Generating many Majorana modes via periodic driving: A superconductor model

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Realizing Majorana modes (MMs) in condensed-matter systems is of vast experimental and theoretical interests, and some signatures of MMs have been measured already. To facilitate future experimental observations and to explore further applications of MMs, generating *many* MMs at ease in an experimentally accessible manner has become one important issue. This task is achieved here in a one-dimensional *p*-wave superconductor system with the nearest- and next-nearest-neighbor interactions. In particular, a periodic modulation of some system parameters can induce an effective long-range interaction (as suggested by the Baker-Campbell-Hausdorff formula) and may recover time-reversal symmetry already broken in undriven cases. By exploiting these two independent mechanisms at once we have established a general method in generating many Floquet MMs via periodic driving.

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I. INTRODUCTION

The Majorana fermion, a particle which is its own antiparticle,[1] is attracting tremendous attention.[2–5] In addition to its fundamental interest,[6–9] its potential applications in topological quantum computation are also noteworthy.[10] Along with considerable theoretical studies,[7,10–20] the experimental search for Majorana modes (MMs) in condensed-matter systems has become a timely and important research topic. Indeed, following the theoretical results in Refs. [19–25], the zero-bias conductance peaks observed recently[26–29] are regarded as a signature of MMs in one-dimensional (1D) spin-orbit coupled semiconductor nanowires. However, the observed zero-bias peaks can be due to other reasons as well, e.g., the strong disorder in the nanowire[30,31] or smooth confinement potential at the wire end.[32] This being the case, the formation of MMs in these systems are yet to be double-confirmed by other approaches.

To identify MMs and facilitate their experimental observation, the signal strength should be enhanced.[33–35] If *many* MMs are present at the same edge and topologically protected from hybridizing with each other, one may verify if the signal originates from MMs by tuning the actual number of them, with the enhanced signal also more robust against experimental disorder[24,30,36] and contaminations from thermal excitations.[21–25] It is thus constructive to find a general method to form many MMs within one single system. In this respect, two facts are known. First, the formation of many MMs needs the protection of time-reversal symmetry.[35,37,38] Second, a longer-range interaction in a system is helpful to obtain more than two pairs of MMs.[37] As such, the generation of many MMs is equivalent to the following theoretical question: How does one synthesize a long-range interaction in a topologically nontrivial condensed-matter system while maintaining time-reversal symmetry?

As a conceptual advance, our answer to this question is rather simple and general. Given that periodic driving has become one highly controllable and versatile tool in generating different topological states of matter,[39–47] we show that a periodic driving protocol can create many Floquet MMs because it can generically induce effective long-range interactions and may also restore time-reversal symmetry (if it is broken without driving). Note that Floquet MMs are a particular class of MMs associated with the Floquet quasienergy bands of a periodically driven system:[44] They may be used for topological quantum computation as "normal" MMs do.[48]

Specifically, we propose to generate multiple Floquet MMs by switching (periodic quenching) a Hamiltonian from H_1 for the first half period to H_2 for the second one. The Floquet operator U is then

$$U(T) = e^{-\frac{iH_2T}{2\hbar}} e^{-\frac{iH_1T}{2\hbar}} \equiv e^{-\frac{iH_{\text{eff}}T}{\hbar}}, \quad (1)$$

where an effective Hamiltonian H_{eff} for the driven system has been defined. Using the Baker-Campbell-Hausdorff (BCH) formula, one finds that H_{eff} is formally given by

$$H_{\text{eff}} = \frac{H_1}{2} + \frac{H_2}{2} - \frac{iT}{8\hbar}[H_2, H_1]$$
$$- \frac{T^2}{96\hbar^2}[(H_2 - H_1), [H_2, H_1]] + \cdots. \quad (2)$$

Clearly then, even if H_1 or H_2 are short-range Hamiltonians, the engineered H_{eff} may still have long-range hopping or pairing terms via the nested-commutator terms in Eq. (2). This constitutes a main difference from undriven systems. Thus, the remaining job is to design such a protocol so that H_{eff} also possesses time-reversal symmetry. Interestingly, in the first proposal to realize Floquet MMs,[44] no more than two pairs of MMs can be generated precisely because time-reversal symmetry is not restored by the periodic driving therein.

In the following we present our detailed results using a model of a 1D spinless *p*-wave superconductor with the

nearest- and next-nearest-neighbor (NNN) interactions only. Under a periodic modulation of superconducting phases, we not only demonstrate that many Floquet MMs (e.g., 13 pairs in one case) can be generated, but also show that the number of the MMs may be widely tuned by scanning the modulation period. These results also shed more light on the inherent advantages of driven systems in exploring different topological states of matter, which can be useful for other timely topics related to long-range interactions (e.g., fractional Chern insulators[49,50]).

II. STATIC MODEL

We start from the Kitaev model Hamiltonian for a 1D spinless p-wave superconductor

$$H = -\mu \sum_{l=1}^{N} c_l^\dagger c_l - \sum_{a=1}^{2} \sum_{l=1}^{N-a} (t_a c_l^\dagger c_{l+a} + \Delta_a c_l^\dagger c_{l+a}^\dagger + \text{H.c.}),$$

where μ is the chemical potential, t_a and $\Delta_a = |\Delta_a|e^{i\phi_a}$ with $a = 1$ ($a = 2$) describes the nearest- (next-nearest-) neighbor hopping amplitude and pairing potential, respectively, and ϕ_a is the associated superconducting phases. All energy-related parameters are scaled by $|\Delta_1|$ and $\hbar = 1$ is set in our calculations. Majorana operators here refer to $(c_l + c_l^\dagger)$ or $i(c_l - c_l^\dagger)$. Such synthesized MMs may appear as edge modes under an open boundary condition, if the bulk band structure is topologically nontrivial.

The relative phase $\phi = \phi_1 - \phi_2$ determines the topological class of H.[51] For $\phi = 0$ and π, H has time-reversal and particle-hole symmetries. These cases then belong to the so-called "BDI" class characterized by a topological invariant Z. For other values of ϕ, H has particle-hole symmetry only and falls into the so-called "D" class characterized by a topological invariant Z_2. The D class can generate at most one pair of MMs. As to the BDI class, despite its potential in forming many MMs,[38] at most two pairs of MMs can be generated here due to the short-range nature of H.

III. DRIVEN MODEL

We now turn to periodically driven cases under a protocol given by Eq. (1). The emergence of Floquet MMs is directly connected to topological properties of the eigenstates of the Floquet operator $U(T)$. Let $|u\rangle$ be an eigenstate of $U(T)$ with an eigenvalue $e^{-i\epsilon T}$, namely, $U(T)|u\rangle = e^{-i\epsilon T}|u\rangle$. Evidently, the eigenvalue index ϵ is defined only up to a period $2\pi/T$ and hence called "quasienergy." The periodicity in ϵ may lead to a different topological structure in driven systems, with the corresponding topological classification revealed by the homotopy groups.[42] However, if the driven system belongs to a trivial class to this topological structure, then topological properties of the driven system is fully characterized by H_{eff} defined in Eq. (1).[38] This will be the case for our driving protocol proposed below.

As an explicit example, we propose to switch between two Hamiltonians H_1 and H_2 by the following: In the first half period, $H_1 = H(\phi_1, \phi_2)$ with both superconducting phase parameters ϕ_1 and ϕ_2 fixed, whereas in the second half period, we swap ϕ_1 and ϕ_2 so that $H_2 = H(\phi_2, \phi_1)$. Without loss of generality, we choose $\phi_1 = \pi/2$ and $\phi_2 = 0$. Thus,

within each half period, the Hamiltonian is in class D that breaks time-reversal symmetry. In addition to a possible generation of long-range interactions for H_{eff}, this driving protocol is designed to recover time-reversal symmetry. In particular, let $\mathcal{K}$ be a conventional time-reversal operator and $G \equiv e^{-i\frac{\phi_1+\phi_2}{2}\sum_l c_l^\dagger c_l}$ be a gauge transformation operator. Considering a generalized time-reversal operator $\bar{\mathcal{K}} \equiv \mathcal{K}G$, we find

$$\bar{\mathcal{K}}U(T)\bar{\mathcal{K}}^{-1} = e^{\frac{iH_1T}{2\hbar}} e^{\frac{iH_2T}{2\hbar}} = U^\dagger(T). \quad (3)$$

This constitutes a direct proof that our driven system now possesses time-reversal symmetry, and as a result its topological class is switched from class D to class BDI. To further examine this restored time-reversal symmetry, we work in the momentum representation and directly find an analytical H_{eff} from Eq. (1). We define $c_k = \sum_l c_l e^{-ikl}/\sqrt{N}$ and introduce the Nambu representation $C_k = [c_k, c_{-k}^\dagger]^T$. A standard procedure then leads to $H_{\text{eff}} = \sum_{k \in \text{BZ}} C_k^\dagger H_{\text{eff}}(k) C_k$, with $H_{\text{eff}}(k) = E_k \vec{n}(k) \cdot \vec{\sigma}$, where $\vec{\sigma}$ represents the Pauli matrices.[41] The three components of $\vec{n}(k)$ are given by $n_1(k) = 0$, and

$$n_2(k) = \frac{g_{1,k} \sin(s_k T)}{s_k \sin(E_k T)} - \frac{2g_{2,k}\eta_k \sin^2(s_k T/2)}{s_k^2 \sin(E_k T)}, \quad (4)$$

$$n_3(k) = \frac{\eta_k \sin(s_k T)}{s_k \sin(E_k T)} + \frac{2g_{1,k}g_{2,k} \sin^2(s_k T/2)}{s_k^2 \sin(E_k T)}, \quad (5)$$

where $g_{a,k} = |\Delta_a| \sin(ak)$, $s_k = (\eta_k^2 + \sum_a g_{a,k}^2)^{1/2}$, $\eta_k = -\mu - 2\sum_a t_a \cos(ak)$, and $\cos(E_k T) = \cos(s_k T) + 2(g_{2,k}^2/s_k^2)\sin^2(s_k T/2)$. For each value of k, one obtains two values of E_k and hence two values for the quasienergy ϵ. Consistent with the $\bar{\mathcal{K}}$ symmetry, we now have $H_{\text{eff}}^*(-k) = H_{\text{eff}}(k)$. Noting the inherent particle-hole symmetry of H_{eff}, one may construct a chiral symmetry for H_{eff}, a fact consistent with our above result that $\vec{n}(k)$ is in the yz plane for all k. The above analysis makes it clear that our driving protocol changes both the underlying symmetry and the topological class of the system.

Without a gap closing between the two branches of E_k, the topological invariant Z in class BDI can be obtained by the integer winding number $W = \int_{-\pi}^{\pi} \frac{d\theta_k}{2\pi} \in Z$, where $\theta_k = \arctan[n_3(k)/n_2(k)]$. A computational example illustrating W is shown in Fig. 1(a). The number of pairs of MMs under an open boundary condition is then given by $|W|$. As some system parameters continuously change, gap closing and consequently topological phase transitions occur.[7] Figure 1(b) depicts a phase diagram, obtained by explicitly evaluating W. It is seen that $|W|$ ranges from 0 to 3. This indicates that three pairs of MMs can be formed in our driven system. This is beyond the expectation for the undriven model, where the NNN interaction can give at most two pairs of MMs. Therefore, the finding of $|W| = 3$ in some parameter regime is a clear sign that our driving protocol may synthesize some features absent in the static model. The boundaries between different topological phases of our driven system are also interesting on their own right. The solid and dotted lines in Fig. 1(b) depict the topological phase transition points at which W jumps by one. This is found to go with the gap closing at $k = 0$ or $\pm\pi$. The dashed line gives the phase transition points at which W jumps by two. This happens at $k = \pi/2$.
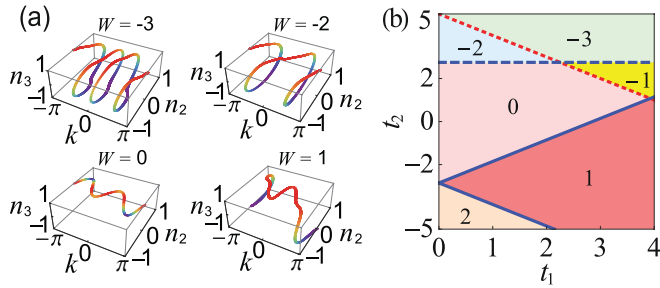
FIG. 1. (Color online) (a) Winding of the $\vec{n}(k)$ [see Eqs. (4) and (5)] for $k \in [-\pi, \pi]$. $W = -3, -2, 0, 1$ correspond to $(t_1, t_2) = (1,5), (1,3), (1,0), (1,-3)$, respectively. Indicated on each panel is the winding number W. The solid and dotted line indicates a gap closing (of E_k) at $k = 0$ or $\pm\pi$, while the dashed line corresponds to a gap closing at $k = \pi/2$. Other parameters are $\mu = -10$, $|\Delta_2| = 2.5$ and $T = 0.2$.

To confirm our theoretical results presented in Fig. 1 we carry out numerical calculations of the quasienergy spectrum ϵ under an open boundary condition. Because $\epsilon = \pi/T$ is equivalent to $\epsilon = -\pi/T$, Floquet MMs have two flavors: one at $\epsilon = 0$ and the other at $\epsilon = \pm\pi/T$. The second flavor is certainly absent in an undriven system.[44] For fixed $t_1 = 1$ and a varying t_2, Fig. 2(a) depicts the formation of both flavors of Floquet MMs, with the second flavor emerging in a wider parameter regime. The total number of pairs of MMs should equal $|W|$ (if the winding number is well defined). For example, Fig. 2(a) shows that two degenerate pairs of MMs at $\epsilon = \pm\pi/T$ and one pair of MMs at $\epsilon = 0$ are formed when $t_1 = 1$ and $t_2 = 4$. This agrees with the $W = -3$ region shown in Fig. 1(b). Likewise, all other details in Fig. 2(a) are fully consistent with our analytical results shown in Fig. 1(b). We have also studied the dynamics of the formed MMs in one full period of driving: They are indeed well localized at two edges. Further, as a comparison with our static model H, we plot in Fig. 2(b) our system's energy spectrum in the absence of driving. It is seen that at most one pair of MMs can be formed only in a very narrow t_2 regime for the large $|\mu|$ case. The parallel driven case is however different: One may still obtain three pairs of MMs. Thus, even in the large $|\mu|$ case, our driving protocol can still generate more MMs than the static case. This is both interesting and useful because, in general, the large $|\mu|$ is preferred for the protection of MMs against strong disorder in actual experiments.
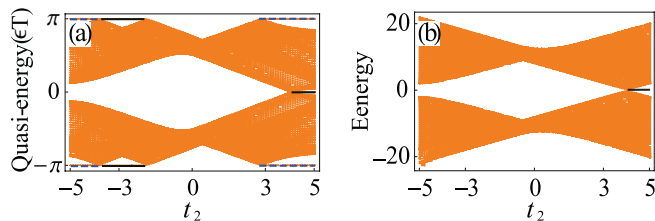

FIG. 2. (Color online) Quasienergy spectrum for a driven case (a) vs energy spectrum for a static case (b) obtained under an open boundary condition. The dashed blue line and solid black line stand for two degenerate pairs and a single pair of MMs, respectively. $t_1 = 1$, $N = 200$, and other parameters are the same as in Fig. 1(b).
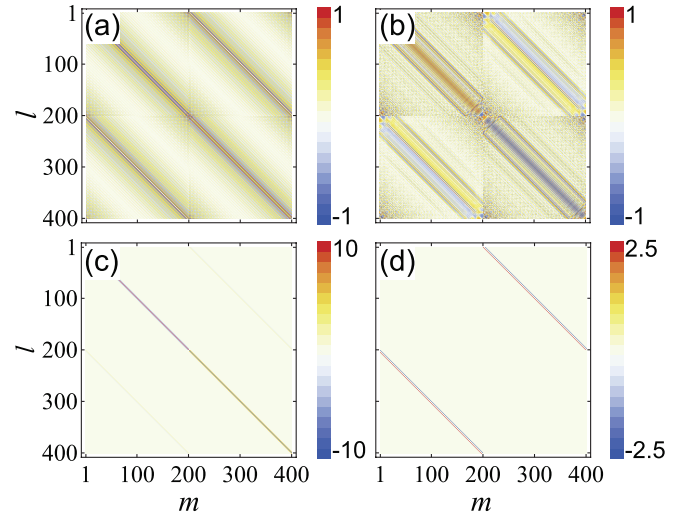

FIG. 3. (Color online) The expansion coefficients of H_{eff} for $T = 0.2$ (a) and 2.0 (b), and of the static H for the real (c) and imaginary (d) parts in the operator basis $(c_1, \ldots, c_N, c_1^\dagger, \ldots, c_N^\dagger)^T$. l and m are the base indices. Other parameters are the same as in Fig. 2.

In efforts to generate even more MMs, we now extend our direct numerical studies to other parameter regimes. Remarkably, the BCH formula in Eq. (2) indicates that as T increases, the nested commutators on the right hand side of Eq. (2) will have heavier weights. An increasing T can then induce longer-range interactions in H_{eff}. This trend is investigated in Fig. 3, where the expansion coefficients of H_{eff} [numerically obtained from Eq. (1)], with H_{eff} expanded as a quadratic function of the operators $(c_1, \ldots, c_N, c_1^\dagger, \ldots, c_N^\dagger)^T$, are shown for two different values of T. For comparison, the expansion coefficients for the static case are also plotted in Figs. 3(c) and 3(d). A few interesting observations can be made from Fig. 3. First, the plotted expansion coefficients of H_{eff} are all real, which is different from the shown static case with both real and imaginary coefficients. This difference reflects the restored time-reversal symmetry for the driven case. Second, in sharp contrast to the results shown in Figs. 3(c) and 3(d), coefficients for quite long-range hopping and pairing interactions (e.g., across more than ten sites) can be appreciably nonzero for H_{eff} in both cases of $T = 0.2$ and $T = 2.0$. The latter case, plotted in Fig. 3(b) with wider stripes, confirms the emergence of longer-range terms with considerable weights as T increases. Third, the diagonal terms in the expansion shown in Figs. 3(a) and 3(b) (which can be understood as an effective chemical potential) are much smaller than the diagonal elements, i.e., $|\mu|$, in Fig. 3(c). This further explains why a driven system may generate many MMs despite a large $|\mu|$ in the undriven model.

Results in Fig. 3 motivate us to explore the formation of Floquet MMs with sufficiently large values of T. There is also one twist as we increase T. That is, the quasienergy gap may be generically closed at $\epsilon = 0$. Consequently the winding number W is no longer well defined. To characterize the topological phases at $\epsilon = \pm\pi/T$, where MMs can still be topologically protected, we resort to another topological invariant,[52]

$$\nu = \frac{1}{2} \sum_{n_3(k)=0, E_k \neq 0} \text{sgn}\{\partial_k [E_k n_3(k)]\} \text{sgn}[E_k n_2(k)], \quad (6)$$

TABLE I. Number of MMs localized at each boundary for different T. Other parameters are the same as in Fig. 2(a).

t_2	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8
$T = 0.5$	2	4	4	3	3	2	0	0	0	1	1	2	2	4	4	4	3
$T = 1.0$	6	6	7	7	6	3	3	2	1	1	2	5	5	6	7	7	6
$T = 2.0$	13	13	12	11	9	8	8	1	1	3	4	7	11	10	13	13	12

which reduces to the winding number W when the gap at $\epsilon = 0$ is also open. We present in Table I the number of pairs of MMs we obtain, for an increasing T and for different choices of t_2. For $T = 0.5$, the best observation is the generation of four pairs of MMs but no MMs for $|t_2| \lesssim |t_1|$. For $T = 1.0$, it is possible to achieve seven pairs. For $T = 2.0$, as many as 13 pairs of MMs can be formed. Interestingly, in cases with large T such as $T = 1.0$, our driving protocol can also form several pairs of MMs for $|t_2| < |t_1|$ or even with $t_2 = 0$. Note that as more MMs are generated by increasing T, the bulk quasienergy gap at $\varepsilon = \pm \pi / T$ decreases in general, leading to a larger "penetration length" (into the bulk) for the synthesized MMs. Considering the necessary protection of MMs by a nonzero bulk gap, one may not wish to push our driving protocol too far.

It is noted that the finite switching time in the practice to our ideal step-driving scheme has no qualitative change to our results. However, it may influence quantitatively the range of the synthesized interaction as well as the numbers of the generated MMs. To simulate the smooth switching we have separated each of the two half periods of our driving scheme into 15 intermediate staircaselike changes and confirmed numerically that longer-range interactions as well as more MMs can be generated. As a final remark, the number of the MMs characterized by the topological invariant depends on the topological properties of the Floquet states, which are determined by all the physical parameters in the driven model.

IV. CONCLUSIONS

A periodic driving has the capacity to restore time-reversal symmetry and to induce an effective long-range interaction. With these two mechanisms working at once, the generation of many MMs is achieved using a standard p-wave superconductor model under a certain periodic modulation.

In terms of a possible experimental confirmation of our predictions, our model may be realized with cold atoms or molecules in a designed optical lattice, as clean systems with negligible perturbations. Explicitly, the nearest-neighbor and NNN hopping (t_a) can be realized by a simple zigzag chain lattice,[34] with the hopping strength adjustable by the lattice geometry. The chemical potential (μ) is controllable through the optical trap potential or a radio frequency detuning. The pairing terms (Δ_a) may be induced by a Raman induced dissociation of Cooper pairs forming an atomic BCS reservoir and the associated superconducting phases (ϕ_a) can be tuned by complex Rabi frequencies.[44] Another experimental realization is to use the recently proposed quantum-dot-superconductor arrays with a zigzag geometry.[53] Here μ can be gate controlled. Δ_a can be proximity induced and ϕ_a can be tuned via applying fluxes on the superconducting islands. The MMs formed in our system may be probed using techniques analogous to what is being used for undriven systems,[34] but now with the hope of some enhanced signals if a measurement exploits the simultaneous generation of *many* MMs. The generation of a tunable number of many MMs is also expected to offer another dimension for experimental studies.

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