Orientation peculiarities of dc Josephson tunneling between *d*-wave superconductors with charge density waves

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The theory of the stationary Josephson tunnel current I_c was devised for junctions involving superconductors partially gapped by biaxial or unidirectional charge density waves (CDWs) and possessing a superconducting order parameter of *d*-wave symmetry. Specific calculations were carried out for symmetric junctions between two identical CDW superconducting electrodes and nonsymmetric ones composed of a CDW superconductor and a conventional isotropic *s*-wave superconductor. Two kinds of superconducting pairing symmetries were studied, namely, that appropriate to cuprates ($d_{x^2-y^2}$) and the d_{xy} one. The corresponding calculations were also carried out for the extended *s*-wave symmetry of the superconducting order parameter. Allowances were made for the directionality of tunneling. In all the cases studied, the dependences of I_c on the angle γ between the chosen crystal direction and the normal to the junction plane were found to be significantly influenced by CDWs. It was shown in particular that the *d*-wave driven periodicity of $I_c(\gamma)$ in the CDW-free case is transformed into double-period beatings depending on the parameters of the system. The results of calculations testify that the orientation-dependent patterns $I_c(\gamma)$ measured for CDW superconductors allow the CDW configuration (unidirectional or checkerboard) and the symmetry of superconducting order parameter to be determined. The predicted effects can be used to indirectly reveal CDWs in underdoped cuprates where pseudogaps are observed. The break-junction technique is an appropriate tool for this purpose.

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I. INTRODUCTION

The original Bardeen-Cooper-Schrieffer (BCS) theory is well known to be based on the assumption of spatially isotropic (*s*-wave) spin-singlet Cooper pairing.[1] However, this kind of pairing is not unique and a variety of superconducting (many kinds of metallic conductors) or superfluid (He^3) order parameters of other types are in principle possible.[2–13] As for high-T_c superconducting cuprates, the majority of researchers believe, on the basis of experimental evidence, that the order parameters in both hole- and electron-doped materials possess $d_{x^2-y^2}$ symmetry.[14–24] For instance, recent direct measurements of the dc Josephson current in a π-design superconducting quantum interference device (SQUID) made of epitaxial $Sr_{1-x}La_xCuO_2$ film showed a purely $d_{x^2-y^2}$ behavior with no traces of the subdominant *s*-wave order-parameter component.[25]

In contrast, there is a large body of tunnel[26–32] and NMR (Ref. 33) data that testify that the extended and even the isotropic *s*-wave scenario cannot be excluded from consideration. This alternative is also supported by a thorough analysis.[34–40] In particular, it is well known[41] that, in the absence of directionality, the Josephson current between *d*- and *s*-wave superconductors has to be strictly equal to zero. However, the dc Josephson current between $Bi_2Sr_2CaCu_2O_{8+\delta}$ and Pb,[42] $Bi_2Sr_2CaCu_2O_{8+\delta}$ and Nb,[43] $YBa_2Cu_3O_{7-\delta}$ and PbIn,[44] $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$ and Pb (Ref. 26) superconductors was experimentally found to differ from zero. Hence either a subdominant *s*-wave component of the superconducting order parameter does exist in cuprate materials or the introduction of tunnel directionality is inevitable to reconcile any theory dealing with tunneling of quasiparticles from (to) high-T_c oxides and the experimental results. In this paper, the necessity of taking tunnel directionality[45–52] into account is put forth; however, this assumption alone cannot help in solving the intricate issue of the actual superconducting order-parameter symmetry.

In addition, one may imagine that more exotic states are realized. For instance, it may be the superconducting pairing with the combined $d_{x^2-y^2}+id_{xy}$ symmetry, especially in tunnel structures,[27] near the rough surfaces,[53,54] and in an external magnetic field,[55–58] i.e., in situations routinely studied in experiments. Another possibility involves the *d*-wave superconducting order parameter with a substantial admixture of higher harmonics.[59]

To distinguish between the indicated possibilities, various investigation techniques are needed. In particular, Josephson tunneling measurements involving ac and dc effects comprise a powerful tool to achieve this objective,[8,38,41,60–69] although the intrinsically inhomogeneous nature of oxide samples might complicate the issue.[70–81] There is one more obstacle to inferring the cuprate order-parameter symmetry directly from coherent tunneling measurements, namely, the appearance of unidirectional (stripelike[82–85]) or checkerboard (biaxial) charge density wave (CDW) structures in many high-T_c oxide materials.[77,86–93] It seems, however, that this very circumstance can be made beneficial for studying both superconducting and dielectric (CDW) orderings in cuprates using tunnel spectroscopy and Josephson current measurements.[76,94–97] It may concern other superconductors with coexisting order parameters as well. For instance, one can indicate new superconducting families of pnictides and chalcogenides, where spin density waves (SDWs) and possibly also CDWs were found along with superconductivity.[12,98–110] The microscopic origin of CDWs in cuprates is most probably the electron-phonon one (i.e., the electron spectrum of the material is gapped as a consequence of Peierls instability[111–113]), although our basic phenomenological equations are similar for partial excitonic insulators. The latter state can also be realized in systems with a nested Fermi surface (FS).[114,115]

In this paper, we calculate dc Josephson currents I_c between two d-wave (or extended s-wave) superconductors or one d-wave (extended s-wave) superconductor and its conventional isotropic counterpart suggesting that the CDW gapping distorts the FSs of the high-T_c electrodes. All coherent effects are regarded as being due to the Cooper pairing, so the more exotic cases of the CDW phase-dependent tunnel currents[116,117] (interesting *per se*) are neglected. We assume a two-dimensional FS with CDW-gapped nested sections in accordance with the microscopic theory[118–124] and observations.[81,85,89,92,110,112,125–133] The results obtained show that the interference between CDWs and superconductivity can substantially alter the angle and temperature T dependences of the current I_c, not to mention the amplitude of the current, which should be substantially suppressed by the dielectric gapping.[134] In particular, the CDW sectors on the FS hot spots should lead to the nonmonotonic character of the I_c dependences on the angle between the crystal-lattice axes and the normal to the tunnel-junction plane. Thus Josephson current measurements probing superconducting properties may also reveal the partial CDW insulating background.

II. SOME ESSENTIAL PECULIARITIES OF CUPRATES

As is well known, the coherent properties of a collective state with Cooper pairing can be revealed by measuring the dc or ac Josephson tunnel currents between two superconducting electrodes because the currents depend on the phase difference between superconducting order parameters in them.[20,135–137] It is no wonder that manifestations of coherent pair tunneling are more complex for superconductors with anisotropic order parameters than for those with an isotropic energy gap. In particular, it is true for d-wave superconductors, where the order parameter changes its sign on the FS.[5,20,41,61,64,138,139] As indicated above, high-T_c oxides are usually considered as $d_{x^2-y^2}$ materials. Since cuprates are considered here as the main objects of study, the $d_{x^2-y^2}$ scenario will be considered as the basic one.

In addition to the unconventional, nonisotropic order-parameter symmetry, high-T_c oxides reveal another important peculiarity. Namely, the so-called pseudogap is observed both below and above T_c.[77,81,86–88,90,140–150] Pseudogapping manifests itself in resistive, magnetic, optical, photoemission [angle-resolved photoemission spectroscopy (ARPES)] and tunnel [scanning tunnel microscopy (STM) and break-junction] measurements as a depletion in the electron density of states, in analogy to what is observed in quasi-one-dimensional compounds above the mean-field phase-transition temperature.[151,152] Despite large theoretical and experimental efforts, the pseudogap nature still remains unknown.[77,80,81,90,91,122,129,144,153–169] Some researchers associate them with precursor order-parameter fluctuations, which might be of either a superconducting or some other competing (CDWs, SDWs, etc.) origin. Another viewpoint consists in relating pseudogaps to those competing orderings, but treating them on an equal footing with superconductivity as well-developed states for which an allowance can be made in the mean-field approximation, fluctuation effects being noncrucial. We believe that the available observations support the latter viewpoint (see, e.g., recent experimental evidence of CDW formation in various cuprates[89,93,117,149,170–179]). Although undoped cuprates are antiferromagnetic insulators,[180] the CDW seems to be a more suitable candidate responsible for the pseudogap phenomenon.[77,86–88,90]

Direct evidence that the CDW is a superconductivity destructor was obtained recently for $YBa_2Cu_3O_{7-\delta}$ by x-ray diffraction[175,177] for c-axis $Bi_{2+x}Sr_{2-y}CuO_{6+\delta}$ mesas by intrinsic tunneling[178] and for $Bi_2Sr_2CaCu_2O_{8+\delta}$ by the ARPES (Ref. 150) methods. However, a skepticism concerning the CDW competition with Cooper pairing still remains. Namely, the coexistence of two phenomena is qualified as a simple intertwining.[181] Moreover, a conclusion based on the analysis of ARPES data for $Bi_2Sr_{2-x}La_xCuO_{6+\delta}$ and $La_{2-x}Sr_xCuO_4$ was made[182] that, at least in these one-layer compounds, two pseudogaps of different magnitude coexist with superconductivity and spoil the intrinsic $d_{x^2-y^2}$ behavior. It seems that this conclusion is ill founded since, according to STM measurements of various high-T_c oxides,[73,183–186] pseudogap values in cuprate samples are so strongly dispersed that it is impossible to unambiguously resolve pseudogap species.

Anyway, the type of dominant instability competing with superconductivity in cuprates is not known with certainty. For instance, neutron-diffraction studies of a number of various high-T_c oxides revealed a nonhomogeneous magnetic ordering (usually associated with SDWs) in the pseudogap state.[187,188] However, whatever the nature of the pseudogap, the latter competes with Cooper pairing in underdoped and even overdoped high-T_c oxide samples.[77,81,86–90,92,93,123,131,132,149,189–191] As for the mechanism of Cooper pairing *per se* in unconventional superconductors, it is also not known. The controversy concerns spin fluctuations[192,193] versus conventional phonons[194–197] as a bosonic glue keeping electrons together.

III. MODEL PARAMETERS FOR CDW SUPERCONDUCTORS

Bearing in mind that the main objects to which our theory is applied are cuprates, the FS of the model CDW superconductor (CDWS) is approximated as a two-dimensional one.[118–124,198] The FS is gapped by both superconducting (d-wave BCS) and dielectric (CDW) instabilities, which additionally compete with each other for the quasiparticle states at the FS. Both Cooper and electron-phonon pairings are nonisotropic in momentum space. In Fig. 1, a sketch of a typical Brillouin zone for a high-T_c superconductor is exhibited. The bold solid curves (pockets) illustrate the FS of this superconductor in the double-normal state, i.e., in the absence of both gapping distortions.

Charge density waves are accompanied by the appearance of dielectrically gapped (dielectrized) FS sections. A necessary condition for that is the nesting of the electron spectrum on them in a parent CDW metal,[117,199–201]

$$\xi(\mathbf{p}) = -\xi(\mathbf{p}+\mathbf{Q}), \tag{1}$$

where $\mathbf{Q}$ is the CDW vector. The rest of the FS remains nondielectrized. For instance, the $\mathbf{Q}$ vectors in Fig. 1 connect those FS sections where the nesting phenomenon is the most probable. Then, possible orientations of the CDW

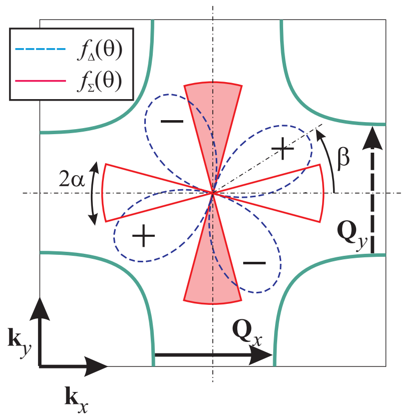

FIG. 1. (Color online) Schematic diagram of the two-dimensional Fermi surface appropriate to cuprates and angular profiles of the d-wave superconducting $f_\Delta(\theta)$ and dielectric $f_\Sigma(\theta)$ order parameters. Charge density waves are described by one $\mathbf{Q}_x$ or two $\mathbf{Q}_x$ and $\mathbf{Q}_y$ CDW vectors. The corresponding CDW configuration function $f_\Sigma(\theta)$ possesses either one (nontinted) or two sector pairs, respectively. Here 2α is a CDW sector opening and β is a mismatch angle between superconducting lobes and CDW sectors (see the text for further details).

vectors turn out to be coupled rather strongly with the inverse lattice, namely, they are directed along the $\mathbf{k}_x$ and $\mathbf{k}_y$ axes in momentum space.[77,123,202] In cuprates, CDWs constitute a system with a fourfold ($N = 4$, the checkerboard pattern) or a twofold ($N = 2$, the unidirectional pattern) symmetry.[59,77,90,91,127,131,133,177,203–207] This situation is described (see Fig. 1) by the availability of two CDW vectors $\mathbf{Q}_x$ and $\mathbf{Q}_y$ or only one $\mathbf{Q}_x$. The fourfold symmetry is often broken in various high-T_c oxides revealing unidirectional CDWs or nematicity in nanoscale clusters.[208] This phenomenon was considered as a manifestation of the pseudogap formation.[133,209–211]

As a matter of fact, CDW distortions in cuprates are short ranged and form random patches. The inhomogeneity of electronic properties in high-T_c oxides is most probably intrinsic (not necessarily being of the pseudogap origin) and may substantially influence observations.[70,73,76,77,95,189,212–214] Nevertheless, in this paper we restrict ourselves to the approximation where the CDWs constitute spatially homogeneous patterns.

We consider CDWSs in the framework of a model that is an extension of the Bilbro-McMillan one, initially developed for the CDW s-wave superconductors.[215] In this model, the magnitude of the CDW order parameter Σ is assumed to be uniform (the s-wave symmetry) within the FS dielectrized sections, each of width 2α.[77,90,91,96] Note that a similar separation into hot (in our terms, dielectrically gapped) and cold spots in cuprates was introduced some time ago to treat antiferromagnetic fluctuations.[198,216] To fix the position of dielectrized sections on the FS, we introduce the angular factor $f_\Sigma(\theta)$ (see Fig. 1),

$$f_\Sigma(\theta) = \begin{cases} 1 & \text{for} |\theta - \beta + k\Omega| < \alpha \text{ (dielectrized sections)} \\ 0 & \text{otherwise (nondielectrized sections),} \end{cases} \tag{2}$$

where k is an integer number and

$$\Omega = \begin{cases} \pi/2 & \text{for checkerboard CDWSs } (N = 4) \\ \pi & \text{for unidirectional CDWSs } (N = 2). \end{cases} \tag{3}$$

In essence, the parameter N equals the number of FS dielectrized sections. Then the actual dielectric order-parameter Σ distribution over the FS takes the factorized form

$$\bar{\Sigma}(T,\theta) = \Sigma(T) f_\Sigma(\theta). \tag{4}$$

Here $\Sigma(T)$ is the conventional T dependence of the CDW order parameter.

In known high-T_c superconductors, the lobes of the superconducting order parameter are also directed along the $\mathbf{k}_x$ and $\mathbf{k}_y$ axes (see Fig. 1), which corresponds to the $d_{x^2-y^2}$ type of superconducting order-parameter symmetry. (According to the adopted terminology, CDW sectors emerge in antinodal directions of superconducting lobes.) For definiteness, we select the direction of either of two CDW vectors (in the checkerboard geometry) or the single one (in the unidirectional geometry)—in other words, the bisectrix of a CDW sector—as a reference one in the crystal. From the physical viewpoint, it is the direction of a basis vector in the inverse lattice; let it be the vector $\mathbf{k}_x$ (see Fig. 1). This circumstance is connected with experimental conditions of crystal growing. Then the $d_{x^2-y^2}$ type of superconducting order-parameter symmetry corresponds to the value $\beta = 0^\circ$ of the mismatch angle between the bisectrices of the CDW sector and the superconducting order-parameter lobe (for definiteness, we chose it as positive). However, intensively discussed in the literature is, e.g., the d_{xy} symmetry, when the superconducting lobes are rotated in the inverse lattice by 45°. Moreover, we suppose that the angle β can acquire other values as well (formally, they can be arbitrary), e.g., if a superconductor is subjected to a nonuniform deformation.[91] If those directions are different, the angle β will be reckoned from the bisectrix of the CDW sector to that of the nearest positive superconducting order-parameter lobe. Of course, the sign of β has no importance for thermodynamic properties of CDWSs. However, as will be evident from what follows, it manifests itself in the orientation dependences of the tunnel current.

Following this convention, we can write the superconducting order-parameter profile over the FS in a factorized form similar to formula (4) for the CDW order parameter,

$$\bar{\Delta}(T,\theta) = \Delta(T) f_\Delta(\theta). \tag{5}$$

Here $\Delta(T)$ is the T-dependent magnitude of the superconducting order parameter and the angular factor $f_\Delta(\theta)$ is described in momentum space by the sign-alternating expression

$$f_\Delta^d(\theta) = \cos 2(\theta - \beta) \tag{6}$$

in the case of d-wave symmetry (see Fig. 1). We also intend to analyze, on an equal footing, the case of extended s-wave symmetry described by the angular function

$$f_\Delta^{s^{\rm ext}}(\theta) = |\cos 2(\theta - \beta)|, \tag{7}$$

for which the superconducting gap profile is the same as in Fig. 1, but all four lobes have the same sign (for definiteness, let it be positive).We emphasize that neither the $f_\Delta(\theta)$ nor the $f_\Sigma(\theta)$ angular profile depends on T.

The described quantities (α,N,β) together with the zero-temperature values of order parameters for the parent pure d-wave superconductor Δ_0 and the CDW metal Σ_0 constitute a full set of parameters describing the partially gapped CDW d-BCS superconductor. The number of energy-dependent parameters of the problem can be reduced by normalizing them by the parameter Δ_0. Thus we introduce the dimensionless parameter $\sigma_0 = \Sigma_0/\Delta_0$ describing the relative strength of the CDW and BCS pairings, the dimensionless temperature $t = T/\Delta_0$, and the dimensionless order parameters $\sigma(t) = \Sigma(T)/\Delta_0$ and $\delta(t) = \Delta(T)/\Delta_0$.

In what follows it is convenient to introduce the notation $S_{\mathrm{CDW}N}^{\mathrm{sym}_\beta}$ for a partially gapped CDW superconductor, which reflects a certain symmetry $\mathcal{S}$ of the superconducting order parameter (see below) with the mismatch angle β between the superconducting lobes and CDW sectors, as well as the checkerboard ($N = 4$) or unidirectional ($N = 2$) CDW configuration. The special case of d-wave symmetry with $\beta = 0^\circ$, as for cuprates, will be denoted in the conventional manner $d_{\beta=0^\circ} = d_{x^2-y^2}$. For the case $d_{\beta=45^\circ}$, which we also intend to analyze, the corresponding notation is d_{xy}. All intermediate β values might be possible only in the case of internal deformations, when the crystal symmetry inherent to cuprates is broken; they will not be analyzed below. As indicated in the Introduction, we shall also consider the cases of extended s-wave symmetry for the superconducting order parameter, for which we shall use the notations $s_{x^2-y^2}^{\mathrm{ext}}$ and s_{xy}^{ext}. We also introduce the notations S_{BCS}^s and S_{BCS}^d for pure s- and d-wave superconductors, respectively.

The solution of the self-consistent problem for the CDW d-wave superconductors in the general case (with arbitrary allowed parameter values[77,90,91,96,217]) demonstrates a mutual depression of both order parameters in comparison with their corresponding pure independent uncoupled counterparts. Moreover, the competition between the Σ and Δ order parameters gives rise to an interesting phenomenon of T-reentrant Σ.[77,90,91,96,217] The phenomenon of T-dependent reentrance was really observed in $Bi_2Sr_2CaCu_2O_{8+\delta}$.[150] The reentrance found by us is a subtle effect existing due to the difference between the order-parameter symmetries. This result has nothing in common with the electron concentration reentrance obtained in the framework of the Landau theory for the competing d-wave Cooper pairing and d-density wave.[218]

However, all those issues remain beyond the scope of this paper. As shown in the following section, the key quantities are the angular profiles of the superconducting order parameter [see Eq. (5)] and the total gap in momentum space. In particular, the order parameters demonstrate a peculiar interference on the FS dielectrized sections; namely, the total gap that emerges there equals

$$\bar{D}(T,\theta) = \sqrt{\bar{\Sigma}^2(T,\theta) + \bar{\Delta}^2(T,\theta)}. \tag{8}$$

This formula remains also valid, but becomes trivial, for the FS nondielectrized sections, since $\Sigma = 0$ there. As a result, a variety of momentum-dependent gap profiles over the FS ("gap roses")[90,91] arise. These roses, distorting the pristine FS in the normal state (the solid curve in Fig. 1), form the profile that is registered in ARPES experiments. In this work we omit the consideration of the issues dealing with the temperature effects or the effects associated with the modification of CDWS parameters, e.g., by doping the CDWSs, on the dc Josephson current, leaving them to be analyzed elsewhere. We are interested here only in the angular dependences of dc Josephson current, which could help in determining the pairing symmetry in high-T_c superconductors. Therefore, the specific calculations will be carried out only for zero temperature $T = 0$. Several examples of gap roses relevant to the paper's subject are illustrated in Appendix A.

Thus the mixed-state CDW d-wave superconductor demonstrates neither pure d-wave nor pure s-wave angular behavior. This fact was indicated, e.g., while analyzing ARPES spectra of $Bi_{1.5}Pb_{0.55}Sr_{1.6}La_{0.4}CuO_{6+\delta}$.[191] Of course, any admixture of Cooper pairing with a symmetry different from $d_{x^2-y^2}$ (Refs. 16,27,32,53–58,219, and 220) may alter the results. Moreover, the superconducting order-parameter symmetry might be doping dependent.[221] Other possibilities for predominantly d-wave superconductivity coexisting with CDWs lie somewhere between those pure s and d extremes.

IV. TUNNEL CURRENT: THEORY

A. General issues

We would like to point out that, owing to the quasi-two-dimensional character of the FS for high-T_c oxides, we consider the simplest geometry of the tunnel junction between two CDWSs. Namely, their c axes are assumed to be parallel to each other and the junction plane.

The dc Josephson critical current through a tunnel junction between two superconductors, whatever their order-parameter symmetry, in the tunnel Hamiltonian approximation is given by the general equation[20,135–137]

$$I_c(T) = 4eT\sum_{\mathbf{pq}}|\widetilde{T}_{\mathbf{pq}}|^2\sum_{\omega_n}\mathsf{F}^+(\mathbf{p};\omega_n)\mathsf{F}'(\mathbf{q};-\omega_n). \tag{9}$$

Here $\widetilde{T}_{\mathbf{pq}}$ are the matrix elements of the tunnel Hamiltonian, which correspond to various combinations of FS sections for superconductors taken on different sides of tunnel junction; $\mathbf{p}$ and $\mathbf{q}$ are the transferred momenta; $e > 0$ is the elementary electrical charge; and $\mathsf{F}(\mathbf{p};\omega_n)$ and $\mathsf{F}'(\mathbf{q};-\omega_n)$ are Gor'kov Green's functions for superconductors to the left and right, respectively, of the tunnel barrier (hereafter all primed quantities will be associated with the right-hand-side electrode). The internal summation is carried out over the discrete fermionic "frequencies" $\omega_n = (2n+1)\pi T,\ n = 0,\pm 1,\pm 2,\ldots$. The external summation in Eq. (9) for the Josephson tunnel current takes into account the anisotropy of the electron spectrum $\xi(\mathbf{p})$ in a superconductor in the manner suggested long ago for all kinds of anisotropic superconductors,[222] the directionality of tunneling,[45–52] and the dielectric electron-hole (CDW) gapping of the nested FS sections (if any).[134] Let us assume for definiteness that $\mathsf{F}(\mathbf{p};\omega_n) \equiv \mathsf{F}_{\mathrm{CDWS}}(\mathbf{p};\omega_n)$ corresponds to the high-T_c oxide superconductor (assumed to be CDW gapped) with a d-wave or extended s-wave order parameter. At the same time, $\mathsf{F}'(\mathbf{q};\omega_n)$ may correspond to either the identical high-T_c oxide $[\mathsf{F}'(\mathbf{q};\omega_n) \equiv \mathsf{F}'_{\mathrm{CDWS}}(\mathbf{q};\omega_n)]$ or an s-wave isotropic superconductor of the original BCS model $[\mathsf{F}'(\mathbf{q};\omega_n) \equiv \mathsf{F}'_{S_{\mathrm{BCS}}^s}(\mathbf{q};\omega_n)]$ with the order parameter $\bar{\Delta}_{s\mathrm{BCS}}(T)$.[1] Thus we restrict the consideration to two representative

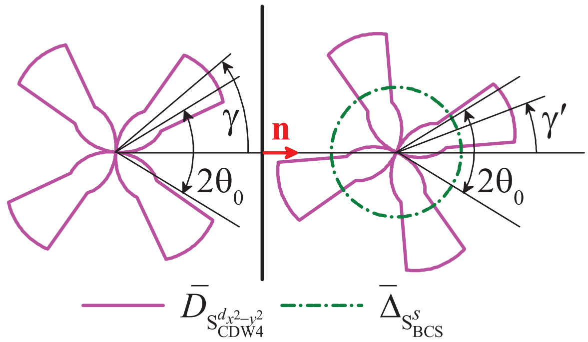

FIG. 2. (Color online) Geometries of symmetric ($S_{\rm CDW4}^{d_{x^2-y^2}}$-$I$-$S_{\rm CDW4}^{d_{x^2-y^2}}$) and nonsymmetric ($S_{\rm CDW4}^{d_{x^2-y^2}}$-$I$-$S_{\rm BCS}^{s}$) tunnel junctions describing the gap rose orientations in both electrodes (angles γ and γ') with respect to the normal $\mathbf{n}$ to the junction plane. The tunnel directionality is described by the effective angle $2\theta_0$. See the text for notation and further details.

cases: (i) the symmetric junction $S_{{\rm CDW}N}^{\rm sym}$-I-$S_{{\rm CDW}N}^{\rm sym}$ involving identical high-T_c superconductors separated by an insulator interlayer I (this case will be denoted by $J_{\rm sj}$-$S_{{\rm CDW}N}^{\rm sym}$) and (ii) the nonsymmetric $S_{{\rm CDW}N}^{\rm sym}$-I-$S_{\rm BCS}^{s}$ one ($J_{\rm nj}$-$S_{{\rm CDW}N}^{\rm sym}$). Figure 2 illustrates both of those cases for the checkerboard CDW scenario ($N = 4$) and the order-parameter mismatch angle $\beta = 0^\circ$.

We consider the gap rose of the CDWS to the left from the junction plane to be oriented at the angle γ with respect to the normal $\mathbf{n}$ to the plane. The orientation direction of the whole CDWS is determined by the direction of the bisectrix of the positive superconducting order-parameter lobe in the parent BCS superconductor (see Sec. III and Appendix A). The actual order-parameter profiles are governed by the crystal-lattice geometry. At the same time, the junction plane is created artificially and generally speaking the normal $\mathbf{n}$ to it may not coincide with any crystal axis. In the symmetric-junction geometry, the same superconductor is to the right of the junction, but its gap rose may be oriented at a different angle γ' with respect to the normal. In the case of nonsymmetric-junction geometry, the gap rose of the right-hand-side superconductor $S_{\rm BCS}^{s}$ is an isotropic circle.

B. Tunnel directionality

Specifying the dc Josephson current (9), we introduce two kinds of directionality. The first one involves the factors $|\mathbf{v}_{g,{\rm nd}} \cdot \mathbf{n}|$ and $|\mathbf{v}_{g,d} \cdot \mathbf{n}|$,[49,64,223] where $\mathbf{v}_{g,{\rm nd}} = \nabla \xi_{\rm nd}$ and $\mathbf{v}_{g,d} = \nabla \xi_d$ are the normal-state quasiparticle group velocities for proper FS sections. Those factors can be considered as proportional to the number of electron attempts to penetrate through the barrier.[138] They were introduced decades ago in the framework of a general problem dealing with tunneling in heterostructures.[224–226] In the framework of the phenomenological approach adopted here (cf. approaches in Refs. 41,61,65,138,139, and 227), this multiplier can be factorized into $\cos\theta$, where θ is the angle at which the pair/quasiparticle transmits through the barrier, and an angle-independent coefficient, which can be in the usual way incorporated into the junction normal-state resistance R_N (see below).

In addition, in agreement with previous studies,[45,46,48,49,51,228] the tunnel matrix elements $\widetilde{T}_{\mathbf{pq}}$ in Eq. (9) should also make allowances for the tunnel directionality (the angle-dependent probability of penetration through the barrier).[63,64,97,223] The problem is rather difficult. Even if the barrier were uniformly rectangular over the whole junction plane and the WKB approximation were sufficient for calculations, the situation would not be simple.[63] In reality, in this case, for a particle traversing the barrier at an angle θ, the barrier penetration coefficient would be proportional to $\exp(-h_b d)$, where h_b is the effective barrier height for the particle (the difference between the particle and barrier top energies), $d = L/\cos\theta$ is the relevant tunnel path, and L is the junction thickness. We do not know the actual θ dependences for realistic junctions from microscopic considerations. Therefore, similarly to what was proposed in Ref. 45, we simulate the barrier-associated directionality by the phenomenological function

$$w(\theta) = \exp\left[-\ln 2 \times \left(\frac{\tan\theta}{\tan\theta_0}\right)^2\right] \tag{10}$$

so that the cone opening of effective tunnel angles is equal to $2\theta_0$ (this parameter depends on L) (see Fig. 2). The barrier transparency is normalized by the maximum value obtained for the tunneling in the normal direction to the junction plane and included into the junction resistance R_N; hence $w(\theta = 0) = 1$. The multiplier $\ln 2$ in Eq. (10) was selected to provide $w(\theta = \theta_0) = \frac{1}{2}$.

C. Green's functions

In accord with previous treatments of partially gapped CDW s-wave superconductors[86–88,94,134,213,215,229–235] and their generalization to their d-wave counterparts[77,90,91,96,217,236] and in line with the basic theoretical framework for unconventional superconductors,[5,65] the anomalous Gor'kov Green's functions for d- or extended s-wave superconductors are assumed to be different for angular sectors with coexisting CDWs and superconductivity (dielectrized sections of the FS) and the purely superconducting rest of the FS (nondielectrized sections)

$$\mathsf{F}_{{\rm CDWS},{\rm nd}}(\mathbf{p};\omega_n) = \frac{\bar{\Delta}(T,\theta)}{\omega_n^2 + \bar{\Delta}^2(T,\theta) + \xi_{\rm nd}^2(\mathbf{p})}, \tag{11}$$

$$\mathsf{F}_{{\rm CDWS},d}(\mathbf{p};\omega_n) = \frac{\bar{\Delta}(T,\theta)}{\omega_n^2 + \bar{D}^2(T,\theta) + \xi_d^2(\mathbf{p})}. \tag{12}$$

Here the angle θ is reckoned from the tilt angle γ (γ') in the case of the left- (right-) hand-side electrode. The quasiparticle spectra $\xi_d(\mathbf{p})$ and $\xi_{\rm nd}(\mathbf{p})$ correspond to hot and cold spots of the cuprate FS, respectively (see, e.g., Refs. 92,112,120,141, 149,190,237, and 238). It is evident that Eq. (11) is a particular case of expression (12) since, owing to formula (8), the gap $\bar{D}(T,\theta)$ on the nondielectrized FS section equals $\Delta(T,\theta)$. The difference between the Green's functions for d- or extended s-wave superconductors consists in the proper choice of the order-parameter angular profile $f_\Delta(\theta)$ [see Eqs. (6) and (7)]. The Gor'kov Green's function for the conventional s-wave superconductor has the standard form

$$\mathsf{F}_{s\mathrm{BCS}}(\mathbf{p};\omega_n) = \frac{\Delta_{s\mathrm{BCS}}(T)}{\omega_n^2 + \Delta_{s\mathrm{BCS}}^2(T) + \xi_{s\mathrm{BCS}}^2(\mathbf{p})}. \quad (13)$$

It is also a particular case of Eq. (12) in which, besides the absence of dielectric gapping, the dependence on the angle θ is also absent. The d-wave $\Delta(T)$ and s-wave $\Sigma(T)$ gap functions are calculated self-consistently on the basis of BCS-like electron-electron and electron-hole attractions characterized by the strengths Δ_0 and Σ_0, respectively.[77,90,91,96] As indicated in Sec. III, the quantities Δ_0 and Σ_0 are zero T values of the relevant energy gaps in the absence of a competing interaction. In particular, at $\Delta = 0$, the dependence $\Sigma(T)$ is given by the s-wave Mühlschlegel function.[1,239] In contrast, at $\bar{\Sigma} = 0$, the dependence $\Delta(T)$ is determined by the weak-coupling equation for the d-wave superconductor.[240] The self-consistent dependences $\Delta(T)$ and $\Sigma(T)$ for extended s-wave superconductors are identical to those for d-wave superconductors.

D. The dc Josephson current

Substituting Eqs. (11)–(13) into Eq. (9), carrying out standard transformations,[135,136] assuming the coherent character of tunneling,[38,45,63] as opposed to the noncoherent approximation[241,242] valid for isotropic superconductors, and making some simplifications, we arrive at the following formula for the dc Josephson current across the tunnel junction:

$$I_c(T,\gamma,\gamma') = \frac{1}{2eR_N} \times \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} d\theta \cos\theta\, w(\theta) P(T,\theta,\gamma,\gamma'), \quad (14)$$

where[134,222]

$$P(T,\theta,\gamma,\gamma') = \bar{\Delta}\bar{\Delta}' \int_{\min\{\bar{D},\bar{D}'\}}^{\max\{\bar{D},\bar{D}'\}} \frac{dx \tanh\frac{x}{2T}}{\sqrt{(x^2-\bar{D}^2)(\bar{D}'^2-x^2)}}. \quad (15)$$

Here, for brevity, we indicated the dependences $\bar{\Delta}(T,\theta-\gamma)$, $\bar{\Delta}'(T,\theta-\gamma')$, $\bar{D}(T,\theta-\gamma)$, and $\bar{D}'(T,\theta-\gamma')$ without their arguments. Formula (14) is applicable for both the symmetric (with an accuracy to the electrode orientation) and nonsymmetric junctions. The parameter R_N is the normal-state resistance of the tunnel junction determined by $|\widetilde{T}_{\mathbf{pq}}|^2$without the factorized multiplier $w(\theta)$. Integration over the angle variable θ is carried out within the interval $-\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$, i.e., over the FS hemicircle turned towards the junction plane. The barrier permeability $w(\theta)$ is described by formula (10). Formula (14) was obtained in the weak-coupling approximation,[241] i.e., the reverse influence of the energy gaps on the initial FS was neglected.

At $w(\theta) \equiv 1$ (the tunneling directionality associated with the θ-dependent barrier transmittance is neglected), setting $f_\Delta^{(\prime)} = 1$ (actually, it is a substitution of an isotropic s-wave superconductor for the d-wave or extended s-wave ones) and $f_\Sigma^{(\prime)} = 0$ (the absence of CDW gapping), as well as replacing $\cos\theta$ by 1 (the absence of tunnel directionality associated with momentum projection), Eq. (14) expectedly reproduces the basic Ambegaokar-Baratoff result for tunneling between s-wave superconductors.[135–137,241,242] In contrast, if the directionality and dielectric gapping are excluded but f_Δ and f'_Δ are retained, we return to the Sigrist-Rice model.[60] Note that we restricted ourselves to the classical tunnel junction[135,136,241] being a strong-barrier limit of the more general model.[243] This means that Andreev-Saint-James-reflection processes[41,244–246] were disregarded.

In this paper we do not consider the T dependences of the dc Josephson current. Therefore, all further calculations will be carried out for zero temperature $T = 0$. In this case, formula (15) can be expressed in terms of complete elliptic functions[135,247]

$$P(\theta,\gamma,\gamma') = \frac{\bar{\Delta}\bar{\Delta}'}{\max\{\bar{D},\bar{D}'\}} K\left(\sqrt{1-\left(\frac{\min\{\bar{D},\bar{D}'\}}{\max\{\bar{D},\bar{D}'\}}\right)^2}\right). \quad (16)$$

Here again the reduced notations should be understood as follows: $\bar{\Delta} = \bar{\Delta}(T=0,\theta-\gamma)$, $\bar{\Delta}' = \bar{\Delta}'(T=0,\theta-\gamma')$, $\bar{D} = \bar{D}(T=0,\theta-\gamma)$, and $\bar{D}' = \bar{D}'(T=0,\theta-\gamma')$. In the general case $T \neq 0$, integration has to be carried out numerically.

V. PRELIMINARY CALCULATIONS

A. Reference CDW superconductors: The choice of parameters

The formulated problem does not allow the final results to be derived in an analytical form. Therefore, we must confine ourselves to numerical calculations. Issues associated with the influence of inherent CDWS parameters—the relative strength of superconducting and CDW pairing σ_0 and the degree of FS dielectrization α—as well as the temperature t on the dc Josephson current will not be analyzed in the present paper. Here we consider only various orientation dependences in order to check whether they can be useful to establish the type of superconducting pairing symmetry and CDW configuration in high-T_c superconductors. We confine the consideration to the case $t = 0$. Specific values of the problem parameters indicated above were estimated on the basis of the available experimental data.

First of all, these are the parameters σ_0 and α for the reference CDWS. According to the ARPES measurements for underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Refs. 81,112,150,248, and 249) and $(Bi,Pb)_2(Sr,La)_2CuO_{6+\delta}$,[81,159,191] we may estimate the value of the parameter α (the angular width of the nodal area affected by electron spectrum dielectrization) to be about 15°. The same experiments together with STM (Refs. 73,142,207, and 250–253) and break-junction[95,254] studies lead to the ratio between the critical superconducting T_c and pseudogap T_s temperatures approximately equal to or less than 1:3. For instance, tunnel measurements of nearly optimally doped $Bi_2Sr_{2-x}La_xCuO_{6+\delta}$ ($x = 0.4$) mesa structures revealed $T_s \approx 165$ K, the onset superconducting critical temperature $T_{c0} \approx 40$ K, and the true critical temperature $T_c \approx 30$ K.[253] The ratio 1:3 can be satisfactorily reproduced in the framework of our CDWS model if we set $\sigma_0 = 1.3$ at $N = 4$ and $\beta = 0$, i.e., for the $S_{\mathrm{CDW4}}^{d_{x^2-y^2}}$ superconductor. More specifically, $t_c = T_c/\Delta_0 \approx 0.23$ and $t_s = T_s/\Delta_0 \approx 0.74$ so that $t_c{:}t_s \approx 1{:}3.2$. The zero-temperature values of the order parameters are shown in Table I. Note that the corresponding calculated

TABLE I. Critical temperatures and zero-temperature order-parameter values for $S^{\rm sym}_{{\rm CDW}N}$ superconductors with $\sigma_0 = 1.3$ and $\alpha = 15^\circ$.

Symmetry	N	t_c	t_s	$\delta(0)$	$\sigma(0)$
$d_{x^2-y^2}$, $s^{\rm ext}_{x^2-y^2}$	4	0.23	0.74	0.62	1.16
$d_{x^2-y^2}$, $s^{\rm ext}_{x^2-y^2}$	2	0.38	0.74	0.87	1.00
d_{xy}, $s^{\rm ext}_{xy}$	4	0.45	0.74	0.92	1.27
d_{xy}, $s^{\rm ext}_{xy}$	2	0.46	0.74	0.96	1.27

ratio between the zero-temperature order parameters $\sigma(0)$ and $\delta(0)$ for the $d_{x^2-y^2}$ superconducting order-parameter symmetry and the checkerboard CDWs is approximately equal to 1.9, whereas the direct measurements[255] of the relaxation dynamics of photoexcited quasiparticles in $Bi_2Sr_2CaCu_2O_{8+\delta}$ reveals $\Sigma(0) = 41$ meV, $\Delta(0) = 24$ meV so that $\Sigma(0)/\Delta(0) \approx 1.7$, which confirms the adequacy of the selected parameters.

This combination of problem parameters—an $S^{\rm sym}_{{\rm CDW}N}$ superconductor with the superconducting pairing symmetry $\mathcal{S} = d_{x^2-y^2}$, the checkerboard CDW configuration ($N = 4$), and the parameters $\sigma_0 = 1.3$ and $\alpha = 15^\circ$—is selected below as the reference one. We intend to illustrate the influence of such parameters as $\mathcal{S}$ and N on the orientation current characteristics, leaving the parameters σ_0 and α constant. Table I also quotes the relevant parameter values for other $S^{\rm sym}_{{\rm CDW}N}$ possible reference superconducting states considered in this paper. The corresponding zero T gap roses can be found in Appendix A. Although the considered order-parameter symmetries are most likely not realized in cuprates, they might appear in other superconductors and hence deserve to be examined.

To detect the influence of the CDW on the tunnel current, we also need a corresponding CDW-free reference object. As will be seen below, in this case the best such objects for the $S^{\rm sym}_{{\rm CDW}N}$ superconductor are the pure $S^{\rm sym}_{\rm BCS}$ ones with $\delta_{\rm BCS}(0)$ equal to $\delta(0)$ of the former (one should bear in mind that, in the general case, $\delta(0) < \delta_0 = 1$). We shall refer to this BCS superconductor as the BCS counterpart. It involves no CDW sectors and we define its orientation by the bisectrix of the positive lobe of the superconducting order parameter.

We also select Nb as an $S^s_{\rm BCS}$ superconductor. Then the ratio between its superconducting gap[256] and those in typical high-T_c oxides[257] gives the value $\delta_{S^s_{\rm BCS}}(0) = 0.1$.

B. Selection of θ_0

The angle of effective tunneling in Eq. (10) has a high degree of uncertainty. First, in the framework of the tunnel Hamiltonian approach,[135,136] the very mechanism of tunneling and in particular the link between the initial and final positions of a tunneling pair on the FSs of both electrodes are not clear. Therefore, we confine the analysis to the quasiclassical model of tunneling (the WKB approximation[224]). Here the path of a pair under the barrier equals $L = d/\cos\theta$, where d is the interlayer thickness and θ is the angle at which the pair traverses the barrier. It is clear that, for large enough d, the aperture of effective tunneling can be made arbitrarily small (of course, with a drastic reduction of the overall tunnel current magnitude). In contrast, for very thin junctions, the angles of effective tunneling can vary significantly within the interval $-90^\circ < \theta < 90^\circ$ and effectively smear the current peculiarities stemming from the existence of FS dielectrized and nondielectrized sections [see Eq. (14)].

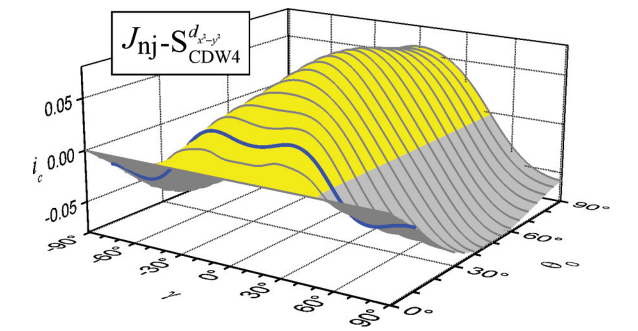

FIG. 3. (Color online) Influence of the directional tunneling angle θ_0 on the angular dependences of the normalized dc Josephson current $i_c(\gamma)$ in the $J_{\rm nj}$-$S^{d_{x^2-y^2}}_{\rm CDW4}$ junction between the reference CDW d-wave and BCS s-wave superconductors (see Sec. V A). Here $\delta_{S^s_{\rm BCS}}(0) = 0.1$. Positive and negative i_c values are marked by light and dark colors, respectively.

As was said above (see Sec. IV B), formula (10) was selected as a phenomenological one, which provides a reasonable behavior of the barrier transmission coefficient for various angles of pair passage. We made a number of calculations for $S^{\rm sym}_{{\rm CDW}N}$ superconductors with various sets of model parameters (see Sec. V A), different tunnel junctions, and various values of the directional tunneling parameter θ_0 in Eq. (10). Figure 3 illustrates how effectively the peculiarities in the orientation dependence of the dc Josephson current are averaged out when the cone of effective tunneling grows (the specific calculations were carried out for the $J_{\rm nj}$-$S^{d_{x^2-y^2}}_{\rm CDW4}$ junction). The figure testifies that, for the selected set of reference parameters, the angle θ_0 must be rather small to effectively reveal the CDW role, e.g., $\theta_0 = 10^\circ$, as shown in the figure by a bold curve. The results of calculations for other $S^{\rm sym}_{{\rm CDW}N}$ superconductors and experimental setups (see Appendix B) confirm that it is the best choice for all scenarios.

VI. RESULTS OF CALCULATIONS

A. Symmetric junctions: CDW configuration

Symmetric Josephson junctions ($J_{\rm sj}$-$S^{\rm sym}_{{\rm CDW}N}$) demonstrate more diversified characteristics due to the availability of a larger number of problem parameters and their possible combinations as compared to the nonsymmetric $J_{\rm nj}$-$S^{\rm sym}_{{\rm CDW}N}$ case. Hence let us begin with the former junctions. The relevant electrode parameters are $\alpha = \alpha'$, $\beta = \beta'$, $N = N'$, $\sigma_0 = \sigma_0'$, and (in the general case) $\gamma \neq \gamma'$. Accordingly, the notation i_c for the quantity $\frac{2eR_N}{\Delta_0^2} I_c(T=0)$ is introduced.

As said above, this paper is devoted to the analysis of orientation dependences of the dc Josephson current, i.e., the dependences of i_c on the angles γ and γ' (see Fig. 2), for various kind of electrodes in order to determine whether they can be useful in establishing the superconducting pairing symmetry and the availability of CDWs in high-T_c superconductors. For this purpose, we will confine the scope of the presented material by reporting the results obtained for

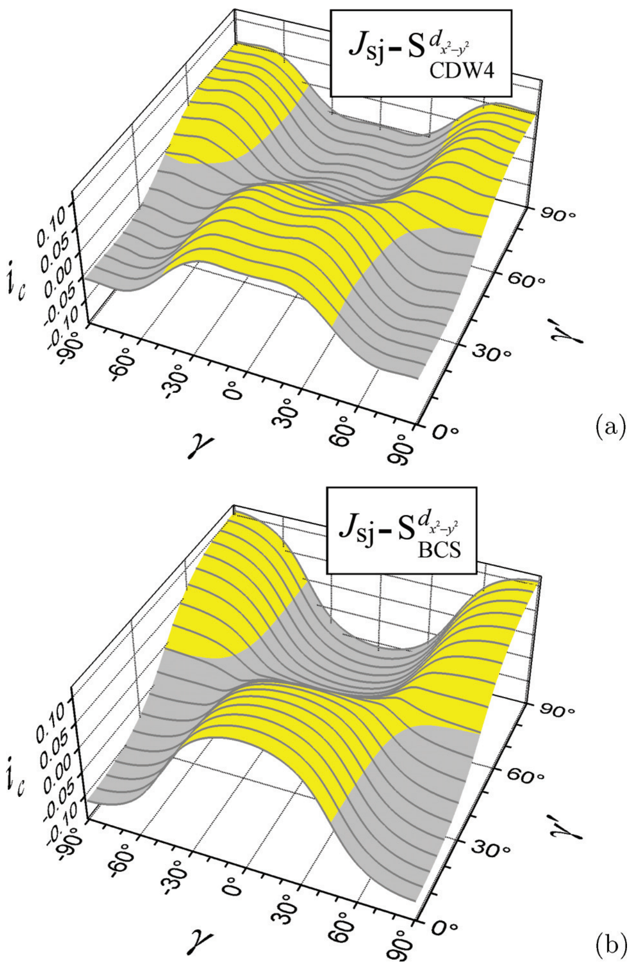

FIG. 4. (Color online) Dependences $i_c(\gamma,\gamma')$ for the (a) J_{sj}-$S^{d_{x^2-y^2}}_{CDW4}$ and (b) J_{sj}-S^{d}_{BCS} junctions with (a) the reference CDWS superconductor and (b) its d-wave BCS counterpart (see Sec. V A).

model $S^{\mathrm{sym}_\beta}_{\mathrm{CDW}N}$ CDWSs with the common parameters $\sigma_0 = 1.3$ and $\alpha = 15^\circ$. The sets of other parameters are $\mathcal{S} = (d,s^{\mathrm{ext}})$, $\beta = (0^\circ,45^\circ)$, and $N = (2,4)$.

First, let us consider the angular dependences describing the influence of different crystal orientations relative to each other and the junction plane. In Fig. 4(a) the dependences $i_c(\gamma,\gamma')$ for the J_{sj}-$S^{d_{x^2-y^2}}_{CDW4}$ junction are depicted. The role of CDW dielectrization becomes clear if we make a comparison with the analogous dependence, but for the J_{sj}-S^{d}_{BCS} junction containing the electrodes of the corresponding S^{d}_{BCS} counterpart [Fig. 4(b); see Sec. V A]. Note that, in the latter case, both angles γ and γ' are reckoned from the junction-plane normal **n** and determine the orientation of the positive superconducting order-parameter lobes (see Fig. 2). One can see that, against the background of smooth oscillations with a period of 180°, the dependence $i_c(\gamma)$ demonstrates some distortions, which are the most appreciable at the minima and maxima.

An analogous comparison is even more spectacular and illustrative for the cases of symmetric junctions with an $S^{d_{x^2-y^2}}_{CDW2}$ superconductor and its S^{d}_{BCS} counterpart (Fig. 5). The reason is that in this case the superconducting and CDW order parameters have different rotational symmetries: fourfold and twofold, respectively [see the gap rose in Fig. 10(b)]. When

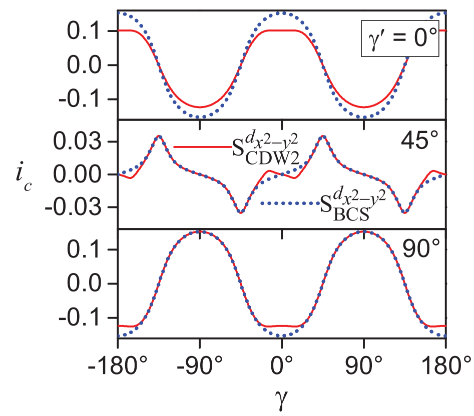

FIG. 5. (Color online) Dependences $i_c(\gamma)$ for the J_{sj}-$S^{d_{x^2-y^2}}_{CDW2}$ and J_{sj}-S^{d}_{BCS} junctions with the reference CDWS superconductor and its d-wave BCS counterpart (see Sec. V A), respectively, for various γ'.

both electrodes are oriented at angles close to 90°, only the FS nondielectrized sections make a significant contribution to the current owing to the tunnel directionality. Here, however, the $\bar{D}(0,\theta)$ gap profile in the $S^{d_{x^2-y^2}}_{CDW2}$ superconductor is close to the $\bar{\Delta}(0,\theta)$ profile in its S^{d}_{BCS} counterpart and the $i_c(\gamma)$ dependences in both cases are almost indistinguishable. However, if either of the electrodes is oriented in such a way that its CDW sector is perpendicular to the junction plane, the equivalence disappears. Now the period of the $i_c(\gamma)$ dependence becomes equal to 180°. Hence the angular dependence $i_c(\gamma)$ may be not only a useful tool for the determination of whether the CDW exists in a high-T_c superconductor (by the deviations from a smooth periodic behavior), but also an indicator of the CDW configuration, checkerboard or unidirectional.

An interesting configuration may arise if a single-crystalline high-T_c sample is cut at an arbitrary angle with respect to the crystal axes. Later on, both pieces are used as electrodes in the same tunnel junction. In essence, we would obtain a correlated rotation of crystals on both junction sides. Accordingly, this means a correlated rotation of the electrodes' CDWs with the orientation mismatch angle $\Delta\gamma = \gamma - \gamma' = 0$. The results of the numerical simulation of such an experiment are presented in Fig. 6 for the J_{sj}-$S^{d_{x^2-y^2}}_{CDW4}$ [Fig. 6(a)] and J_{sj}-$S^{d_{x^2-y^2}}_{CDW2}$ [Fig. 6(b)] junctions. Besides the case $\Delta\gamma = 0$, the figure also exhibits the corresponding dependences for the orientation mismatch angles $\Delta\gamma = 45^\circ$, 90°, and 135°. Actually, these dependences are nothing but the cross sections of the corresponding surfaces $i_c(\gamma,\gamma')$ [for Fig. 6(a) it is the surface depicted in Fig. 4(a)] by the relevant planes $\gamma' - \gamma = \mathrm{const}$.

One should note that if $\Delta\gamma = 0$, the sign of integrand in formula (14) is always positive because the signs of Δ and Δ' [see the coefficient before the integral in Eq. (15)] are identical at any θ. Therefore, the current is also positive for any γ, which is very similar to the case of $s^{\mathrm{ext}}_{x^2-y^2}$-wave pairing (see below). A symmetric situation with an accuracy to the current sign is observed when the orientation mismatch angle $\Delta\gamma = 90^\circ$ because now $\bar{\Delta}(\theta) = -\bar{\Delta}'(\theta)$. It is also clear that dependences $i_c(\gamma)$ for $\Delta\gamma = 0$ and 90° should differ from each other only by sign because the dependences $\bar{D}(\theta)$ and $\bar{D}'(\theta)$ are identical.

A similar result is obtained for the J_{sj}-$S^{d_{x^2-y^2}}_{CDW2}$ junction [Fig. 4(b)]; however, now the dependences $\bar{D}(\theta)$ and $\bar{D}'(\theta)$

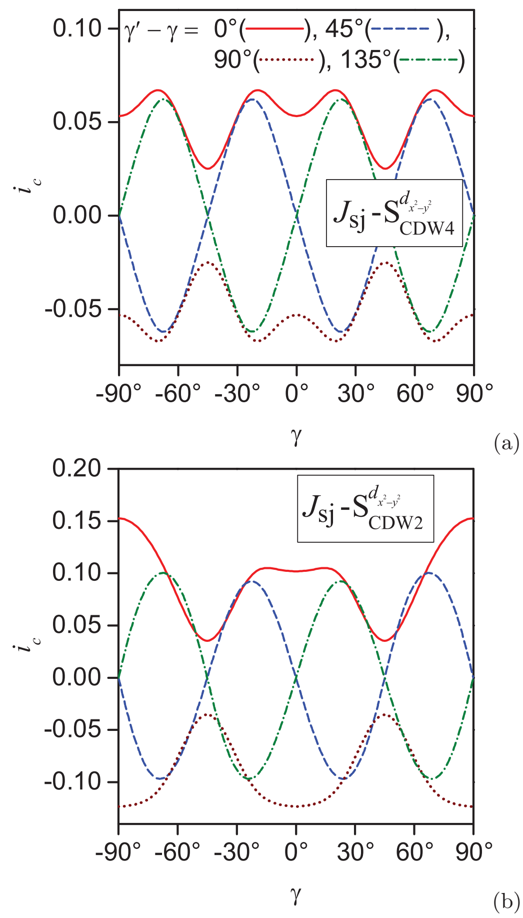

FIG. 6. (Color online) Dependences $i_c(\gamma)$ for the (a) $J_{\rm sj}$-$S^{d_{x^2-y^2}}_{\rm CDW4}$ and (b) $J_{\rm sj}$-$S^{d_{x^2-y^2}}_{\rm CDW2}$ junctions for correlated rotations of CDWs in electrodes at various fixed differences $\gamma' - \gamma$.

have the twofold rotational symmetry and are identical only when $\Delta\gamma = 0$. Therefore, the dependence $i_c(\gamma)$ for $\Delta\gamma = 0$ is not a specular reflection of that for $\Delta\gamma = 90^\circ$.

The considered feature of sign preservation by the current within definite vicinities of mismatch angles $\Delta\gamma = 0$ and 90° is possible only due to the tunnel directionality. All other angles lead to the intermediate situations of variable sign. To get a better idea of them, the latter (unidirectional) case is illustrated in Appendix C in more detail.

B. Symmetric junctions: Superconducting pairing symmetry

An interesting question consists in checking whether we can use the orientation dependences of the stationary Josephson current to discern the pairing symmetry of high-T_c superconductors in the presence of CDWs. In Fig. 7 the dependences $i_c(\gamma)$ are depicted for $J_{\rm sj}$-$S^{\rm sym}_{\rm CDW4}$ junctions with various values of the parameter $\mathcal{S}$. One can see that the curves are qualitatively different and therefore may provide the required information.

At the same time, we see again that, in general, the CDW-induced peculiarities in the orientation dependences $i_c(\theta)$ are rather weak for the selected reference parameter set and can be easily overlooked against the large background superconducting behavior. This situation is illustrated in Fig. 8 for the $J_{\rm sj}$-$S^{d_{xy}}_{\rm CDW2}$ junction. In particular, Fig. 8(b) demonstrates that the discussed peculiarities, interesting *per se*, are observable only at certain misorientations between the electrodes, being effectively absorbed at other angles.

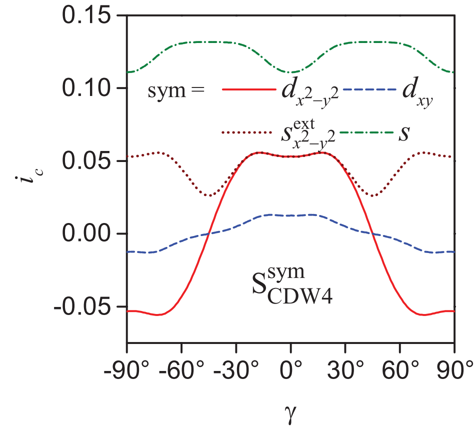

FIG. 7. (Color online) Dependences $i_c(\gamma)$ for $J_{\rm sj}$-$S^{\rm sym}_{\rm CDW4}$ junctions with various symmetries of the superconducting order parameter with $\gamma' = 0$.

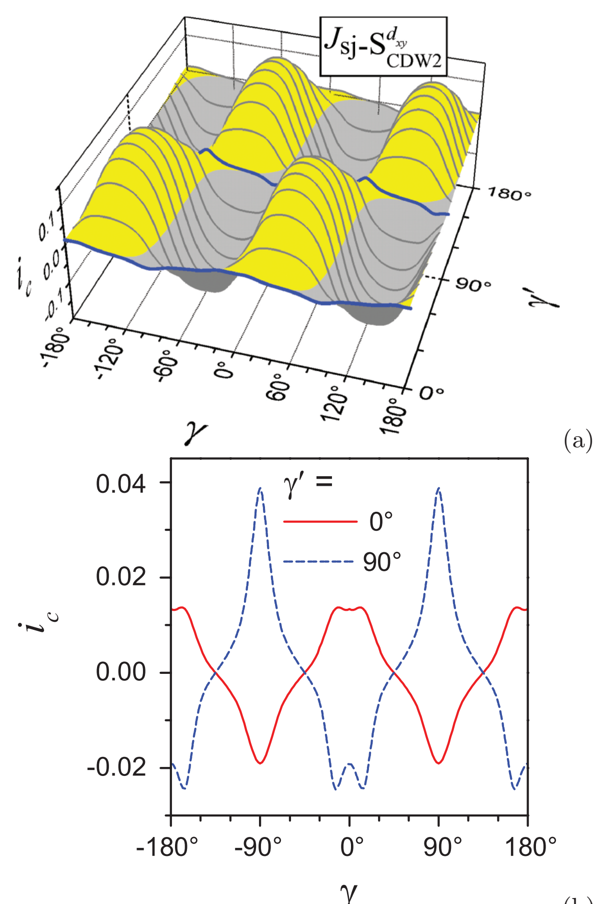

FIG. 8. (Color online) (a) Dependence $i_c(\gamma,\gamma')$ for the $J_{\rm sj}$-$S^{d_{xy}}_{\rm CDW2}$ junctions. (b) Cross sections at $\gamma' = 0$ and 90° corresponding to the bold curves in (a).

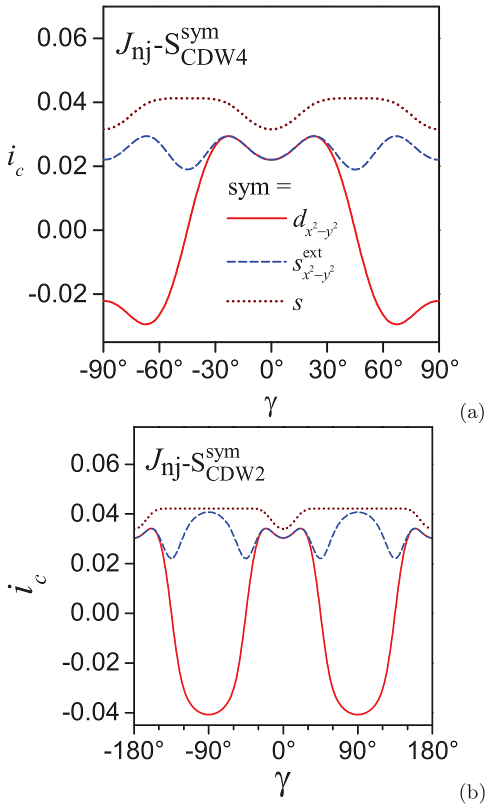

FIG. 9. (Color online) Dependences $i_c(\gamma)$ for the (a) $J_{\rm nj}$-$S^{\rm sym}_{\rm CDW4}$ and (b) $J_{\rm nj}$-$S^{\rm sym}_{\rm CDW2}$ junctions for various types of superconducting order-parameter symmetries.

C. Nonsymmetric junctions

Nonsymmetric Josephson junctions ($J_{\rm nj}$-$S^{\rm sym}_{{\rm CDW}N}$), where one electrode is a pure $S^s_{\rm BCS}$ superconductor, are described by a shorter list of parameters. In particular, $S^s_{\rm BCS}$ is characterized by a single parameter $\delta_{\rm BCS}$ and is isotropic (the orientation angle γ' loses its sense). As a result, the corresponding orientation dependences $i_c(\gamma)$ are not so diversified as in the symmetric case. Nevertheless, $J_{\rm nj}$-$S^{\rm sym}_{{\rm CDW}N}$ junctions are rather important because they correspond to the STM geometry with a superconducting tip, which can be practically realized.

Generally speaking, it is not strange that $i_c(\gamma)$ dependences should be qualitatively similar to those obtained in the symmetric setup at fixed orientations of the right-hand-side electrode. Therefore, we confine the subject of this section to the influence of the possible superconducting pairing symmetries in the CDWS electrode on the overall angle dependences. The results of relevant calculations are depicted in Fig. 9 for the cases of checkerboard [Fig. 9(a)] and unidirectional [Fig. 9(b)] CDW configurations. The calculations were carried out for the reference set of parameters. One can see that the character of the dependences changes drastically from one pairing symmetry to another and the distinction is more pronounced in the case of unidirectional CDWs.

VII. CONCLUSION

Our calculations showed that CDWs induced by the electron-hole pairing and appropriate to high-T_c cuprates can be probed and studied by means of the coherent superconducting dc Josephson tunneling. The interplay between CDW manifestations, tunnel directionality, and possible unconventional symmetry of the superconducting order parameter may lead to an involved behavior with several superimposed periods in the angular dependences. Symmetric junctions (break junctions and mesas) are preferable in comparison with nonsymmetric ones (STM) in revealing CDW effects.

The results obtained confirm that the dc Josephson current is always suppressed by the electron-hole (dielectric) CDW pairing, which, in agreement with the totality of experimental data, is assumed here to compete with its superconducting electron-electron (Cooper) counterpart. We emphasize that, as concerns the quasiparticle current, the interpretation of the results may be much more ambiguous. In particular, the states in the nodal region of the FS in d-wave superconductors are also engaged in CDW gapping[77,90,91,96,217,236,258] so that the tunnel spectroscopy (or ARPES) experiences the overall energy gaps being larger than their superconducting constituent.

We demonstrated that the emerging CDWs distort the dependence of the critical Josephson I_c on the angle γ between a certain crystal axis and the normal **n** to the junction plane, whatever the symmetry of superconducting order parameter is. At the same time, if an s-wave contribution to the actual order parameter in a cuprate sample is dominant up to the complete disappearance of the d-wave component, the $I_c(\gamma)$ dependences for junctions involving CDW superconductors are no longer constant as in the CDW-free case. This prediction can be verified for CDW superconductors with *a fortiori* s-wave order parameters (such materials are quite numerous[77,86–88,90,91]).

Once more returning to the problem of true underlying order-parameter symmetry in superconducting cuprates, one should recognize the leading role of phase-sensitive measurements based on the Josephson effect in solving the problem concerned, as well as in consequent applications of the results.[17,18,21,22,24,259] We are not going to analyze SQUID interferometry, SQUID magnetometry, or tricrystal experiments, which have little to do with our calculations. As for the Josephson single-junction tunneling, it was realized both in the a-b plane and in the c-axis direction of various high-T_c oxide structures.

In particular, Josephson currents between the s-wave superconductor Pb and $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$ single crystals along their c axes demonstrated conspicuous magnitudes and conventional Fraunhofer patterns in the magnetic field.[26] Meanwhile, purely d-wave superconductors would have revealed the absence of the Josephson tunneling current in this configuration.[38] In contrast, more peculiar experiments involving c-axis tunneling between Pb and a twinned $YBa_2Cu_3O_{7-\delta}$ crystal clearly showed the predominantly $d_{x^2-y^2}$-wave pairing symmetry with a noticeable admixture of the s-wave component.[27] Measurements of Josephson currents in the a-b plane of $YBa_2Cu_3O_{7-\delta}$ films with nanocracks showed that neither the pure d- nor the pure s-symmetrical superconducting order parameter can describe the angular (in-plane) dependences of the critical current.[260] At the same time, the $d + is$ combination with comparable amplitudes of the constituents was satisfactory to describe currents at large $T \leqslant T_c$, whereas all applied basic theoretical models failed at low temperatures.

Another attempt to elucidate the symmetry of the order parameter in $YBa_2Cu_3O_{7-\delta}$ ceramics was carried out for epitaxial films, where the intergrain current was measured.[261] No interference effects appropriate to $d_{x^2-y^2}$ superconductivity were detected, so the isotropic pairing was considered as the most probable one.

Josephson currents between superconductors with different purported order-parameter symmetries were measured in zigzag structures between the electron-doped high-T_c cuprate $Nd_{2-x}Ce_xCuO_{4-y}$ and Nb.[262] The $d_{x^2-y^2}$ symmetry was found to occur and even dominate at various doping levels and temperatures.

Finally, the most popular layered high-T_c oxide Bi_2Sr_2 $CaCu_2O_{8+x}$ was also tested by Josephson measurements. In particular, c-axis-directed currents were studied for $Bi_2Sr_2CaCu_2O_{8+x}$/Au/Nb junctions[43] and $Bi_2Sr_2CaCu_2O_{8+x}$/Pb ones.[42] The authors of Ref. 43 made an inference that both the s- and $d_{x^2-y^2}$-wave contributions to the overall order parameter of $Bi_2Sr_2CaCu_2O_{8+x}$ do exist, the former possessing a smaller T_c and a smaller magnitude than its anisotropic counterpart. Nevertheless, the conclusion about the critical temperatures seems questionable from the theoretical viewpoint. At the same time, the manifestation of the s-wave component is beyond any doubt. The experiments on $Bi_2Sr_2CaCu_2O_{8+x}$/Pb junctions revealed a conspicuous (although tiny $\Delta_s/\Delta_d \approx 10^{-3}$) s-wave contribution as well as a typical Fraunhofer pattern when the magnetic field was switched on.[42] Those results were explained by the involvement of the Pb order-parameter anisotropy;[263] however, this model seems unconvincing and cannot explain the results obtained for Nb.[43]

Josephson measurements in all-$Bi_2Sr_2CaCu_2O_{8+x}$ tunnel structures were made as well, so any crucial influence of the counterelectrode superconducting order-parameter symmetry was excluded. Specifically, the c-axis superconducting tunneling currents between the whiskers were measured.[264,265] The results of measurements[264] testified to the d-like angular current dependence, which, nevertheless, differed from the conventional $d_{x^2-y^2}$-wave pattern. In contrast, twist junctions[265] demonstrated an incoherent normal and Josephson tunneling independent of the twist angle and with an undoubted s-wave component. Those results confirmed the earlier ones obtained while studying c-axis bicrystal twist junctions,[29] which were quite correctly interpreted on the basis of a substantial s-wave contribution to the $Bi_2Sr_2CaCu_2O_{8+x}$ superconducting order parameter.[34,38,266]

On the basis of the foregoing synopsis, we are forced to conclude that the problem of the cuprate order parameter, in particular, in connection with the Josephson effect, is far from being solved. That is why we included several most frequently discussed types of superconducting pairing symmetries into consideration while performing our calculations presented in this work.

In this paper our approach was purely theoretical. We did not discuss unavoidable experimental difficulties if one tries to fabricate Josephson junctions suitable to check the results obtained here. We are fully aware that the emerging problems can be solved using the already accumulated knowledge concerning the nature of grain boundaries in high-T_c oxides.[17–20,23,267–274] Note that required junctions can be created at random in an uncontrollable fashion using the break-junction technique.[254] This method allows us to comparatively easily detect the CDW (pseudogap) influence on the tilt-angle dependences.

To summarize, measurements of the Josephson current between an ordinary superconductor and a d-wave or extended s-wave one or between two unconventional superconductors (first of all, high-T_c oxides) would be useful to detect a possible CDW influence on the electron spectrum of the latter. Similar studies of iron-based superconductors with doping-dependent SDWs would also be of benefit (see, e.g., recent reviews in Refs. 106 and 109) since CDW and SDW superconductors have certain similar properties.[86–88]

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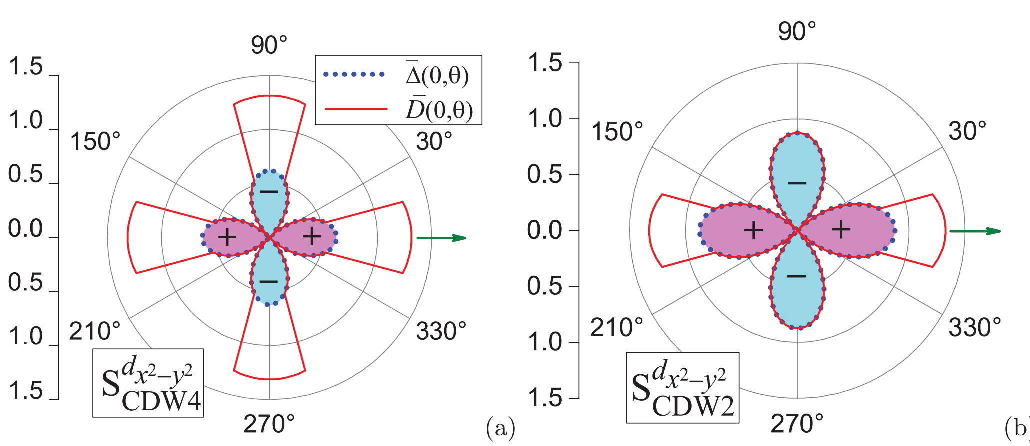

FIG. 10. (Color online) Gap roses for (a) $S^{d_{x^2-y^2}}_{CDW4}$ and (b) $S^{d_{x^2-y^2}}_{CDW2}$ superconductors with reference parameters.

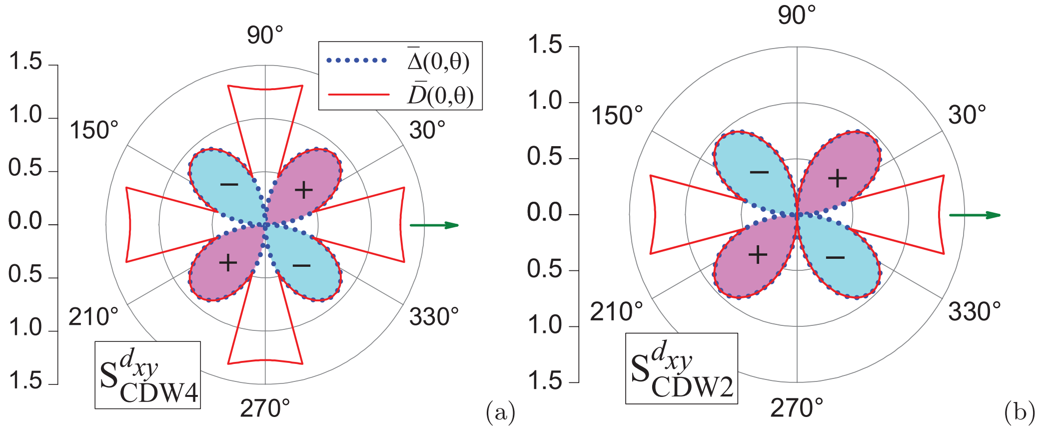

FIG. 11. (Color online) Same as in Fig. 10, but for the case of d_{xy}-wave symmetry.

APPENDIX A: GAP ROSES

Due to a competitive interaction between Cooper and electron-hole pairings, both leading to the electron spectrum gapping, the actual values of the relevant order parameters $\Delta(T)$ and $\Sigma(T)$ are smaller than the corresponding parent values Δ_0 and Σ_0 (see Sec. III). Besides, the dielectric (CDW) order parameter occupies only some sections (dielectrized sections) of the FS. As a result, a rather involved allocation of the combined energy gap $\bar{D}(T,\theta)$ arises on the FS [see Eq. (8)]. For the calculation of the dc Josephson current [see Eqs. (14) and (15)], two quantities are important: $\bar{D}(T,\theta)$ (the so-called gap rose) and $\bar{\Delta}(T,\theta)$. They can be extracted, e.g., from the ARPES measurements. The overall angular patterns differ substantially from the pure d-wave dependence $\cos 2\theta$.[90,91,182,217]

Both $\bar{D}(T,\theta)$ and $\bar{\Delta}(T,\theta)$ are anisotropic in momentum space. Therefore, tunnel directionality will inevitably result in the dependence of the stationary Josephson current i_c on the orientation of both CDWS crystal axes with respect to the junction plane, the angles γ and γ' (see Fig. 2). In this paper we analyze the dependences $i_c(\gamma,\gamma')$ for the cases when the electrodes are $S^{d_{x2-y2}}_{CDW4}$, $S^{d_{x2-y2}}_{CDW2}$, $S^{d_{xy}}_{CDW4}$, and $S^{d_{xy}}_{CDW2}$ superconductors. We restrict ourselves to the case $T=0$ and make specific calculations for a CDWS with the parameters $\sigma_0 = 1.3$ and $\alpha = 15^\circ$. The corresponding $\bar{\Delta}(T,\theta)$ and $\bar{D}(T,\theta)$ profiles over the two-dimensional FS are exhibited in Figs. 10 and 11. On the nondielectrized sections, both profiles coincide.

Note that, in all cases, the influence of FS dielectrization on superconductivity is revealed as a decrease of the actual $\delta(T=0)$ value (the amplitude of superconducting order-parameter lobes) with respect to the parent one $\delta_0(T=0)=1$. The figures demonstrate that this effect is stronger if the dielectrized FS fraction is larger (the checkerboard CDW configuration $N=4$ versus the unidirectional one $N=2$). At the same time, this influence is more pronounced in the case of $d_{x^2-y^2}$-wave superconducting order-parameter symmetry, inherent to cuprates, because in the d_{xy} case, the dielectrized sections are located around the nodal points of the superconducting order

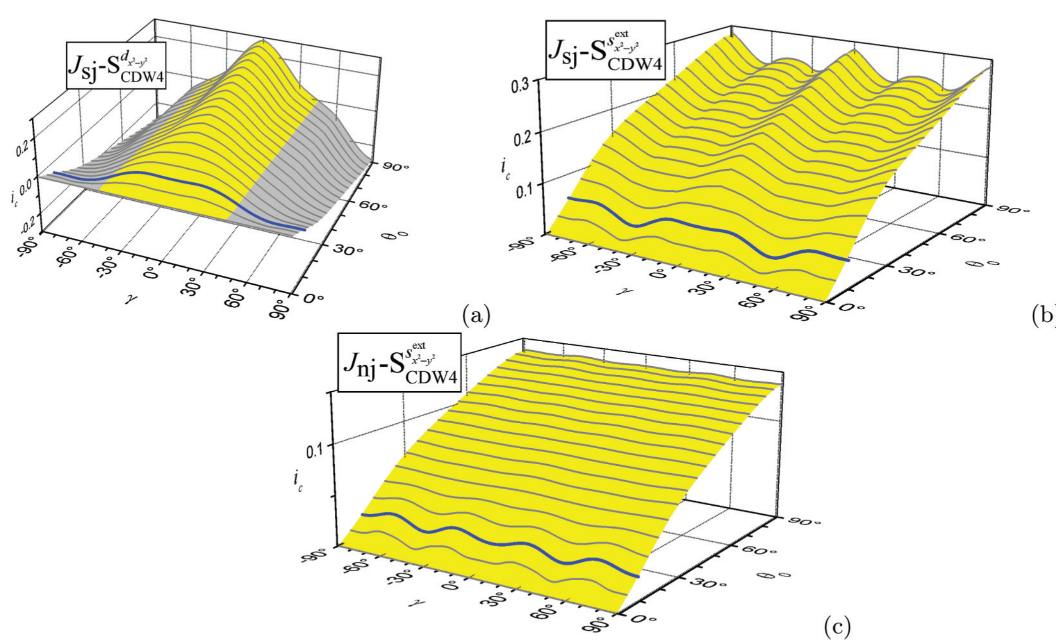

FIG. 12. (Color online) Same as in Fig. 3, but for the (a) J_{sj}-$S^{d_{x2-y2}}_{CDW4}$, (b) J_{sj}-$S^{s^{ext}_{x2-y2}}_{CDW4}$, and (c) J_{nj}-$S^{s^{ext}_{x2-y2}}_{CDW4}$ junctions.

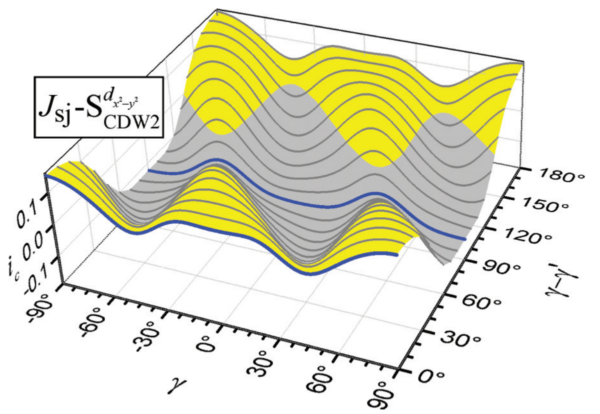

FIG. 13. (Color online) Three-dimensional representation of the dependence $i_c(\gamma,\gamma'-\gamma)$ corresponding to the coordinated rotation of CDWs in both electrodes for the J_{sj}-$S^{d_{x^2-y^2}}_{CDW2}$ junction.

parameter, whereas the superconducting order-parameter lobes are the most developed on the nondielectrized sections, so their interplay is minimal.

The vector in each panel defines the orientation of the corresponding CDWS, so the angles γ and γ' in Eq. (14) are the angles between the vector **n** normal to the junction plane (see Fig. 2) and the indicated vectors on the left- and right-hand sides of the junction, respectively.

In the case of s-extended symmetry of the superconducting order parameter, i.e., if the electrodes are $S^{s^{ext}_{x^2-y^2}}_{CDW4}$, $S^{s^{ext}_{x^2-y^2}}_{CDW2}$, $S^{s^{ext}_{xy}}_{CDW4}$, and $S^{s^{ext}_{xy}}_{CDW2}$ CDWSs, the gap roses remain the same, but all $\bar{\Delta}$ lobes have the same sign (we took it to be positive for definiteness).

APPENDIX B: SELECTION OF EFFECTIVE TUNNELING ANGLE

This appendix includes some illustrative sets (see Fig. 12) of $i_c(\gamma,\theta_0)$ dependences calculated, in addition to that shown in Sec. V B (Fig. 3), for the J_{sj}-$S^{d_{x^2-y^2}}_{CDW4}$ [Fig. 12(a)], J_{sj}-$S^{s^{ext}_{x^2-y^2}}_{CDW4}$ [Fig. 12(b)], and J_{nj}-$S^{s^{ext}_{x^2-y^2}}_{CDW4}$ [Fig. 12(c)] junction setups with the same reference set of parameters (see Sec. V A). One can see that for all those cases, the value of the effective tunneling angle $\theta_0 = 10^\circ$ in formula (10) is the best choice for the demonstration of the influence of the CDW on the orientation dependences of the dc Josephson current $i_c(\gamma)$. The corresponding choice is marked as the bold curve. Of course, it is not known which θ_0 is inherent to any created junction. However, one might expect that the interlayer variation readily realized, e.g., in break junctions,[95,254] would produce a broad spectra of θ_0.

APPENDIX C: J_{sj}-$S^{d_{x^2-y^2}}_{CDW2}$ JUNCTIONS

In this appendix we plot the dependences $i_c(\gamma)$ for the J_{sj}-$S^{d_{x^2-y^2}}_{CDW2}$ junction described in Sec. VI A [Fig. 6(b)] with the varying orientation γ' of the right-hand-side electrode, but in a more detailed representation (see Fig. 13). The curves corresponding to the characteristic cases $\gamma' = 0$ and 90° are shown in bold. In essence, this dependence is a counterpart of that exhibited in Fig. 4(a), but for $N = 2$ and rotated by 45°.

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