

Effective spin diffusion in spin-polarized equilibrium and quasiequilibrium Fermi liquids

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We calculate the effective spin diffusion coefficient of weak ferromagnet and quasiequilibrium paramagnetic systems using Landau Fermi liquid theory. We find that the behavior of the diffusion coefficient of the quasiequilibrium system is determined primarily by the internal magnetic field, which, in turn, depends upon the nonequilibrium magnetization and the antisymmetric Landau parameters. We also show that this is qualitatively similar to the diffusion coefficient of the weak ferromagnetic system. We discuss its implication for the spin-polarized state created in cold atom systems.

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I. INTRODUCTION

Polarized nonequilibrium (PNEQ) spin systems have attracted a great deal of research interest recently. These systems can be prepared in liquid helium,¹⁻⁴ spintronic materials,^{5,6} and atomic gases.⁷⁻¹⁰ In spintronics, the PNEQ system has been used to study spin transport in paramagnetic metals, semiconductors, and metal alloys.⁵ The PNEQ system has also been utilized in atomic gases in order to explore the superfluid state for varying magnetization fractions.⁷⁻¹⁰

In this paper we study theoretically the effects of the presence of nonequilibrium magnetization on the effective spin diffusion coefficient D_{eff} for systems that can be characterized as Fermi liquids or gases. The D_{eff} of a Fermi liquid can be measured in a spin-echo experiment. The decay rate in the spin-echo measurements is directly proportional to the real part of the spin diffusion coefficient, which, as we show in what follows, depends on the nonequilibrium magnetization. More importantly, the spin echo experiment indirectly confirms the existence and nature of the spin collective modes in a Fermi liquid, since both the dispersion relation of the modes and the magnetization-dependent diffusion coefficient are derived from the same Landau kinetic equation. Thus, if the dependence of D_{eff} on the nonequilibrium magnetization, which we derive below, agrees with the measured behavior observed in spin echo experiments, the existence of the gapless mode in the PNEQ system, which is reported in Ref. 11, will be confirmed.

We are going to consider paramagnetic systems in the presence of an external magnetic field and a weak ferromagnetic system as examples of equilibrium Fermi liquids. The theory for spin diffusion is well known for the paramagnetic Fermi liquids.^{12,13} Here, we consider a system that is created in a PEQ state, and on top of which a nonequilibrium magnetization \vec{m}_{NEQ} is imposed. Thus the system we consider is a hybrid one (PEQ + PNEQ). We show that the effect of \vec{m}_{NEQ} on the diffusion coefficient can be extracted theoretically from a PEQ + PNEQ system by changing the nonequilibrium magnetization, while keeping the external magnetic field constant. Any deviation of the diffusion coefficient of the polarized equilibrium state will be due to the nonequilibrium magnetization. We propose that this should be experimentally observable with current techniques.

The temperature dependence of the effective diffusion coefficient D_{eff} , in the equilibrium phases of liquid He³ and the dilute mixtures of He³ in superfluid He⁴ in a small magnetic field has been studied by Leggett and Rice.¹² In their paper,¹² the authors have shown that D_{eff} is complex and the real part as a function of temperature has a maximum separating the hydrodynamic regime from a collisionless regime. One of the main results of the paper was that the position and the magnitude of the maximum of D_{eff} are determined by the Larmor frequency, $\omega_L = 2B$, where B is the external magnetic field. In a follow up paper, Leggett¹⁴ went beyond the earlier calculation of Leggett and Rice¹² by deriving a result for the nonequilibrium case. For the nonequilibrium phase, he considered the case where the longitudinal spin relaxation time, T_1 , and the transverse spin relaxation time, T_2 , are infinite or at least much longer than the other relevant time scales, e.g., the diffusion lifetime and the time to make the measurement. In this limit, the magnetization of the system need not be at the equilibrium value for the external field that is applied to the system. To account for this, Leggett introduces what he calls the equivalent Larmor frequency ω_L^* ,¹⁴ which is related to our m_{NEQ} . It is in this sense that we introduced the PNEQ paramagnetic Fermi liquids in our paper¹¹ and in the current paper. The main difference between our papers and Leggett's¹⁴ is that we have explicitly calculated the change in the Fermi energy of spin σ , ε_F^σ , and the spin wave spectrum, $\omega_0^\pm(q)$, in leading order in m_{NEQ} , in the QEQ limit.

Here, we will see that the D_{eff} for the QEQ paramagnetic systems is qualitatively the same as for the weak ferromagnetic metal. For a weak ferromagnetic metal, we derive the D_{eff} from the hydrodynamic equation developed by Bedell and Blagoev.¹⁵ The motivation that was behind the current calculation is rooted in our previous observation that the spin collective modes of the PNEQ system are qualitatively similar to that of the weak ferromagnetic Fermi liquids.¹¹

II. DERIVATION OF SPIN DIFFUSION COEFFICIENT

Let us assume that we have a PEQ state that is created due to the application of an external field $\vec{B} = B\hat{z}$, and on top of this system let us impose a nonequilibrium state that is created by polarized spin injection or optical pumping. In what follows, we consider the case in which the direction

of the polarization of the equilibrium and nonequilibrium magnetization is the same, i.e., \hat{z} direction. In this case, the magnetization $\vec{m} = m_z = n^\uparrow - n^\downarrow$ (where n^\uparrow and n^\downarrow are the densities of up and down spins) of the system will be the sum of the magnetization due to the $\vec{m}_{\text{EQ}} = n_{\text{EQ}}^\uparrow - n_{\text{EQ}}^\downarrow = \frac{\gamma\hbar}{2} \frac{N(0)\vec{B}}{(1+F_0^a)}$ for small polarization, where γ is the gyromagnetic ratio, $N(0)$ is the quasiparticle density of states at the Fermi energy, and F_0^a is the $l = 0$ antisymmetric Fermi liquid parameter,^{16,17} and $\vec{m}_{\text{NEQ}} = n_{\text{NEQ}}^\uparrow - n_{\text{NEQ}}^\downarrow$ where the nonequilibrium density of up and down spins is controlled by the external source (such as the intensity of the polarized light in spin pumping) used to create this nonequilibrium spin population. We note that the total magnetization $\vec{m} = \vec{m}_{\text{EQ}} + \vec{m}_{\text{NEQ}}$ is an approximation valid only for small nonequilibrium polarization. A more accurate calculation would require a self-consistent approach taking into account the change in the distribution function due to the nonequilibrium polarization. We will assume that the spin relaxation times, T_1 and T_2 , are very long compared to the diffusion time or the time to make the spin echo measurements. Under these conditions the total magnetization is conserved.

Next we also show that the current \vec{j} of the hybrid system discussed above is also equal to the currents due to the equilibrium state \vec{j}_{EQ} and the nonequilibrium state \vec{j}_{NEQ} . To prove so, we utilize the spin conservation law. Starting from the definition of the local polarization density distribution function, $\vec{m}_{\vec{p}}(\vec{r}, t) = \vec{m}_{\text{EQ},\vec{p}}(\vec{r}, t) + \vec{m}_{\text{NEQ},\vec{p}}(\vec{r}, t)$ of the hybrid system, to derive the spin conservation law, we need to sum the equation for the time evolution of the magnetization distribution function,^{16,17} i.e.,

$$\begin{aligned} \frac{\partial \vec{m}_{\vec{p}}(\vec{r}, t)}{\partial t} + \frac{\partial}{\partial r_i} \left[\frac{\partial \varepsilon_{\vec{p}}}{\partial p_i} \vec{m}_{\vec{p}}(\vec{r}, t) + \frac{\partial \hbar_{\vec{p}}}{\partial p_i} n_{\vec{p}}(\vec{r}, t) \right] \\ = -\vec{m}_{\vec{p}}(\vec{r}, t) \times \vec{h}_{\vec{p}}(\vec{r}, t) \end{aligned} \quad (1)$$

over the momenta, where

$$\vec{h}_{\vec{p}}(\vec{r}, t) = -\frac{\gamma\hbar}{2} \vec{B} + \sum_{\vec{p}'} f_{\vec{p}\vec{p}'}^a \vec{m}_{\vec{p}'}(\vec{r}, t) \quad (2)$$

is the effective local field, $f_{\vec{p}\vec{p}'}^a$ is the quasiparticle interaction, $\varepsilon_{\vec{p}}$ is the quasiparticle energy, and $n_{\vec{p}}(\vec{r}, t) = n_{\text{EQ},\vec{p}}(\vec{r}, t) + n_{\text{NEQ},\vec{p}}(\vec{r}, t)$ is the quasiparticle density distribution function. The repeated index i in Eq. (1) refers to the Cartesian coordinates of space \vec{r} and it is to be summed. In what follows, we set $\frac{\gamma\hbar}{2} = 1$. After summation over the quasiparticle momentum \vec{p} , we get

$$\frac{\partial \vec{m}(\vec{r}, t)}{\partial t} + \frac{\partial}{\partial r_i} \vec{j}_i(\vec{r}, t) = \vec{m}(\vec{r}, t) \times \vec{B}, \quad (3)$$

where

$$\vec{m}(\vec{r}, t) = \sum_{\vec{p}} \vec{m}_{\vec{p}}(\vec{r}, t) \quad (4)$$

and

$$\vec{j}_i(\vec{r}, t) = \sum_{\vec{p}} \left[\frac{\partial \varepsilon_{\vec{p}}}{\partial p_i} \vec{m}_{\vec{p}}(\vec{r}, t) + \frac{\partial \hbar_{\vec{p}}}{\partial p_i} n_{\vec{p}}(\vec{r}, t) \right] \quad (5)$$

is the spin current along \hat{i} direction. Equation (3) represents the net spin conservation law. For the details of this formalism, we would like to refer authors to Refs. 16 and 17.

Assuming that the spin polarization is nonzero only for \vec{p} near the Fermi surface, the current equation can also be written as^{16,17}

$$\vec{j}_i(\vec{r}, t) = \sum_{\vec{p}} v_{\vec{p}i} [\vec{m}_{\text{EQ},\vec{p}}(\vec{r}, t) + \vec{m}_{\text{NEQ},\vec{p}}(\vec{r}, t)] \left(1 + \frac{F_1^a}{3} \right), \quad (6)$$

where $\vec{v}_{\vec{p}} = \nabla_{\vec{p}} \varepsilon_{\vec{p}}^0$ is the equilibrium quasiparticle velocity and F_1^a is the $l = 1$ component of the antisymmetric Fermi liquid parameter. From this equation it is clear that the total spin current of the system is the sum of the corresponding contributions of the equilibrium and nonequilibrium states.

In a similar fashion, we can show that in the presence of a transverse magnetic perturbation (applied perpendicular to the direction of the applied equilibrium magnetic field, i.e., \hat{z}), the transverse current is also the sum of the transverse current due to equilibrium and nonequilibrium states. For a transverse perturbation $\vec{B}^\pm = B_x \hat{i} \pm B_y \hat{j}$, we define $m_{\vec{p}}^\pm(\vec{r}, t) = m_{\vec{p}}^x \pm i m_{\vec{p}}^y$ as the transverse component of $\vec{m}_{\vec{p}}(\vec{r}, t)$. For excitations close to the Fermi surface, we introduce $m_{\vec{p}}^\pm(\vec{r}, t) = -\frac{\partial n_{\vec{p}}^0}{\partial \varepsilon_{\vec{p}}^0} v_{\vec{p}}^\pm(\vec{r}, t)$ and $v_{\vec{p}}(\vec{r}, t)$ is the distortion of the Fermi surface. Using these definitions, the transverse component of the spin current is given by

$$j_i^\pm(\vec{r}, t) = \left(1 + \frac{F_1^a}{3} \right) \int \frac{d^3 p}{(2\pi)^2} v_{\vec{p}i} \left[-\frac{\partial n_{\vec{p}}^0}{\partial \varepsilon_{\vec{p}}^0} v_{\vec{p}}^\pm(\vec{r}, t) \right]. \quad (7)$$

This equation shows that if the transverse magnetization of the two systems adds algebraically, then the transverse spin current does, as well.

We now are in a position to rederive the spin diffusion coefficient in order to show the nonequilibrium effects on the diffusion. Initially, we follow Ref. 12 for the spin current equation, which is obtained by multiplying Eq. (1) by the Fermi velocity. The transverse component of the resulting spin current satisfies the following equation:

$$\begin{aligned} \frac{\partial j_i^\pm(\vec{r}, t)}{\partial t} + c_s^2 \frac{\partial m^\pm(\vec{r}, t)}{\partial r_i} \\ = \mp i \omega_L j_i^\pm(\vec{r}, t) \pm \frac{2}{N(0)} \left(F_0^a - \frac{F_1^a}{3} \right) m_z j_i^\pm(\vec{r}, t) \\ - \left(1 + \frac{F_1^a}{3} \right) \frac{j_i^\pm(\vec{r}, t)}{\tau_D}, \end{aligned} \quad (8)$$

where the last term represents the collision contribution to the spin current, ω_L is the Larmor precession frequency, $c_s^2 = \frac{1}{3}(1 + F_0^a)(1 + \frac{F_1^a}{3})v_F^2$ is the spin-wave velocity, v_F is the Fermi velocity, and τ_D is spin diffusion life time.

We would like to pause here a moment and compare this equation with the similar equation for spin current in the ferromagnetic system as derived in Ref. 15. The two equations differ only on the definition of the spin-wave velocity given by c_s^2 and the origin of the magnetization. In the PEQ + PNEQ system, $c_s^2 = \frac{1}{3}(1 + F_0^a)(1 + \frac{F_1^a}{3})v_F^2$, whereas in the ferromagnetic system it is given by $c_s^2 = \frac{1}{3}(1 + F_0^a)(1 + \frac{F_1^a}{3})v_F^2$. The major difference in the magnetization is that the magnetization of the PNEQ system is imposed and maintained externally, whereas in the ferromagnetic system

it is coming from the ground state of the ferromagnet. The surprising observation is that although the ferromagnetic and PNEQ systems are completely different the equations of motion for the magnetization distribution function are qualitatively the same. In what follows, we will track the effect of this difference for the PNEQ system and the ferromagnetic system and will point out wherever the correction is to be made.

For a PEQ + PNEQ system in a nearly steady state condition, the transverse component of the spin current [see Eq. (8)] can be written as

$$c_s^2 \frac{\partial m^\pm(\vec{r}, t)}{\partial r_i} = \pm \frac{2(F_0^a - \frac{F_1^a}{3})}{N(0)} m_z j_i^\pm(\vec{r}, t) - \left(1 + \frac{F_1^a}{3}\right) \frac{j_i^\pm(\vec{r}, t)}{\tau_D}, \quad (9)$$

where we have used the fact that in the quasisteady state the spin current also precesses around the external field with the Larmor precession frequency.^{12,13} Solving this equation with the transverse component of the net spin conservation law, i.e.,

$$\frac{\partial m^\pm(\vec{r}, t)}{\partial t} \pm i\omega_L m^\pm(\vec{r}, t) + \frac{\partial j_i^\pm(\vec{r}, t)}{\partial r_i} = 0, \quad (10)$$

we get an expression for the transverse magnetization:

$$\frac{\partial m^\pm(\vec{r}, t)}{\partial t} \pm i\omega_L m^\pm(r, t) - D^\pm \nabla^2 m^\pm(\vec{r}, t) = 0, \quad (11)$$

where

$$D^\pm = \frac{v_F^2 \tau_D (1 + F_0^a)}{3 \left[1 \pm i 2 \tau_D \left(\frac{m_{\text{NEQ}} + m_{\text{NEQ}}}{N(0)} \right) \left(\frac{F_1^a - F_0^a}{1 + \frac{F_1^a}{3}} \right) \right]} = \frac{v_F^2 \tau_D (1 + F_0^a)}{3 \left[1 \pm i \tau_D \omega_L \left(\frac{1}{1 + F_0^a} - \frac{1}{1 + \frac{F_1^a}{3}} \right) \pm i \tau_D \frac{2m_{\text{NEQ}}}{N(0)} \left(\frac{F_1^a - F_0^a}{1 + \frac{F_1^a}{3}} \right) \right]}. \quad (12)$$

Defining $\lambda = \frac{1}{1 + F_0^a} - \frac{1}{1 + \frac{F_1^a}{3}}$, $K^\pm = \pm \omega_L$, and $\omega_1^\pm = \pm \frac{2m_{\text{NEQ}}(\frac{F_1^a}{3} - F_0^a)}{N(0)}$ (which is the internal field of the nonequilibrium system created due to the fermionic interaction), Eq. (12) can be written as

$$D^\pm = \frac{v_F^2 \tau_D (1 + F_0^a)}{3 \left(1 + i \lambda K^\pm \tau_D + i \frac{\omega_1^\pm \tau_D}{1 + \frac{F_1^a}{3}} \right)}. \quad (13)$$

In the absence of nonequilibrium magnetization, the spin diffusion coefficient reduces to

$$D_{\text{EQ}}^\pm = \frac{v_F^2 \tau_D (1 + F_0^a)}{3(1 + i \lambda K^\pm \tau_D)}, \quad (14)$$

which is identical to the results obtained by Bedell and Meltzer.¹³ On the other hand, for a purely nonequilibrium state (in the absence of the equilibrium magnetization) the spin diffusion coefficient is given by

$$D_{\text{NEQ}}^\pm = \frac{v_F^2 \tau_D (1 + F_0^a)}{3 \left(1 + i \frac{\omega_1^\pm \tau_D}{1 + \frac{F_1^a}{3}} \right)}. \quad (15)$$

For the ferromagnetic system, we only need to replace $(1 + F_0^a)$ by $(|1 + F_0^a|)$, and keep in mind that $\omega_1^\pm = \pm \frac{2m_{\text{EQ}}(\frac{F_1^a}{3} - F_0^a)}{N(0)}$ is the equilibrium internal field of the ferromagnetic system. In what follows, the real part of the D_{NEQ}^\pm is what we are interested in, and this is given by $\text{Re}(D_{\text{NEQ}}^\pm) = \frac{v_F^2 (1 + F_0^a) \tau_D}{1 + (\tau_D \omega_{\text{eff}}^\pm)^2}$, where $\omega_{\text{eff}}^\pm = \lambda K^\pm + \frac{\omega_1^\pm}{1 + \frac{F_1^a}{3}}$.

III. RESULT AND DISCUSSION

We see in Eq. (13) that the diffusion coefficient is dependent on ω_L , which represents the contribution of the equilibrium state, and ω_1 , which represents the contribution of the nonequilibrium state. As discussed above, the nonequilibrium magnetization is rotationally invariant. So for practical purposes it is desirable to retain a finite polarized equilibrium magnetization, so that there exists in the system a fixed quantization axis for the polarization. For such a situation, we now discuss, in general, terms how one could experimentally extract the effect of the nonequilibrium magnetization on the real part of the diffusion coefficient. For simplicity, in what follows we restrict the discussion to positive distortions of the Fermi surface, i.e., when the distortion is describe by ν^+ (the negative distortion effects follow from the positive distortion effects trivially).

We propose that using a traditional spin-echo experimental setup one could first measure the $\text{Re}(D^+)$ behavior for a given ω_L at $m_{\text{NEQ}} = 0$. Then for the same ω_L one could measure $\text{Re}(D^+)$ for varying $m_{\text{NEQ}} \neq 0$. Any deviation in $\text{Re}(D^+)$ would reveal the effect of the nonequilibrium magnetization on spin diffusion. From Eq. (12), we see theoretically that there is a nontrivial effect coming from m_{NEQ} , and we expect this to be borne out in experiment.

To better understand theoretically the effect of m_{NEQ} on spin diffusion, we can evaluate the real part of the diffusion coefficient $\text{Re}(D^+)$ as a function of diffusion life time τ_D . (For the purpose of numerical calculation, ω_L is normalized with the Fermi energy, and m_{NEQ} is used in the form of a polarization fraction, i.e., m_{NEQ}/n , where n is the total particle density). In Fig. 1, we show the behavior of $\text{Re}(D^+)$ as a function of the diffusion lifetime, which is normalized with respect to Fermi energy, for two different values of the Larmor frequency, $\omega_L = 0.1, 0.05$, and a fixed value of the nonequilibrium magnetization, $m_{\text{NEQ}} = 0.1$. One can see that $\text{Re}(D^+)$ decreases for all τ_D with a corresponding increase in either ω_L or m_{NEQ} . We also see that for low τ_D , in agreement with the literature,¹² $\text{Re}(D)$ increases initially, attains a maximum value, and decreases towards zero for $\tau_D \rightarrow \infty$. This behavior is characteristic of a spin system that undergoes a transition from a collision-dominated regime (low τ_D) to a collision-less regime (high τ_D).

It is important to emphasize the significant effect a varying m_{NEQ} has on $\text{Re}(D^+)$. For example, for $\omega_L = 0.05$, the height of the maxima of $\text{Re}(D^+)$ decreases by about 100% on ramping up m_{NEQ} from 0 to 0.1 (compare the labeled long-dashed and short dashed curve). This is a significant change which should be readily observable in a spin echo experiment. The unlabeled long-dashed and short-dashed curves correspond to $\omega_L = 0.1$, $m_{\text{NEQ}} = 0$ and $\omega_L = 0.1$, $m_{\text{NEQ}} = 0.1$,

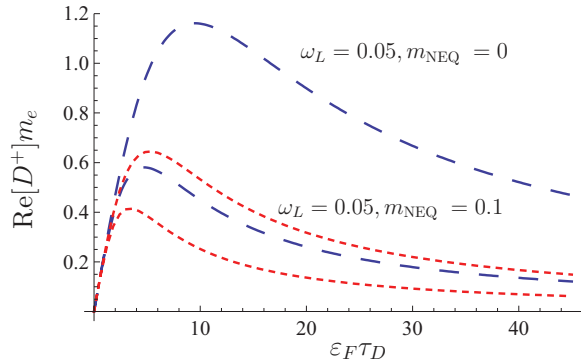


FIG. 1. (Color online) The real part of the diffusion coefficient $\text{Re}[D^+]$ is shown as a function of the diffusion lifetime τ_D for different values of the normalized Larmor frequency $\omega_L/\varepsilon_F = 0.1, 0.05$ and fixed value of $m_{\text{NEQ}}/n = 0.1$. The unlabeled long-dashed and unlabeled short-dashed curve correspond to $\omega_L = 0.1, m_{\text{NEQ}} = 0$ and $\omega_L = 0.1, m_{\text{NEQ}} = 0.1$, respectively. For $\omega_L = 0.05$, increasing m_{NEQ} by 0.1 reduces the diffusion coefficient by 100%. The positions of the maxima are determined by both the m_{NEQ} and ω_L . m_e is the bare mass of the electron.

respectively, where we see that the change in $\text{Re}(D)$ due to change in $m_{\text{NEQ}} = 0.1$ is almost 50%. Thus it is clear that the effect of change in m_{NEQ} is more drastic for smaller ω_L .

The position of the maxima in Fig. 1 is determined not only by ω_L , but also by m_{NEQ} . An increase in ω_L or m_{NEQ} increases the total polarization of the system, and thus makes the system more susceptible to particle collisions, as signified by the overall shift of the maxima of $\text{Re}(D^+)$ towards lower τ_D values.

Motivated by the fact that the effect of m_{NEQ} on $\text{Re}(D^+)$ is bigger for smaller values of ω_L , we calculated $\text{Re}(D^+)$ for $\omega_L = 0.001$, and for $m_{\text{NEQ}} = 0.01, 0.05, 0.1$. In the limiting case of $\omega_L \rightarrow 0^+$, the system can be considered to be in a pure nonequilibrium state. The numerical calculation of $\text{Re}(D^+)$ for such a system is shown in Fig. 2. We see that an increase in the nonequilibrium polarization decreases the magnitude of $\text{Re}(D)$ significantly. The positions of the maxima are again determined by the nonequilibrium magnetization. We would like to note here that the nonequilibrium polarization well above 10% has been obtained experimentally^{4,10} making it feasible to observe the effects discussed above.

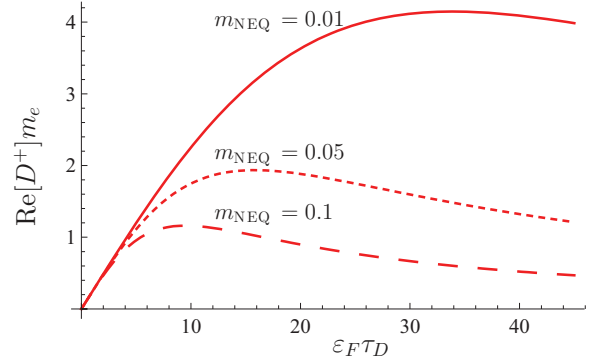


FIG. 2. (Color online) The spin diffusion coefficient of a PNEQ system when the equilibrium magnetization is very small. The diffusion coefficient decreases with the increase in m_{NEQ} . The position of the maxima is determined by the internal field. m_e is the bare mass of the electron. We take $\omega_L = 0.001$.

We would like to comment on the implication of the current study in the cold atom system. From the calculations of the spin-wave dispersion,¹¹ the equation for the spin current and the diffusion coefficient we have shown that the polarized nonequilibrium system mimics the behavior of a weak ferromagnetic system. In our formalism, the difference is only in the origin of the magnetization and the magnitude of the Fermi liquid parameters characterizing these systems. This observation might be very important for cold-atom physics because PNEQ states have been created in number of cold atom systems.^{7-10,18} The observation that a paramagnetic ground state which, when forced to have finite nonequilibrium polarization, can mimic the behavior of a weak ferromagnet suggests that one needs to be careful while interpreting the experimental data.

IV. CONCLUSION

We studied the nonequilibrium magnetization dependence of the real part of the spin diffusion coefficient of a polarized nonequilibrium system. We predicted that this coefficient changes significantly with the variation of the nonequilibrium magnetization. We propose that a spin echo experiment should be able to test our predictions. We also showed that the polarized nonequilibrium system mimics the behavior of a weak ferromagnetic system.

¹M. Bouchiat, T. Carver, and C. Varnum, *Phys. Rev. Lett.* **5**, 373 (1960).

²F. D. Colegrove, L. D. Scheerer, and G. K. Walters, *Phys. Rev.* **132**, 2561 (1963).

³B. Castaing and P. Nozières, *J. Phys. (Paris)* **40**, 257 (1979).

⁴M. Chapellier, G. Frossati, and F. B. Rasmussen, *Phys. Rev. Lett.* **42**, 904 (1979).

⁵I. Zutic, J. Fabian, and S. D. Sarma, *Rev. Mod. Phys.* **76**, 323 (2004), and references therein.

⁶A. G. Mal'shukov, C. S. Tang, C. S. Chu, and K. A. Chao, *Phys. Rev. Lett.* **95**, 107203 (2005).

⁷A. J. Leggett, *Rev. Mod. Phys.* **73**, 307 (2001).

⁸W. Ketterle, *Rev. Mod. Phys.* **74**, 1131 (2002).

⁹A. Cho, *Science* **310**, 1892 (2005).

¹⁰M. W. Zwierlein, A. Schirotzek, C. H. Schunck, and W. Ketterle, *Science* **311**, 492 (2006), and references therein.

¹¹K. S. Bedell and H. P. Dahal, *Phys. Rev. Lett.* **97**, 047204 (2006).

- ¹²A. J. Leggett and M. J. Rice, *Phys. Rev. Lett.* **20**, 586 (1968).
- ¹³K. S. Bedell and D. E. Meltzer, *Phys. Rev. B* **33**, 4543 (1986).
- ¹⁴A. J. Leggett, *J. Phys. C: Solid State Phys.* **3**, 448 (1970).
- ¹⁵K. S. Bedell and K. B. Blagoev, *Phil. Mag. Lett.* **81**, 511 (2001).
- ¹⁶G. Baym and C. Pethick, *Landau Fermi-liquid Theory* (Wiley, New York, 1991).
- ¹⁷G. Baym and C. Pethick, in *The Physics of Liquid and Solid Helium, Part II*, edited by K. H. Benemann and J. B. Ketterson (Wiley, New York, 1978).
- ¹⁸G. B. Jo, Y. R. Lee, J. H. Choi, C. A. Christensen, T. H. Him, J. H. Thywissen, D. E. Pritchard, and W. Ketterle, *Science* **325**, 1521 (2009).