

Anomalous energy transport across topological insulator superconductor junctions

Jie Ren^{1,*} and Jian-Xin Zhu^{1,2,†}

¹Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

²Center for Integrated Nanotechnologies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

(Received 28 October 2012; revised manuscript received 1 April 2013; published 15 April 2013)

We study the nonequilibrium energy transport across a topological insulator superconductor junction by deriving an interfacial heat current formula through the scattering wave approach. Several anomalous thermal properties are uncovered, such as thermal energy's Klein tunneling, asymmetric Kapitza resistance, and negative differential thermal resistance. We expect these findings will have potential applications for energy control in various hybridized mesoscopic systems.

DOI: [10.1103/PhysRevB.87.165121](https://doi.org/10.1103/PhysRevB.87.165121)

PACS number(s): 44.10.+i, 05.60.Gg, 73.40.-c, 74.78.-w

Topological insulators (TIs), characterized by a bulk gap and a gapless surface mode with a Dirac-like linear dispersion, are presently one of the most interesting topics in condensed-matter physics.^{1,2} Their conducting surface states in the insulating gap are topologically protected by time-reversal symmetry, hence they are robust to disorders and perturbations, potentially leading to various device applications. With the help of doping-induced superconductivity in TIs³ or by depositing superconducting materials on TIs due to the proximity effect,⁴⁻⁶ the interplay between the superconducting ordering and the gapless chiral surface state has triggered much interest.²

On the one hand, such topological insulator superconductor (TI/S) junctions have been used to create chiral Majorana fermions for topological quantum computations and for the study of their impacts on electronic tunneling properties.⁷⁻¹¹ However, the thermal properties of such systems have not yet attracted an equal amount of attention. On the other hand, hybrid superconducting circuits are widely used for quantum computing and simulations,^{12,13} where managing heat dissipation at cryogenic temperatures becomes important for the right device operation. Thus, in view of the fact that hybrid topological superconductor junctions could be a natural candidate for quantum simulations and computing, the bottleneck in the future will be efficiently managing heat dissipation/refrigeration and controlling energy transport in such systems. Therefore, understanding the thermal properties of such hybrid mesoscopic structures at cryogenic temperatures will be crucial for future quantum and nanotechnology.^{14,15} It even has the potential to open up a rich variety of thermal device concepts in superconducting-circuit-based hybrid systems, just like thermal diodes, thermal transistors, thermal logic gates, and thermal memories in dielectric phononics.¹⁶

In this paper, we study the nonequilibrium energy transport across a TI/S junction interface and uncover its anomalous thermal properties, such as thermal energy's Klein tunneling, asymmetric Kapitza resistance, and negative differential thermal conductance (NDTC). Among them, Kapitza resistance measures the interfacial thermal resistance when thermal energy flows through the interface between two different materials.^{17,18} Asymmetric Kapitza resistance is one unusual thermal property by which the interface acts as a good thermal conductor if a positive thermal bias is applied, while with a negative thermal bias it exhibits poor thermal conduction, thus

effectively acting as a thermal insulator. As such, it functions as a thermal rectifier or diode (for a review, see Refs. 16 and 19). The NDTC, another unusual thermal transport phenomenon in which the heat current across a thermal conductor decreases when the temperature bias increases (for a review, see Ref. 16), is an essential element for the construction of thermal transistors and thermal logic gates, and it has been shown to exist in many anharmonic lattice systems. These concepts were usually restricted to pure phononic systems, where the thermal energy is carried by quantized lattice vibrations, i.e., phonons. Here, we report similar findings in superconducting hybridized mesoscopic junctions, which could extend the conceptual thermal devices in dielectrics into superconductor-based hybrid systems and could have great potential applications at cryogenic temperatures²⁰ in the near future.

As depicted in Fig. 1, we consider a two-dimensional (2D) TI/S junction attached to the respective thermal reservoirs at T_L and T_R , with an insulating barrier locating at $x \in [0, d]$. The 2D TI junction part ($x < 0$) could be formed on the surface of 3D topological insulators, the S junction part ($x > d$) could be induced via the proximity effect, and the barrier (B) part could be constructed by gating voltage or doping. The bulk of TI is a bad insulator of electrons, and when doped or with other disorders, both electrons and phonons will be strongly scattered but with the surface electronic state unaffected (so called topologically protected). Thus, the bulk contribution to the energy transport will be seriously suppressed and negligible. Moreover, the phonon contribution is not only suppressed seriously by the doping and disorders in the bulk, but also blocked by the barrier layer located at the interface between the TI and S sides. When the TI and S sides have a large lattice mismatch, the phonon contribution will be further reduced. Therefore, in the present work we only focus on the thermal transport contributed by the surface metallic states.

We assume the same Fermi level in both TI and S, and employ the Bogoliubov-de Gennes (BdG) equation $\hat{H}\psi = E\psi$ to study the thermal transport properties. The Hamiltonian of the surface state on the topological superconductor is given by^{1,2}

$$\hat{H} = \begin{pmatrix} \hat{H}_0(\mathbf{k}) - E_F & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{H}_0^*(-\mathbf{k}) + E_F \end{pmatrix} \quad (1)$$

acting on a Nambu basis $(\psi_{e\uparrow}, \psi_{e\downarrow}, \psi_{h\uparrow}, \psi_{h\downarrow})$, with $\hat{H}_0(\mathbf{k}) = \hbar v_F(k_x \hat{\sigma}_y - k_y \hat{\sigma}_x) + U_0 \Theta(x) \Theta(d - x)$ and $\hat{\Delta} = i \hat{\sigma}_y \Delta(\theta, T)$

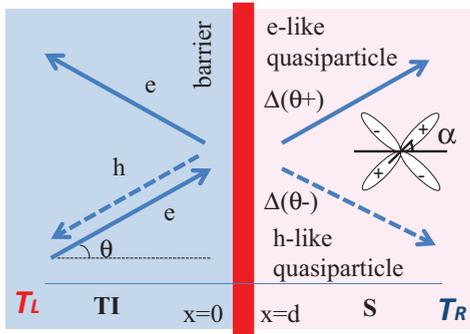


FIG. 1. (Color online) Schematic illustration of the reflections and transmissions at the TI/S interfaces.

$\Theta(x-d)$. Here, v_F is the Fermi velocity, $\hat{\sigma}_{x(y)}$ denote Pauli matrices, U_0 is the barrier potential, and $\Delta(\theta, T)$ depicts the order parameter with a given pairing symmetry and temperature dependencies. Throughout the work, we consider $E_F \gg (|\Delta|, E)$ to satisfy the mean-field nature of the BdG approach, i.e., the superconducting coherence length $\hbar v_F/|\Delta|$ is much larger compared to the Fermi wavelength $\hbar v_F/E_F$. Adjusting E_F can be achieved via doping or gate voltage.

Since in superconductor junctions the quasiparticles are carriers of thermal energy, we need to obtain the transmission (equivalently the reflection) coefficients of quasiparticles in order to investigate the thermal transport properties. Considering the conservation law for particle current, we can simplify the problem by merely considering the particle (including both electron and hole) current in the side of TI. Defining $f = \begin{pmatrix} \psi_{e\uparrow} \\ \psi_{e\downarrow} \end{pmatrix}$ and its hole counterpart $g = \begin{pmatrix} \psi_{h\uparrow} \\ \psi_{h\downarrow} \end{pmatrix}$, we have the probability density for finding either an electron or a hole, $P = |f|^2 + |g|^2$. By using the BdG equation $i\hbar\partial_t \begin{pmatrix} f \\ g \end{pmatrix} = \hat{H} \begin{pmatrix} f \\ g \end{pmatrix}$ with Eq. (1), and considering the continuity equation $\partial_t P + \nabla \cdot J_P = 0$, we obtain the x component of the particle current:

$$J_P^x = v_F(f^\dagger \hat{\sigma}_y f - g^\dagger \hat{\sigma}_y g). \quad (2)$$

Note that the hole current is a time-reversed counterpart of the electron contribution so that it naturally obtains an opposite sign compared with the electron current. If we express the whole wave function at the TI side, $\Psi_{\text{TI}} := \begin{pmatrix} f \\ g \end{pmatrix}$, in terms of the normal reflection amplitude b and Andreev reflection²¹ (for electron-hole conversion) amplitude a , which will be defined explicitly below, and substitute $\begin{pmatrix} f \\ g \end{pmatrix}$ into Eq. (2), we then obtain the particle current as

$$J_P^x(E, \theta) = 2v_F \cos \theta [1 - |a(E, \theta)|^2 - |b(E, \theta)|^2]. \quad (3)$$

This expression of the particle current has a clear physics picture that $\tilde{\kappa}(E, \theta) := 1 - |a(E, \theta)|^2 - |b(E, \theta)|^2$ denotes the transmission of energy carriers with energy E and incident angle θ , $v_F \cos \theta$ denotes the effective velocity in the x direction, and 2 is the spin degeneracy. Considering the carrier's energy E , the Fermi occupation difference between two sides of the interface $f_L - f_R = \frac{1}{e^{E/\hbar k_B T_L} + 1} - \frac{1}{e^{E/\hbar k_B T_R} + 1}$, and summation over all possible incidence angles and momenta $\sum_k \int_{-\pi/2}^{\pi/2} d\theta = \frac{1}{2\pi} \int dE \int_{-\pi/2}^{\pi/2} d\theta (dE/dk)^{-1} = \int dE \int_{-\pi/2}^{\pi/2} d\theta \frac{1}{2\pi \hbar v_F}$, we

arrive at the energy current expression:

$$J_Q = \frac{2}{h} \int_{-\infty}^{\infty} dE E \kappa(E) [f_L - f_R], \quad (4)$$

with $\kappa(E) = \int_{-\pi/2}^{\pi/2} d\theta \cos \theta \tilde{\kappa}(E, \theta) = \int_{-\pi/2}^{\pi/2} d\theta \cos \theta (1 - |a|^2 - |b|^2)$. Note that this expression is general and is obtained before solving the reflection coefficients a, b . In fact, similar expressions of the energy current in 1D topological-trivial metal superconductor junctions have been obtained by a rigorous derivation through linear response of entropy production²² or by a heuristic argument.²³ The latter was then applied to the superconducting graphene systems.^{24,25} It is worth emphasizing that all the transports considered in this work are charge-neutral, i.e., the carriers transport only thermal energy without charge current. In fact, if we follow the same procedure for charge transport, we will arrive at a similar expression for electric current: $J_e = \frac{2e}{h} \int dE \int d\theta \cos \theta \tilde{\sigma}(E, \theta) [f_L - f_R]$, with $\tilde{\sigma} = 1 + |a|^2 - |b|^2$, similar to the BTK formula.^{21,26} One can then see that due to the even symmetry of $\tilde{\sigma}$ [$\tilde{\sigma}(E, \theta) = \tilde{\sigma}(-E, \theta)$] and the odd symmetry of $f_L - f_R$ with respect to E , J_e vanishes as zero. This is also a consequence of the particle-hole symmetry in our system.

We now proceed to determine the scattering coefficients of Andreev reflection amplitude a and normal reflection amplitude b by imposing the boundary conditions on the wave functions at the interfaces of the barrier. Diagonalizing Eq. (1) straightforwardly yields the wave functions in the TI, barrier, and S regions. In the TI region ($x < 0$), for electrons and holes traveling in the $\pm x$ direction with a conserved transverse momentum k_y and energy E measured from E_F , the wave functions are given as

$$\begin{aligned} \psi_{\text{TI}}^{e\pm} &= (1, \pm i e^{\pm i\theta}, 0, 0) e^{i(\pm k_x x + k_y y)}, \\ \psi_{\text{TI}}^{h\pm} &= (0, 0, i, \pm e^{\pm i\theta}) e^{i(\mp k_x x + k_y y)}, \end{aligned}$$

where $k_x = E_F \cos \theta / (\hbar v_F)$ and $\theta = \arcsin(\hbar v_F k_y / E_F)$ is the angle of incidence. Note that we have used the mean-field condition $E_F \gg E$. In the barrier region ($0 < x < d$), we employ the thin barrier limit:²⁷ $d \rightarrow 0$, $U_0 \rightarrow \infty$ but with a constant product $Z \equiv U_0 d / (\hbar v_F)$, characterizing the strength of the insulating barrier. We then obtain

$$\begin{aligned} \psi_{\text{B}}^{e\pm} &= (1, \pm i, 0, 0) e^{i(\pm Zx/d + k_y y)}, \\ \psi_{\text{B}}^{h\pm} &= (0, 0, i, \pm 1) e^{i(\mp Zx/d + k_y y)}. \end{aligned}$$

In the S region ($x > d$), the electron (hole) -like quasiparticles are mixtures of electrons and holes. Thus, the transmitted wave functions have the forms

$$\begin{aligned} \psi_{\text{S}}^{e+} &= (1, i e^{i\theta}, -i \Gamma_+ e^{i(\theta - \phi_+)}, \Gamma_+ e^{-i\phi_+}) e^{i(k_x x + k_y y)}, \\ \psi_{\text{S}}^{h+} &= (\Gamma_-, -i \Gamma_- e^{-i\theta}, i e^{-i(\theta + \phi_-)}, e^{-i\phi_-}) e^{i(-k_x x + k_y y)}, \end{aligned}$$

where $e^{i\phi_{\pm}} = \Delta(\theta_{\pm}, T) / |\Delta(\theta_{\pm}, T)|$ with $\theta_+ = \theta$, $\theta_- = \pi - \theta$, and $\Gamma_{\pm} = v_{\pm} / u_{\pm}$, with $u_{\pm}^2 = \frac{1}{2} [1 + \sqrt{E^2 - |\Delta(\theta_{\pm}, T)|^2 / |E|}] = 1 - v_{\pm}^2$. For a d -wave pairing symmetry, $\Delta(\theta, T) = \Delta(T) \cos(2\theta - 2\alpha)$ with $\Delta(T) = \Delta_0 \tanh[(\pi k_B T_c / \Delta_0) \sqrt{0.953(T_c/T - 1)}]$.²⁸ Here T_c is the critical temperature, Δ_0 denotes the superconducting gap at zero temperature, and α is the angle between the normal direction of the barrier interface and the x axis of the $d_{x^2-y^2}$ -wave superconductor. By taking

into account the boundary conditions

$$\Psi_{\text{TI}}|_{x=0} = \Psi_{\text{B}}|_{x=0}, \quad \Psi_{\text{B}}|_{x=d} = \Psi_{\text{S}}|_{x=d},$$

with $\Psi_{\text{TI}} = \psi_{\text{TI}}^{e+} + b\psi_{\text{TI}}^{e-} + a\psi_{\text{TI}}^{h-}$, $\Psi_{\text{S}} = t_e\psi_{\text{S}}^{e+} + t_h\psi_{\text{S}}^{h+}$, and $\Psi_{\text{B}} = r_1\psi_{\text{B}}^{e+} + r_2\psi_{\text{B}}^{e-} + r_3\psi_{\text{B}}^{h+} + r_4\psi_{\text{B}}^{h-}$, the Andreev and normal reflection coefficients are found to be

$$a = \frac{-\cos^2 \theta \Gamma_+ e^{i(\theta - \phi_+)}}{\cos^2 \theta + \sin^2 Z \sin^2 \theta (1 - \Gamma_+ \Gamma_- e^{i(\phi_- - \phi_+)})},$$

$$b = \frac{\sin Z \sin \theta (\cos Z \cos \theta - i \sin Z) (1 - \Gamma_+ \Gamma_- e^{i(\phi_- - \phi_+)})}{-e^{-i\theta} [\cos^2 \theta + \sin^2 Z \sin^2 \theta (1 - \Gamma_+ \Gamma_- e^{i(\phi_- - \phi_+)})]}.$$

Finally, using the obtained coefficients a and b , we get

$$\tilde{\kappa}(E, \theta) := 1 - |a(E, \theta)|^2 - |b(E, \theta)|^2$$

$$= \tilde{\kappa}_{\text{TI}} \frac{1 - \tilde{\kappa}_{\text{TI}} |\Gamma_+|^2 + (\tilde{\kappa}_{\text{TI}} - 1) |\Gamma_+ \Gamma_-|^2}{|1 + (\tilde{\kappa}_{\text{TI}} - 1) \Gamma_+ \Gamma_- e^{i(\phi_- - \phi_+)}|^2}, \quad (5)$$

where

$$\tilde{\kappa}_{\text{TI}} := \tilde{\kappa}(|E| \gg \Delta_0, \theta) = \frac{\cos^2 \theta}{\cos^2 \theta + \sin^2 Z \sin^2 \theta} \quad (6)$$

is reminiscent of the relativistic Klein tunneling,²⁹ as a consequence of the spin-orbit coupling in TIs. The barrier becomes transparent for the thermal energy transport at the resonance condition, $Z := \frac{U_0 d}{\hbar v_F} = n\pi, n = 0, \pm 1, \dots$ (such that $\sin Z = 0$), or at the normal incidence ($\theta = 0$). Equations (5) and (6) are one of the main results, which enable us to uncover in the following the anomalous thermal properties, such as thermal energy's Klein tunneling, asymmetric Kapitza resistance, and NDTC.

In the linear-response regime, $T_L = T + \delta T/2, T_R = T - \delta T/2, \delta T \rightarrow 0$, we have the thermal conductance:

$$G := \frac{J_Q}{\delta T} = \frac{2}{h} \int_{-\infty}^{\infty} dE \int_{-\pi/2}^{\pi/2} d\theta \frac{E^2 \cos \theta \tilde{\kappa}(E, \theta)}{4k_B T^2 \cosh^2\left(\frac{E}{2k_B T}\right)}. \quad (7)$$

As illustrated in Fig. 2(a), the oscillatory dependence of G on the barrier strength Z shows the Klein tunneling behavior of energy: thermal conductance anomalously increases when the barrier increases within $[(n + 1/2)\pi, (n + 1)\pi]$, consistent with the behavior of $\sin^2 Z$ in Eq. (6). G also has an oscillatory dependence on α , the rotation angle of the superconducting order parameter. As detailed in Fig. 2(b), when $Z = 0$, G monotonically decreases as α rotates from 0 to $\pi/4$, but when Z increases to $\pi/2$, G first increases and then decreases. This nonmonotonic behavior is due to the competition of Klein tunneling and the superconducting order parameter (gap), which we explain below.

In the S part, thermal energy is carried by the quasiparticles, which only transport beyond the gap Δ , so that the smaller Δ is, the larger is the thermal conductance. In other words, the rotation angle α adjusts G through adjusting the angle dependence of the superconducting gap. To further understand the competition between Klein tunneling and the order parameter Δ , we define the angle-resolved thermal conductance G_θ as $G = \int_{-\pi/2}^{\pi/2} d\theta G_\theta$ and plot it as a function of the incidence angle θ in Figs. 2(c1)–(c3). For the case of $Z = 0$, when $\alpha = 0$, G_θ has two peaks around $\theta = \pm\pi/4$ where Δ is gapless, while when α rotates to $\pi/4$, two peaks becomes a single peak around $\theta = 0$. As a result, G , the angle integration of

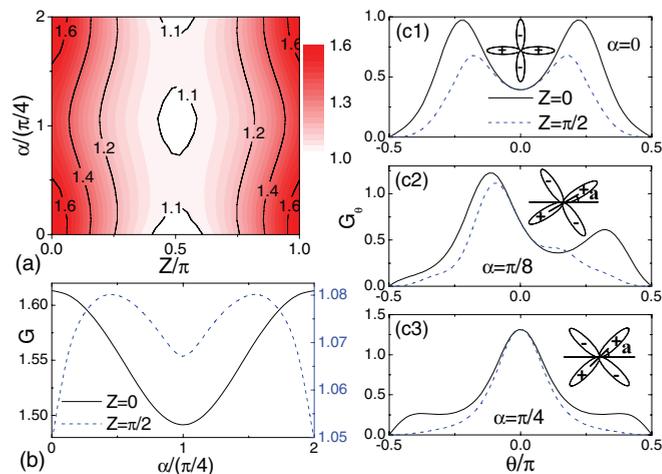


FIG. 2. (Color online) Thermal energy's Klein tunneling. (a) Thermal conductance as a function of the barrier strength Z and the rotation angle α of the d -wave order parameter. (b) Intersections of (a) for $Z = 0$, and $\pi/2$. (c) Angle-resolved thermal conductance G_θ for different α and Z . Solid lines are for $Z = 0$ while dashed ones are for $Z = \pi/2$. Parameters are $T = 70$ K, $T_c = 116$ K, and $\Delta_0 = 30$ meV. Thermal conductance G is in the unit of $\Delta_0 k_B/h$.

G_θ , decreases as α increases from 0 to $\pi/4$. For α changing from $\pi/4$ to $\pi/2$, the behavior is symmetrically reversed. When the barrier Z increases, the Klein tunneling comes into play. For the case of $Z = \pi/2$, when $\alpha = 0$, although there are still two peaks for thermal conductance, their intensity is suppressed dramatically. When $\alpha = \pi/4$, the gapless angle coincides with the normal incidence angle, the barrier becomes transparent. Therefore, the Klein tunneling helps to keep the single conductance peak (at $\theta = 0$) intensity unchanged. At the intermediate regime $\alpha = \pi/8$, one peak near $\theta = 0$ is preserved while the other peak far from $\theta = 0$ is repressed. As a consequence, the thermal conductance G increases first and then decreases within $\alpha \in [0, \pi/4]$ [see Fig. 2(b)]. It is also interesting to notice that at certain angles, the angle-resolved thermal conductance G_θ can be even enhanced by the nonzero barrier strength [see Fig. 2(c2)]. This anomalous behavior is a consequence of the competition of thermal Klein tunneling and the orientation angle of the d -wave superconductor. We note that these predictions would be validated by the present techniques of angle-resolved thermal transport measurements (for a review, see Ref. 30).

The asymmetric Kapitza resistance is essentially a non-linear response behavior. It is a consequence of different temperature responses of different materials at two sides of the interface.¹⁶ The inset of Fig. 3 shows typical J_Q behaviors at $T = 70$ K for $Z = 0$ (solid line) and $\pi/2$ (dashed line), via varying temperature bias. As a measure of the asymmetry, we define the rectification ratio $R = |J_-/J_+|$, where J_+ refers to thermal current when $\delta T = T_L - T_R > 0$ while J_- refers to thermal current after switching the temperature bias $T_L \leftrightarrow T_R$. As shown in Fig. 3, when temperature bias is apart from 0, R deviates further from 1. Except for the low-temperature case (e.g., $T = 35$ K), the other three examples show that increasing bias does not always increase the rectification ratio at the large bias regime. The results also indicate

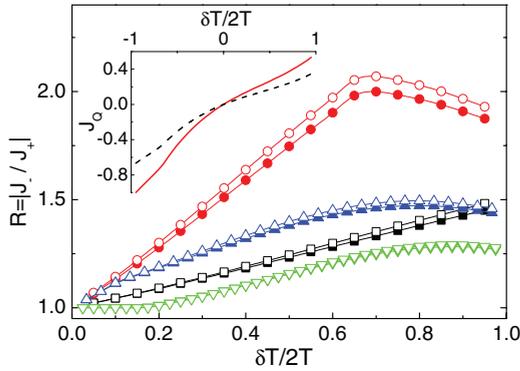


FIG. 3. (Color online) Rectification ratios as a function of temperatures. Values apart from 1 indicate asymmetric Kapitza resistances. \blacksquare ($T = 35$ K), \bullet ($T = 70$ K), \blacktriangle ($T = 104$ K), \blacktriangledown ($T = 140$ K) are denoted for $Z = 0$. Their hollow counterparts are for cases of $Z = \pi/2$. Inset shows one example of the J_Q profiles depending on temperature bias for $Z = 0$ (solid) and $\pi/2$ (dashed), at $T = 70$ K. $\alpha = 0$. Other parameters are the same as in Fig. 2. The energy current J_Q is in the unit of Δ_0^2/h .

that although the insulating barrier changes the J_Q profiles quite noticeably, it does not change the rectification ratio significantly, which is even slightly enhanced by the barrier. In addition, R has a nonmonotonic temperature dependence so that increasing T first increases and then decreases R , as exemplified by the highest curves for $T = 70$ K in Fig. 3. This is reasonable that the asymmetric Kapitza resistance in our system results from the different temperature responses of the topological insulator and superconductor at the sides of the interface: At higher temperature, the superconducting gap diminishes so that the superconductor tends to the topological insulator and both sides of the interface tend to have the same temperature response, which explains the reduction in R . At lower temperatures, although two sides of the interface have distinct temperature responses, the bias δT cannot be larger than T such that small bias reduces R . Therefore, the optimal R appears at intermediate temperature.

The superconductor is usually a bad thermal conductor since its gap Δ forbids the existence of quasiparticles, which are responsible for the energy transport. Increasing the temperature T_R at the S part could increase the energy transmission $\kappa(E)$ [in Eq. (4)] by diminishing the superconducting gap to allow more thermal energy carriers, while simultaneously decreasing the temperature bias $\delta T = T_L - T_R$ as well as the occupation difference $f_L - f_R = \frac{1}{e^{E/k_B T_L} + 1} - \frac{1}{e^{E/k_B T_R} + 1}$. Once the increased $\kappa(E)$ is able to compensate for the loss in $f_L - f_R$ due to the decreased δT , we expect to observe NDTC, i.e., the energy current increases as the bias δT decreases. To characterize this anomalous behavior, we define the differential thermal conductance (DTC): $dJ_Q/d\delta T$, for given finite δT . Note that this is a nonlinear quantity similar to the nonlinear differential electrical conductance dI/dV , and it is different from the previously defined thermal conductance G in Eq. (7), which is a linear quantity at $\delta T \rightarrow 0$.

As a showcase, we calculate DTC in Fig. 4 with varying T_R in the S region but fixing T_L as a reference temperature. Indeed, NDTC appears as we expect, although it is absent for the low T_L case (e.g., $T_L = 70$ K $>$ T_R). Increasing T_L

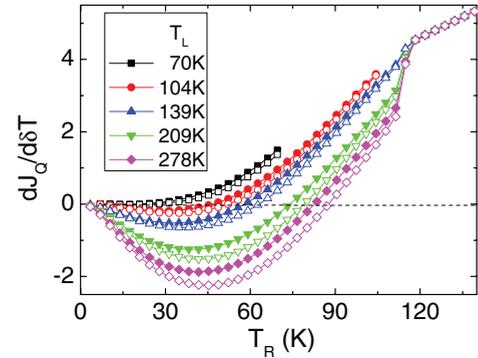


FIG. 4. (Color online) NDTC for various temperatures at $\alpha = 0$ (filled symbols) and $\alpha = \pi/4$ (hollow symbols). $Z = 0$. Other parameters are the same as in Fig. 2. The DTC $dJ_Q/d\delta T$ is in the unit of $\Delta_0 k_B/h$.

enhances the regime of NDTC. In addition, tuning the angle of superconducting pairing symmetry α can also slightly enhance NDTC, as shown in Fig. 4. However, when T_R increases across a threshold and approaches T_L , NDTC disappears. When the superconducting part is replaced with a topological insulator, there exists no NDTC. In fact, when $T_R > T_c$ (see 116 K in Fig. 4), the S part fades into nonsuperconducting TI, which causes the collapse of DTC beyond T_c . In this case, we can have a constant thermal transmission $\kappa(E) \approx \kappa$, where κ depends on the barrier strength. Then from Eq. (4), we have DTC: $dJ_Q/d\delta T = -dJ_Q/dT_R = \frac{2\kappa}{h} \int dE \frac{E^2}{4k_B T_R^2 \cosh^2[E/(2k_B T_R)]} = \frac{2\pi^2 k_B^2}{3h} \kappa T_R$, which explains the linear behavior beyond the critical temperature in Fig. 4.

Finally, we would like to point out that although we exemplified the anomalous thermal transport by a d -wave superconductor, in principle, when replaced with an s -wave superconductor, the results will be qualitatively the same. The asymmetric Kapitza and NDTC are consequences of the different temperature responses of both the nonsuperconducting TI side and the superconducting side. The thermal energy's Klein tunneling results from the Dirac-like linear dispersion of the materials. When competing with the orientation angle of the d -wave symmetry, the behavior of angle-resolved thermal transport becomes rich.

In summary, using the scattering wave approach, we have derived an interfacial heat current formula in a TI/S junction. With the help of this formula, we have studied the nonequilibrium energy transport across this interfacial system and have uncovered several anomalous thermal properties for the TI/S interface, such as thermal energy's Klein tunneling, asymmetric Kapitza resistance, and negative differential thermal resistance.

The asymmetric Kapitza resistance and NDTC have already been discussed in dielectric phonon systems at high (room) temperatures.¹⁶ But previous studies about the asymmetric resistance and NDTC focus on pure phononic systems and do not involve any superconductors or topological insulators. Here, we have uncovered the anomalous thermal transport in a hybrid topological insulator superconductor system.

One immediate advantage of this kind of hybrid system is that the thermal energy's Klein tunneling renders highly efficient heat dissipation even when the TI/S interface is

not perfect so as to produce a large barrier. However, in a normal metal superconductor system, the heat dissipating ability will be severely reduced by the large barrier induced by the imperfect interface.

Since hybrid superconductor and topological insulator systems are crucial for future quantum and nanotechnology at cryogenic temperatures, we believe that understanding the anomalous heat transport in such hybrid systems would be useful for managing heat dissipation in future cryogenic

devices and might even reveal the potential applications of such hybrid systems for the smart energy control at mesoscopic scales.

This work was supported by the National Nuclear Security Administration of the US DOE at LANL under Contract No. DE-AC52-06NA25396, the LDRD Program at LANL (J.R.), and in part by the Center for Integrated Nanotechnologies—a US DOE Office of Basic Energy Sciences user facility (J.X.Z.).

*renjie@lanl.gov

†jxzhu@lanl.gov

¹M. Z. Hasan and C. L. Kane, *Rev. Mod. Phys.* **82**, 3045 (2010).

²X.-L. Qi and S.-C. Zhang, *Rev. Mod. Phys.* **83**, 1057 (2011).

³Y. S. Hor, A. J. Williams, J. G. Checkelsky, P. Roushan, J. Seo, Q. Xu, H. W. Zandbergen, A. Yazdani, N. P. Ong, and R. J. Cava, *Phys. Rev. Lett.* **104**, 057001 (2010).

⁴L. Fu and C. L. Kane, *Phys. Rev. Lett.* **100**, 096407 (2008).

⁵L. Fu and C. L. Kane, *Phys. Rev. Lett.* **102**, 216403 (2009).

⁶X.-L. Qi, T. L. Hughes, and S.-C. Zhang, *Phys. Rev. B* **82**, 184516 (2010).

⁷A. R. Akhmerov, J. Nilsson, and C. W. J. Beenakker, *Phys. Rev. Lett.* **102**, 216404 (2009).

⁸Y. Tanaka, T. Yokoyama, and N. Nagaosa, *Phys. Rev. Lett.* **103**, 107002 (2009).

⁹K. T. Law, P. A. Lee, and T. K. Ng, *Phys. Rev. Lett.* **103**, 237001 (2009).

¹⁰J. Linder, Y. Tanaka, T. Yokoyama, A. Sudbø, and N. Nagaosa, *Phys. Rev. Lett.* **104**, 067001 (2010).

¹¹P. A. Iosevich and M. V. Feigel'man, *Phys. Rev. Lett.* **106**, 077003 (2011).

¹²J. Q. You and F. Nori, *Phys. Today* **58**(11), 42 (2005).

¹³J. Q. You and F. Nori, *Nature (London)* **474**, 589 (2011).

¹⁴F. Giazotto, T. T. Heikkilä, A. Luukanen, A. M. Savin, and J. P. Pekola, *Rev. Mod. Phys.* **78**, 217 (2006).

¹⁵V. Chandrasekhar, *Supercond. Sci. Technol.* **22**, 083001 (2009).

¹⁶N. Li, J. Ren, L. Wang, G. Zhang, P. Hänggi, and B. Li, *Rev. Mod. Phys.* **84**, 1045 (2012).

¹⁷G. L. Pollack, *Rev. Mod. Phys.* **41**, 48 (1969).

¹⁸E. T. Swartz and R. O. Pohl, *Rev. Mod. Phys.* **61**, 605 (1989).

¹⁹N. A. Roberts and D. G. Walker, *Int. J. Therm. Sci.* **50**, 648 (2011).

²⁰M. Nahum *et al.*, *Appl. Phys. Lett.* **65**, 3123 (1994); A. M. Clark *et al.*, *ibid.* **86**, 173508 (2005).

²¹G. E. Blonder, M. Tinkham, and T. M. Klapwijk, *Phys. Rev. B* **25**, 4515 (1982).

²²R. A. Riedel and P. F. Bagwell, *Phys. Rev. B* **48**, 15198 (1993).

²³A. Bardas and D. Averin, *Phys. Rev. B* **52**, 12873 (1995).

²⁴T. Yokoyama, J. Linder, and A. Sudbø, *Phys. Rev. B* **77**, 132503 (2008).

²⁵M. Salehi, M. Alidoust, and G. Rashedi, *J. Appl. Phys.* **107**, 123916 (2010); M. Salehi, M. Alidoust, Y. Rahnavard, and G. Rashedi *ibid.* **108**, 083917 (2010).

²⁶Y. Tanaka and S. Kashiwaya, *Phys. Rev. Lett.* **74**, 3451 (1995).

²⁷S. Bhattacharjee and K. Sengupta, *Phys. Rev. Lett.* **97**, 217001 (2006).

²⁸J. Tao, R. P. Prasankumar, E. E. M. Chia, A. J. Taylor, and J.-X. Zhu, *Phys. Rev. B* **85**, 144302 (2012).

²⁹O. Klein, *Z. Phys.* **53**, 157 (1929); M. I. Katsnelson, K. S. Novoselov, and A. K. Geim, *Nat. Phys.* **2**, 620 (2006).

³⁰Y. Matsuda, K. Izawa, and I. Vekhter, *J. Phys.: Condens. Matter* **18**, R705 (2006).