Nonreciprocal light diffraction by a vortex magnetic particle of spherical shape

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We report a theoretical study of light diffraction by a spherical magnetic particle with a vortex magnetization distribution. It is shown that the intensity of the diffracted light involves a nonreciprocal contribution. This contribution depends on the vorticity of particle magnetization. It appears due to the excitation of an electric quadrupole, magnetic dipole, and the addition to the electric dipole moment in the particle, which depend on the particle magnetization vorticity. The dependence of the nonreciprocal contribution on particle parameters and geometry of diffraction is analyzed separately for the specified moments. The calculation of the effect for a cobalt particle and two linear polarizations of the incident light fits the data of recent experimental studies in the lattice of triangle magnetic particles by an order of magnitude.

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I. INTRODUCTION

Remarkable achievements in microtechnologies and nanotechnologies open possibilities in fabrication of optical media with interesting properties. Photonic three-dimensional crystals exhibit extraordinary optical effects, such as photonic band gap,[1,2] negative refractive index,[3,4] and ultraslow light propagation.[5] Surface plasmons in planar metal structures lead to enhanced magneto-optical effects,[6–8] extraordinary light transmission through subwavelength holes,[9] and effective generation of the second harmonic.[10,11] Interesting objects actively investigated in the past decade are planar chiral structures.[12–16] Such a structure is ordinarily a two-dimensional regular lattice of nonmagnetic particles that possesses the time and spatial inversion symmetries.[17] Nevertheless, light transmission through a planar chiral structure experiences polarization rotation, which is different for the light traveling in opposite directions due to the absence of mirror symmetry and strong anisotropy in the orientation of the structure elements. The term "nonreciprocal" is often used for this effect, and some speculations appear on the time-reversal symmetry violation.[14–16] Note that the intensity of light diffracted by a planar chiral structure does not change with a reversal of the propagation direction, in accordance with the reciprocity law. The nonreciprocal intensity effects can appear only in the presence of a magnetic field[18–20] or in magnetic systems.[21–25] Such effects have been observed recently in a two-dimensional lattice of magnetic vortices.[26] The particles had a triangular shape, which allowed manipulation of their vorticity by an uniform external magnetic field, thus making it identical for all particles.

The problem of scattering of electromagnetic waves by a particle of an arbitrary size with gyrotropic dielectric permittivity and magnetic permeability has been addressed in a general form[27] and for magnetic particles specifically.[28,29] However, the gyrotropy terms are always supposed to be constant over the particle, which corresponds to uniform magnetization. This paper is devoted to theoretical calculations of the diffraction of light by a magnetic vortex that turns out to be a nonuniform magnetic particle. Our theory describes the mechanisms underlying the effect of nonreciprocal light scattering by such a particle and also allows us to estimate its magnitude and compare the theoretical data with the experiment.[26] Note that this effect is assumed to be simply summed over the lattice of particles (i.e., the collective phenomena are neglected), thus making it possible to focus on the light diffraction by a single particle.

In Sec. II we consider some phenomenological arguments in favor of the nonreciprocal light diffraction by a vortex particle. Then a microscopic model of the effect is proposed in the approximation of a spherical particle that is small compared to the wavelength. The main assumptions are outlined in Sec. III. Further, we consider two approaches to the problem. A simple Born approximation (permittivity is close to unity) that helps reveal the existence of the effect is described in Sec. IV. Yet, within this approximation the effect occurs only for one linear polarization of the incident light. Section V is devoted to calculation for the arbitrary permittivity, based on the first-order perturbation theory with respect to the particle size-to-wavelength ratio. We show that the nonreciprocal term appears due to excitation of the electric quadrupole, magnetic dipole, and a small addition to the electric dipole moment, that emit light interfering with the main magnetization-independent electrodipole radiation, which agrees to well-known mechanisms.[30] The analysis of the results is given in Sec. VI. We perform simple estimations for the parameters of cobalt and an appropriate particle size and compare our calculations to the experimental results.[26] A summary of our results is outlined in Sec. VII.

II. PHENOMENOLOGICAL CONSIDERATIONS

We begin with some phenomenological arguments in favor of the nonreciprocal light diffraction by the vortex particle.[26] If we consider the light scattering cross section summed over the polarizations of incident and diffracted light, the reciprocity law takes a simple form

$$\sigma(\mathbf{k},\mathbf{k}',\mathbf{M}(\mathbf{r})) = \sigma(-\mathbf{k}',-\mathbf{k},-\mathbf{M}(\mathbf{r})), \tag{1}$$

where σ is the differential cross section for the diffracted light, $\mathbf{k}$ and $\mathbf{k}'$ are the wave vectors of the incident and diffracted beams, $\mathbf{M}(\mathbf{r})$ is the magnetization spatial distribution. The term "nonreciprocal effect" implies the following inequality:

$$\sigma(\mathbf{k},\mathbf{k}',\mathbf{M}(\mathbf{r})) \neq \sigma(-\mathbf{k}',-\mathbf{k},\mathbf{M}(\mathbf{r})), \tag{2}$$

which can be transformed using Eq. (1) to

$$\sigma(\mathbf{k},\mathbf{k}',\mathbf{M}(\mathbf{r})) \neq \sigma(\mathbf{k},\mathbf{k}',-\mathbf{M}(\mathbf{r})), \tag{3}$$

thus making it possible to observe the effect by inverting the magnetization direction instead of swapping the source and the detector.[26]

For systems without center of inversion the scattering cross section may contain the term $((\mathbf{k}+\mathbf{k}')\cdot\mathbf{C})$, where $\mathbf{C}$ is a vector. Being linear in the wave vector, this term leads to the nonreciprocal effects described by (2) and (3). The $\mathbf{C}$ vector should be a *polar vector* that also changes its sign under the time reversal. For a magnetic scatterer of a centrosymmetrical shape and material $\mathbf{C}$ can be chosen in the simplest form $\mathbf{C} = \alpha\langle[\mathbf{r}\times\mathbf{M}(\mathbf{r})]\rangle$, which is a toroidal moment of the particle associated with the magnetic vorticity,[31] the brackets mean the spatial averaging over the scatterer, α is a constant. According to these considerations, the diffraction of unpolarized light by a particle with the vortex magnetization distribution is nonreciprocal and the scattering cross section has a contribution depending on the vorticity,

$$\sigma(\mathbf{k},\mathbf{k}',\mathbf{M}(\mathbf{r})) = \cdots + \alpha[(\mathbf{k}+\mathbf{k}')\cdot\langle\mathbf{r}\times\mathbf{M}(\mathbf{r})\rangle]. \tag{4}$$

The weak point of this phenomenological consideration is the polarization dependence of the nonreciprocal effect. Indeed, the existence of the effect for unpolarized light means it exists for at least one linear polarization, but the question about the contribution of different polarizations to it arises. However, the experiments currently are performed exactly for a linearly polarized light.[26]

III. MAIN ASSUMPTIONS AND DEFINITIONS

As has already been mentioned, we assume that the diffracting particle has a spherical shape with radius a. The incident wave is assumed to be monochromatic (all fields change in time as $e^{i\omega t}$, ω is the wave circular frequency) and plane. Its wavelength λ is much bigger than a,

$$\lambda \gg a. \tag{5}$$

The particle has a vortex magnetic moment (see Fig. 1)

$$\mathbf{M} = \mathbf{e}_\phi = \mathbf{e}_y cos\phi - \mathbf{e}_x \sin\phi. \tag{6}$$

Here $\mathbf{e}_\phi$ is the unit vector in spherical coordinates, $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors in Cartesian coordinates, ϕ is azimuthal angle

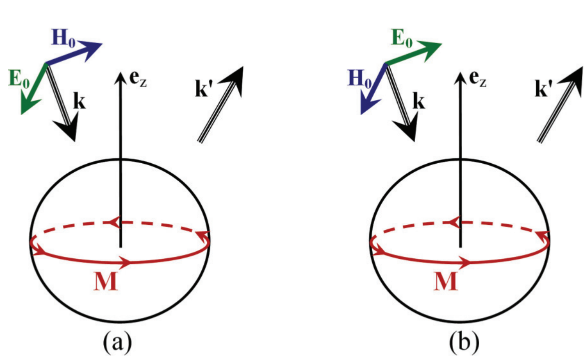

FIG. 1. (Color online) The geometry of light diffraction by a vortex magnetic particle. (a) The incident light is s polarized. (b) The incident light is p polarized.

in spherical coordinates, magnetic moment $\mathbf{M}$ is normalized to unity.

The magnetic permeability tensor is unit, while the dielectric permittivity tensor is assumed to have a locally gyrotropic form

$$(\hat{\epsilon})_{jl} = \varepsilon\delta_{jl} + i\gamma e_{jlm} M_m, \tag{7}$$

where δ_{jl} is the Kronecker δ, e_{jlm} is the completely antisymmetric tensor (the Levi-Civita tensor). Since $\mathbf{M}$ is normalized to unity, the γ coefficient includes its magnitude. Expression (7) is valid if the magnitude of quasiclassic electron oscillations is small compared to the scale of the magnetic moment variation. In our case it is equal to the particle radius a, hence, we get applicability criterion

$$\frac{eE_0\lambda}{mc^2}\lambda \ll a, \tag{8}$$

where e is the absolute electron charge, m is the electron mass, E_0 is the magnitude of the electric field of the incident wave, c stands for the light velocity. The first term in the left-hand part of (8) is the ratio of the energy the electron gains while oscillating in the electric field of the wave, to its rest energy. In the optical range this ratio is very small for the existing sources, which allows simultaneous fulfillment of conditions (5) and (8).

Another assumption used in our symmetry considerations (Sec. II) and elsewhere throughout the paper is that the diffracted wave is plane. In fact, the wave diffracted by a small particle is spherical at a large distance from the latter. But on a scale much smaller than the distance between the particle and the measuring point the wave may be considered plane.[32] Its intensity, indeed, depends on the distance from the particle.

Since the experiments on light diffraction by the lattice of magnetic vortices have been carried out for the linear polarizations of incident light, we also use these polarizations in theoretical calculations. The following designations for the linear polarizations are used throughout the paper: the wave polarized so that its electric field vector lies in the plane defined by the $\mathbf{k}$ vector and the axis of the magnetic vortex $\mathbf{e}_z$ is termed p polarized (see Fig. 1); the wave with the electric field perpendicular to that plane is s polarized.

One more important approximation consists in restricting ourselves to the first order in the magnetic moment, which mathematically corresponds to the first order in a small parameter γ/ε [see Eq. (7)]. Our choice of approximation is determined by both the fact that the phenomenon described by Eq. (4) is linear in $\mathbf{M}$ and that γ/ε is a very small parameter for the existing ferromagnets.[33,34]

IV. BORN APPROXIMATION

The simplest calculation is based on the assumption that permittivity tensor $\hat{\epsilon}$ is almost equal to unit tensor $\hat{1}$,

$$\hat{\epsilon} = \hat{1} + \hat{\delta\epsilon}. \tag{9}$$

The Maxwell equations can be solved in this case for a scattering particle of an arbitrary shape.[32] According to the well-known theory, the electric field of diffracted wave $\mathbf{E}_{\text{scat}}$

can be written as

$$\mathbf{E}_{\text{scat}} = -\frac{e^{ik'R_0}}{4\pi R_0}\left[\mathbf{k}' \times \left(\mathbf{k}' \times \int_V \hat{\delta\epsilon}\mathbf{E}_0 e^{i\mathbf{q}\mathbf{r}} dV\right)\right]. \tag{10}$$

Here $\mathbf{E}_0$ is the electric field of the incident wave, $\mathbf{q}$ stands for $\mathbf{k} - \mathbf{k}'$, V is the volume of a scattering particle, R_0 is the distance between the scattering particle and the point where $\mathbf{E}_{\text{scat}}$ is measured. According to our assumptions (Sec. III), R_0 is much bigger than the linear dimension of scatterer $V^{1/3}$.

We can now use (7) and (9) to determine $\hat{\delta\epsilon}$ and calculate the diffracted electric field. It is convenient to use the matrix form to represent the diffraction coefficients for different linear polarizations,

$$\hat{S} = \frac{k_0^2}{4\pi R_0}\begin{pmatrix} S_{ss} & S_{sp} \\ S_{ps} & S_{pp} \end{pmatrix}, \tag{11}$$

where the diagonal terms represent the diffraction without a change of polarization, while the off-diagonal terms stand for the diffraction to another polarization. The term k_0 in Eq. (11) is the absolute value of the wave vector $k_0 = \omega/c$. The matrix components in (11) are

$$S_{ss} = \int_V (\varepsilon - 1)e^{i\mathbf{q}\mathbf{r}} dV \tag{12}$$

$$S_{sp} = i\gamma\left(\mathbf{k}\cdot\int_V \mathbf{M}(\mathbf{r})\, e^{i\mathbf{q}\mathbf{r}} dV\right) \tag{13}$$

$$S_{ps} = -i\gamma\left(\mathbf{k}'\cdot\int_V \mathbf{M}(\mathbf{r})\, e^{i\mathbf{q}\mathbf{r}} dV\right) \tag{14}$$

$$S_{pp} = (\mathbf{k}\cdot\mathbf{k}')\int_V (\varepsilon - 1)e^{i\mathbf{q}\mathbf{r}} dV - - i\gamma\left[(\mathbf{k}\times\mathbf{k}')\cdot\int_V \mathbf{M}(\mathbf{r})e^{i\mathbf{q}\mathbf{r}} dV\right] \tag{15}$$

When analyzing Eqs. (12)–(15) we should first note that the linear polarizations do not mix without magnetization. Since we are interested in the intensity effect that is linear in $\mathbf{M}$, only the diagonal terms of matrix (11) should be taken into account. The matrix component S_{ss} does not depend on the magnetic moment, hence, we come to a conclusion that the s polarized light diffraction does not feature nonreciprocal properties, while for the p-polarized light there is a nonreciprocal contribution in the diffraction intensity.

Finally, taking into account Eq. (5), we can expand the term $e^{i\mathbf{q}\mathbf{r}}$ into a Tailor series and neglect all but the first two terms ($e^{i\mathbf{q}\mathbf{r}} \approx 1 + i\mathbf{q}\mathbf{r}$). These correspond to the electrodipole and electroquadrupole terms that are of the zero and the first order in a/λ. Calculation of the intensity for $\mathbf{M}$ as given by (6) leads to

$$I_s = \frac{k_0^4 a^6}{9R_0^2}|\varepsilon - 1|^2 \tag{16}$$

$$I_p = \frac{k_0^4 a^6}{9R_0^2}\left\{|\varepsilon - 1|^2(\mathbf{n}\cdot\mathbf{n}')^2 - [(\mathbf{n}+\mathbf{n}')\cdot\mathbf{e}_z] \times \frac{3k_0 a}{16}[(\mathbf{n}\cdot\mathbf{n}') - (\mathbf{n}\cdot\mathbf{n}')^2]\mathrm{Re}[(\varepsilon - 1)^*\gamma]\right\}. \tag{17}$$

Here I_p and I_s are the intensities of the diffracted light for p and s polarizations of incident light, $\mathbf{n}$ and $\mathbf{n}'$ are the wave vectors of the incident and diffracted waves normalized to unity: $\mathbf{n} = \mathbf{k}/k_0$, $\mathbf{n}' = \mathbf{k}'/k_0$, $\mathbf{e}_z$ is the unit vector along the vortex axis.

In order to find the intensity of diffracted light for the unpolarized incident light (we label it I_{np}), the intensity should be averaged over the polarizations. Formally, it leads to $I_{\text{np}} = \frac{1}{2}(I_s + I_p)$, where $I_{\text{np}} \sim \sigma$. From Eqs. (16) and (17) it is clear that the above calculation is consistent with the phenomenological considerations.

In the conclusion of this section, we point out that the nonreciprocal light diffraction was of the same order of magnitude for both s and p polarizations of incident light in the experiment.[26] But within the used approach the nonreciprocal contribution exists only for the p-polarized light. In the next section we propose a more general theory that has no such shortcoming.

V. PERTURBATION THEORY

A more exact solution of the problem is gained through the perturbation theory based on condition (5). To account for the electroquadrupole and magnetodipole terms, we need to expand the solution up to the first order in a/λ. The correction to the electrodipole term will be gained naturally. Unlike in some simple models[35] we cannot use the quasistatic Maxwell equations here. Hence, our approach is based on the approximations of the Maxwell equations

$$\mathrm{rot}\,\mathbf{E} = ik_0\mathbf{H}, \tag{18}$$

$$\mathrm{rot}\,\mathbf{H} = -ik_0\hat{\epsilon}\mathbf{E}, \tag{19}$$

$$\mathrm{div}\,(\hat{\epsilon}\mathbf{E}) = 0, \tag{20}$$

$$\mathrm{div}\,\mathbf{H} = 0, \tag{21}$$

which follow from the estimation

$$\mathrm{rot}\,\mathbf{E} \sim \mathbf{E}/a, \tag{22}$$

$$\mathrm{rot}\,\mathbf{H} \sim \mathbf{H}/a. \tag{23}$$

Next, we assume that the permittivity tensor is not large. Taking into account that $\gamma/\varepsilon \ll 1$, we impose the condition on ε: $|\varepsilon| \ll \lambda/a$ [$\lambda/a \gg 1$ due to (5)]. Introducing a typical scale of fields in the particle $L \sim \lambda/\sqrt{|\varepsilon|}$ (for real ε L is the wavelength in the medium, for imaginary ε it is the skin depth), we have

$$a \ll L^2/\lambda. \tag{24}$$

This condition along with Eq. (5) allows us to derive the equations for the fields of zero and first order in a/λ from Eqs. (18)–(21). These equations are derived inside and outside the particle. The solutions are connected via the boundary conditions

$$\mathbf{E}_\tau^{\text{in}} = \mathbf{E}_\tau^{\text{out}}, \tag{25}$$

$$E_n^{\text{in}} = (\hat{\epsilon}\mathbf{E})_n^{\text{out}}, \tag{26}$$

$$\mathbf{H}^{\text{in}} = \mathbf{H}^{\text{out}}. \tag{27}$$

First, let us find the electric field inside the particle. As is well known, in zero order in a/λ and in γ/ε the electric and magnetic fields inside the particle have the form

$$\mathbf{E}_{\text{in}}^{0,0} = \frac{3}{\varepsilon+2}\mathbf{E}_0, \tag{28}$$

$$\mathbf{H}_{\text{in}}^{0,0} = \mathbf{H}_0 = \mathbf{n}\times\mathbf{E}_0. \tag{29}$$

The first upper index in $\mathbf{E}^{0,0}$ and $\mathbf{H}^{0,0}$ is used to denote the order in a/λ, the second index specifies the order in γ/ε.

The equation for $\mathbf{E}^{1,0}$ follows from Eqs. (18) and (22)

$$\operatorname{rot}\mathbf{E}^{1,0} = ik_0\mathbf{H}^{0,0}. \tag{30}$$

By solving this equation with a plane wave boundary condition at the infinity we arrive at

$$\mathbf{E}_{\text{in}}^{1,0} = \frac{i}{2}[(\mathbf{k}\cdot\mathbf{r})\mathbf{E}_0 - (\mathbf{E}_0\cdot\mathbf{r})\mathbf{k}] + \frac{5}{2}\frac{i}{2\varepsilon+3}[(\mathbf{k}\cdot\mathbf{r})\mathbf{E}_0 + (\mathbf{E}_0\cdot\mathbf{r})\mathbf{k}]. \tag{31}$$

We now calculate the fields linear in the magnetic moment. The equation for $\mathbf{H}^{0,1}$ is

$$\operatorname{rot}\mathbf{H}^{0,1} = 0, \tag{32}$$

$$\operatorname{div}\mathbf{H}^{0,1} = 0, \tag{33}$$

and, taking into account the zero boundary condition at the infinity, we have $\mathbf{H}^{0,1} = 0$.

Finally, the equations for $\mathbf{E}^{0,1}$ and $\mathbf{E}^{1,1}$ inside the particle are very similar:

$$\operatorname{rot}\mathbf{E}^{l,1} = 0, \tag{34}$$

$$\operatorname{div}\{\varepsilon\mathbf{E}^{l,1} + i\gamma[\mathbf{E}^{l,0}\times\mathbf{M}(\mathbf{r})]\} = 0, \tag{35}$$

where $l = 0,1$. Using Eq. (34), the solution can be found as $\mathbf{E}^{l,1} = -\nabla\psi^l(\mathbf{r})$, where ψ is the scalar function that satisfies the equation

$$\Delta\psi^l = i\frac{\gamma}{\varepsilon}\operatorname{div}[\mathbf{E}^{l,0}\times\mathbf{M}]. \tag{36}$$

We should mention here that the equation for ψ^l outside the particle is simply

$$\Delta\psi^l = 0. \tag{37}$$

The right-hand part of Eq. (36) may be treated as a charge density distribution. Solving this equation we have

$$\psi_{\text{in}}^l = -\frac{i}{4\pi}\frac{\gamma}{\varepsilon}\int_V \frac{\operatorname{div}[\mathbf{E}^{l,0}(\mathbf{r}')\times\mathbf{M}(\mathbf{r}')]}{|\mathbf{r}-\mathbf{r}'|}dV'. \tag{38}$$

Substitution of Eqs. (6), (28), and (31) in Eq. (38) yields a partial solution to the nonhomogeneous equation (36) inside the particle. The ψ_{out}^l function could be considered zero since the equation defining it is already homogeneous. It should be mentioned that the integral over the particle in the right-hand part of Eq. (38) cannot be expressed as elementary functions.

In order to satisfy boundary conditions at the edge of the particle we should add a complete solution to a homogeneous equation

$$\Delta\varphi^l = 0. \tag{39}$$

The electric field is thereby defined as follows:

$$\mathbf{E}^{l,1} = -\nabla\psi^l - \nabla\varphi^l. \tag{40}$$

The solution to (39) is well known[32] and reads

$$\varphi_{\text{out}}^l = \sum_{m=0}^{\infty}\sum_{n=-m}^{m} G_{mn}^l Y_{mn}\frac{a^{m+2}}{r^{m+1}}, \tag{41}$$

$$\varphi_{\text{in}}^l = \sum_{m=0}^{\infty}\sum_{n=-m}^{m} H_{mn}^l Y_{mn}\frac{r^{m}}{a^{m-1}}, \tag{42}$$

inside and outside the particle correspondingly, taking into account that φ is finite at $r=0$ and $\varphi\to 0$ at $r\to\infty$. Here Y_{mn} are spherical functions. The coefficients H_{mn}^l that we need to calculate the electric field inside the particle are found from boundary conditions at the edge of the particle,

$$H_{mn}^l = \left.\frac{i\gamma\int Y_{mn}^*\big[(\mathbf{E}_{\text{in}}^{l,0}\times\mathbf{M})_r - \epsilon(\psi^l)'_r - \frac{l+1}{a}\psi^l\big]d^2\Omega}{(\epsilon+1)m+1}\right|_{r=a}. \tag{43}$$

The integral here is taken over the boundary of the particle. Substitution of $l = 0,1$ into Eq. (43) leads to the coefficients H_{mn}^0 and H_{mn}^1 that define the electric field in zeroth and first order in a/λ. Practically, we need to calculate only H_{1n}^0, H_{2n}^0, and H_{1n}^1 in order to find the electrodipole, electroquadrupole, and magnetodipole moments of the particle.

The next step of our solution procedure is calculation of the electric current density $\mathbf{j}$ that is related to the electric field[32]

$$\mathbf{j} = -\frac{i\omega}{4\pi}(\hat{\epsilon}-\hat{1})\mathbf{E}. \tag{44}$$

Substitution of (7) in (44) yields

$$\mathbf{j}^{l,0} = -\frac{i\omega}{4\pi}(\varepsilon-1)\mathbf{E}^{l,0}, \tag{45}$$

$$\mathbf{j}^{l,1} = -\frac{i\omega}{4\pi}(\varepsilon-1)\mathbf{E}^{l,1} + \frac{\gamma\omega}{4\pi}(\mathbf{E}^{l,0}\times\mathbf{M}). \tag{46}$$

Knowing the electric current density distribution over the particle, we can calculate the electric field of the diffracted wave far from the diffracting particle[36]

$$\mathbf{E}_{\text{scat}} = \frac{ik_0^2 e^{ik_0R_0 - i\omega t}}{\omega R_0}\int_V \mathbf{j}(\mathbf{r})\,e^{-i\mathbf{k}'\mathbf{r}}dV. \tag{47}$$

We expand the expression $e^{-i\mathbf{k}'\mathbf{r}}$ in this formula in the Tailor series and keep the first two terms ($e^{-i\mathbf{k}'\mathbf{r}} \approx 1 - i\mathbf{k}'\mathbf{r}$). This expansion corresponds to the inclusion of the electroquadrupole, magnetodipole, and the first-order correction to the electrodipole term. All of these terms have the same order of magnitude in a/λ. The waves radiated by them all interfere with the zero-order electrodipole radiation. There is no point in separating them here.

Finally, for the electric field of the diffracted wave we have

$$\mathbf{E}_{\text{scat}} = \frac{ik_0^2 e^{ik_0R_0 - i\omega t}}{\omega R_0}\int_V [(\mathbf{j}^{0,0}+\mathbf{j}^{1,0}+\mathbf{j}^{0,1}+\mathbf{j}^{1,1}) - i(\mathbf{r}\cdot\mathbf{k}')(\mathbf{j}^{0,0}+\mathbf{j}^{0,1})]dV, \tag{48}$$

where $j^{0,0}, j^{1,0}, j^{0,1}$, and $j^{1,1}$ are determined by Eqs. (45) and (46). We have earlier noted that the integral defining the electric field inside the particle cannot be expressed as elementary functions; however, the double integral that appears in Eq. (48) is taken by changing the integration order. Thus, $\mathbf{E}_{scat}$ turns out to be expressed as elementary functions, indeed.

The last step of the calculation is to find the intensity of the diffracted light. Again, for two linear polarizations of the incident light we get

$$I_s = \frac{k_0^4 a^6}{R_0^2}\left|\frac{\varepsilon-1}{\varepsilon+2}\right|^2\left\{1-[(\mathbf{n}+\mathbf{n}')\cdot\mathbf{e}_z]\frac{\pi k_0 a}{32}\mathrm{Re}\left(\frac{6\gamma}{2\varepsilon+3}\right)\right\}, \tag{49}$$

$$I_p = \frac{k_0^4 a^6}{R_0^2}\left|\frac{\varepsilon-1}{\varepsilon+2}\right|^2\left[(\mathbf{n}\cdot\mathbf{n}')^2-\{[\mathbf{n}+\mathbf{n}']\cdot\mathbf{e}_z\}\frac{\pi k_0 a}{32}\mathrm{Re}\left(\frac{6\gamma}{2\varepsilon+3}\left\{(\mathbf{n}\cdot\mathbf{n}')+\frac{5}{\varepsilon-1}[(\mathbf{n}\cdot\mathbf{n}')-(\mathbf{n}\cdot\mathbf{n}')^2]\right\}\right)\right]. \tag{50}$$

Equations (49) and (50) manifest that the nonreciprocal contribution exists for both s- and p-polarized incident light. For the p-polarized light they show a somewhat different dependence on the angle between $\mathbf{n}$ and $\mathbf{n}'$ from that yielded by the Born approximation. These formulas are completely consistent with the phenomenological considerations (Sec. II).

VI. DISCUSSION

In this section we give the calculations of I_s and I_p dependence on different system parameters and its analysis. First, we take the constants and parameters that correspond to the experiment.[26] The wavelength is $\lambda = 632.8$ nm. The particle is made of cobalt. At this wavelength cobalt absorbs light quite well, so the permittivity ϵ is defined by the complex constants $\varepsilon = -12.6 + 18.6i, \gamma = 0.749 - 0.602i$.[33] The incident light propagates along the vortex axis ($\mathbf{n} = -\mathbf{e}_z$).

Generally, the particles used in the experiment[26] are flat (their thickness is much less than their lateral size), whereas our theory is developed for a sphere. The most important scale in this theory is the scale of variation of the Foucault current density that the electromagnetic wave generates in the particle. In a sphere it is obviously equal to the particle radius a. In a flat disk the vortex electric current changes its direction at the particle thickness length near edges (see Fig. 2). Therefore the important scale here is the particle thickness. So we take the particle diameter $2a = 30$nm.

The dependence of calculated absolute and relative nonreciprocal terms for s-polarized incident light defined as $\Delta I_s = I_s(-\mathbf{M}) - I_s(\mathbf{M})$ and $\Delta I_s/I_s = 2(I_s(-\mathbf{M}) - I_s(\mathbf{M}))/(I_s(-\mathbf{M}) + I_s(\mathbf{M}))$ on the angle between $\mathbf{n}$ and $\mathbf{n}'$ is depicted in Fig. 3. Contributions of the electroquadrupole, magnetodipole, and correction to the electrodipole term are shown separately in the same figures. Figures 3(a) and 3(b) are identical, which is caused by the fact that I_s is independent of the angle in the electrodipole approximation if the magnetic moment is neglected. These pictures show that all three contributions are of the same order of magnitude.

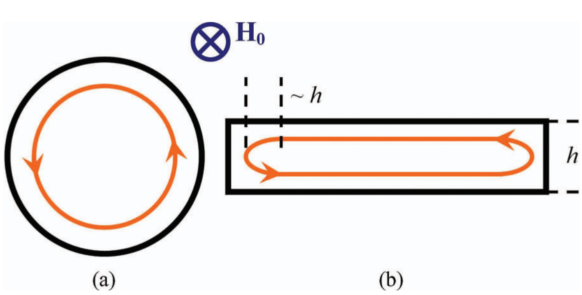

FIG. 2. (Color online) Foucault current density scale in (a) sphere and (b) disk.

The dependence of calculated absolute and relative nonreciprocal terms for p-polarized incident light defined as $\Delta I_p = I_p(-\mathbf{M}) - I_p(\mathbf{M})$ and $\Delta I_p/I_p = 2(I_p(-\mathbf{M}) - I_p(\mathbf{M}))/(I_p(-\mathbf{M}) + I_p(\mathbf{M}))$ on the angle between $\mathbf{n}$ and $\mathbf{n}'$ is represented in Fig. 4. We see that $\Delta I_p/I_p$ tends to infinity at $\pm\pi/2$ which are the Brewster angles for small particle. Indeed, Fig. 4(a) shows that ΔI_p tends to zero at these angles. However, it could be convenient to observe the effect experimentally in the vicinity of the Brewster angle. Analyzing different contributions into ΔI_p, we see that the relative value of the electrodipole term remains constant over the angle, while the electroquadrupole and magnetodipole relative values lead to the divergence of $\Delta I_p/I_p$ at the Brewster angles. These two terms become dominating close to these angles, but far from them all three terms are of the same order of value.

Comparison of Figs. 3 and 4 leads to the conclusion that relative nonreciprocal contributions have equal values for s- and p-polarized incident light at zero angle. The relative value grows for p polarization when $\mathbf{n}'$ is deflected from $\mathbf{n}$

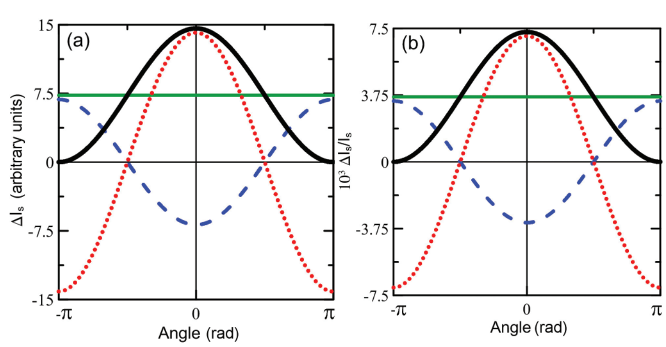

FIG. 3. (Color online) The dependence of nonreciprocity on the angle between $\mathbf{n}$ and $\mathbf{n}'$. The incident light is s polarized; $\mathbf{n}$ is parallel to the vortex axis $\mathbf{n} = -\mathbf{e}_z$. (a) Absolute value ΔI_s. (b) Relative value $\Delta I_s/I_s$. Solid bold line (black online) represents the overall ΔI_s, Solid thin gray line (green online) shows the electrodipole contribution into it, dashed line (blue online) displays the electroquadrupole term, dotted line (red online) is the magnetodipole contribution.

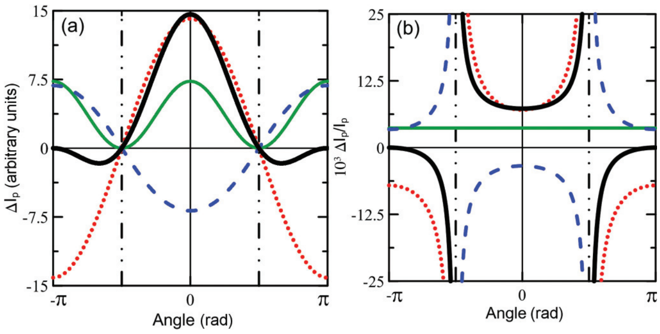

FIG. 4. (Color online) Same as Fig. 3 for p-polarized incident wave. Vertical dot-and-dash lines correspond to the asymptotes of relative nonreciprocity at $\pm\pi/2$.

and decreases for s polarization, which explains the fact that the effect is greater for p-polarized incident light than for s polarized.

The linear dependence of relative nonreciprocal contribution into light diffraction for s- and p-polarized incident light on particle size a is shown in Fig. 5 for the 30° angle between $\mathbf{n}$ and $\mathbf{n}'$. The important scales L^2/λ and $1/k_0$ [see (5) and (24)] calculated for the chosen parameters are approximately 25 and 100 nm. We see that $2a = 30$ nm (that have been chosen as a good approximation of the experimental setup) is of the order of value of L^2/λ here.

A plot of different contributions into the relative nonreciprocal intensity term versus the dielectric constant ε for s- and p-polarized incident light is depicted in Fig. 6. It is important to note that ε is assumed real here, while the magnetic contribution into $\hat{\epsilon}$ is defined by constant $\gamma = 0.749 - 0.602i$ that does not depend on ε. This leads to the divergence of the relative nonreciprocal contribution for p polarization at $\varepsilon = 1$ [Fig. 6(b)]. The absolute value of the intensity tends to zero at this value of ε. Another divergence is observed for both polarizations at $\varepsilon = -3/2$ and is explained by a quadrupole resonance. However, adding absorption (which is quite strong at a quadrupole resonance) will lead to degrading

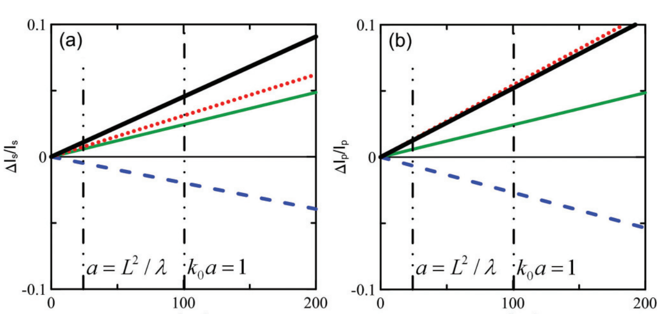

FIG. 5. (Color online) The dependence of relative nonreciprocity $\Delta I/I$ on the particle size a. The incident light is (a) s polarized, (b) p polarized. $\mathbf{n}$ is parallel to the vortex axis $\mathbf{n} = -\mathbf{e}_z$, angle between $\mathbf{n}$ and $\mathbf{n}'$ is 30°. Solid bold line (black online) represents the overall $\Delta I/I$, Solid thin gray line (green online) shows the electrodipole contribution into it, dashed line (blue online) displays the electroquadrupole term, dotted line (red online) is the magnetodipole contribution. Vertical dot-and-dash lines correspond to the size of particle at which $a = L^2/\lambda$, $k_0 a = 1$ consequently [see (5) and(24)].

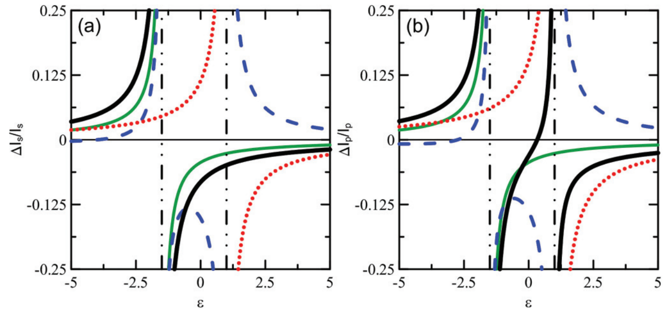

FIG. 6. (Color online) The dependence of relative nonreciprocity $\Delta I/I$ on the dielectric constant ε. The incident light is (a) s polarized, (b) p polarized. $\mathbf{n}$ is parallel to the vortex axis $\mathbf{n} = -\mathbf{e}_z$, angle between $\mathbf{n}$ and $\mathbf{n}'$ is 30°. Solid bold line (black online) represents the overall $\Delta I/I$, Solid thin gray line (green online) shows the electrodipole contribution into it, dashed line (blue online) displays the electroquadrupole term, dotted line (red online) is the magnetodipole contribution. Dot-and-dash lines correspond to the asymptotes at $\varepsilon = -3/2$ and $\varepsilon = 1$.

of this singularity. Also the electroquadrupole term of zero order in $\mathbf{M}$ is big here and thus should be taken into account. It is important to note that there is no peculiarity at the dielectric resonance $\varepsilon = -2$, which could be explained by exactly the same singularity both in linear in magnetic moment nonreciprocal term and in zero-order intensity.

Finally, we perform the calculation of the diffracted light intensity for the direction of $\mathbf{n}'$, particle size and dielectric constants that correspond to the parameters of experimental measurement.[26] The diffracted light is deflected from the vortex axis so that the angle between $\mathbf{n}$ and $\mathbf{n}'$ is 30°, other constants are specified above. For these parameters we have $\Delta I_s/I_s \approx 6.8 \times 10^{-3}, \Delta I_p/I_p \approx 7.7 \times 10^{-3}$. The experimental value of the effect is 2×10^{-3}. So, we come to a conclusion that although this calculation does not agree to the experiment exactly it turns out to be a satisfactory fit. The difference could be attributed to the triangle shape of the particle. Another possible reason is that the particle radius a is nearly equal to L^2/λ [see (24)].

VII. CONCLUSION

We have solved the problem of light diffraction by a spheric magnetic particle with the vortex magnetization distribution up to the first order in the a/λ parameter. Our results show that the vorticity-dependent nonreciprocal contribution in the intensity of the diffracted light takes place for both s and p polarizations of the incident light. The nonreciprocal term arises due to the interference of the zero-order electrodipole radiation and the first-order electrodipole, electroquadrupole, and magnetic dipole radiation that linearly depends on the magnetic moment of the particle. The estimations of the effect value for the parameters of cobalt yield a relative value $\Delta I/I \sim 7 \times 10^{-3}$ that fits the experiment[26] by an order of magnitude.

It should be noted that our approach can easily be expanded to the next-order calculation, but generally it is not necessary in order to gain an insight into the nature of these phenomena. The method could also be easily modified to describe different distributions of magnetization.

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