Spin orbit magnetism and unconventional superconductivity

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We find an exotic spin excitation in a magnetically ordered system with spin orbit magnetism in two dimensions, where the order parameter has a net spin current and no net magnetization. Starting from a Fermi liquid theory, similar to that for a weak ferromagnet, we show that this excitation emerges from an exotic magnetic Fermi liquid state that is protected by a generalized Pomeranchuk condition. We derive the propagating mode using the Landau kinetic equation and find that the dispersion of the mode has a $\sqrt{q}$ behavior in leading order in two dimensions. We find an instability toward superconductivity induced by this exotic mode, and a further analysis based on the forward scattering sum rule strongly suggests that this superconductivity has p-wave pairing symmetry. We perform similar studies in the three-dimensional case, with a slightly different magnetic system, and find that the mode leads to a Lifshitz-like instability, most likely toward an inhomogeneous magnetic state in one of the phases.

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I. INTRODUCTION

The Landau Fermi-liquid theory is a very successful theory in condensed-matter physics. It provides a phenomenological framework for describing thermodynamics, transport, and collective modes of itinerant fermionic systems. In the Landau theory, the interactions among quasiparticles are described by the Landau parameters $F_l^{s,a}$, where l denotes the orbital angular momentum partial-wave channel, and s and a denote spin-symmetric and -antisymmetric channels, respectively. It has been proved by Pomeranchuk that for the Fermi surface to be stable, the Landau parameters should satisfy the relation $F_l^{s,a} > -(2l+1)$. Whenever the relation is violated, there will exist an instability of the Fermi surface known as a Pomeranchuk instability,[1] such as the Stoner ferromagnetism when $F_0^a \to -1^+$ or phase separation when $F_0^s \to -1^+$. In 1959, Abrikosov and Dzyaloshinskii[2] developed a ferromagnetic Fermi-liquid theory (FFLT) of itinerant ferromagnetism based on Landau Fermi-liquid theory, whose microscopic foundations were established later by Dzyaloshinskii and Kondratenko.[3] Further studies have been made of this state using a generalized Pomeranchuk instability based on the FFLT of Blagoev *et al.*[4] and Bedell and Blagoev.[5]

Recently, Pomeranchuk instabilities in higher angular momentum partial-wave channels have been studied by many authors, such as the quantum nematic Fermi-liquid phase as a result of an instability in the F_2^s channel[6–11] and the so-called α and β phases the in F_1^a channel by Wu *et al.*[12,13] In these papers, mean-field theory is used based on the microscopic Hamiltonian to demonstrate the instabilities of the disordered phase and to classify the possible phases of the ordered state. The Goldstone modes are studied within the random-phase-approximation (RPA) approach. Among these instabilities, the F_1^a channel is especially interesting since the order parameter, which is proportional to the spin current, is closely related to the spin orbit coupling. In fact, the spin-orbit-coupled Fermi liquid is well studied.[14–16] In this system, due to the broken spin rotation symmetry, Landau parameters with more general forms are calculated,[16] and new collective modes induced by spin orbit coupling are studied.[15]

In this paper, we study a system that dynamically generates the spin-orbit coupling through the generalized Pomeranchuk instability in the F_1^a channel. This system was first studied by Wu and Zhang,[12] where mean-field theory was used based on the microscopic Hamiltonian to demonstrate the instabilities of the disordered phase and to classify the possible phases of the ordered state and Goldstone modes were studied within the RPA approach. Here, we use the traditional Fermi-liquid theory, similar to the FFLT, in the weak magnetic limit to study the generalized Pomeranchuk instability in the F_1^a channel in two-dimensional (2D) and three-dimensional (3D) systems. We start from the state with an ordered phase, using the Landau kinetic equation to study the collective modes. In this symmetry-broken phase we find an exotic collective mode. We further find a superconducting instability induced by this mode. We also carry out a similar calculation in a 3D system with a slightly different model and find the mode leads to a Lifshitz-like instability toward an inhomogeneous magnetic state.

II. GENERALIZED POMERANCHUK INSTABILITY

Similar to what was done in the weakly ferromagnetic system,[4] we expand the deviation of the energy around the ordered ground state in the spirit of Landau up to second order in the deviations, $\delta n_{\mathbf{p}\sigma}$ of the momentum distribution function:

$$\delta\left(\frac{\Omega}{V}\right) = \frac{1}{V}\sum_{\mathbf{p}\sigma}\left(\varepsilon^0_{\mathbf{p}\sigma} - \mu\right)\delta n_{\mathbf{p}\sigma} + \frac{1}{2V}\sum_{\mathbf{p}\sigma,\mathbf{p}'\sigma'} f^{\sigma\sigma'}_{\mathbf{p}\mathbf{p}'}\delta n_{\mathbf{p}\sigma}\delta n_{\mathbf{p}'\sigma'} + \cdots, \tag{1}$$

where $\varepsilon^0_{\mathbf{p}\sigma}$ is the quasiparticle energy and $f^{\sigma\sigma'}_{\mathbf{p}\mathbf{p}'}$ are the quasiparticle interactions in the presence of the internal field. In the limit of a weakly ordered system, we can treat the quasiparticle interaction as rotationally invariant in spin space;[17] then

$$f^{\sigma\sigma'}_{\mathbf{p}\mathbf{p}'} = f^s_{\mathbf{p}\mathbf{p}'} + f^a_{\mathbf{p}\mathbf{p}'}\sigma\cdot\sigma' + O\left(m_1^2\right). \tag{2}$$

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Given the distribution function for the ordered state, we can calculate the free energy, and the minimization of the free energy leads to the generalized Pomeranchuk condition.

III. MODEL IN TWO DIMENSIONS

In two dimensions, we start with the model

$$\mathbf{m}_{\mathbf{p}}^0(\mathbf{r}) = -\frac{1}{N(0)}\frac{\partial n_{\mathbf{p}}^0}{\partial \varepsilon_{\mathbf{p}}^0} m_1(\hat{\mathbf{z}}\times\hat{\mathbf{p}}). \tag{3}$$

This model defines a spin orbit magnetism (SOM) state with zero net magnetization but nonzero spin current proportional to m_1, which can be seen as

$$\sigma^0(\mathbf{r}) = 2\sum \mathbf{m}_{\mathbf{p}}^0(\mathbf{r}) = 0, \tag{4}$$

$$\begin{aligned}\mathbf{j}_{\sigma,\mathbf{i}} &= 2\sum_{\mathbf{p}} v_{\mathbf{p},\mathbf{i}}\mathbf{m}_{\mathbf{p}}^0(\mathbf{r})\left(1+\frac{F_1^a}{2}\right)\\ &= \frac{1}{2}v_f\left(1+\frac{F_1^a}{2}\right)m_1(\hat{\mathbf{z}}\times\hat{\mathbf{i}}).\end{aligned} \tag{5}$$

To understand the instability to this ground state, we first use Eq. (1) to calculate the free-energy change based on this model using $[\delta n_{\mathbf{p}}] = \mathbf{m}_{\mathbf{p}}^0\cdot\vec{\sigma}$ in spin space:

$$\delta\left(\frac{\Omega}{V}\right) = \frac{1}{N(0)}\left(1+\frac{F_1^a}{2}\right)m_1^2 + \beta m_1^4 + \cdots, \tag{6}$$

which means that this ground state is protected by a generalized Pomeranchuk condition in the F_1^a channel since we work in the ordered state. Here, $\beta > 0$ is a phenomenological parameter making sure the model is valid and this term is the next-leading-order term allowed by symmetry. The minimum of the free energy for $F_1^a < -2$ leads to the equilibrium order parameter (ground-state spin current) $\overline{m}_1 \sim |1+\frac{F_1^a}{2}|^{\frac{1}{2}}$, and in the limit $F_1^a \to -2^-$, $\overline{m}_1$ is small, i.e., in the weakly ordered limit.

A. Collective modes

In this section, we study an important feature of this new exotic magnetic Fermi liquid (EMFL) ground state, the collective modes, which exhibit exotic dispersion relations.

1. *Hydrodynamic-like approach*

We investigate the free oscillation of the momentum dependent magnetization $\delta\mathbf{m}_{\mathbf{p}}$. These oscillations of $\delta\mathbf{m}_{\mathbf{p}}$ can be determined from the linearized Landau kinetic equation in the spin channel:[18]

$$\begin{aligned}&\frac{\partial\delta\mathbf{m}_{\mathbf{p}}(\mathbf{r},t)}{\partial t} + \mathbf{v}_{\mathbf{p}}\cdot\nabla\left[\delta\mathbf{m}_{\mathbf{p}}(\mathbf{r},t) - \frac{\partial n_{\mathbf{p}}^0}{\partial\varepsilon_{\mathbf{p}}^0}\delta\mathbf{h}_{\mathbf{p}}(\mathbf{r},t)\right]\\ &\quad= -2\left[\mathbf{m}_{\mathbf{p}}{}^0(\mathbf{r},t)\times\delta\mathbf{h}_{\mathbf{p}}(\mathbf{r},t) + \delta\mathbf{m}_{\mathbf{p}}(\mathbf{r},t)\times\mathbf{h}_{\mathbf{p}}{}^0(\mathbf{r},t)\right]\\ &\qquad + I[\mathbf{m}_{\mathbf{p}}],\end{aligned} \tag{7}$$

where $\mathbf{h}_{\mathbf{p}}^{\mathbf{0}} = -\mathbf{B} + 2\sum_{\mathbf{p}'} f_{\mathbf{pp}'}^a\mathbf{m}_{\mathbf{p}}'$ and $\delta\mathbf{h}_{\mathbf{p}} = -\delta\mathbf{B} + 2\sum_{\mathbf{p}'} f_{\mathbf{pp}'}^a\delta\mathbf{m}_{\mathbf{p}}'$ are the effective equilibrium field and its fluctuation, respectively. To study the free oscillations when $\mathbf{B}=0$ we set $\delta\mathbf{B}=0$. At low temperature the collision integral $I[\mathbf{m}_{\mathbf{p}}]$ is negligible, and it can be ignored in what follows.

To derive the dispersion relations, we do a Fourier transformation of Eq. (7), plug in our model, Eq. (3), and set $\delta\mathbf{m}_{\mathbf{p}}(\mathbf{q}) = (-\frac{1}{N(0)})\frac{\partial n_{\mathbf{p}}^0}{\partial\varepsilon_{\mathbf{p}}^0}\vec{\nu}_{\mathbf{p}}(\mathbf{q}) = \sum_l(-\frac{1}{N(0)})\frac{\partial n_{\mathbf{p}}^0}{\partial\varepsilon_{\mathbf{p}}^0}\vec{\nu}_l(\mathbf{q})e^{il\phi_{\mathbf{p}}}$. Finally, Eq. (7) becomes

$$\begin{aligned}&\sum_{l,m}\left[\omega - \mathbf{q}\cdot\mathbf{v}_{\mathbf{p}}\left(1+\frac{F_{|l|}^a}{a_l}\right)\right]\vec{\nu}_l(\mathbf{q})e^{il\phi_{\mathbf{p}}}\\ &\quad= 2m_1 i\sum_{l,m}\left(\frac{f_1^a}{2} - \frac{f_{|l|}^a}{a_l}\right)(\hat{\mathbf{z}}\times\hat{\mathbf{p}})\times\vec{\nu}_l(\mathbf{q})e^{il\phi_{\mathbf{p}}},\end{aligned} \tag{8}$$

where $a_l = \delta_{l,0} + 2(1-\delta_{l,0})$.

Projecting Eq. (7) to each component of $e^{il\phi_{\mathbf{p}}}$, we take the $l = 0,1,-1$ component of the equation and keep the expansion of F_l^a only up to the $l=1$ term. The equations for the $l=0$, $l=1$, and $l=-1$ momenta are

$$\omega\vec{\nu}_0 - \frac{qv_f}{2}\left(1+\frac{F_1^a}{2}\right)e^{i\phi_{\mathbf{q}}}\vec{\nu}_1 - \frac{qv_f}{2}\left(1+\frac{F_1^a}{2}\right)e^{-i\phi_{\mathbf{q}}}\vec{\nu}_{-1} = 0, \tag{9}$$

$$\omega\vec{\nu}_1 - \frac{qv_f}{2}(1+F_0^a)e^{-il\phi_{\mathbf{q}}}\vec{\nu}_0 - m_1 i\left(f_0^a - \frac{f_1^a}{2}\right)\vec{\nu}_0\times\mathbf{L}_1 = 0, \tag{10}$$

$$\omega\vec{\nu}_{-1} - \frac{qv_f}{2}(1+F_0^a)e^{il\phi_{\mathbf{q}}}\vec{\nu}_0 - m_1 i\left(f_0^a - \frac{f_1^a}{2}\right)\vec{\nu}_0\times\mathbf{L}_2 = 0, \tag{11}$$

where $\mathbf{L}_1 = (i,1,0)$ and $\mathbf{L}_2 = (-i,1,0)$ are two complex vectors. Considering each component of the vectors, we can solve these nine equations and get the dispersion relation of the collective modes. The dispersion relations for the gapless modes are given by

$$\omega_c = \pm\frac{1}{2}\sqrt{|2+F_1^a|(2f_0^a - f_1^a)m_1v_fq - |2+F_1^a|(1+F_0^a)v_f^2q^2} \to \pm\frac{1}{2}\sqrt{|2+F_1^a|(2f_0^a - f_1^a)m_1v_fq}. \tag{12}$$

In this hydrodynamic-like approach, the truncation of the Fermi-surface distortions up to $l=1$ is reasonable since if we include the $l=2$ distortion terms, we will find that $\frac{|\vec{\nu}_{\pm2}|}{|\vec{\nu}_{\pm1}|} = \frac{qv_f}{2\omega}(1+\frac{F_1^a}{2})$, which is very small for small momentum transfer. In this sense, the inclusion of $\vec{\nu}_{\pm2}$ will not qualitatively change the dispersion of the collective modes.

To determine if the mode in this EMFL is propagating or Landau damped, we need to consider the particle-hole (p-h) continuum. The continuum can be determined from Eq. (7), and for two dimensions we find that $\omega_{\mathrm{ph}}^{\pm} = \mathbf{q}\cdot\mathbf{v}_{\mathbf{p}} \pm m_1|f_1^a|$.

This mode is very exotic since it propagates with a $\sqrt{q}$ dispersion relation for small momentum, unlike the magnons

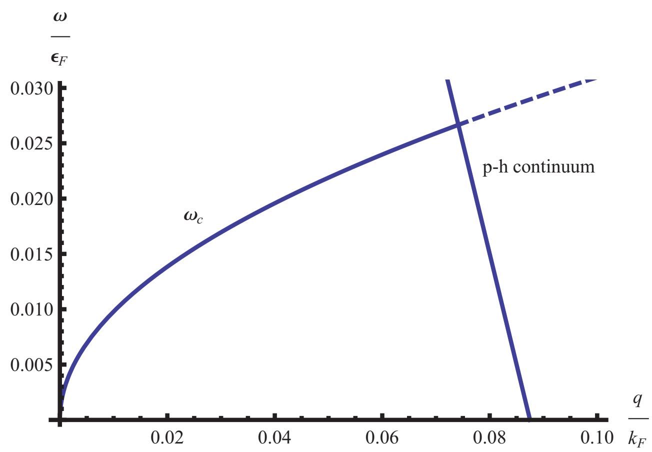

FIG. 1. (Color online) Collective mode together with the p-h continuum in a 2D system. The dashed line indicates that the collective mode merges inside the continuum. Here, we take $F_0^a = 0.1$, $F_1^a = -2.1$, $m_1 = 0.12n$, and n is the particle density.

found in the ferromagnetic and antiferromagnetic phases. We realize that, due to the $\sqrt{q}$ dispersion, this mode will have higher-order temperature dependence in, e.g., the specific heat, making it difficult to detect in low-temperature specific-heat measurements. Given that it is separated from the p-h continuum, it may be possible using neutron scattering to detect this spin mode. Taking reasonable values of the Landau parameters and the order parameter, we evaluate the dispersion relation of the collective mode and p-h continuum. The result is presented in Fig. 1.

In Fig. 1, we show the collective mode together with the p-h continuum. Clearly, we can see that this gapless mode can propagate for small momentum and merges into the continuum for relatively large momentum.

2. Dynamical response function

We can check the validity of the hydrodynamic approach, for studying the collective modes, by calculating the dynamical spin response function using $\vec{\chi} = -\frac{\vec{\nu}_0}{\delta B}$. Here we use the Landau kinetic equation [Eq. (7)],[19] where we keep $\delta\mathbf{B}$ in the equation. The details of the calculation are given in the Appendix. By solving for the poles of the spin response function, we can also get the dispersion of the collective mode:

$$\omega_c = \pm\frac{1}{2}\sqrt[4]{1 - \frac{f_0^a}{2f_0^a - f_1^a}}\sqrt{\left|2 + F_1^a\right|\left(2f_0^a - f_1^a\right)m_1 v_f q}, \tag{13}$$

which is consistent with the result we found in the previous hydrodynamic-like approach with $\sqrt{q}$ dispersion in leading order. In comparing Eqs. (11) and (12), the leading-order behavior is not exactly the same. This is because in the previous hydrodynamic-like approach, we truncated the Fermi-surface distortion at $l = 1$. In the calculation of the response function, we truncate the Landau parameters at $l = 1$, but we keep the Fermi-surface distortion to all orders. Although the inclusion of the higher-order distortions will not dramatically change the leading-order $\sqrt{q}$ behavior, which is already shown above, it can still slightly modify the prefactor.

B. Superconductivity instability

In this EMFL state it is possible that the new spin wave mode could give rise to a superconducting instability. The response function for this mode for small momentum and energy transfer is approximately given by

$$\chi \sim \frac{2N(0)}{F_0^a - \frac{F_1^a}{2}}\frac{\omega_c^2}{\omega^2 - \omega_c^2}. \tag{14}$$

The structure resembles that of the response function of a phonon, which makes it possible that the spin-fluctuation-mediated interaction can cause the pairing of two quasiparticles and lead to superconductivity. Since this pairing is caused by spin fluctuations, we expect that the superconductivity is unconventional, in the sense that the pairing symmetry is different from the normal s-wave phonon-mediated superconductors. It is actually p wave, which is demonstrated below by the argument from the forward scattering sum rule.

Within the framework of Landau Fermi-liquid theory, based on the forward scattering sum rule,[4,18] we can demonstrate the instability towards superconductivity and analyze the pairing symmetry of it. In Fermi-liquid theory, the scattering amplitude for small momentum transfer can be expanded as $N(0)a_{\mathbf{pp}'}^{\sigma\sigma'} = \sum_l (A_l^s + A_l^a\sigma\sigma')P_l(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$.[18] In the case of weak magnetic ordering the quasiparticle scattering amplitude A_l^α can be expressed by Landau parameters as $A_l^\alpha = \frac{F_l^\alpha}{1+F_l^\alpha/a_l}$, where $\alpha = a,s$ and a_l has the same definition as above.[18] The forward scattering sum rule states that the triplet scattering of two quasiparticles with the same momenta must vanish. Therefore, to the leading order of m_1, we have $\sum_l(A_l^s + A_l^a) = 0$. Since in our model, we only consider the interaction up to $l = 1$, we can truncate the equation up to $l = 1$; then

$$A_0^a + A_0^s + A_1^a + A_1^s = 0. \tag{15}$$

In our magnetically ordered state close to the phase transition, $F_1^a \to -2^-$, and it follows that $A_1^a \to +\infty$, which requires at least one of the first three terms in Eq. (15) to diverge as $-\infty$ when approaching the transition point. First, the diverging of A_1^s implies the vanishing of the effective mass, and since we assume a finite density of state on the Fermi surface, it will not occur in our system. Then only A_0^s and A_0^a are left to satisfy Eq. (15). Taking A_0^s as an example, let $A_0^s \to -A_1^a$ diverge to $-\infty$, which indicates instabilities in both spin and charge sectors, respectively. This leads to phase separation at the point of the magnetic phase transition. We can now look at the scattering amplitude in both spin singlet and triplet channels, where the expansion is still truncated up to $l = 1$:

$$a_{\mathbf{pp}'}^{\text{singlet}} = A_0^s - 3A_0^a + \left(A_1^s - 3A_1^a\right)(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}'), \tag{16}$$

$$a_{\mathbf{pp}'}^{\text{triplet}} = A_0^s + A_0^a + \left(A_1^s + A_1^a\right)(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}'). \tag{17}$$

In the magnetically ordered state close to the transition, consider the scattering of a pair of quasiparticles with opposite momentum; the scattering amplitude becomes

$$a^{\text{singlet}} = 2A_1^a \to +\infty, \tag{18}$$

$$a^{\text{triplet}} = -2A_1^a \to -\infty. \tag{19}$$

Obviously, we see a strong repulsion in the singlet channel and a strong attraction in the triplet channel, indicating an instability towards p-wave superconductivity. The same scenario happens if we let A_0^a diverge.

IV. MODEL IN THREE DIMENSIONS

We also study the same model in a 3D system, where the Fermi-surface distortion is very different from that in the 2D case. In the 2D system, since the quasiparticle momentum $\mathbf{p}$ lives in the xy plane, the magnitude of $\mathbf{m_p}$ is independent of the direction of $\mathbf{p}$, which means the Fermi-surface distortion is isotropic and there is a constant gap between the two branches of the Fermi surface with different spin polarizations. In a 3D system, however, the gap will depend on the direction of $\mathbf{p}$, and there are nodes located at the north and south poles of the Fermi surface, which makes the p-h continuum very different from that in the 2D case. The p-h continuum is no longer gapped at zero momentum; instead, it sweeps a finite region at zero momentum, which will Landau damp the $\sqrt{q}$ mode, and it will not propagate at all. In order to avoid this problem, we introduce an additional Ferromagnetic order in our model, which will gap out the p-h continuum at zero momentum, so that a small window will be opened to let the collective mode propagate. So the new model becomes

$$\mathbf{m}_{\mathbf{p}}^0(\mathbf{r}) = -\frac{1}{N(0)}\frac{\partial n_{\mathbf{p}}^0}{\partial \varepsilon_{\mathbf{p}}^0}[m_0\hat{\mathbf{z}} + m_1(\hat{\mathbf{z}}\times\hat{\mathbf{p}})], \quad (20)$$

which defines a state with magnetization proportional to m_0 and spin current proportional to m_1, similar to the 2D case except for the nonzero magnetization.

Similar to what we do in the 2D case, we can also calculate the free-energy change based on this model:

$$\delta\left(\frac{\Omega}{V}\right) = \frac{1+F_0^a}{N(0)}m_0^2 + \frac{2}{3N(0)}\left(1+\frac{F_1^a}{3}\right)m_1^2 + o\left(m_0^2, m_1^2\right), \quad (21)$$

which means that this ground state is also protected by generalized Pomeranchuk conditions.

A. Competing of two phases

Since there are multiple order parameters, it is necessary to study the competition between the different order parameters. This is the leading-order result, and to study the competition between these two orders, we need to add higher-order terms to the free energy, so that we have

$$\delta\left(\frac{\Omega}{V}\right) = \frac{1+F_0^a}{N(0)}m_0^2 + \frac{2}{3N(0)}\left(1+\frac{F_1^a}{3}\right)m_1^2 + Bm_0^2m_1^2 + Cm_0^4 + Dm_1^4 + \cdots, \quad (22)$$

where B,C,D are phenomenological parameters and they satisfy $C,D > 0$, $B > -2\sqrt{CD}$. By minimizing the free energy, we can get three kinds of phase diagrams for different values of B ($-2\sqrt{CD} < B < 0$, $0 < B < \sqrt{CD}$, and $B > \sqrt{CD}$).

These three kinds of phase diagrams are similar, and we take the case $B > \sqrt{CD}$ as an example; Fig. 2 shows the schematic phase diagram in this case. The boundary between phase II and phase IV is described by the equation $1 + \frac{F_1^a}{3} = \frac{D}{B}(1 + F_0^a)$, and the boundary between phase III and phase IV is described by the equation $1 + \frac{F_1^a}{3} = \frac{B}{4C}(1 + F_0^a)$. The phase diagram consists of four different phases, as listed in Table I. Among the four phases, phases I and II are paramagnetic and ferromagnetic phases; phase III is the SOM phase we introduced above with net spin current but no net magnetization, and phase IV can be regarded as the mixed order of phases II and III with both net magnetization and net spin current. The phase diagram indicates that by properly tuning the Landau parameters $F_{0/1}^a$, phases III and IV can be realized.

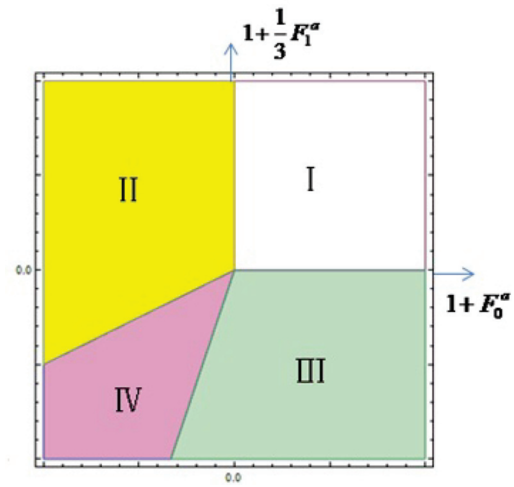

FIG. 2. (Color online) Schematic phase diagram in the case $B > \sqrt{CD}$. Phase boundaries between phases II, III, and IV are described by the equations $1 + \frac{F_1^a}{3} = \frac{D}{B}(1 + F_0^a)$ and $1 + \frac{F_1^a}{3} = \frac{B}{4C}(1 + F_0^a)$.

B. Collective modes

Using the same hydrodynamic-like approach as in the 2D case, we can study the fate of the collective modes in different phases.

1. The FM phase

In phase II, where $m_0 > 0, m_1 = 0$, for small momentum, the two modes lead to the dispersions

$$\omega_1 \to 2D_0 + \frac{A_0A_1}{6D_0}q^2, \quad (23)$$

$$\omega_2 \to \frac{A_0A_1}{6D_0}q^2, \quad (24)$$

TABLE I. Four phases with different values of order parameters.

	m_0	m_1
Phase I (PM)	=0	=0
Phase II (FM)	>0	=0
Phase III (SOM)	=0	>0
Phase IV(mixed order)	>0	>0

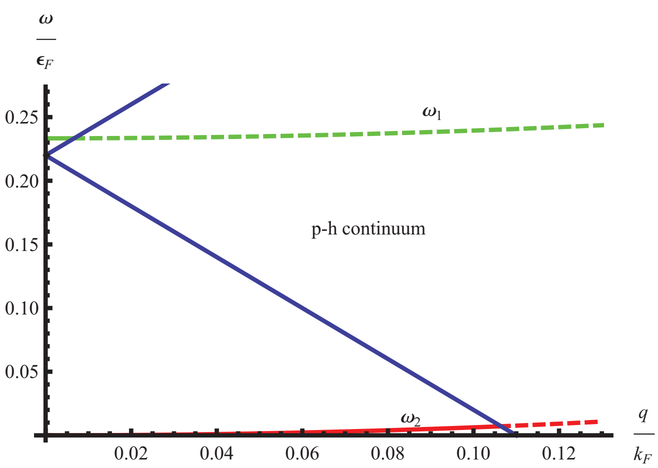

FIG. 3. (Color online) Dispersions of the collective modes with the p-h continuum in phase II. The dashed line indicates that the collective modes merge inside the continuum. Here, we take $F_0^a = -1.1$, $F_1^a = 1$, $m_1 = 0.15n$, and n is the particle density.

where

$$A_0 = \left(1 + F_0^a\right)v_f, \quad A_1 = \left(1 + \frac{F_1^a}{3}\right)v_f,$$
$$D_0 = m_0\left(f_0^a - \frac{f_1^a}{3}\right), \quad D_1 = m_1\left(f_0^a - \frac{f_1^a}{3}\right),$$

with the particle-hole continuum

$$\omega_{\rm ph} = \mathbf{q}\cdot\mathbf{v_p} \pm 2m_0|f_0|. \tag{25}$$

In Fig. 3, we show these two modes (one gapless, the other gapped) together with the particle-hole continuum, which was already studied before,[5] and we reproduce them here in phase II.

2. *The SOM phase*

In phase III, with $m_0 = 0, m_1 > 0$, for small momentum, the modes become

$$\omega_{1,2} = \sqrt{\tfrac{1}{3}A_0A_1q^2 \pm \tfrac{2}{3}|A_1D_1|q_\perp}, \tag{26}$$

and the particle-hole continuum becomes

$$\omega_{\rm ph} = \mathbf{q}\cdot\mathbf{v_p} \pm \tfrac{2}{3}\left|m_1 f_1^a \sin\theta_{\mathbf{p}}\right|. \tag{27}$$

In Fig. 4, we show the two modes together with the particle-hole continuum, and we find that these two modes are always sitting inside the particle-hole continuum for small momentum and become Landau damped, as we expected when we generalized the original model to the 3D system.

3. *The mixed phase*

We find two modes in phase IV:

$$\omega_1 \to 2D_0 + \left(\frac{A_0A_1}{6D_0} + \frac{A_1^2D_1^2}{36D_0^3}\right)q_\perp^2, \tag{28}$$

$$\omega_2 \to \frac{2|A_1|\sqrt{q - \left|\frac{2D_1}{A_0}\right|}}{\sqrt{\frac{9A_0D_0^2+6A_1D_1^2}{D_1^3}}}, \tag{29}$$

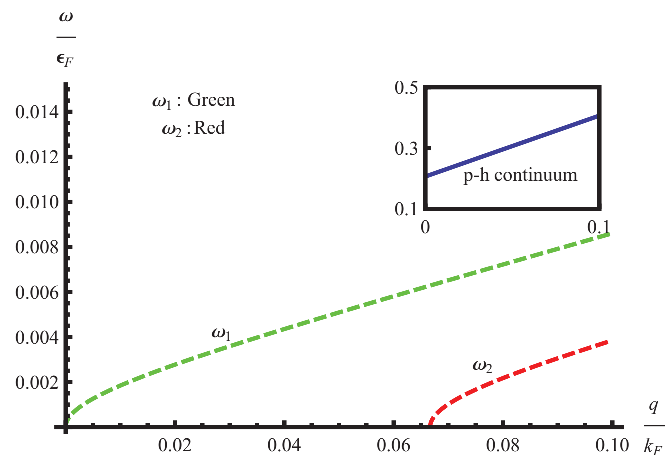

FIG. 4. (Color online) Dispersions of the collective modes with the p-h continuum in phase III. The dashed line indicates that the collective modes merge inside the continuum. Here, the p-h continuum is so large that it completely encloses the two modes, and its boundary is shown in the inset with the same axes. We take $F_0^a = -1.1$, $F_1^a = -3.1$, $m_1 = 0.15n$, and n is the particle density.

and the p-h continuum is

$$\omega_{\rm ph} = \mathbf{q}\cdot\mathbf{v_p} \pm \tfrac{2}{3}\sqrt{9\left(m_0 f_0^a\right)^2 + \left(m_1 f_1^a\right)^2 \sin^2(\theta_p)}.$$

Here, since $m_0 > 0$, the p-h continuum is gapped, which opens up a window for the modes to propagate. We evaluate the collective modes and p-h continuum with reasonable values of Landau parameters and order parameters, and the result is presented in Fig. 5.

In Fig. 5, we show the gapless and gapped modes outside the p-h continuum. Clearly, we can see that $\omega_2^2 < 0$ for small q, which is a very exotic feature. This indicates a Lifshitz-like instability[20] of the ground state towards some inhomogeneous magnetic state such as a spiral phase.[21]

V. SUMMARY

In summary, using Landau Fermi-liquid theory, we studied the collective modes in the spin orbit order magnetic state in the f_1^a channel, in both 2D and 3D systems. In both cases, the $\sqrt{q}$ dispersion is found in leading order. In the 2D system, we also calculate the spin density response function, which gives a consistent result($\sqrt{q}$ dispersion) for the collective mode, suggesting that the hydrodynamic description captures the essential physics of the state. This exotic mode can play a role in the formation of Cooper pairs of two quasiparticles since it has similar structure to the phonon propagator, so we expect an instability toward superconductivity close to the magnetic phase transition. A further argument based on forward scattering sum rules confirms the instability again and strongly indicates a p-wave pairing symmetry. In a 2D system, the model describes one general structure of spin-orbital coupling, and it is actually closely related the Rashba Hamiltonian[22] in the 2D semiconductor heterostructures. Therefore, we expect

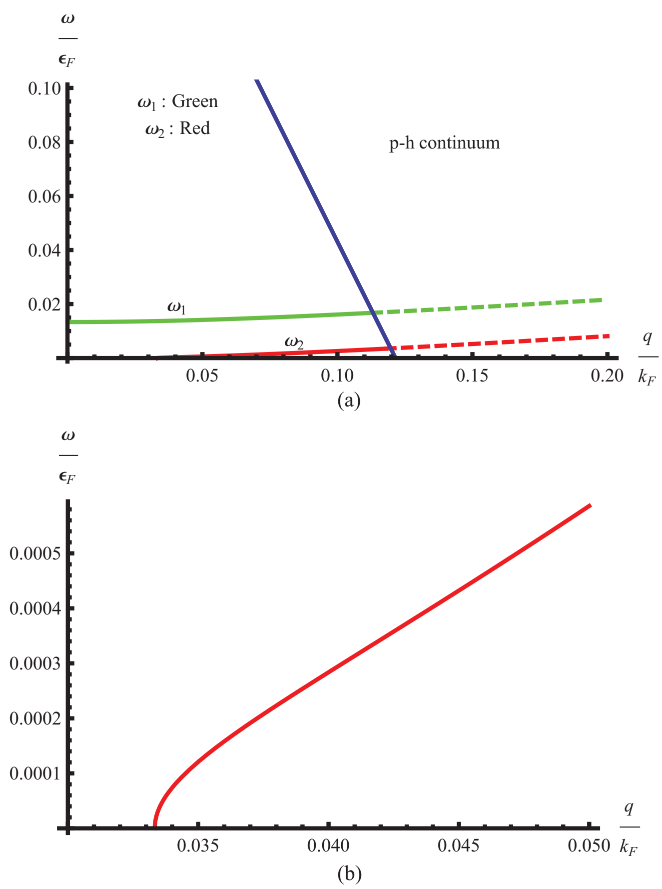

FIG. 5. (Color online) (a) Dispersions of the collective modes with the p-h continuum in phase IV. The dashed line indicates that the collective modes merge inside the continuum. (b) Close-up of mode ω_2. Here, we take $F_0^a = -1.1$, $F_1^a = -3.1$, $m_0 = 0.15n$, $m_1 = 0.075n$, and n is the particle density.

that this model can describe 2D or quasi-2D systems with spin-orbital coupling. In three dimensions, a Ferromagnetic order is added to the ground state to avoid the Landau damping, and the collective mode leads to a Lifshitz-like instability towards an inhomogeneous magnetic state in one of the phases.

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APPENDIX: CALCULATION OF DYNAMICAL RESPONSE FUNCTION

We can calculate the spin response function (i.e., spin-spin correlation function) based on the Landau kinetic equation [Eq. (7)]. After the Fourier transformation, keeping $\delta\mathbf{B}$ in the equation, instead of getting equations of $\vec{\nu}_l$ [Eq. (8)], we get equations of $\vec{\nu}_\mathbf{p}$:

$$[(\omega - \mathbf{q}\cdot\mathbf{v_p}) - im_1 f_1^a(\hat{\mathbf{z}}\times\hat{\mathbf{p}})\times]\vec{\nu}_\mathbf{p} = [N(0)\mathbf{q}\cdot\mathbf{v_p} - 2im_1(\hat{\mathbf{z}}\times\hat{\mathbf{p}})\times][\delta\mathbf{h_p} - \delta\mathbf{B}]. \quad (A1)$$

Here, we let $\delta\mathbf{h_p} = 2\sum_{\mathbf{p}'}(-\frac{1}{N(0)})\frac{\partial n^0_{\mathbf{p}'}}{\partial\varepsilon^0_{\mathbf{p}'}} f^a_{\mathbf{pp}'}\vec{\nu}_{\mathbf{p}'}$. Equation (A1) can be written in matrix form as

$$\mathbf{M}_1\cdot\vec{\nu}_\mathbf{p} = \mathbf{M}_2\cdot(\delta\mathbf{h_p} - \delta\mathbf{B}), \quad (A2)$$

where

$$\mathbf{M}_1 = \begin{pmatrix} \omega - \mathbf{q}\cdot\mathbf{v_p} & 0 & -im_1 f_1^a\hat{p}_x \\ 0 & \omega - \mathbf{q}\cdot\mathbf{v_p} & -im_1 f_1^a \hat{p}_y \\ im_1 f_1^a\hat{p}_x & im_1 f_1^a\hat{p}_y & \omega - \mathbf{q}\cdot\mathbf{v_p} \end{pmatrix},$$

$$\mathbf{M}_2 = \begin{pmatrix} N(0)\mathbf{q}\cdot\mathbf{v_p} & 0 & -2im_1\hat{p}_x \\ 0 & N(0)\mathbf{q}\cdot\mathbf{v_p} & -2im_1\hat{p}_y \\ 2im_1\hat{p}_x & 2im_1\hat{p}_y & N(0)\mathbf{q}\cdot\mathbf{v_p} \end{pmatrix}.$$

Keeping the Landau parameters up to $l = 1$, we have $\delta\mathbf{h_p} = f_0^a\vec{\nu}_0 + \frac{1}{2}f_1^a e^{i\phi_\mathbf{p}}\vec{\nu}_1 + \frac{1}{2}f_1^a e^{-i\phi_\mathbf{p}}\vec{\nu}_{-1}$; then the equation becomes

$$\vec{\nu}_\mathbf{p} = \mathbf{K}\cdot\left(f_0^a\vec{\nu}_0 + \tfrac{1}{2}f_1^a e^{i\phi_\mathbf{p}}\vec{\nu}_1 + \tfrac{1}{2}f_1^a e^{-i\phi_\mathbf{p}}\vec{\nu}_{-1} - \delta\mathbf{B}\right), \quad (A3)$$

where $\mathbf{K} = \mathbf{M}_1^{-1}\cdot\mathbf{M}_2$. Considering the continuity equation [Eq. (9)], the equation becomes

$$\vec{\nu}_\mathbf{p} = \mathbf{K}\left\{\left[\frac{qv_f}{2\omega}\left(1+\frac{F_1^a}{2}\right)f_0^a e^{i\phi_\mathbf{p}} + \frac{1}{2}f_1^a e^{i\phi_\mathbf{p}}\right]\vec{\nu}_1 + \left[\frac{qv_f}{2\omega}\left(1+\frac{F_1^a}{2}\right)f_0^a e^{-i\phi_\mathbf{p}} + \frac{1}{2}f_1^a e^{-i\phi_\mathbf{p}}\right]\vec{\nu}_{-1} - \delta\mathbf{B}\right\}. \quad (A4)$$

We set the external field in the z direction and perform operation $\frac{2}{\delta B}\sum_\mathbf{p}(-\frac{1}{N(0)})\frac{\partial n^0_\mathbf{p}}{\partial\varepsilon^0_\mathbf{p}}e^{\pm i\phi_\mathbf{p}}$ on Eq. (A4), and we get two equations:

$$\vec{\chi}_{j^+} = \left[\frac{qv_f}{2\omega}\left(1+\frac{F_1^a}{2}\right)f_0^a e^{i\phi_\mathbf{q}}\mathbf{K}_1 + \frac{1}{2}f_1^a\mathbf{K}_2\right]\cdot\vec{\chi}_{j^-} + \left[\frac{qv_f}{2\omega}\left(1+\frac{F_1^a}{2}\right)f_0^a e^{-i\phi_\mathbf{q}}\mathbf{K}_1 + \frac{1}{2}f_1^a\mathbf{K}_0\right]\cdot\vec{\chi}_{j^+} - \mathbf{K}_1\cdot\hat{\mathbf{z}}, \quad (A5)$$

$$\vec{\chi}_{j^-} = \left[\frac{qv_f}{2\omega}\left(1+\frac{F_1^a}{2}\right)f_0^a e^{-i\phi_\mathbf{q}}\mathbf{K}_{-1} + \frac{1}{2}f_1^a\mathbf{K}_{-2}\right]\cdot\vec{\chi}_{j^+} + \left[\frac{qv_f}{2\omega}\left(1+\frac{F_1^a}{2}\right)f_0^a e^{i\phi_\mathbf{q}}\mathbf{K}_{-1} + \frac{1}{2}f_1^a\mathbf{K}_0\right]\cdot\vec{\chi}_{j^-} - \mathbf{K}_{-1}\cdot\hat{\mathbf{z}}, \quad (A6)$$

where $\vec{\chi}_{j^\pm}$ are spin-spin current correlation functions defined as $\vec{\chi}_{j^\pm} = \frac{\vec{\nu}_{\pm1}}{\delta\mathbf{B}}$ and they are related to the spin-spin correlation function $\vec{\chi} = \frac{\vec{\nu}_0}{\delta\mathbf{B}}$ through the continuity equation as

$$\omega\vec{\chi} - \frac{qv_f}{2}\left(1+\frac{F_1^a}{2}\right)e^{i\phi_\mathbf{q}}\vec{\chi}_{j^-} - \frac{qv_f}{2}\left(1+\frac{F_1^a}{2}\right)e^{-i\phi_\mathbf{q}}\vec{\chi}_{j^+} = 0, \quad (A7)$$

and these $\mathbf{K}_l$ matrices are defined as $\mathbf{K}_l = 2\sum_{\mathbf{p}}(-\frac{1}{N(0)})\frac{\partial n^0_{\mathbf{p}}}{\partial \varepsilon^0_{\mathbf{p}}} e^{il\phi_{\mathbf{p}}}\mathbf{K}$.

By solving Eqs. (A5)–(A7) for the response functions and determining their poles, we get the dispersion of the collective mode:

$$\omega_c = \pm\frac{1}{2}\sqrt[4]{1 - \frac{f^a_0}{2f^a_0 - f^a_1}}\sqrt{|2 + F^a_1|(2f^a_0 - f^a_1)}m_1 v_f q. \tag{A8}$$

[1] I. I. Pomeranchuk, Sov. Phys. JETP **8**, 361 (1959).
[2] A. A. Abrikosov and I. E. Dzyaloshinskii, Sov. Phys. JETP **35**, 535 (1959).
[3] I. E. Dzyaloshinskii and P. S. Kondratenko, Zh. Eksp. Teor. Fiz. **70**, 1987 (1976) [Sov. Phys. JETP **43**, 1036 (1976)].
[4] K. B. Blagoev, J. R. Engelbrecht, and K. S. Bedell, Philos. Mag. Lett. **78**, 169 (1998); Phys. Rev. Lett. **82**, 133 (1999).
[5] K. S. Bedell and K. B. Blagoev, Philos. Mag. Lett. **81**, 511 (2001).
[6] V. Oganesyan, S. A. Kivelson, and E. Fradkin, Phys. Rev. B **64**, 195109 (2001).
[7] M. Garst and A. V. Chubukov, Phys. Rev. B **81**, 235105 (2010).
[8] M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010).
[9] P. Wölfle and A. Rosch, J. Low Temp. Phys. **147**, 165 (2007).
[10] C. A. Lamas, D. C. Cabra, and N. Grandi, Phys. Rev. B **80**, 075108 (2009).
[11] C. A. Lamas, D. C. Cabra, and N. E. Grandi, Int. J. Mod. Phys. B **25**, 3539 (2011).
[12] C. Wu and S. C. Zhang, Phys. Rev. Lett. **93**, 036403 (2004).
[13] C. Wu, K. Sun, E. Fradkin, and S. C. Zhang, Phys. Rev. B **75**, 115103 (2007).
[14] D. S. Saraga and D. Loss, Phys. Rev. B **72**, 195319 (2005).
[15] Y. Li and C. Wu, Phys. Rev. B **85**, 205126 (2012).
[16] T. Fujita and K. F. Quader, Phys. Rev. B **36**, 5152 (1987).
[17] K. S. Bedell and C. Sanchez-Castro, Phys. Rev. Lett. **57**, 854 (1986).
[18] G. Baym and C. Pethick, *Landau Fermi-Liquid Theory* (Wiley, New York, 1991).
[19] D. Pines and P. Nozieres, *The Theory of Quantum Liquids* (Perseus, New York, 1966).
[20] P. M. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge University Press, Cambridge, 1995).
[21] S. A. Brazovskii, Zh. Eksp. Teor. Fiz. **68**, 175 (1975) [Sov. Phys. JETP **41**, 85 (1975)].
[22] E. I. Rashba, Sov. Phys. Solid State **2**, 1109 (1960).