Multipair dc Josephson resonances in a biased all-superconducting bijunction

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An all-superconducting bijunction consists of a central superconductor contacted to two lateral superconductors, such that nonlocal crossed Andreev reflection is operating. Then new correlated transport channels for the Cooper pairs appear in addition to those of separated conventional Josephson junctions. We study this system in a configuration where the superconductors are connected through gate-controllable quantum dots. Multipair phase-coherent resonances and phase-dependent multiple Andreev reflections are both obtained when the voltages of the lateral superconductors are commensurate, and they add to the usual local dissipative transport due to quasiparticles. The two-pair resonance (quartets) as well as some other higher order multipair resonances are π shifted at low voltage. Dot control can be used to dramatically enhance the multipair current when the voltages are resonant with the dot levels.

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I. INTRODUCTION

One of the most striking manifestations of macroscopic quantum coherence is the Josephson effect:[1] a dc current flows when a phase difference is imposed on a junction bridging two superconductors with a narrow insulating, metallic, or semiconducting region. When applying a constant voltage bias to this same junction, an oscillatory current arises[2] and the application of an rf irradiation leads to the observation of Shapiro steps with zero differential resistance[2,3] and phase coherence.[4] More generally, the microscopic origin of these effects is Andreev reflections of electrons and holes at the boundaries of the two superconductors. The same mechanism participates in the appearance of a subgap structure in highly transparent voltage-biased junctions, a feature understood to be due to dissipative quasiparticle emissions called multiple Andreev reflections (MAR),[5,6] which were observed in atomic point contact experiments.[7]

Nonlocal quantum mechanical phenomena[8] and entanglement are nowadays investigated in condensed matter physics, in particular, in superconducting circuits.[9] Multiterminal superconducting hybrid devices with one superconducting arm and two normal-metal electrodes have also been studied in the last decade[10] with the aim of detecting nonlocal entangled electron pairs.[11] There is now convincing experimental data on nonlocal current and noise detection which points in this direction.[12–14] Yet, there is also a growing interest in three-terminal all-superconducting hybrid structures,[15–18] so far mainly in regimes dominated by phase-insensitive processes. A recent calculation for a superconductor/normal conductor/superconductor junction, where the N region is tunnel coupled to another superconductor, also showed resonances ascribed to voltage-induced Shapiro steps.[19]

The present work shows that a nondissipative phase-coherent Josephson signal of Cooper pair transport could be observed in a device consisting of three superconductors driven out of equilibrium. This effect relies on a combination of both direct Andreev reflections and nonlocal crossed Andreev reflections (CAR),[10] and is thus directly tied to nonlocal entangled electron processes as well as Josephson physics. Here, the "bijunction" which we propose consists of a central superconductor S_0 coupled via two adjustable quantum dots to two lateral superconductors S_a and S_b, biased at voltages V_a and V_b (Fig. 1). As the coherence length of S_0 (which is grounded at $V_0 \equiv 0$) is assumed to be larger than the distance between the dots, this bijunction cannot be simply considered as two separated junctions in parallel. Each junction consists of a quantum dot, made with, e.g., carbon nanotubes[13,20] or nanowires,[14] and labeled D_α ($\alpha = a,b$). The dots introduce additional degrees of freedom (position of energy levels, coupling widths) which provide full control of the junctions. Equilibrium calculations[21] in a similar three-terminal device involving normal-metal interfaces showed that a bijunction could be a source of spatially correlated pairs of Cooper pairs (referred to as "nonlocal quartets") transmitted into S_a and S_b simultaneously.

This paper reports on calculations of out-of-equilibrium transport in a biased $S_aD_aS_0D_bS_b$ bijunction, with the following main results:

(i) At commensurate voltages $nV_a + mV_b = 0$ (m and n integers), dc Josephson resonances appear, which correspond to the phase-coherent transport of n pairs to S_a, and m pairs to S_b, from S_0.

(ii) The Josephson current-phase relation of quartet resonances ($n = m = 1$) and that of some higher-order resonances are π shifted at low bias. This new mechanism for producing a π shift is of particular importance for future interferometry experiments.

(iii) Gate and/or bias voltages can be tuned to enhance the multipair resonances by orders of magnitude as compared to the adiabatic regime, making them easily observable in experiments.

(iv) At larger biases, a dc quasiparticle-pair interference term, corresponding to phase-dependent MAR, emerges from the dissipative Josephson component.

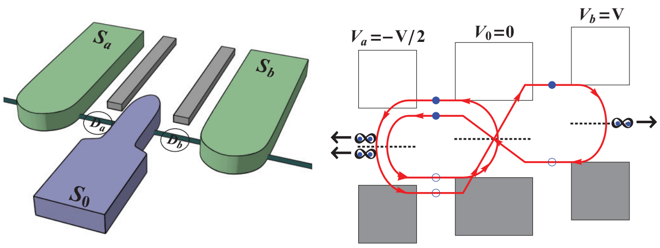

FIG. 1. (Color online) A Josephson bijunction (left). Superconductors S_α ($\alpha = a,b$) are biased at voltages V_α, while S_0 is grounded. The distance between the two quantum dot junctions is comparable to the coherence length. The right panel shows the energy diagram for the $S_a D_a S_0 D_b S_b$ bijunction and a higher-order diagram associated with a "sextet" current with three pairs emitted, two in S_a and one in S_b, with $V_a = -V_b/2$.

The structure of this paper is the following. In Sec. II, we explain qualitatively the multipair Josephson resonances from a simple adiabatic argument. The following sections are concerned with an exact out-of-equilibrium calculation, valid at arbitrary voltages. Section III details the Hamiltonian formalism which we have used to perform the calculations. Section IV shows and discusses results obtained in the regime where the quantum dots have a behavior similar to metallic junctions. The next section shows results for the opposite regime where the dots present a narrow resonance. Finally, Sec. VI presents the conclusions and perspectives of this work.

II. ADIABATIC ARGUMENT

A simple phase argument[21] suggests the existence of quartet resonances in a bijunction. Starting with an equilibrium situation, the current-phase relation of a single tunnel junction $S_a S_0$ with phases φ_a in S_a and φ_0 in S_0 is $I_c \sin(\varphi_a - \varphi_0)$, to which higher-order harmonics can also contribute. In a $S_a S_0 S_b$ bijunction (with phase φ_b in S_b), there exists in addition a quartet and a pair cotunneling supercurrent. The dc quartet supercurrent can be viewed as a nonlocal second-order harmonic

$$I_Q = I_{Q0} \sin(\varphi_a + \varphi_b - 2\varphi_0), \tag{1}$$

while the pair cotunneling corresponds to a dc Josephson effect between S_a and S_b through S_0:[21]

$$I_{\rm PC} = I_{\rm PC0} \sin(\varphi_a - \varphi_b). \tag{2}$$

More generally, assuming large enough transparencies, multipair currents $I_{a/b}$ in electrodes S_a/S_b are obtained when differentiating the Josephson free energy with respect to the superconducting phases (assuming $\varphi_0 \equiv 0$)

$$I_{a/b} = \sum_{n,m} I_{a/b,(n,m)} \sin(n\varphi_a + m\varphi_b). \tag{3}$$

When voltages $V_{a/b}$ are applied to $S_{a/b}$, φ_a and φ_b acquire a time dependence, and in the special case where

$$nV_a + mV_b = 0, \tag{4}$$

the adiabatic approximation yields

$$d\,[n\varphi_a(t) + m\varphi_b(t)]\,/dt = 0. \tag{5}$$

The corresponding current component

$$I_{a/b,(n,m)} \sin[n\varphi_a(t) + m\varphi_b(t)] \tag{6}$$

and its higher harmonics are constant in time despite the applied voltages, thus leading to a dc current signaling the existence of a multipair resonance. An example of such a resonance is provided in Fig. 1 from a diagrammatic point of view, showing the case $2V_a + V_b = 0$ to lowest order. The voltage constraint allows one to close a resonance path provided by one Andreev reflection in S_b and S_0, two in S_a, as well as two CAR amplitudes in S_0. Note that in general these multipair resonances must coexist with the usual ac components

$$I_{a,(1,0)} = I_{0a} \sin\,\varphi_a(t), \quad I_{b,(0,1)} = I_{0b} \sin\,\varphi_b(t), \tag{7}$$

and with the MAR dc currents discussed in Ref. 16. While this low-bias argument suggests the possibility of multipair resonances also at higher voltages, an exact out-of-equilibrium calculation at arbitrary voltage is still lacking, and it is discussed below.

III. HAMILTONIAN FORMALISM

The model Hamiltonian of the $S_a D_a S_0 D_b S_b$ bijunction is written as

$$\hat{\mathcal{H}} = \sum_j \hat{\mathcal{H}}_j + \hat{\mathcal{H}}_D + \hat{\mathcal{H}}_T, \tag{8}$$

where $\hat{\mathcal{H}}_j$ is the Hamiltonian for the lead S_j ($j = 0,a,b$), expressed with the Nambu spinors

$$\hat{\mathcal{H}}_j = \sum_k \Psi^\dagger_{jk}\,(\xi_k\,\sigma_z + \Delta\,\sigma_x)\,\Psi_{jk}, \qquad \Psi_{jk} = \begin{pmatrix} \psi_{jk,\uparrow} \\ \psi^\dagger_{j(-k),\downarrow} \end{pmatrix}, \tag{9}$$

with the Pauli matrices acting in the Nambu space. $\hat{\mathcal{H}}_D$ is the Hamiltonian of the two dots, with a single noninteracting level in each dot:

$$\hat{\mathcal{H}}_D = \sum_{s,\alpha=a,b} \varepsilon_\alpha d^\dagger_{\alpha s} d_{\alpha s}. \tag{10}$$

$\hat{\mathcal{H}}_T$ is for the tunneling between the dots and the electrodes:

$$\hat{\mathcal{H}}_T(t) = \sum_{jk\alpha} \Psi^\dagger_{jk} t_{j\alpha} e^{i\sigma_z \varphi_j/2} \mathbf{d}_\alpha + \text{H.c.}, \tag{11}$$

where $\mathbf{d}_\alpha = (d_{\alpha\uparrow} d^\dagger_{\alpha\downarrow})$ is the Nambu spinor for dot α, and $t_{j\alpha}$ is the tunneling amplitude between lead j and dot α.

The phases are specified by the applied voltages $\varphi_j(t) = \varphi_j^{(0)} + 2eV_j t/\hbar$. The "bare" phases $\varphi_j^{(0)}$, which are usually unimportant in an out-of-equilibrium setup, are relevant here in the transport calculations. The superconducting gaps Δ are assumed identical and the couplings are taken symmetric. The width of the superconducting region S_0 is assumed to be negligible, a situation which corresponds to the maximum coupling between the two junctions forming the bijunction.

As the leads degrees of freedom are quadratic, they can be integrated out by averaging the evolution operator over these leads. We use for this a Keldysh path-integral technique. The Green's function $\hat{G}$ of the dots which is nonperturbative in $\hat{\mathcal{H}}_T$, is obtained from a Dyson equation[6] involving the free dots Green function, and electrode self-energies with both local and nonlocal propagators.[23] The details for a multiterminal structure with two quantum dots have been given in Ref. 24. Due to the presence of the two dots, the Green function of the dots is a 2×2 matrix in the dots space:

$$\check{G}^{\eta\eta'}_{\alpha\beta}(t,t') = -i\langle T_C\{\mathbf{d}^{\eta}_{\alpha}(t)\mathbf{d}^{\dagger\eta'}_{\beta}(t')\}\rangle, \tag{12}$$

where η,η' are Keldysh indices. The self-energy is also a 2×2 matrix in the dots space:

$$\check{\Sigma} = \begin{pmatrix} \hat{\Sigma}_{aa} & \hat{\Sigma}_{ab} \\ \hat{\Sigma}_{ba} & \hat{\Sigma}_{bb} \end{pmatrix}, \tag{13}$$

and the component $\Sigma_{\alpha\beta}$ (with $\alpha,\beta = a,b$) is given by a sum over the leads j:

$$\hat{\Sigma}_{\alpha\beta}(t_1,t_2) = \sum_j \Gamma_{j,\alpha\beta} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t_1-t_2)} e^{-i\sigma_z(V_j t_1 + \varphi_j^{(0)}/2)}[\omega 1 - \Delta_j \sigma_x] e^{+i\sigma_z(V_j t_2+\varphi_j^{(0)}/2)}$$
$$\otimes \left[-\frac{\Theta(\Delta_j - |\omega|)}{\sqrt{\Delta_j^2 - \omega^2}}\tau_z + i\,\mathrm{sign}(\omega)\frac{\Theta(|\omega| - \Delta_j)}{\sqrt{\omega^2 - \Delta_j^2}} \begin{pmatrix} 2f_\omega - 1 & -2f_\omega \\ +2f_{-\omega} & 2f_\omega - 1 \end{pmatrix} \right], \tag{14}$$

where $\Gamma_{j,\alpha\beta} = \pi\nu(0)t^*_{j\alpha}t_{j\beta}$ and f_ω is the Fermi function.

The average current from electrode j can then be computed using a Meir-Wingreen-type formula[22] generalized to superconductors:[23,24]

$$\langle I_{j\alpha}\rangle(t) = \frac{1}{2}\mathrm{Tr}\left\{(\tau_z \otimes \sigma_z)\int_{-\infty}^{+\infty} dt'(\check{G}(t,t')\check{\Sigma}_j(t',t) - \check{\Sigma}_j(t,t')\check{G}(t',t))_{\alpha\alpha}\right\}, \tag{15}$$

where τ_z acts in Keldysh space, and σ_z in Nambu space, and the trace is taken in the Nambu-Keldysh space. For arbitrary voltages V_a and V_b, the time dependence of the system is described in terms of two independent Josephson frequencies $\omega_a = 2eV_a/\hbar$ and $\omega_b = 2eV_b/\hbar$, the Green function $\hat{G}(t,t')$ is a function of two times, and solving the Dyson equation is a daunting task. However, when the voltages V_a and V_b applied to superconductors a and b are commensurate ($nV_a + mV_b = 0$, with n and m integers), the time dependence of the system is periodic, with a period $T = |m|2\pi/\omega_a = |n|2\pi/\omega_b$, where $\omega_{a,b} = 2e|V_{a,b}|/\hbar$ are the Josephson frequencies. As in the study of standard multiple Andreev reflection (MAR) between two superconductors,[23] it is then convenient to introduce the double Fourier transforms with summation over discrete domains in frequency:

$$\check{G}(t,t') = \sum_{n,m=-\infty}^{+\infty} \int_F \frac{d\omega}{2\pi} e^{-i\omega_n t + i\omega_m t'}\check{G}_{nm}(\omega), \tag{16}$$

$$\check{\Sigma}(t,t') = \sum_{n,m=-\infty}^{+\infty} \int_F \frac{d\omega}{2\pi} e^{-i\omega_n t + i\omega_m t'}\check{\Sigma}_{nm}(\omega), \tag{17}$$

where $\omega_n = \omega + n\tilde{V}$, the frequency integration is performed over a finite domain $F \equiv [-\tilde{V}/2,\tilde{V}/2]$, and $\tilde{V}$ is the smallest common mutiple of $|V_a|$ and $|V_b|$. The advantage of this representation is that the Dyson equation for the full Green function $\check{G}_{nm}(\omega)$ is now a matrix equation:

$$\check{G}_{nm}(\omega) = \left[\check{G}^{-1}_{0,nm}(\omega) - \check{\Sigma}_{nm}(\omega)\right]^{-1}, \tag{18}$$

where $\check{G}_{0,nm}$ is the dots Green function without coupling to the superconducting leads. This equation can be solved by limiting the discrete Fourier transforms to a cutoff energy E_c, which gives finite matrices in Eq. (18). The cut-off energy E_c must be chosen large compared to all the relevant energies in the system. E_c defines a finite number of frequency domains $n_{\max}$. As the width of each domain is $\sim V$, one has $n_{\max} \sim \Delta/V$, which implies that obtaining numerically the full Green function becomes very expensive at very low voltage. Typical values which we have used in our calculations are in the range $E_c \sim 5$ to 10Δ.

From Eq. (14), we find that the self-energy in the double Fourier representation is (writing explicitly the 2×2 matrix of Nambu space)

$$\hat{\Sigma}_{\alpha\beta,nm}(\omega_n) = \sum_j \Gamma_j \begin{pmatrix} \delta_{n,m}\hat{X}_j(\omega_n - \sigma_j V/2) & \delta_{n-\sigma_j,m}\hat{Y}_j(\omega_n - \sigma_j V/2)e^{-i\varphi_j^{(0)}} \\ \delta_{n+\sigma_j,m}\hat{Y}_j(\omega_n + \sigma_j V/2)e^{+i\varphi_j^{(0)}} & \delta_{n,m}\hat{X}_j(\omega_n + \sigma_j V/2) \end{pmatrix}, \tag{19}$$

where $\hat{X}$ and $\hat{Y}$ are matrices in the Keldysh space:

$$\hat{X}_j(\omega) = \left[-\frac{\Theta(\Delta_j - |\omega|)\omega}{\sqrt{\Delta_j^2 - \omega^2}}\hat{\tau}_z + i\frac{\Theta(|\omega| - \Delta_j)|\omega|}{\sqrt{\omega^2 - \Delta_j^2}} \begin{pmatrix} 2f_\omega - 1 & -2f_\omega \\ +2f_{-\omega} & 2f_\omega - 1 \end{pmatrix} \right], \tag{20}$$

and $\hat{Y}_j(\omega) = -\Delta_j \hat{X}_j(\omega)/\omega$. The expression of the Fourier transform of the current from dot α to lead j is

$$\langle I_{j\alpha}\rangle(\omega') = \sum_{n,l} 2\pi\delta(\omega' - (n-l)V) \times \frac{1}{2}\int_F \frac{d\omega}{2\pi}\mathrm{Tr}\left(\sigma_z\hat{\tau}_z \sum_m [\check{G}_{nm}(\omega)\check{\Sigma}_{j,ml}(\omega) - \check{\Sigma}_{j,nm}(\omega)\check{G}_{ml}(\omega)]_{\alpha\alpha}\right). \quad (21)$$

The dc current, which we study in the following sections, is obtained by taking $\omega' = 0$ in the last equation.

IV. METALLIC JUNCTION REGIME

We first consider the regime in which each dot mimics a metallic junction, achieved by placing energy levels out of resonance $\epsilon_\alpha > \Delta$ and choosing large couplings $\Gamma_\alpha > \Delta$ ($\alpha = a,b$, and $\Gamma_\alpha = \sum_j \pi\nu(0)|t_{j\alpha}|^2$, where $t_{j\alpha}$ are tunneling couplings defined in Eq. (11), and $\nu(0)$ is the normal density of states of the electrodes at the Fermi energy). We compute the dc currents $\langle I_{a/b}\rangle$ for different ratios of the voltages, satisfying $nV_a + mV_b = 0$. The results for the largest resonances ($|n| + |m| \leqslant 3$ in $nV_a + mV_b = 0$) are shown in the left panel of Fig. 2. One clearly sees that the resonances are easily distinguished from the phase-independent background current. The resonant multipair dc current $\langle I_a^{\mathrm{MP}}\rangle$ is a function of the combination $n\varphi_a^{(0)} + m\varphi_b^{(0)}$, which implies a simultaneous crossing of n pairs from S_0 to S_a and m pairs from S_0 to S_b. The upper right panel in Fig. 2 shows as an example the phase dependence for $n = 2$ and $m = 1$, which is indeed a sinusoidal function of the combination $2\varphi_a^{(0)} + \varphi_b^{(0)}$. The existence of dc phase-coherent resonances despite large nonzero voltages is the result of new coherent modes connecting the three superconductors.

One of the lowest-order (and larger) resonances corresponds to quartets ($V_a = -V_b$), i.e., to the correlated transmission of two pairs from S_0 to S_a and S_b, respectively. The "dual" lowest-order resonance corresponds to $V_b = V_a$, where pairs cross from S_a to S_b by cotunneling through S_0. The sign of the multipair resonances is nontrivial. In particular, the quartet resonance is negative, which means that the current $\langle I_a(\varphi_a^{(0)} + \varphi_b^{(0)})\rangle$ is of π type, as shown in the lower right panel of Fig. 2. Similar sign changes of the multipair current-phase relation are also obtained for certain high-order resonances. The π shift is understood from a simple argument. It is related to the internal structure of a Cooper pair via the antisymmetry of its wave function, similarly to the π-junction behavior of a magnetic junction formed by a quantum dot with a localized spin.[25] Starting from two Cooper pairs in S_0, the production of a nonlocal quartet consists in forming two nonlocally entangled singlets in the dots D_a and D_b. These two split pairs correspond to two CAR amplitudes, as those apparent in Fig. 1. A nonlocal singlet is obtained by the operator $\frac{1}{\sqrt{2}}(d_{a\uparrow}^\dagger d_{b\downarrow}^\dagger - d_{a\downarrow}^\dagger d_{b\uparrow}^\dagger)$ acting on the empty dots. Applying this operator twice to describe a nonlocal quartet state leads to $\Psi_{Q,D_a,D_b} = -|\uparrow\downarrow\rangle_a|\downarrow\uparrow\rangle_b$, which is recast as the opposite of the product of a pair in D_a and another one in D_b. A similar reasoning can be applied in order to explain the anomalous sign of higher-order harmonics.

V. RESONANT DOTS REGIME

We now investigate the possibility for optimizing the multipair resonances by tuning the dot levels, with $\varepsilon_a = -\varepsilon_b = -0.4\Delta$ inside the gap, choosing small values of the couplings $\Gamma_a = \Gamma_b = 0.1\Delta$. We focus on the quartet resonance

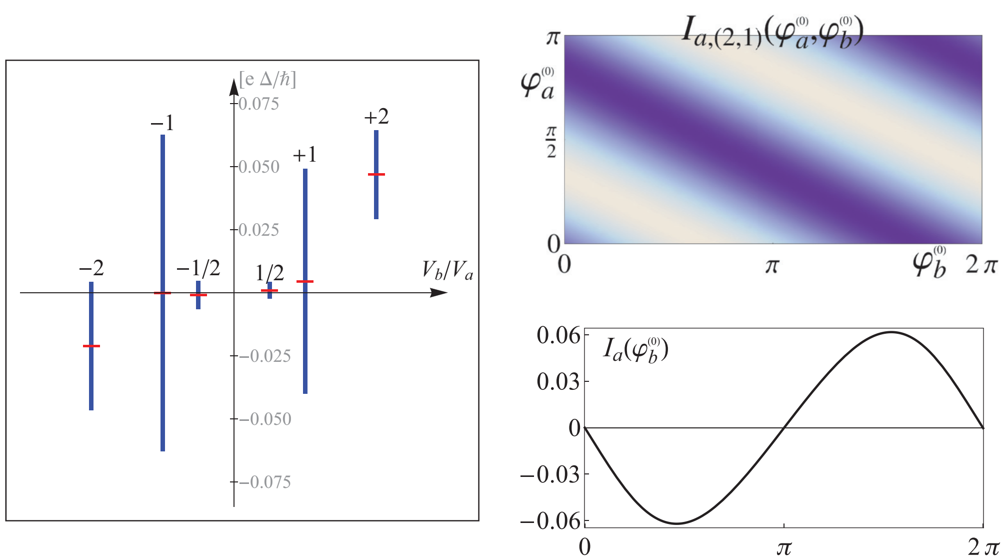

FIG. 2. (Color online) "Broad" dots regime (metallic junctions): $|\varepsilon_{a,b}| = 6\Delta$, $\Gamma_{a,b} = 4\Delta$. Left: amplitudes of the phase-dependent dc current $\langle I_a(\varphi_b^{(0)})\rangle$ for the main resonances (with $|n| + |m| \leqslant 3$), in units of $e\Delta/\hbar$, centered around the values of the phase-independent current (small horizontal bars). Horizontal axis is V_a/V_b, with $V_b/\Delta = 0.3$. Upper right: current $\langle I_a\rangle$, for the resonance $2V_a + V_b = 0$, as a function of the phases $\varphi_a^{(0)}$ and $\varphi_b^{(0)}$, showing the dependence in $2\varphi_a^{(0)} + \varphi_b^{(0)}$. Lower right: the current-phase relation $\langle I_a(\varphi_b^{(0)})\rangle$ at $\varphi_a^{(0)} = 0$ for the resonance $V_a + V_b = 0$, which shows the π-phase behavior.

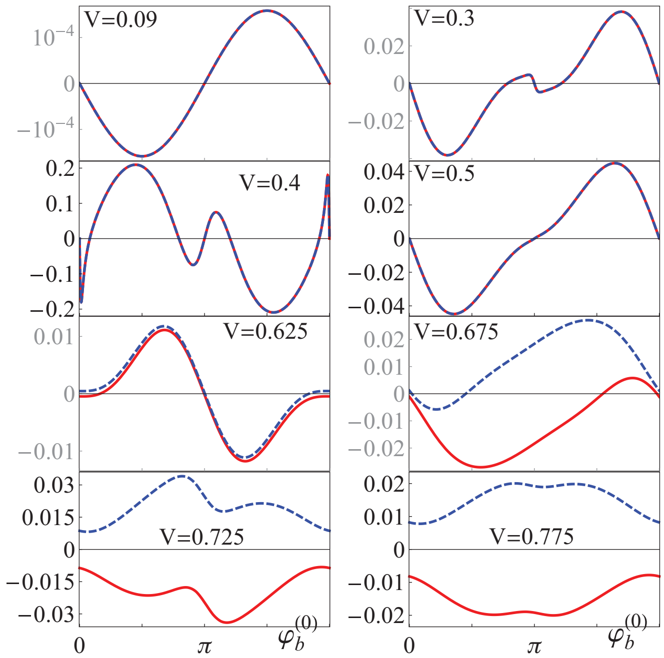

FIG. 3. (Color online) Current-phase relations $I_a(\varphi_b^{(0)})$ (red, full curve) and $I_b(\varphi_b^{(0)})$ (blue, dashed curve), in units of $e\Delta/\hbar$, in the quartet configuration $V_a = -V_b$, for the resonant dots regime: $\varepsilon_a = -\varepsilon_b = 0.4\Delta$ and $\Gamma = 0.1\Delta$. Note the different y scales in the different panels.

$V_a = -V_b$ for specificity (similar behavior is observed for the other resonances).

When the bias is small enough ($V_b \lesssim 0.1\Delta$ here), the system is in the adiabatic regime, and the current does not change when V_b is varied. We independently checked with a Matsubara formalism calculation (not shown) that this current is the same as the one obtained here at equilibrium ($V_b = 0$). The current-phase relations $\langle I_a(\varphi_b^{(0)})\rangle$ and $\langle I_b(\varphi_b^{(0)})\rangle$ for $V = 0.09\Delta$ are shown in the first panel of Fig. 3. These average currents are identical, and thus are made only from a quartet component. They show a purely harmonic function of the phase $\varphi_b^{(0)}$, and suggest a π-junction behavior for the quartet resonance near equilibrium.

When V_b increases and the nonadiabatic regime is reached, drastic changes appear in the current-phase relations, as shown in the next panels of Fig. 3. There, both the sign and the (nonsinusoidal) shape of the current-phase relation changes rapidly with V_b as it approaches the dot energy $|\varepsilon_b|$. The amplitude of the quartet current near the resonance is ~ 1000 times larger than the one in the adiabatic regime. This resonant effect of the dot levels is most apparent by plotting the critical current I_c^Q [the maximum of the absolute value of the phase-dependent part of $I_a(\varphi_b^{(0)})$] as a function of V_b. This is shown in the left panel of Fig. 4. I_c^Q sharply increases and reaches a maximum around $V_b \simeq \varepsilon_b$. The large increase in the quartet current is due to a double resonant effect: First, as the dots have opposite energies $\varepsilon_a = -\varepsilon_b$, the formation of a quartet in the double dot as a pair in D_a and a pair in D_b is resonant (this is true for any voltage V_b); second, when $V_b \simeq \varepsilon_b$ the tunneling of a pair from D_a to S_a, and from D_b to S_b, is also resonant.

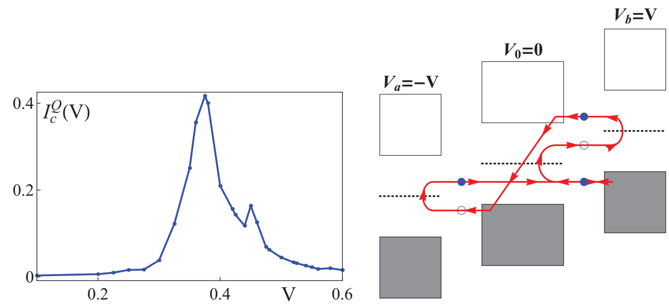

FIG. 4. (Color online) Left panel: critical current I_c^Q in units of $e\Delta/\hbar$ as a function of the voltage in the quartet configuration, for resonant dots (same parameters as in Fig. 3). Right panel: lowest-order diagram contributing to the phase-dependent MAR current.

Increasing V_b further, e.g., $V_b \gtrsim 2\Delta/3$ in the present case, we see from the lower panels of Fig. 3 that the currents $\langle I_a(\varphi_b^{(0)})\rangle$ and $\langle I_b(\varphi_b^{(0)})\rangle$ start to deviate substantially. This implies the existence of another phase-sensitive process different from the one responsible for multipair resonances. We call this current contribution I^{phMAR} (for phase-sensitive MAR), as it is the result of the combination of a multipair process with MAR. The lowest-order diagram contributing to I^{phMAR} is shown in the right panel of Fig. 4. It can be seen as the interference of the amplitudes of two MAR processes at the S_aS_0 and S_bS_0 interfaces, each promoting a quasiparticle from an energy $\sim -\Delta$ in superconductor S_b to an energy $\sim +\Delta$ in superconductor S_0. This diagram has a threshold at $V = 2\Delta/3$, corresponding to the observed value at which I^{phMAR} becomes noticeable. However, unlike the usual MAR processes found in single junctions, this process (and similar ones of higher order) has the striking property of being phase dependent.

Addressing more general resonances, the total dc current can be decomposed into three components, as inspired by Josephson's work.[2] Defining $\bar{I} = (\langle I_a\rangle, \langle I_b\rangle)$, $\bar{\varphi} = (\varphi_a, \varphi_b)$, $\bar{V} = (V_a, V_b)$, and $\bar{\epsilon} = (\epsilon_a, \epsilon_b)$ one has $\bar{I}(\bar{\varphi}, \bar{V}, \bar{\epsilon}) = \bar{I}^{\text{MP}}(\bar{\varphi}, \bar{V}, \bar{\epsilon}) + \bar{I}^{\text{phMAR}}(\bar{\varphi}, \bar{V}, \bar{\epsilon}) + \bar{I}^{qp}(\bar{V}, \bar{\epsilon})$. Assuming electron-hole symmetry to hold, for instance, with flat normal-metal density of states in the leads, one can show that in the situation studied here, the dc current obeys the relation

$$\bar{I}(\bar{\varphi}^{(0)}, \bar{V}, \bar{\epsilon}) = -\bar{I}(-\bar{\varphi}^{(0)}, -\bar{V}, -\bar{\epsilon}). \tag{22}$$

Then the following properties hold: (i) The pure quasiparticle current $\bar{I}^{qp}$ is phase insensitive and odd in voltages. (ii) The coherent multipair current $\bar{I}^{\text{MP}}$ is a function of $n\varphi_a^{(0)} + m\varphi_b^{(0)}$; it is *odd* in phases and *even* in voltages, just like the nondissipative Josephson term. It satisfies $m\langle I_a\rangle = n\langle I_b\rangle$. (iii) The component $\bar{I}^{\text{phMAR}}$ is *even* in phases and *odd* in voltages, like the dissipative ("cos φ") Josephson component, but it becomes dc in a bijunction. This $\bar{I}^{\text{phMAR}}$ component is also a function of $n\varphi_a^{(0)} + m\varphi_b^{(0)}$.

VI. CONCLUSIONS

We have shown by nonperturbative out-of-equilibrium calculations that coherent multipair and phase-dependent

MAR processes appear in a superconducting bijunction. These are due to crossed Andreev reflection processes, through the formation of several entangled nonlocal pairs, and lead to signatures in the dc current with very specific phase and voltage dependence. A natural extension of the present work should focus on the role of local Coulomb interaction on the dots. In the metallic junction regime and in the resonant regime near the dot resonance, a self-consistent mean-field treatment could be applied (as done in Ref. 26 for a three-terminal normal-superconducting setup with resonant dots). We expect that the same physical mechanisms would qualitatively produce the same effects. A more complex treatment would be required away from resonance, where interactions would have a larger impact and the Kondo mechanism could play an important role.

From an experimental standpoint, multipair resonances can be directly detected by transport measurements where one probes the nonlocal conductance $d\langle I_a\rangle/dV_b$ as a function of V_a,V_b. The phase coherence of the multipair current, and its actual dependence in $\varphi_{a/b}^{(0)}$, however, are more difficult to probe directly. One way would be to design specific superconducting quantum interference device geometries or microwave reflectivity experiments.[27]

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