S. G. Louie, Phys. Rev. B **66**, 020513 (2002).

[46] V. G. Kogan and R. Prozorov, Rep. Prog. Phys. **75**, 114502 (2012).