Zero-differential conductance of two-dimensional electrons in crossed electric and magnetic fields

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An electronic state with zero-differential conductance is found in nonlinear response to an electric field *E* applied to two dimensional Corbino discs of highly mobile carriers placed in quantizing magnetic fields. The state occurs above a critical electric field $E>E_{th}$ at low temperatures and is accompanied by an abrupt dip in the differential conductance. The proposed model considers a *local* instability of the electric field *E* as the origin of the observed phenomenon. Comparison between the observed electronic state and the state with zero differential resistance, occurring in Hall bar geometry, indicates that the nonlinear response of edge states and/or skipping orbits is not essential in the studied samples. The result confirms that quantal heating is the dominant nonlinear mechanism leading to electronic states with both zero differential resistance and conductance.

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The recent interest in a comprehensive study of the nonlinear magnetotransport in two-dimensional (2D) electron systems was stimulated by an observation of the Zener tunneling of highly mobile 2D electrons between Landau levels, which is induced by the Hall electric field in GaAs/AlGaAs heterojunctions.¹ The effect was originally found in Hall bar geometry and appeared as oscillations of magnetoresistance $r_{xx}(B)$ induced by dc electric current I_{dc} . Positions of the oscillations in magnetic field *B* obeyed the following relation: $\gamma R_c e E_H = l \hbar \omega_c$, where $\gamma \approx 2$, *l* is an integer, ω_c is the cyclotron frequency, R_c is the cyclotron radius and E_H is Hall electric field. Later, the Zener oscillations of the magnetoresistance r_{xx} were found in highly doped GaAs quantum wells,^{[2](#page-3-0)} in double quantum wells,^{[3](#page-3-0)} and in hole gas.⁴ Very recently the Zener oscillations were detected in the differential conductance of Corbino discs, where the Hall electric field E_H is absent.^{[5](#page-3-0)}

Another intriguing nonlinear phenomenon, that is observed in 2D electron systems placed in crossed electric and quantizing magnetic fields, is the electronic state with zero differential resistance $(ZDR state)$.^{[6](#page-3-0)} The experimental data have demonstrated that in the Hall bar geometry the initial decrease of the longitudinal differential resistance r_{xx} with applied dc current I_{dc} terminates at $I_{dc} = I_{th}$ corresponding to $r_{xx} = 0$. At $I_{dc} > I_{th}$ the differential resistance stays at zero value in a broad range of electric currents $I_{dc} > I_{th}$, significantly exceeding the threshold value I_{th} . The initial drop of the resistance is associated with a quantal heating induced by the spectral diffusion of 2D electrons in crossed electric and magnetic fields.^{7–10} The transition into the ZDR state is attributed to the local instability of the electric current at $I_{dc} > I_{th}$.^{[11](#page-3-0)} The local instability is considered to be the origin of another spectacular phenomenon: the zero resistance state observed in highly mobile 2D electron systems under a microwave irradiation.^{12–14} We note that an uncertainty of the microwave field distribution in studied samples limits the quantitative comparison of the nonlinear response with theories. Data presented below are obtained in the lowfrequency domain, where the distribution of the electric field is considered to be quite well determined.

Recently a strong nonlinear response of two-dimensional electrons was observed in a geometry in which a nonlocal electron transport, associated with the propagation of the edge states or/and skipping orbits, $15-22$ plays the dominant role.[23](#page-3-0) The observation of the nonlocal nonlinear response has raised a question regarding a possibility of the significant contribution of the edge states and/or skipping orbits to the nonlinear transport of 2D electrons observed in the Hall bar geometry^{24–35} and, thus, the applicability of the currently accepted theoretical approach^{[7](#page-3-0)} to the observed nonlinearity. We should note that in the Hall bar geometry a separation between the local and the nonlocal contributions to the electron conductance is a challenging problem.

A convenient geometry in which the nonlocal contributions of the edge states and/or skipping orbits to the electron conductance can be significantly suppressed is the Corbino geometry. In this geometry the edge states are localized near the edges of the inner and outer contacts and do not propagate through the Corbino ring. Thus experiments in the Corbino geometry provide information on the bulk nonlinear response. A comparison of the nonlinear response of Corbino disks with the response of Hall bar samples may shed a light on the amount of the nonlocal contributions to the nonlinear resistance in the Hall bar geometry. Below we investigate the nonlinear response of Corbino disks and compare it with experiments on Hall bar samples.

The paper presents a study of nonlinear transport properties of 2D electron Corbino disks with inner radius $r_1 = 0.9$ mm and outer radius $r_2 = 1$ mm. The Corbino disks were fabricated from selectively doped heterojunction GaAs/AlAs. The heterojunction was a single GaAs quantum well sandwiched between AlAs/GaAs superlattice barriers.^{[36](#page-4-0)} The width of the quantum well was 13 nm. The structure was grown by molecular beam epitaxy on (100) GaAs substrate. AuGe eutectic was used to provide electric contacts to the 2D electron gas. The contacts were made by thermal diffusion after the AuGe deposition and photolithography. Differential conductance $g_{12} = I_{ac}/V_{ac}$ was measured using ac current *Iac* with frequency from 10 Hz to 1 KHz. An ac voltage

FIG. 1. (Color online) Dependence of conductance *g*¹² of a 2D electron Corbino disk on magnetic field at temperature $T = 1.6$ K at different dc electric fields as labeled. The inset shows the electric scheme for measurements of differential conductance *g*12.

Vac was applied between contacts 1 and 2, shown in the inset of Fig. 1. The amplitude of the voltage was kept fixed and was below 1 mV during experiments. The measurements were taken at temperatures $T = 1.6$ K and $T = 4.2$ K in magnetic fields $B < 1$ T. Three samples with electron density $n = 8 \times 10^{15}$ m⁻² and mobility $\mu = 150$ m²/Vs at $T = 4.2$ K were studied and have demonstrated the same results. The paper presents data for one of these samples.

Figures 1 and 2 present dependence of the differential conductance $g_{12}(B)$ of 2D electrons in the Corbino disk on the magnetic field *B* taken at $T = 1.6$ K at different electric fields as labeled. For the studied samples the width of the conducting o-ring was much less than the averaged radius of the o-ring : $\Delta r = r_2 - r_1 \ll (r_2 + r_1)/2$. Due to this property the dc electric field between contacts was nearly independent of the radius *r* and equal to $E_{dc} = V_{12}/\Delta r$. At $E_{dc} = 0$ the magnetoconductance $g_{12}(B)$ demonstrates Shubnikov–de Haas (SdH) oscillations in magnetic fields exceeding 0.3 T as shown in Fig. 1. An application of the electric field $E_{dc} = 250$ V/m decreases the amplitude of the quantum oscillations significantly, and at strong magnetic fields the conductance of the structure approaches values that are very close to zero. Shown in Fig. 2, further increase of the dc electric field produces additional peaks in the dependence $g_{12}(B)$, which are labeled by arrows. As shown recently, these maxima result from Zener tunneling between Landau levels, which is induced by applied electric field E_{dc} ^{[5](#page-3-0)} Positions of the maxima obey the following relation: $\gamma R_c e E_{dc} = l \hbar \omega_c$, shown in the inset of Fig. 2.

Figure $3(a)$ presents dependencies of $g_{12}(E_{dc})$ for different magnetic fields as labeled and the temperature $T =$ 1.6 K. At magnetic field $B = 0.261$ T the initial drop of the differential conductance with the E_{dc} is due to the intralevel quantal heating.^{[7,10](#page-3-0)} The increase of the differential

FIG. 2. (Color online) Dependence of conductance *g*¹² of a 2D electron Corbino disk on magnetic field at temperature $T = 1.6$ K at different dc electric fields as labeled. Arrows indicate the positions of the maximum B_l at $l = 1$ in different electric fields. The inset presents the dependence of B_1^2 on dc electric field E_{dc} . The solid line corresponds to the relation $\gamma e E_{dc} R_c = \hbar \omega_c$. At $\gamma = 2$ the electron effective mass $m_e \approx 0.070$, which is in accord with another experiment (Ref. [37\)](#page-4-0).

conductance at higher electric field is related to interlevel electron transitions.^{1,[38](#page-4-0)} In Fig. [3\(a\)](#page-2-0) the maximum marked by the arrow corresponds to Zener tunneling between Landau levels at $l = 1$. At higher magnetic field $B = 0.847$ T the differential conductance demonstrates similar behavior at small electric fields, but at higher dc biases the conductance retains values near zero, $g_{12} \approx 0$, in a broad range of the electric field E_{dc} . This is the zero differential conductance state (ZDCS). Figuer $4(a)$ reveals that the transition into the ZDC state is associated with one or few sharp "spikes" of the differential conductance into the region with negative values. As shown in the figure, the state with $g_{12} = 0$ does not occur at $T = 4.2$ K.

Figure [3\(b\)](#page-2-0) presents *V*-*I* dependencies of the 2D Corbino disk at temperature $T = 1.6$ K for two different magnetic fields as labeled. The figure shows that, when the 2D electron systems enters the state with zero differential conductance, the electric current *Idc* saturates and becomes independent of the electric field E_{dc} . A comparison between the dependencies $g_{12}(E_{dc})$ and $I_{dc}(V_{dc})$ taken at temperature $T = 1.6$ K and magnetic field $B = 0.847$ T indicates that the electric current I_{dc} reaches a saturation value I_s at electric field $E_{dc} > E_{th}$.

Similar to the case of the Hall bar geometry, 6 we consider that, in the studied Corbino disks, the $g_{12} = 0$ state occurs due to a *local* instability of the electric field E_{dc} .^{[11](#page-3-0)} The dominant nonlinear mechanism, leading to the instability, is a peculiar Joule heating (quantal heating), which occurs in systems with a discrete spectrum.^{[7,10](#page-3-0)} The instability develops at the conditions of a negative differential conductivity corresponding to the

FIG. 3. (Color online) (a) Dependence of differential conductance *g*¹² on dc electric field *Edc* at different magnetic fields as labeled. The arrow indicates a maximum corresponding to Zener transition at $l = 1$; $T = 1.6$ K. (b) Dependence of electric current I_{dc} on dc voltage V_{dc} at temperature $T = 1.6$ K in different magnetic fields as labeled. The inset in the upper left corner shows suggested N-shaped dependence $J_{dc}(E_{dc})$ indicating two electric fields E_1 and E_2 corresponding to the same value J_{dc} . The inset in the lower right corner insert shows the possible distribution of the electric field corresponding to the electron state with zero differential conductance in a 2D Corbino disk.

negative slope of the N-shaped *V* -*I* dependence shown in the inset of Fig. $3(b)$. Shown in Fig. $4(a)$, regions with the negative differential resistance further support this interpretation. In the case of the N-shaped *V* -*I* dependence, a spatially uniform distribution of the electric field is not stable and typically should evolve into a structure containing domains of a weak E_1 and a strong E_2 as shown in the inset of Fig. $3(b).^{39}$ $3(b).^{39}$ $3(b).^{39}$ At these conditions both moving and static domains may occur. In the first case, in a conductor with a fixed voltage applied there are oscillations of the electric current. This is known as Gunn effect. 40 In the case of static domains the constant electric current saturates with the applied voltage. 41 There is a similarity between nonlinear transport in Gunn diodes^{[40](#page-4-0)} and in the 2D electron systems presented in this paper. We note however that despite the similarity the nonlinear mechanisms

FIG. 4. (Color online) (a) Dependence of differential conductance g_{12} on electric field E_{dc} in magnetic field $B = 0.847$ T at different temperatures as labeled. (b) Dependence of the differential resistance r_{xx} on dc bias I_{dc} in a Hall bar sample fabricated from the same quantum well as in (a). The dependence is taken at magnetic field $B = 0.841$ T at different temperatures as labeled.

leading to the local instability of the electric field *Edc* are different in these two systems.

The presented nonlinear response of Corbino disks is obtained in the regime where the edge states and/or skipping orbits are localized near the contacts and do not participate in the electron transport through the systems. It is important to compare the obtained results with the nonlinear response of Hall bar samples, where the electron transport near the edge may provide significant contributions.²³ Below we compare the threshold electric field $E_{th} = 96$ V/m corresponding to the transition into the state with zero differential conductance, shown in Fig. $4(a)$, with the Hall electric field corresponding to the transition into the state with zero differential resistance (ZDRS) in a Hall bar sample fabricated from the same quantum well. Figure $4(b)$ presents the dependence of the differential resistance of the Hall bar sample on the applied dc bias *Idc* taken under the same experimental conditions. The transition to the ZDR state occurs at Hall electric field $E_{th}^H = 118/V/m$, corresponding to the threshold dc bias $I_{dc} = 9.3 \mu A$. The comparison demonstrates quite similar values of the electric

fields, at which both ZDRS and ZDCS transitions occur. Furthermore we note that samples with comparable physical parameters demonstrate comparable threshold fields. In particular, shown in Fig. $2(a)$ of Ref. 6, the threshold electric current $I_{th} = 6.7 \mu A$ corresponds to the ZDRS transition obtained at $B = 0.784$ T, $T = 1.94$ K on sample N1 with electron density $n = 8.2 \times 10^{15}$ m⁻² and mobility $\mu =$ 85 m2*/*Vs. Taking into account that the Hall resistance of the sample N1 at $B = 0.784$ T is $R_H = B/ne = 597 \Omega$, one can evaluate the Hall electric field E_H corresponding to the current I_{th} : $E_{th}^H = R_H \times I_{th}/W = 80$ V/m, where $W = 50$ μ m is the width of the sample N1. The sample demonstrates similar value of the threshold electric field. Thus in the studied Hall bar samples the edge states and/or skipping orbits do not provide a considerable contribution to the nonlinear response, and thus the accepted model of the nonlinearity^{7,10} holds for these systems.

In summary the paper presents experimental study of the effect of dc electric field on the conductance of Corbino disks of highly mobile two-dimensional electrons placed in crossed electric and quantizing magnetic fields. Experimental data shows that at low temperature the differential conductance of the Corbino disks reaches zero value in a broad range of applied dc voltages. It indicates the presence of the zero differential conductance state in which the electric current does not depend on the voltage. The results are in accord with the data obtained in the Hall bar geometry indicating that the nonlinearity leading to the ZDC and ZDR states occurs inside 2D electron systems. It provides significant support for the model of the local nonlinearity based on the quantal Joule heating in systems with a discrete or modulated spectrum. Finally, both the zero differential conductance and zero differential resistance states are observed in systems with a modest electron mobility, broadening significantly the

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class of electron systems in which the quantal heating is

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