Pomeranchuk-nematic instability in the presence of a weak magnetic field

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We analyze a two-dimensional Pomeranchuk-nematic instability, trigger by the Landau parameter $F_2 < 0$, in the presence of a small magnetic field. Using Landau Fermi-liquid theory in the isotropic phase, we analyze the collective modes near the quantum critical point $F_2 = -1, \omega_c = 0$ (where ω_c is the cyclotron frequency). We focus on the effects of parity symmetry breaking on the Fermi-surface deformation. We show that the linear response approximation of the Landau-Silin equation is not sufficient to study the critical regime and it is necessary to compute corrections at least of order ω_c^2. Identifying the slowest oscillation mode in the disordered phase, we compute the phase diagram for the isotropic/nematic phase transition in terms of F_2 and ω_c.

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I. INTRODUCTION

The isotropic-nematic quantum phase transition was proposed as a possible mechanism to explain the anisotropic behavior of several strongly correlated systems. Some interesting examples are quantum Hall liquids, high-T_c superconductors, and heavy fermions systems. An interesting review can be found in Ref. 1.

This transition can be understood as an instability of a Fermi surface under the influence of a strongly attractive two-body potential in the forward-scattering channel, with d-wave symmetry (or, equivalently, with angular momentum $\ell = 2$). From the point of view of the Landau Fermi-liquid theory, it is triggered by a Pomeranchuk instability produced by a large negative value of the Landau parameter F_2 in the charged sector. As a consequence of the transition, the Fermi surface is deformed, getting an ellipsoidal component. The Goldstone modes, related to rotational symmetry breaking, are dissipative overdamped excitations, characterized by the dynamical exponent $z = 3$. The order parameter theory was developed using different techniques: mean field theory,[2] multidimensional bosonization.[3,4] and Landau Fermi-liquid theory.[5] While the collective bosonic excitations are reasonably well understood, the fate of the fermionic spectrum is still under debate.[4,6,7]

From an experimental point of view, the study of Fermi-surface deformations can be performed by means of at least two independent techniques: angle-resolved photoemission spectroscopy[8] and the observation of quantum oscillations,[9] for instance, the de Haas–van Alphen effect. The use of the latter resides in the ability to reconstruct Fermi-surface shapes from the information contained in quantum oscillations of different observables when an externally applied magnetic field is varied.

The application of a strong magnetic field suppresses any Pomeranchuk instability since it opens a gap in the spectrum due to Landau-level quantization. However, for small magnetic fields, the Landau levels form a dense set near the Fermi energy and strong attractive interactions mix all levels in a nontrivial way. Experimentally, nematic instability has been observed in the bilayer ruthenate compound $Sr_3Ru_2O_7$ at finite magnetic field,[10] which suggests that a metamagnetic quantum critical point can be reached by changing the direction of the applied magnetic field.[9,11,12] Therefore, it is important to understand the critical behavior when the quantum critical point is reached by lowering the magnetic field.

With this motivation, we present a study of a two-dimensional Pomeranchuk-nematic instability in the presence of a small magnetic field, applied perpendicular to the two-dimensional fermionic system. We have considered an isotropic and homogeneous charged Fermi liquid subject to a small magnetic field $k_B T \ll \hbar\omega_c \ll \epsilon_F$, where $\omega_c = eB/m^\star$ is the cyclotron frequency and ϵ_F is the Fermi energy of the system. We have focused on a simplified model where only the attractive two-body d-wave interaction is present. Using a semiclassical approach, we have studied collective excitations of the fermionic system using the Landau-Silin equation.[13,14] Studying the oscillatory slowest mode, we can compute the transition line where the isotropic phase gets unstable. The main result is presented in Fig. 1, where we depict the phase diagram for the nematic-Pomeranchuk instability. In this figure, the horizontal axis is the usual Landau control parameter $\alpha = 1 + F_2$, while the vertical axis is the adimensional magnetic field $(\omega_c/\epsilon_F)^2$. We observe a maximum value of the magnetic field above which no Pomeranchuk instability is possible. Moreover, we have observed a reentrant behavior of the isotropic phase for greater values of the interaction parameter. We have also analyzed the behavior of collective modes near the quantum critical point ($F_2 \to -1, \omega_c \to 0$). Since the magnetic field breaks parity symmetry, the collective mode dynamics mixes symmetric as well as antisymmetric modes. Then the Fermi-surface deformation is not an ellipsoid but has a definite parity given by the direction of the magnetic field and the momentum $\mathbf{q}$ of the periodic perturbation.

The paper is structured as follows. In Sec. II we briefly review the Landau theory of charged Fermi liquids and the Landau-Silin equation to describe the collective modes of a Fermi liquid subject to an external magnetic field. In Sec. III we set up our model and deduce the phase diagram of Fig. 1. In Sec. IV we show the collective modes near the nematic quantum critical point. Finally, we discuss our results and we point out possible future developments in Sec. V.

II. SEMICLASSICAL APPROXIMATION

Following the standard Fermi-liquid approach,[15] we start by writing down the energy functional for a two-dimensional

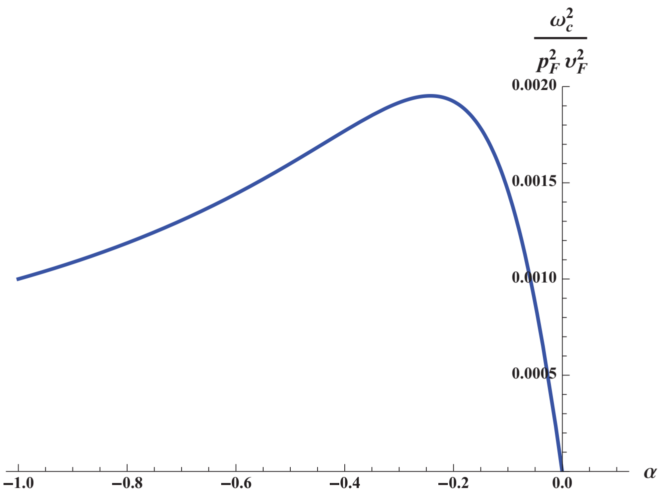

FIG. 1. (Color online) Phase diagram for the Pomeranchuk instability $\ell = 2$ in the presence of a small magnetic field. The external part of the transition curve represents an isotropic Fermi liquid, while the inner part is an anisotropic liquid phase. The adimensional control parameters are $\alpha = 1 + F_2$ and ω_c/ϵ_F, where $\omega_c = \frac{eB}{m^\star}$ is the cyclotron frequency and $\epsilon_F = v_F p_F$ is the Fermi energy. We have plotted Eq. (30) by setting the interaction range $\kappa p_F = 10$.

system of spinless quasiparticles of effective mass $m^\star$, in an electromagnetic field defined by the vector potential $\mathbf{A} = \mathbf{A}(\mathbf{r},t)$,

$$E[n] = \sum_{\mathbf{p}} \frac{(\mathbf{p} + e\mathbf{A})^2}{2m^\star} n(\mathbf{p},\mathbf{r}) + \sum_{\mathbf{p},\mathbf{p}'} \int d\mathbf{r}\, d\mathbf{r}' \times f_{\mathbf{p}+e\mathbf{A};\mathbf{p}'+e\mathbf{A}}(\mathbf{r}-\mathbf{r}') n(\mathbf{p},\mathbf{r}) n(\mathbf{p}',\mathbf{r}) + O(n^3), \quad (1)$$

where $n(\mathbf{p},\mathbf{r})$ is the phase-space density at momentum $\mathbf{p}$ and position $\mathbf{r}$, e is the quasiparticle charge, and $f_{\mathbf{p}+e\mathbf{A};\mathbf{p}'+e\mathbf{A}}(\mathbf{r}-\mathbf{r}')$ is the Landau amplitude characterizing finite-range two-particle interactions. The Landau interaction function should depend on the electromagnetic vector potential to guarantee gauge invariance.

In order to compute a semiclassical evolution equation, we define the effective single-particle Hamiltonian

$$H_{\rm eff}(\mathbf{p},\mathbf{r}) = \frac{\delta E[n]}{\delta n(\mathbf{p},\mathbf{r})}, \quad (2)$$

which generates the following time evolution equation:

$$\frac{\partial n(\mathbf{p},\mathbf{r},t)}{\partial t} = \{H_{\rm eff}, n(\mathbf{p},\mathbf{r},t)\}_{\rm PB} + I_{\rm coll}[n(\mathbf{p},\mathbf{r},t)], \quad (3)$$

where $\{\cdots\}_{\rm PB}$ are Poisson brackets associated with the conjugate variables $\mathbf{r}$ and $\mathbf{p}$ and the effects of quasiparticle scattering are included in the collision integral $I_{\rm coll}[n(\mathbf{p},\mathbf{r},t)]$. By means of Hamilton's equations of motion $d\mathbf{r}/dt = \nabla_{\mathbf{p}} H_{\rm eff}(\mathbf{p},\mathbf{r},t)$ and $d\mathbf{p}/dt = -\nabla_{\mathbf{r}} H_{\rm eff}(\mathbf{p},\mathbf{r},t)$ and using Eqs. (2) and (3), the so-called Landau-Silin kinetic equation is obtained:[14,15]

$$\frac{\partial n(\mathbf{p},\mathbf{r},t)}{\partial t} + \mathbf{v}(\mathbf{p},\mathbf{r},t)\cdot\nabla_{\mathbf{r}} n(\mathbf{p},\mathbf{r},t) - \Big(\mathcal{F}(\mathbf{p},\mathbf{r},t) + \sum_{\mathbf{p}'} \int d\mathbf{r}'\, f_{\mathbf{p}+e\mathbf{A};\mathbf{p}'+e\mathbf{A}}(\mathbf{r}-\mathbf{r}') \nabla_{\mathbf{r}'} n(\mathbf{p}',\mathbf{r}',t)\Big) \cdot \nabla_{\mathbf{p}} n(\mathbf{p},\mathbf{r},t) = I_{\rm coll}[n(\mathbf{p},\mathbf{r},t)], \quad (4)$$

where $\mathcal{F}(\mathbf{p},\mathbf{r},t) = e[\mathbf{E}(\mathbf{r},t) + \mathbf{v}(\mathbf{p},\mathbf{r},t)\times\mathbf{B}(\mathbf{r},t)]$ is the Lorentz force and $\mathbf{v}(\mathbf{p},\mathbf{r},t) = \nabla_{\mathbf{p}} H_{\rm eff}$ is the quasiparticle velocity, including interactions. The Landau-Silin transport equation (4) resembles the conventional classical Boltzmann equation. However, the effective Lorentz force $\mathcal{F}(\mathbf{k},\mathbf{r},t)$ depends self-consistently on the quasiparticle distribution function $n(\mathbf{p},\mathbf{r},t)$. This evolution equation is the cornerstone of the present work.

In this paper we study the effect of an external magnetic field B, applied perpendicular to the plane of the system. We assume that the cyclotron energy $\hbar\omega_c = \hbar eB/m^\star \ll \varepsilon_F$. For simplicity throughout the paper we choose $\hbar \equiv 1$. In general, the scattering mechanisms described by the collision integral can be studied by applying the relaxation-time τ approximation. We consider that the typical collective mode frequencies are greater than the collision quasiparticle frequency. Of course, this is not true at criticality. However, to determine the position of the transition line, it is enough to consider $I_{\rm coll} \to 0$. To set up the kinetic equation of the Fermi-surface collectives modes, let us consider a constant isotropic equilibrium distribution n_p^0 and a small perturbation δn such that $n(\mathbf{p},\mathbf{r},t) = n_p^0 + \delta n(\mathbf{p},\mathbf{r},t)$. For these conditions, the linear expansion of Eq. (4) in δn provides the transport equation

$$\frac{\partial \delta n}{\partial t} + \mathbf{v}_{\mathbf{p}}^0 \cdot \nabla_{\mathbf{r}} \delta\bar{n} - e\big[\mathbf{v}_{\mathbf{p}}^0 \times \mathbf{B}\big]\cdot\nabla_{\mathbf{p}}\delta\bar{n} = 0, \quad (5)$$

where

$$\delta\bar{n}(\mathbf{p},\mathbf{r},t) = \delta n(\mathbf{p},\mathbf{r},t) - \left(\frac{\partial n^0}{\partial\varepsilon^0}\right) \times \sum_{\mathbf{p}'} \int d\mathbf{r}'\, f_{\mathbf{p}+e\mathbf{A};\mathbf{p}'+e\mathbf{A}}(\mathbf{r}-\mathbf{r}')\delta n(\mathbf{p}',\mathbf{r}',t) \quad (6)$$

is the deviation from local equilibrium.

It is important to point out that although Eq. (5) is linear in δn_p, it is highly nonlinear in the magnetic field since it enters the definition of the Landau interaction [Eq. (6)]. Usually, to compute collective plasma modes in charged Fermi liquids this last contribution is neglected, resulting in a true linear response theory.[16] However, as we will show, this approximation is not consistent with the study of Pomeranchuk instabilities.

At low temperatures $k_B T \ll \epsilon_F$ the electron dynamics is confined to a small region around the Fermi surface. Then it is more convenient to define $\delta n(\mathbf{p},\mathbf{r},t) = -(\partial n_{\mathbf{p}}^0/\partial\varepsilon_{\mathbf{p}})\nu_{\mathbf{p}}(\mathbf{r},t)$, where $\nu_{\mathbf{p}}(\mathbf{r},t)$ measures local Fermi-surface deformation. Finally, Fourier transforming in the space variable $\mathbf{r}$, the kinetic equation (5) becomes

$$\frac{\partial \nu_{\mathbf{p}}(\mathbf{q},t)}{\partial t} + \big[i\mathbf{v}_F^0\cdot\mathbf{q} - e\big(\mathbf{v}_F^0 \times \mathbf{B}\big)\cdot\nabla_{\mathbf{p}}\big] \times [\nu_{\mathbf{p}}(\mathbf{q},t) + \delta\varepsilon_{\mathbf{p}}(\mathbf{q},t)] = 0, \quad (7)$$

where $\mathbf{v}_F^0$ is the Fermi velocity and the expression

$$\delta\varepsilon_{\mathbf{p}}(\mathbf{q},t) = \frac{1}{V^2}\sum_{\mathbf{p}'}\left(\frac{\partial n^0_{\mathbf{p}'}}{\partial \varepsilon_{\mathbf{p}'}}\right)\int d\mathbf{r}\,d\mathbf{r}'\,e^{i\mathbf{q}\cdot\mathbf{r}} \times f_{\mathbf{p}+e\mathbf{A},\mathbf{p}'+e\mathbf{A}}(\mathbf{r}-\mathbf{r}')\nu_{\mathbf{p}'}(\mathbf{r}',t) \tag{8}$$

describes the combined effect of interactions and magnetic field, with V being the space volume.

Equations (7) and (8) are the starting point of our analysis. They describe the dynamics of Fermi-surface deformations, given an initial condition $\nu^{\text{in}}(\mathbf{q},0)$, representing a small density fluctuation with wave vector $\mathbf{q}$. In the following section we set up our model and study the Pomeranchuk instability in the nematic channel.

III. POMERANCHUK-NEMATIC INSTABILITY

For simplicity we consider a two-dimensional circular Fermi surface. The interaction Landau function depends essentially on the angle between two Fermi momenta and can be expanded in Landau parameters as

$$f_{\mathbf{p}+e\mathbf{A},\mathbf{p}'+e\mathbf{A}}(\mathbf{r}-\mathbf{r}') \to f_{p_F,p'_F}(r) = \sum_\ell f_\ell(r)e^{i\ell\varphi}, \tag{9}$$

where $\cos\varphi = \mathbf{p}_F\cdot\mathbf{p'}_F/p_F^2$. Moreover, we can expand the deformation of the Fermi surface in Fourier coefficients

$$\nu_{\mathbf{p}}(\mathbf{q},t) = \sum_\ell \nu_\ell(q,t)e^{i\ell\theta}, \tag{10}$$

where $\cos\theta = \mathbf{p}_F\cdot\mathbf{q}/p_F q$.

To study the Pomeranchuk-nematic instability it is sufficient to consider a simplified model defined by $f_2(r)\neq 0$, while $f_\ell(r)=0$ for all $\ell\neq 2$. The presence of other interaction channels does not modify our results qualitatively, provided they are all stable.[4,5] We will consider a short-range but nonlocal interaction $f_2(r)$, whose Fourier transform is given by

$$\tilde{f}_2(q) = \frac{f_2}{1+|F_2|(\kappa q)^2}, \tag{11}$$

where $F_2 = N(0)f_2$ is the usual adimensional Landau parameter with angular momentum $\ell = 2$ [$N(0)$ is the density of states at the Fermi surface] and κ defines an effective interaction range $\xi = \sqrt{|F_2|}\kappa$. Our approach is valid provided $p_F^{-1} \ll \xi \ll q^{-1}$, i.e., when the interaction range is much larger than the interparticle distance, however shorter than the typical scale of long-range perturbations.

In the absence of a magnetic field, the collective dynamics of the Fermi surface, given by Eq. (7) with $\mathbf{A}=0$, reduces to

$$\frac{\partial \nu_\ell(q,t)}{\partial t} + \frac{i v_F q}{2}[\alpha_{\ell-1}\nu_{\ell-1}(q,t)+\alpha_{\ell+1}\nu_{\ell+1}(q,t)] = 0, \tag{12}$$

where we have defined $\alpha_\ell = 1+F_\ell$ and F_ℓ are adimensional Landau parameters. In our model $\alpha_2\equiv\alpha = 1+F_2$ and $\alpha_\ell = 1$ for all $\ell\neq 2$.

We can gain more physical insight by defining symmetric and antisymmetric variables

$$\nu_\ell^\pm(q,t) = \tfrac{1}{2}[\nu_\ell(q,t)\pm\nu_{-\ell}(q,t)] \tag{13}$$

in terms of which the Fermi-surface deformations are parametrized as

$$\nu(\mathbf{q},\theta,t) = \sum_{\ell=0}^{\infty}\nu_\ell^+(q,t)\cos(\ell\theta) + \sum_{\ell=1}^{\infty}\nu_\ell^-(q,t)\sin(\ell\theta). \tag{14}$$

Eliminating in Eq. (12) odd components in favor of even ones, we obtain the coupled oscillator equations[5]

$$\frac{\partial^2\nu_\ell^\pm(q,t)}{\partial t^2} + \left(\frac{v_F q}{2}\right)^2[A_\ell\nu_\ell^\pm(q,t) + C_{\ell-1}\nu_{\ell-2}^\pm(q,t)+C_{\ell+1}\nu_{\ell+2}^\pm(q,t)] = 0, \tag{15}$$

with the adimensional coefficients

$$A_\ell = \alpha_\ell(\alpha_{\ell-1}+\alpha_{\ell+1}),\quad C_\ell = \alpha_{\ell+1}\sqrt{\alpha_\ell\alpha_{\ell-1}}. \tag{16}$$

It is clear from Eq. (15) that the even and odd components of ℓ are decoupled. The same happens with the symmetric and antisymmetric components. The physical reason for that is parity invariance. Hence the ν_2^+ mode is coupled with the even symmetric modes $\nu_0,\nu_4^+,\nu_6^+,\ldots$. Near $F_2=-1$ or $\alpha\sim 0$, the ν_2^+ mode oscillates with frequency

$$\omega_2 = \sqrt{2\alpha}\left(\frac{v_F q}{2}\right), \tag{17}$$

while all the other modes essentially oscillate with $\omega_\ell\sim v_F q/\sqrt{2}$. Then, near $\alpha=0$, $\omega_2\ll\omega_\ell$ with $\ell\neq 2$ showing that, in time scales $\tau\gg(v_F q)^{-1}$, ν_2^+ is a very slow mode, while all other rapid modes can be averaged to zero. Therefore, when $\alpha\to 0$, the Fermi surface has an essentially elliptic form during long periods of time. This is the onset of the Pomeranchuk-nematic instability.

When a magnetic field is applied, parity and time-reversal symmetry are broken. Then the symmetric and antisymmetric modes are no longer decoupled. In linear response theory, we can ignore the contribution of the magnetic field in Eq. (8); then Eq. (7) can be simplify to

$$\frac{\partial\nu_\ell}{\partial t} + \frac{i v_F q}{2}[\alpha_{\ell-1}\nu_{\ell-1}+\alpha_{\ell+1}\nu_{\ell+1}] + i\ell\alpha_\ell\omega_c\nu_\ell = 0, \tag{18}$$

where we have defined the cyclotron frequency $\omega_c = eB/m^\star$. Thus the linear response correction to Eq. (12) is proportional to $\alpha_\ell(\omega_c/v_F q)$, where $\alpha_\ell = 1+F_\ell$. Since $\alpha_\ell\sim 1$ for stable modes ($\ell\neq 2$), this equation is suitable to study collective modes of the Fermi liquid in small magnetic fields. However, near the Pomeranchuk instability ($\alpha_2\equiv\alpha\sim 0$), $(\omega_c/v_F q)^2$ is of the same order of $\alpha(\omega_c/v_F q)$ and cannot be ignored. To see this more clearly, we can compute the oscillation frequency of ν_2^+, using Eq. (18), obtaining

$$\omega_2 \sim \sqrt{2\alpha}\left(\frac{v_F q}{2}\right)\left\{1+4\alpha\left(\frac{\omega_c}{v_F q}\right)^2+\cdots\right\}, \tag{19}$$

where the ellipsis denotes terms of order $\alpha^2(\omega_c/v_F q)^4$. Clearly, for small $\alpha\ll 1$, the frequency is approximately given by Eq. (17) without changing the behavior of the quantum critical point $\alpha = 0$.

Therefore, to study the transition line $\omega_c(\alpha)$, we need to consider quadratic corrections in the magnetic field. To do this, we expand the Landau function f_2 in Eq. (8), keeping linear terms in the vector potential $\mathbf{A}$,

$$f_{\mathbf{p}+e\mathbf{A},\mathbf{p}'+e\mathbf{A}}(\mathbf{r}-\mathbf{r}') = f_2(\mathbf{r}-\mathbf{r}')\frac{(\mathbf{p}\cdot\mathbf{p}')^2}{p_F^4} + 2e\frac{f_2(\mathbf{r}-\mathbf{r}')}{p_F^4}(\mathbf{p}\cdot\mathbf{p}') \times [\mathbf{A}(\mathbf{r})\cdot(\mathbf{p}+\mathbf{p}')]. \tag{20}$$

With this expression, Eq. (8) reduces to

$$\delta\varepsilon_{\mathbf{p}}(\mathbf{q},t) = \delta\varepsilon^0_{\mathbf{p}}(\mathbf{q},t) + \delta\varepsilon^A_{\mathbf{p}}(\mathbf{q},t), \tag{21}$$

where the first term has no contribution from the magnetic field and is given by

$$\delta\varepsilon^0_{\mathbf{p}}(\mathbf{q},t) = -i|f_2|\sum_{\mathbf{p}'}\left(\frac{\partial n^0_{\mathbf{p}'}}{\partial\varepsilon_{\mathbf{p}'}}\right)\left(\frac{\mathbf{p}\cdot\mathbf{p}'}{p_F^2}\right)^2\nu_{\mathbf{p}'}(\mathbf{q},t), \tag{22}$$

while the second term is linear in ω_c,

$$\delta\varepsilon^A_{\mathbf{p}}(\mathbf{q},t) = -2i\left(\frac{\omega_c}{v_F p_F}\right)\frac{(\kappa p_F)^2F_2^2}{p_F^4 N(0)}\sum_{\mathbf{p}'}\left(\frac{\partial n^0_{\mathbf{p}'}}{\partial\varepsilon_{\mathbf{p}'}}\right) \times(\mathbf{p}\cdot\mathbf{p}')[(\mathbf{p}+\mathbf{p}')\times\mathbf{q}]\nu_{\mathbf{p}'}(\mathbf{q},t), \tag{23}$$

where we have chosen the symmetric gauge $\mathbf{A} = (1/2)\mathbf{r}\times\mathbf{B}$. This term depends on the interaction range $(\kappa p_F)^2$; then, for ultralocal interactions ($\kappa = 0$), it makes no contribution. In contrast, the vectorial structure of Eq. (23) filters only contributions to the modes $\nu_{\pm1},\nu_{\pm2}$. Therefore, Fourier transforming in $\mathbf{p}$ we find for these modes

$$\frac{\partial\nu_1}{\partial t} + i\left(\frac{v_F q}{2}\right)\left\{\left[1-2(1-\alpha)^2\left(\frac{\omega_c}{v_F q}\right)^2(\kappa q)^2\right]\nu_0 + \left[\alpha - 2(1-\alpha)^2\left(\frac{\omega_c}{v_F q}\right)^2(\kappa q)^2\right]\nu_2\right\} + i\omega_c\nu_1 = 0 \tag{24}$$

and

$$\frac{\partial\nu_2}{\partial t} + i\left(\frac{v_F q}{2}\right)\left\{1-4(1-\alpha)^2\left(\frac{\omega_c}{v_F q}\right)^2(\kappa q)^2\right\}\nu_1 + i\left(\frac{v_F q}{2}\right)\nu_3 + 2i\alpha\omega_c\nu_2 = 0. \tag{25}$$

The equations for the modes ν_{-1} and ν_{-2} are easily obtained from Eqs. (24) and (25) by changing $\ell \to -\ell$ and $\omega_c \to -\omega_c$. The dynamical equations for the rest of the modes are simply given by Eq. (18). Then, building symmetric and antisymmetric mode combinations and deriving the evolution equations to get a second-order system, we get, for ν_2^+,

$$\frac{\partial^2\nu_2^+}{\partial t^2} + \Omega^2\nu_2^+ + \left(\frac{v_F q}{2}\right)^2(\nu_0+\nu_4^+) + \left(\frac{v_F q}{2}\right)\omega_c(2\nu_1^- + 3\nu_3^-) = 0, \tag{26}$$

with

$$\Omega^2 = 2\alpha\left(\frac{v_F q}{2}\right)^2\left[1+8\alpha\left(\frac{\omega_c}{v_F q}\right)^2\right] + \left(\frac{\kappa q}{2}\right)^2(1-\alpha)^2\omega_c^2. \tag{27}$$

As we have anticipated, the magnetic field mixes symmetric and antisymmetric modes. The first contribution to the frequency in Eq. (27) comes from the linear response theory, while the last term, proportional to the interaction range $\kappa q \ll 1$, is the first "correction" coming from Eq. (23).

Near the transition line $\Omega \to 0$, ν_0 and ν_4^+ are very rapid and stable modes, while the coupling with the antisymmetric modes are very weak. Thus they do not modify the transition qualitatively. In the following section we study these couplings in more detail. Therefore, ν_2^+ is unstable when $\Omega = 0$, leading to the condition

$$\left(\frac{\omega_c}{v_F p_F}\right)^2 = -\frac{1}{8}\left(\frac{q}{p_F}\right)^2\frac{\alpha}{\alpha^2+(1-\alpha)^2(\frac{\kappa q}{4})^2}. \tag{28}$$

Near the quantum critical point, we can expand this expression in powers of α,

$$\left(\frac{\omega_c}{v_F p_F}\right)^2 = -2\left(\frac{1}{\kappa p_F}\right)^2\alpha + O(\alpha^2), \tag{29}$$

obtaining a linear critical region governed by the interaction range κp_F. Corrections of order α^2 depend on q. Thus, differently from the usual Pomeranchuk transition, small perturbations with different values of q will contribute to the instability at different values of α. In contrast, the momentum perturbation is limited to the range $r_c^{-1} < q < \kappa^{-1}$. It is simple to show that the extremal line, necessary to built up the complete phase diagram, is reached at $q = 1/\kappa$. Therefore, the transition line is given by

$$\left(\frac{\omega_c}{v_F p_F}\right)^2 = -2\left(\frac{1}{\kappa p_F}\right)^2\frac{\alpha}{16\alpha^2+(1-\alpha)^2}, \tag{30}$$

where the only free parameter is the interaction range $\kappa p_F > 1$. We depict Eq. (30) in Fig. 1. As expected, a magnetic field strongly reduces the phase space for Pomeranchuk instabilities. For small values of the magnetic field, the quantum critical point is shifted to greater attractive values of the interaction $\alpha < 0$ or $F_2 < -1$. Indeed, we observe a maximum value of the magnetic field

$$\left(\frac{\omega_c}{v_F p_F}\right)^2_{\max} \sim 0.2\left(\frac{1}{\kappa p_F}\right)^2 \tag{31}$$

reached at $\alpha_{\max} \sim -1/4$, above which no Pomeranchuk instability is possible. Moreover, we observe a reentrant behavior of the disordered isotropic phase for greater values of the attractive interaction.

It is important to note a clear difference with the case of the usual Pomeranchuk instability. At zero magnetic field, below the critical point $\alpha = 0$, the isotropic Fermi liquid is unstable under nonhomogeneous density perturbations characterized by a wave vector q. Indeed, any value of q, no matter how small, will produce the phase transition. However, in the presence of a magnetic field, there is another length scale given by the cyclotron radius $r_c = v_F/\omega_c$. This scale introduces an infrared cutoff for the relevant fluctuations that could trigger the phase transition. In other words, in the region below the transition line in Fig. 1, the isotropic Fermi liquid is unstable under density fluctuations in a typical length scale $\kappa < q^{-1} < r_c$. In practice, κ is a microscopic length and r_c is very large, therefore the above restriction is not severe.

In contrast, for $q^{-1} \gg r_c$ there is no possible Pomeranchuk transition. This result is quite clear. In the regime $q^{-1} \gg r_c$, the semiclassical approach is no longer valid. It is necessary to treat the complete quantum problem, where the system is gapped due to Landau-level quantization. This is the quantum Hall regime in which there is no Pomeranchuk instability. From an experimental point of view, fluctuations, in particular the wave vector q, are very difficult to control. However, any random inhomogeneous density fluctuation will contain components of $r_c^{-1} < q < \kappa^{-1}$ that, even with a very small amplitude, will trigger the anisotropic/isotropic phase transition. In contrast, it is always possible to imagine (at least from a theoretical point of view) that one could induce a small density fluctuation by applying a modulated test field with a definite wave vector q.

IV. COLLECTIVE MODES NEAR THE QUANTUM CRITICAL POINT

We would like to analyze the behavior of the stable oscillation modes of the Fermi surface near the quantum critical point ($\alpha = 0, \omega_c = 0$). We are interested in the regime $\alpha \ll 1$ and $\omega_c \ll v_F q \ll v_F p_F$.

We will focus on the unstable model ν_2^+. This mode is directly coupled with ν_0, ν_4^+, ν_1^-, and ν_3^- through Eq. (26). The symmetric modes ν_0 and ν_4^+ are stable modes and oscillate very rapidly near the quantum critical point. Therefore, if we are interested in time scales larger than $(v_F q)^{-1}$, we can average them to zero. The antisymmetric modes ν_1^- and ν_3^- couple with ν_2^+ through a magnetic field ω_c as a manifestation of parity symmetry breaking. Then, dismissing the symmetric couplings ν_0 and ν_4^+, the remaining system $(\nu_2^+, \nu_1^-, \nu_3^-)$ is a closed one. Defining the column vector $\nu = (\nu_2^+, \nu_1^-, \nu_3^-)$, the collective modes satisfy

$$\frac{\partial^2 \nu(q,t)}{\partial t^2} + M \cdot \nu(q,t) = 0, \tag{32}$$

where the matrix M takes the following form near the quantum critical point:

$$M = \begin{pmatrix} 2\left(\frac{v_F q}{2}\right)^2\alpha + \omega_c^2\left(\frac{\kappa q}{2}\right)^2 & \left(\frac{v_F q}{2}\right)\omega_c & 3\left(\frac{v_F q}{2}\right)\omega_c \\ \left(\frac{2\omega_c}{v_F q}\right)\left[\left(\frac{v_F q}{2}\right)^2\alpha + \left(\frac{\omega_c \kappa q}{2}\right)^2\right] & \left(\frac{v_F q}{2}\right)^2\alpha + \omega_c^2\left(1 + \left(\frac{\kappa q}{2}\right)^2\right) & \left(\frac{v_F q}{2}\right)^2\alpha + \left(\frac{\omega_c \kappa q}{2}\right)^2 \\ 5\alpha\omega_c\left(\frac{v_F q}{2}\right) & \alpha\left(\frac{v_F q}{2}\right)^2 & \left(\frac{v_F q}{2}\right)^2 \end{pmatrix}. \tag{33}$$

It is instructive to analyze two different paths when approaching the quantum critical point. In the case of zero applied magnetic field ($\omega_c = 0, \alpha \to 0$), the antisymmetric modes completely decouple from the symmetric ones, due to parity symmetry. Then the ν_2^+ frequency coincides with Eq. (17). However, when approaching the quantum critical point lowering the magnetic field ($\alpha = 0, \omega_c \to 0$), the matrix $M_c = \lim_{\alpha\to 0} M$ takes the form

$$M_c = \begin{pmatrix} \omega_c^2\left(\frac{\kappa q}{2}\right)^2 & \left(\frac{v_F q}{2}\right)\omega_c & 3\left(\frac{v_F q}{2}\right)\omega_c \\ \left(\frac{2\omega_c}{v_F q}\right)\left(\frac{\omega_c \kappa q}{2}\right)^2 & \omega_c^2\left[1 + \left(\frac{\kappa q}{2}\right)^2\right] & \left(\frac{\omega_c \kappa q}{2}\right)^2 \\ 0 & 0 & \left(\frac{v_F q}{2}\right)^2 \end{pmatrix}. \tag{34}$$

In order to find the normal modes, we diagonalize M_c, obtaining the eigenvalues

$$\lambda_1 = \left(\frac{\kappa q}{2}\right)^4 \omega_c^2, \quad \lambda_2 = \omega_c^2, \quad \lambda_3 = \left(\frac{v_F q}{2}\right)^2, \tag{35}$$

with the corresponding eigenvector matrix

$$A = \begin{pmatrix} 1 & 2\left(\frac{\omega_c}{v_F q}\right)\left(\frac{\kappa q}{2}\right)^2 & 0 \\ -2\left(\frac{\omega_c}{v_F q}\right)\left(\frac{\kappa q}{2}\right)^2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{36}$$

Thus it is clear from Eq. (36) that the mode ν_3^- decouples for $\alpha = 0$ and it is a rapid mode, oscillating with frequency $v_F q/2$. In contrast, the modes ν_2^+ and ν_1^- are slow modes, coupled by the small quantities $\omega_c/v_F q \ll 1$ and $(\kappa q/2)^2 \ll 1$. The former is related to the cyclotron frequency, which should be smaller than the frequency of a typical perturbation $(v_F q)^{-1}$, while the latter is related to the interaction range, which should be much smaller than the typical length of the Fermi-surface perturbation q^{-1}.

Therefore, very near the Pomeranchuk instability ($\alpha = 0, \omega_c \ll v_F q$), the Fermi surface fluctuates, following the equation

$$\begin{aligned} \delta k_F = {}& \nu_2^i \left\{ \cos(2\theta) + 2\left(\frac{\omega_c}{v_F q}\right)\left(\frac{\kappa q}{2}\right)^2 \sin\theta \right\} \\ & \times \cos\left[\left(\frac{\kappa q}{2}\right)^2 \omega_c t + \varphi_1\right] \\ & + \nu_{-1}^i \left\{ \sin(\theta) - 2\left(\frac{\omega_c}{v_F q}\right)\left(\frac{\kappa q}{2}\right)^2 \cos(2\theta) \right\} \\ & \times \cos[\omega_c t + \varphi_2] \\ & + \nu_{-3}^i \sin(3\theta) \cos\left[\left(\frac{v_F q}{2}\right) t + \varphi_3\right], \end{aligned} \tag{37}$$

where $\nu_2^i, \nu_{-1}^i, \nu_{-3}^i$ and $\varphi_1, \varphi_2, \varphi_3$ are the initial amplitudes and phases, respectively.

We see that there are two slow modes that oscillate with frequencies proportional to ω_c. The slowest mode [λ_1 in Eq. (35)] is related to ν_2^+ and it is responsible for the Pomeranchuk instability when $\omega_c \to 0$. In contrast, the mode associated with the eigenvalue λ_2 is related to the antisymmetric mode ν_1^-. However, this mode is not unstable at the quantum critical point since, when $\omega_c \to 0$, not only its frequency goes to zero, but also its velocity $\partial \nu_1^-/\partial t \to 0$, implying a constant

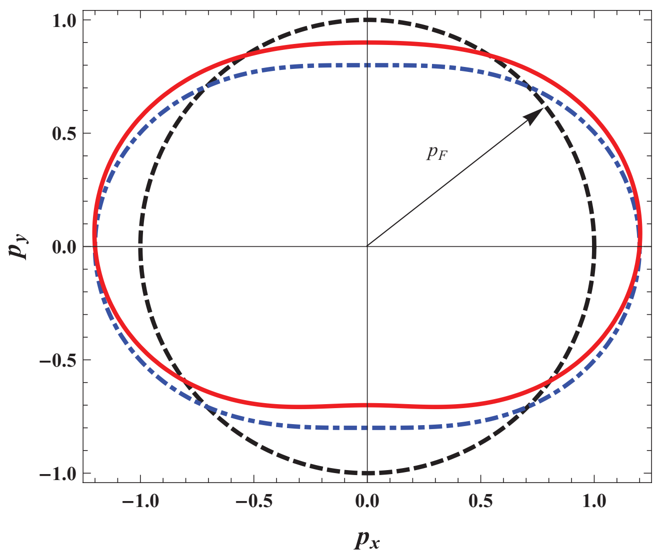

FIG. 2. (Color online) Snapshot of the Fermi-surface deformation, near the quantum critical point $\alpha = 0, \omega_c = 0$. The dash line is the actual Fermi surface, where we have normalized $p_F = 1$. The dash-dotted line is the elliptic deformation without magnetic field $\omega_c = 0$. The continuous line is the deformation in the presence of a small magnetic field. The vector **q** is aligned with the p_x axis; then the parity symmetry breaking in the p_y axis is evident.

mode at the quantum critical point, decoupled from any other symmetric mode.

In Fig. 2 we show a snapshot of the Fermi surface near the Pomeranchuk instability, where we have chosen the initial conditions $\nu_2^i = 0.2$ and $\nu_{-1}^i = \nu_{-3}^i = 0$. The circular dash line is the actual isotropic Fermi surface. The dash-dotted line shows an ellipse, which indicates the usual deformation with nematic symmetry in the absence of a magnetic field, while the continuous line is the deformation of the Fermi surface in the presence of a magnetic field. As expected, we observe a parity breaking in the axis $\mathbf{q} \times \mathbf{B}$ (in this case in p_y since we have chosen **q** pointing in the p_x direction). This is a tiny effect proportional to $(\omega_c/v_F q)(\kappa q/2)^2 \ll 1$. In the figure we have artificially amplified this parameter in order to make the effect of the magnetic field visible.

We have redone all the calculations of this section considering also the couplings with the symmetric modes ν_0 and ν_4^+. In this case, it is not possible to analytically solve the resulting 9×9 linear system. However, making a numerical analysis, we did not find any relevant deviation from the simplified calculation shown. This confirms in some way that the stable rapid modes do not participate in the instability process very near the quantum critical point.

V. CONCLUSION

We have analyzed the behavior of a two-dimensional Fermi liquid subject to an external magnetic field, near a Pomeranchuk instability triggered by the Landau parameter F_2 in the charged sector. We have considered a simple model in which the only interaction is given by the Landau parameter F_2. The presence of other interactions does not modify the results qualitatively, provided they are all stable, i.e., distant from any other Pomeranchuk instability.

We have studied the Fermi-surface stability, approaching the critical region from the isotropic phase, where the Landau theory of Fermi liquids can be used safely. We have studied collective modes using the semiclassical Landau-Silin equation. Usually, this equation is studied in the linear response approximation to analyze plasma modes in charged Fermi liquids. However, near a Pomeranchuk instability this approximation is not sufficient. The reason is that the quantum critical point is controlled by two parameters $\alpha = 1 + F_2$ and $\omega_c/v_F p_F$. The leading-order correction in the magnetic field is proportional to $\alpha(\omega_c/v_F p_F)$. Thus, near the quantum critical point ($\alpha = 0, \omega_c = 0$), corrections proportional to α^2 and $(\omega_c/v_F p_F)^2$ are of the same order and cannot be neglected. Therefore, we need to go to quadratic order in the magnetic field to consistently treat the neighborhood of the quantum critical point.

There are essentially three scales in the theory: the shortest distance scale given by the inverse of the Fermi momentum p_F^{-1}, an interaction range scale κ, and the longest distance scale given by the cyclotron radius $r_c = v_F/\omega_c$. We have found that the isotropic Fermi system could be unstable under inhomogeneous density fluctuations of typical length scale q^{-1} provided the inequality $p_F^{-1} < \kappa \ll q^{-1} \lesssim r_c$ is satisfied.

Identifying the slowest collective mode, it is possible to compute the transition line given in Fig. 1. The transition is completely governed by the interaction range κp_F. We observe an upper limit value for the magnetic field $\omega_c/v_F p_F \sim 1/\kappa p_F$ over which the Pomeranchuk instability is completely suppressed. For smaller values of the magnetic field, we observe that the instability is shift to stronger values of the attractive interaction. Moreover, a reentrant behavior of the isotropic phase is observed for even stronger attractive interactions. Reentrant behavior has posed challenges to microscopic theoretical physics in a variety of condensed matter systems.[17–24] This phenomenon is characterized by the reappearance of a less ordered phase, following a more ordered one, as a control parameter (for example, temperature, pressure, chemical doping, and magnetic field) is varied. It appears that the reentrance phenomenon also occurs, as we report in this paper, in the phase diagram for the Pomeranchuk instability $\ell = 2$ in the presence of a small magnetic field. Basically, the reentrant phenomenon can be produced by the increase of entropy due to disorder or the presence of additional degrees of freedom.

We have also studied collective mode couplings near the critical region. We have identified the ν_2^+ mode as the mainly unstable mode when the quantum critical point is approached. The main contribution to the Fermi-surface deformation has elliptic (nematic) symmetry. However, the magnetic field couples this mode with the antisymmetric ones ν_1^- and ν_3^-. The antisymmetric ν_3^- is a rapid mode oscillating with frequency $v_F q/2$ and does not participate in the instability process. In contrast, ν_1^- is a slow mode, however quicker than ν_2^+, since it oscillates with the cyclotron frequency ω_c. Even though its frequency goes to zero at the quantum critical point, it does not represent a real Pomeranchuk instability since, on the one hand, its coupling with ν_2^+ also goes to zero with the magnetic field and, on the other hand, not only its frequency but also its velocity goes to zero as $\omega_c \to 0$. However, it has an important effect on the Fermi-surface deformation of the unstable mode since its coupling is a direct consequence of parity breaking, producing a contribution that breaks nematic symmetry as shown in Fig. 2. In fact, near the quantum critical point the slowest mode is invariant under the combined transformation $\theta \to \theta + \pi, \omega_c \to -\omega_c$.

In order to have a complete picture of the isotropic-nematic phase transition under the influence of a magnetic field, it is necessary to study the ordered phase. To do that in the context of the Landau theory of Fermi liquids, it is necessary to go beyond the linear approximation in δn_p and study the collision integral $I_{\rm coll}$ in the Landau-Silin equation (4). Conversely, it is possible to face this problem with other approaches, for instance, nonperturbative calculations on specific fermionic models.

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