

Long-range order and spin-liquid states of polycrystalline $\text{Tb}_{2+x}\text{Ti}_{2-x}\text{O}_{7+y}$

T. Taniguchi,¹ H. Kadowaki,¹ H. Takatsu,¹ B. Fåk,² J. Ollivier,³ T. Yamazaki,⁴ T. J. Sato,⁵ H. Yoshizawa,⁵ Y. Shimura,⁴ T. Sakakibara,⁴ T. Hong,⁶ K. Goto,¹ L. R. Yaraskavitch,⁷ and J. B. Kycia⁷

¹*Department of Physics, Tokyo Metropolitan University, Hachioji-shi, Tokyo 192-0397, Japan*

²*SPSMS, UMR-E CEA/UJF-Grenoble-1, INAC, F-38054 Grenoble Cedex 9, France*

³*Institute Laue Langevin, Boîte Postale 156, F-38042 Grenoble, France*

⁴*Institute for Solid State Physics, University of Tokyo, Kashiwa 277-8581, Japan*

⁵*NSL, Institute for Solid State Physics, University of Tokyo, Tokai, Ibaraki 319-1106, Japan*

⁶*Quantum Condensed Matter Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-6393, USA*

⁷*Department of Physics and Astronomy and Guelph-Waterloo Physics Institute, University of Waterloo, Waterloo, ON, Canada N2L 3G1*

(Received 15 November 2012; revised manuscript received 9 January 2013; published 25 February 2013)

Low-temperature states of polycrystalline samples of a frustrated pyrochlore oxide $\text{Tb}_{2+x}\text{Ti}_{2-x}\text{O}_{7+y}$ have been investigated by specific heat, magnetic susceptibility, and neutron scattering experiments. We have found that this system can be tuned by a minute change of x from a spin-liquid state ($x < x_c$) to a partly ordered state with a small antiferromagnetic ordering of the order of $0.1\mu_B$. Specific heat shows a sharp peak at a phase transition at $T_c = 0.5$ K for $x = 0.005$. Magnetic excitation spectra for this sample change from a quasielastic to a gapped type through T_c . The possibility of a Jahn-Teller transition is discussed.

DOI: 10.1103/PhysRevB.87.060408

PACS number(s): 75.10.Kt, 75.40.Cx, 75.70.Tj, 78.70.Nx

Magnetic systems with geometric frustration, a prototype of which is antiferromagnetically coupled Ising spins on a triangle, have been intensively studied experimentally and theoretically for decades.¹ Spin systems on networks of triangles or tetrahedra, such as triangular,² kagomé,³ and pyrochlore⁴ lattices, play major roles in these studies. Subjects that have fascinated many investigators in recent years are classical and quantum spin-liquid states,^{5–8} where conventional long-range order (LRO) is suppressed to very low temperatures. Quantum spin liquids^{6,7} in particular have been challenging both theoretically and experimentally since the proposal of the resonating valence-bond state.⁹ The spin ice materials $R_2\text{Ti}_2\text{O}_7$ ($R = \text{Dy}, \text{Ho}$) are the well-known classical examples,⁵ while other experimental candidates found recently have been studied.^{10–14}

Among frustrated pyrochlore oxides,⁴ $\text{Tb}_2\text{Ti}_2\text{O}_7$ has attracted much attention because it does not show any conventional LRO down to 50 mK and remains in a dynamic spin-liquid state.^{15–17} Theoretical considerations of the crystal-field (CF) states of Tb^{3+} and exchange and dipolar interactions of the system^{18–20} showed that it should undergo a transition into a magnetic LRO state at about 1.8 K within a random-phase approximation.²⁰ The puzzling origin of the spin-liquid state of $\text{Tb}_2\text{Ti}_2\text{O}_7$ is a subject of hot debate.^{4,21–28} An interesting scenario for the spin-liquid state is the theoretical proposal of a quantum spin-ice state.²² More recently, another scenario of a two-singlet spin-liquid state was proposed to explain why inelastic neutron spectra in a low-energy range are observed despite the fact that Tb^{3+} is a non-Kramers ion.^{23,24}

Several experimental puzzles of $\text{Tb}_2\text{Ti}_2\text{O}_7$ originate from the difficulty of controlling the quality of single-crystalline samples, resulting in strongly sample-dependent specific-heat anomalies at temperatures below 2 K.^{18,26,29–33} In contrast, experimental results on polycrystalline samples are more consistent.^{15,16,26} Among the experimental results reported to date, an important clue to solve the puzzles of $\text{Tb}_2\text{Ti}_2\text{O}_7$ seems to be a change of state at about 0.4 K suggested

by specific heat,²⁶ inelastic neutron scattering,²⁶ and neutron spin echo¹⁶ measurements on polycrystalline samples. At this temperature, a few single-crystalline samples show a peak in the specific heat suggesting a phase transition,^{29,30} an issue that has not been pursued seriously. The possibility of a cooperative Jahn-Teller phase transition well below 1 K was inferred many years ago from the observation of an anomalous temperature dependence of the elastic constants above 1 K.³⁴ The two-singlet spin-liquid scenario of Refs. 23,24, and 35 is based on the assumption of a tetragonal lattice distortion in $\text{Tb}_2\text{Ti}_2\text{O}_7$ and the closely related ordered spin-ice compound $\text{Tb}_2\text{Sn}_2\text{O}_7$,³⁶ but the accompanying lattice distortion might be too difficult to observe directly.^{25,37–40} A theoretical study on pyrochlore magnets with non-Kramers magnetic ground doublets, applicable to Pr^{3+} , Tb^{3+} , etc., pointed out the possibilities of quadrupole orderings as well as quantum spin ice.^{41,42}

In the present work, we investigate the hypothesis that the nonstoichiometry x of $\text{Tb}_{2+x}\text{Ti}_{2-x}\text{O}_{7+y}$ is a tuning parameter for a quantum critical point separating a LRO state from a spin-liquid state. We have therefore performed specific heat, magnetization, and neutron scattering experiments on polycrystalline samples of $\text{Tb}_{2+x}\text{Ti}_{2-x}\text{O}_{7+y}$ with different values of x . We find that a minute change of x brings about a systematic change of the specific heat. The ground state goes from LRO with an unknown order parameter for $x > x_c$ to a spin liquid for $x < x_c$.

Polycrystalline samples of $\text{Tb}_{2+x}\text{Ti}_{2-x}\text{O}_{7+y}$ with $-0.015 < x < 0.01$ were prepared by a standard solid-state reaction.¹⁵ The value of x was adjusted by changing the mass ratio of the two starting materials, Tb_4O_7 and TiO_2 , which were heated in air at 1350 °C for several days with periodic grindings to ensure a complete reaction. It was ground into powder and annealed in air at 800 °C for one day. The values of x used in this paper are nominal, and have an offset of about ± 0.002 . The value of y is determined by the oxidizing conditions. X-ray powder-diffraction experiments were carried out using

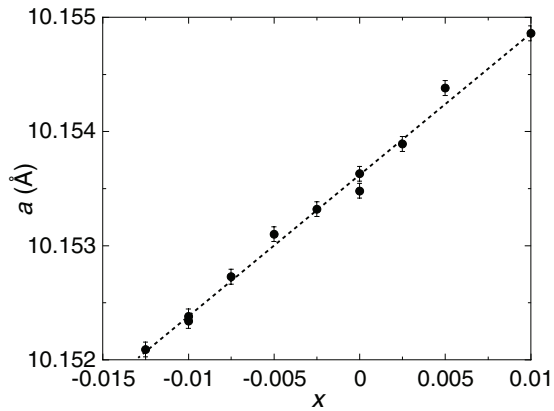


FIG. 1. Lattice constants of polycrystalline $\text{Tb}_{2+x}\text{Ti}_{2-x}\text{O}_{7+y}$ at 25 °C. The dashed line is a guide to the eye.

a RIGAKU-SmartLab powder diffractometer equipped with a Cu $K\alpha 1$ monochromator. The absence of impurity peaks in the powder diffraction patterns shows that the samples are single phase with the pyrochlore structure.⁴³ To measure the x dependence of the lattice constant a at 25 °C, we performed θ - 2θ scans on powder mixtures of $\text{Tb}_{2+x}\text{Ti}_{2-x}\text{O}_{7+y}$ and Si. Figure 1 shows that the lattice constant a , consistent with the previous work for $x = 0$,⁴³ has a smooth variation with x , which ensures a continuous change of the stoichiometry of $\text{Tb}_{2+x}\text{Ti}_{2-x}\text{O}_{7+y}$ for small x .

Specific heat above 0.4 K was measured on a physical-property measurement system. Measurements below 0.4 K were carried out using the quasiadiabatic relaxation method on a dilution refrigerator.⁴⁴ dc magnetization measurements were carried out by a capacitive Faraday magnetometer in a ^3He refrigerator. Neutron powder diffraction measurements were performed on the triple-axis spectrometer CTAX at ORNL. Inelastic neutron scattering measurements were carried out on the time-of-flight spectrometer IN5 operated with $\lambda = 5$ and 10 Å at ILL. For these neutron scattering experiments, samples of $x = 0.005$ and -0.005 with weights of 5 and 9 g were mounted in a ^3He (CTAX) and a dilution refrigerator (IN5), respectively.

In Fig. 2, we show the specific heat C_P of the polycrystalline samples as a function of temperature together with a few previous measurements.^{26,29,45} Earlier work have shown qualitatively similar results.^{46,47} The $C_P(T)$ data show a systematic change by varying x . A sample with $x = 0.005$ shows a clear peak indicating a second-order phase transition at $T_c = 0.5$ K. Samples with $x = 0.0025$ and 0.000 show smaller peaks at 0.43 and 0.4 K, respectively. We note that C_P of the present sample with $x = 0.000$ agrees approximately with our previous measurements,²⁶ the temperature range of which was extended down to 0.2 K in the present work on a sample (nominal $x' = 0$) prepared from a different commercial source of Tb_4O_7 . Our previous interpretation²⁶ of the upturn below 0.5 K as a crossover behavior is incorrect due to the insufficient temperature range. The previous C_P data⁴⁵ (Fig. 2) on a polycrystalline sample with their nominal $x'' = 0$ correspond to our $x = -0.0125$, implying that fine tuning of x requires careful sample preparation. In the inset of Fig. 2, we show a cumulative phase diagram constructed from $C_P(T, x)$

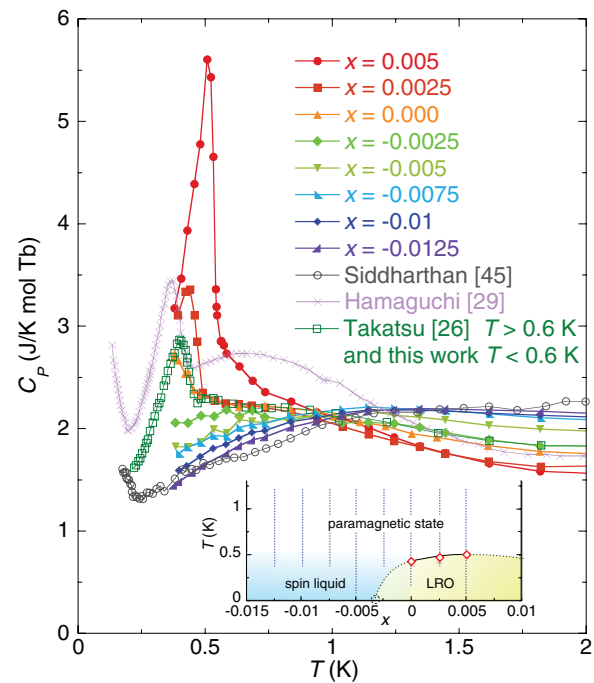


FIG. 2. (Color online) Temperature dependence of the specific heat of polycrystalline $\text{Tb}_{2+x}\text{Ti}_{2-x}\text{O}_{7+y}$. Previous measurements of poly- and single-crystalline samples,^{26,29,45} as well as the present measurements below 0.6 K of a sample prepared in the same manner as in Ref. 26, are plotted for comparison. The inset shows a phase diagram expected from the specific heat, susceptibility, and neutron scattering.

in conjunction with the susceptibility and neutron scattering experiments discussed below.

A peak of $C_P(T)$ in $\text{Tb}_2\text{Ti}_2\text{O}_7$ was first reported for a single-crystalline sample at 0.37 K.²⁹ These $C_P(T)$ data,²⁹ reproduced in Fig. 2, show a significantly different T dependence from any of the polycrystalline samples. The sharp peak at 0.37 K may result from a portion of the sample having a nonstoichiometry parameter around $x = -0.001$, corresponding to a peak slightly lower in temperature than our $x = 0.000$. However, a hump in $C_P(T)$ around 0.75 K for the single crystal does not appear for the polycrystalline samples. We believe that these single- and polycrystalline samples have significant, but presently not well understood, differences in quality.

To check whether T_c is an antiferromagnetic transition, as suggested in Ref. 29, we performed magnetization and neutron powder-diffraction experiments. In Fig. 3, we show the magnetic susceptibility as a function of temperature for three polycrystalline samples with $x = \pm 0.005$ and 0.000. The susceptibilities for $x = 0.005$ and 0.000 show only slight anomalies around the clear peaks of $C_P(T)$ at $T_c = 0.5$ and 0.4 K, respectively. These weak anomalies resemble certain transitions related to magnetic degrees of freedom.

In Fig. 4, we show neutron powder-diffraction patterns for the $x = 0.005$ sample below and above T_c . The pattern below T_c shows neither any clear antiferromagnetic reflections nor any clear changes due to a structural transition. Rough estimates of the upper limits of the antiferromagnetic ordered moment and the structural change are about $0.1\mu_B$ and

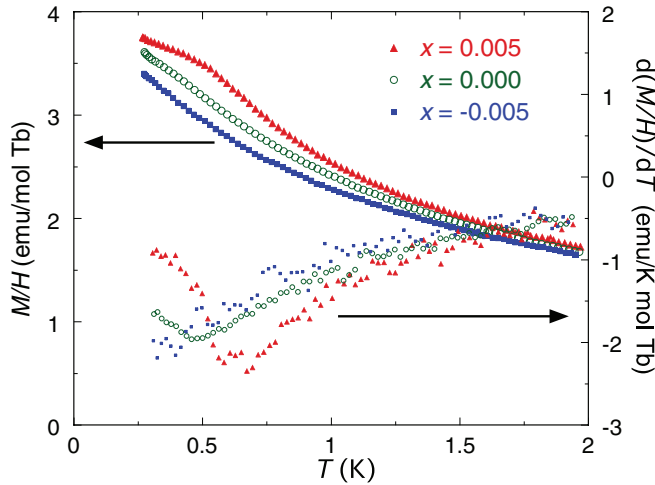


FIG. 3. (Color online) Temperature dependence of the magnetic susceptibility ($H = 0.05$ T) and its derivative with respect to T of polycrystalline $\text{Tb}_{2+x}\text{Ti}_{2-x}\text{O}_{7+y}$ with $x = -0.005, 0.000,$ and 0.005 .

$\Delta a/a < 0.01$ assuming a cubic to tetragonal distortion. The intensity of the sloping paramagnetic scattering, a background for Bragg peaks, decreases slightly as temperature is lowered from 1.2 to 0.28 K. This is brought about by a change in the magnetic excitations. The lack of obvious antiferromagnetism distinctly separates $\text{Tb}_2\text{Ti}_2\text{O}_7$ from the ordered spin-ice compound $\text{Tb}_2\text{Sn}_2\text{O}_7$,^{36,48} in which antiferromagnetic ordering with a moment of $5.9\mu_B$ was observed well below $T_c = 0.87$ K.

To study the spectral change of the magnetic excitations through T_c , we performed inelastic neutron scattering measurements using the spectrometer IN5 (Ref. 49) with an energy resolution of $\Delta E = 0.012$ meV (full width at half-maximum), which is five times better than that in our previous study.²⁶ Figure 5 shows the temperature dependence of an energy spectrum for the $x = 0.005$ sample at $Q = 0.6 \text{ \AA}^{-1}$. It is evident that the spectrum changes from a continuum ($T > T_c$) to a peaked structure at 0.1 meV ($T < T_c$). The

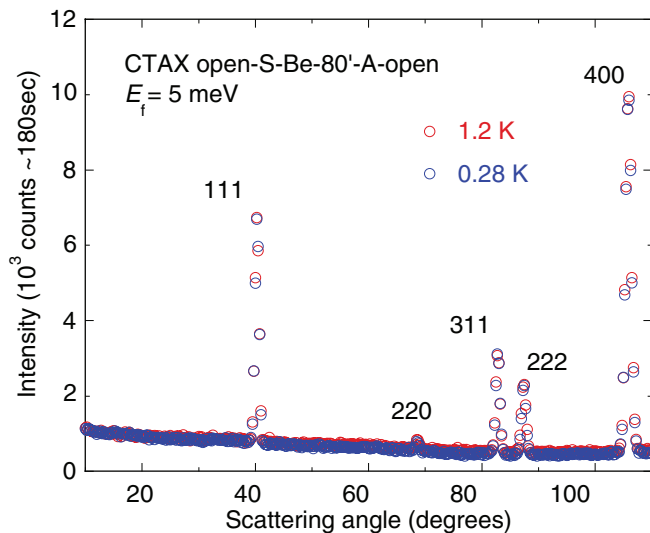


FIG. 4. (Color online) Neutron powder-diffraction pattern of polycrystalline $\text{Tb}_{2+x}\text{Ti}_{2-x}\text{O}_{7+y}$ with $x = 0.005$ taken above and below $T_c = 0.5$ K.

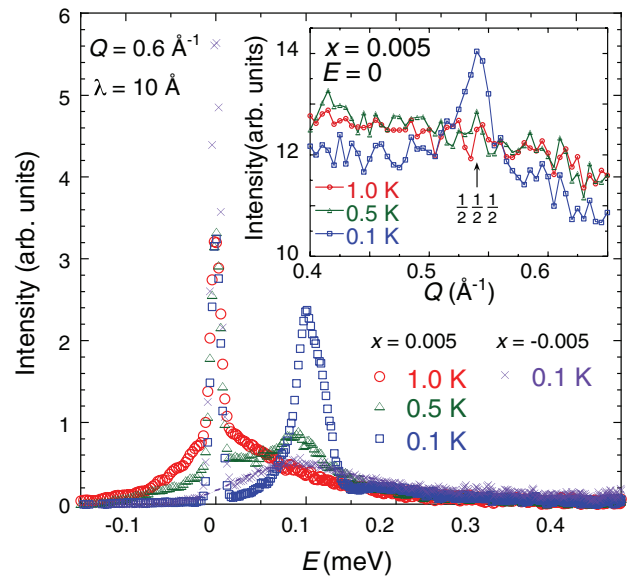


FIG. 5. (Color online) Energy spectra of inelastic neutron scattering for polycrystalline $\text{Tb}_{2+x}\text{Ti}_{2-x}\text{O}_{7+y}$ with $x = 0.005$ and -0.005 . The inset shows the Q dependence of the elastic scattering for the $x = 0.005$ sample around $Q = |(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})|$ above and below T_c . The dashed line is the fit curve.

excitation peak at $T \ll T_c$ is weakly Q -dependent, which may possibly be interpreted as a splitting of the CF ground-state doublet. An energy spectrum of the $x = -0.005$ sample is also shown in Fig. 5 for comparison. Its spectral shape can be approximately expressed by a Lorentzian squared $\text{Im}\chi(E, Q)/E \propto [(\sqrt{2} - 1)E^2 + \Gamma^2]^{-2}$ with $\Gamma = 0.1$ meV (half-width at half-maximum) in $-0.05 < E < 0.3$ meV, revealing quantum spin fluctuations with the same energy scale of 0.1 meV as that of the $x = 0.005$ sample.

The high sensitivity of IN5 enabled us to observe a small Bragg peak for the $x = 0.005$ sample, being undetectable in the CTAX data (Fig. 4). In the inset of Fig. 5, the intensity of the elastic scattering for $|E| < 0.005$ meV is plotted as a function of Q . Below T_c , a clear Bragg peak at $Q = 0.54 \text{ \AA}^{-1}$ is observed, which can be indexed as $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. The Q width of this peak is somewhat larger than the instrumental Q resolution, and corresponds to a correlation length of the order of 100 \AA . Although this peak could be of a nuclear (structural) origin, it is more likely an antiferromagnetic (AFM) reflection. In fact, two recent neutron scattering experiments carried out on single-crystalline samples of $\text{Tb}_2\text{Ti}_2\text{O}_7$ showed magnetic short-range order around the same $Q = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$.^{24,28} A roughly estimated ordered moment for the $x = 0.005$ sample is $0.08\mu_B$ at 0.1 K, where we assume the phase factor $e^{iQ \cdot r} = 1$ in the magnetic structure factor. This ordered moment is much smaller than the magnetic moment $\sim 5\mu_B$ of the ground doublets,^{18,48} which implies that most of the spin fluctuations persist below T_c . In contrast, the entropy change around $T_c = 0.5$ K is $S(T = 0.55) - S(T = 0.38) \simeq 0.25R \ln 2$ (Fig. 2), which is significant. These probably indicate that there is a major order parameter, which is unknown at present.

The present results have provided an answer to the problem of the previously reported transition or crossover at about 0.4 K for the poly- and single-crystalline $\text{Tb}_2\text{Ti}_2\text{O}_7$,^{16,26,29}

and they pose another question: what is the major order parameter associated with T_c ? In the following, we speculatively discuss a few possibilities. A cooperative Jahn-Teller transition due to a magnetoelastic coupling^{50,51} was suggested a long time ago,³⁴ although direct experimental evidence has not been found. Precursor effects of a Jahn-Teller transition were reported using x-ray diffraction on a single-crystalline sample.³⁸ According to Refs. 23,24, and 35, a splitting of the CF ground-state doublet into two singlets can be interpreted as simplest evidence of a Jahn-Teller distortion breaking the local trigonal D_{3d} symmetry of the Tb site. Along these lines, the weakly Q -dependent excitation peak at 0.1 meV (Fig. 5) can be interpreted as the splitting, and the transition is due to a Jahn-Teller effect accompanying a small AFM ordering.⁵¹ A recent theory,^{41,42} exploited to explain quantum fluctuations of pyrochlore magnets with non-Kramers Pr^{3+} , Tb^{3+} , etc., showed possibilities of quadrupole orderings due to an electronic coupling, which are located close to the quantum spin ice state.²² One of these quadrupole orderings^{41,42} may be the order parameter. A resulting structural distortion coupled to the quadrupole ordering could be too small to be observed. Although the major order parameter is unknown at present, the long-standing puzzle of the spin-liquid state of $\text{Tb}_2\text{Ti}_2\text{O}_7$ seems to be reformulated to a novel problem of frustration having spin and other degrees of freedom.

Experimentally, single-crystalline samples with tunable x or y are indispensable for further studies.

In summary, we have investigated the low-temperature states of polycrystalline $\text{Tb}_{2+x}\text{Ti}_{2-x}\text{O}_{7+y}$ samples by specific heat, magnetic susceptibility, and neutron scattering experiments. We have found that this system can be tuned by a minute change of x from a LRO ground state with an unknown major order parameter accompanying a minor AFM ordering for $x > x_c$ to a liquid-type ground state with quantum spin fluctuations for $x < x_c$. Specific heat shows a sharp peak at a second-order phase transition T_c for $x > x_c$. Inelastic neutron scattering of an $x = 0.005$ ($>x_c$) sample shows that a gap opens in the magnetic excitation spectrum below T_c .

We thank M. J. P. Gingras, R. Higashinaka, J. W. Lynn, and K. Matsuhira for useful discussions. This work was supported by KAKENHI NSMIF. The specific heat to 0.4 K and magnetization measurements were performed using the facilities of ISSP, Univ. of Tokyo. Work on CTAX was supported by the US-Japan Cooperative Program on Neutron Scattering. HFIR was partially supported by the US DOE, Office of BES, Division of Scientific User Facilities. The neutron scattering performed using IN5 (France) was transferred from JRR3-HER (proposal 11567) with the approval of ISSP, Univ. of Tokyo, and JAEA, Tokai, Japan.

¹*Introduction to Frustrated Magnetism*, edited by C. Lacroix, P. Mendels, and F. Mila (Springer, Berlin, Heidelberg, 2011).

²G. H. Wannier, *Phys. Rev.* **79**, 357 (1950).

³I. Syôzi, *Prog. Theor. Phys.* **6**, 306 (1951).

⁴J. S. Gardner, M. J. P. Gingras, and J. E. Greedan, *Rev. Mod. Phys.* **82**, 53 (2010).

⁵S. T. Bramwell and M. J. P. Gingras, *Science* **294**, 1495 (2001).

⁶P. A. Lee, *Science* **321**, 1306 (2008).

⁷L. Balents, *Nature (London)* **464**, 199 (2010).

⁸S. Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).

⁹P. W. Anderson, *Mater. Res. Bull.* **8**, 153 (1973).

¹⁰J. S. Helton, K. Matan, M. P. Shores, E. A. Nytko, B. M. Bartlett, Y. Yoshida, Y. Takano, A. Suslov, Y. Qiu, J.-H. Chung, D. G. Nocera, and Y. S. Lee, *Phys. Rev. Lett.* **98**, 107204 (2007).

¹¹T. Itou, A. Oyamada, S. Maegawa, M. Tamura, and R. Kato, *Phys. Rev. B* **77**, 104413 (2008).

¹²B. Fåk, E. Kermarrec, L. Messio, B. Bernu, C. Lhuillier, F. Bert, P. Mendels, B. Koteswararao, F. Bouquet, J. Ollivier, A. D. Hillier, A. Amato, R. H. Colman, and A. S. Wills, *Phys. Rev. Lett.* **109**, 037208 (2012).

¹³K. A. Ross, L. Savary, B. D. Gaulin, and L. Balents, *Phys. Rev. X* **1**, 021002 (2011).

¹⁴L.-J. Chang, S. Onoda, Y. Su, Y.-J. Kao, K.-D. Tsuei, Y. Yasui, K. Kakurai, and M. R. Lees, *Nat. Commun.* **3**, 992 (2012).

¹⁵J. S. Gardner, S. R. Dunsiger, B. D. Gaulin, M. J. P. Gingras, J. E. Greedan, R. F. Kiefl, M. D. Lumsden, W. A. MacFarlane, N. P. Raju, J. E. Sonier, I. Swainson, and Z. Tun, *Phys. Rev. Lett.* **82**, 1012 (1999).

¹⁶J. S. Gardner, A. Keren, G. Ehlers, C. Stock, E. Segal, J. M. Roper, B. Fåk, M. B. Stone, P. R. Hammar, D. H. Reich, and B. D. Gaulin, *Phys. Rev. B* **68**, 180401 (2003).

¹⁷B. D. Gaulin and J. S. Gardner, pp. 177–206 in Ref. 1.

¹⁸M. J. P. Gingras, B. C. den Hertog, M. Faucher, J. S. Gardner, S. R. Dunsiger, L. J. Chang, B. D. Gaulin, N. P. Raju, and J. E. Greedan, *Phys. Rev. B* **62**, 6496 (2000).

¹⁹M. Enjalran and M. J. P. Gingras, *Phys. Rev. B* **70**, 174426 (2004).

²⁰Y.-J. Kao, M. Enjalran, A. Del Maestro, H. R. Molavian, and M. J. P. Gingras, *Phys. Rev. B* **68**, 172407 (2003).

²¹Y. Yasui, M. Kanada, M. Ito, H. Harashina, M. Sato, H. Okumura, K. Kakurai, and H. Kadowaki, *J. Phys. Soc. Jpn.* **71**, 599 (2002).

²²H. R. Molavian, M. J. P. Gingras, and B. Canals, *Phys. Rev. Lett.* **98**, 157204 (2007).

²³P. Bonville, I. Mirebeau, A. Gukasov, S. Petit, and J. Robert, *Phys. Rev. B* **84**, 184409 (2011).

²⁴S. Petit, P. Bonville, J. Robert, C. Decorse, and I. Mirebeau, *Phys. Rev. B* **86**, 174403 (2012).

²⁵B. D. Gaulin, J. S. Gardner, P. A. McClarty, and M. J. P. Gingras, *Phys. Rev. B* **84**, 140402 (2011).

²⁶H. Takatsu, H. Kadowaki, T. J. Sato, J. W. Lynn, Y. Tabata, T. Yamazaki, and K. Matsuhira, *J. Phys.: Condens. Matter* **24**, 052201 (2012).

²⁷T. Fennell, M. Kenzelmann, B. Roessli, M. K. Haas, and R. J. Cava, *Phys. Rev. Lett.* **109**, 017201 (2012).

²⁸K. Fritsch, K. A. Ross, Y. Qiu, J. R. D. Copley, T. Guidi, R. I. Bewley, H. A. Dabkowska, and B. D. Gaulin, arXiv:1210.1242.

²⁹N. Hamaguchi, T. Matsushita, N. Wada, Y. Yasui, and M. Sato, *Phys. Rev. B* **69**, 132413 (2004).

- ³⁰Y. Chapuis, Ph.D. thesis, Université Joseph Fourier, <http://tel.archives-ouvertes.fr/tel-00463643/en/>.
- ³¹Y. Chapuis, A. Yaouanc, P. Dalmas de Réotier, C. Marin, S. Vanishri, S. H. Curnoe, C. Vâju, and A. Forget, *Phys. Rev. B* **82**, 100402 (2010).
- ³²A. Yaouanc, P. Dalmas de Réotier, Y. Chapuis, C. Marin, S. Vanishri, D. Aoki, B. Fåk, L.-P. Regnault, C. Buisson, A. Amato, C. Baines, and A. D. Hillier, *Phys. Rev. B* **84**, 184403 (2011).
- ³³K. A. Ross, Th. Proffen, H. A. Dabkowska, J. A. Quilliam, L. R. Yaraskavitch, J. B. Kycia, and B. D. Gaulin, *Phys. Rev. B* **86**, 174424 (2012), show that a single-crystalline $\text{Yb}_2(\text{Ti}_{2-x}\text{Yb}_x)\text{O}_{7-x/2}$ sample with nominal $x = 0$ has excess Yb atoms affecting a phase transition; M. Revell, L. R. Yaraskavitch, J. D. Mason, K. A. Ross, H. M. L. Noad, H. A. Dabkowska, B. D. Gaulin, P. Henelius, and J. B. Kycia, *Nat. Phys.* **9**, 34 (2013), show an x -dependent effect for $\text{Dy}_2(\text{Ti}_{2-x}\text{Dy}_x)\text{O}_{7-x/2}$. In addition to disorders due to x considered in these papers, chemical pressure effects could be at play, because the off-stoichiometric $x = 0.005$ sample shows the largest peak of $C_P(T)$ (Fig. 2).
- ³⁴L. G. Mamsurova, K. S. Pigal'skiĭ, and K. K. Pukhov, *JETP Lett.* **43**, 755 (1986).
- ³⁵S. Petit, P. Bonville, I. Mirebeau, H. Mutka, and J. Robert, *Phys. Rev. B* **85**, 054428 (2012).
- ³⁶I. Mirebeau, A. Apetrei, J. Rodríguez-Carvajal, P. Bonville, A. Forget, D. Colson, V. Glazkov, J. P. Sanchez, O. Isnard, and E. Suard, *Phys. Rev. Lett.* **94**, 246402 (2005).
- ³⁷T. T. A. Lummen, I. P. Handayani, M. C. Donker, D. Fausti, G. Dhahlenne, P. Berthet, A. Revcolevschi, and P. H. M. van Loosdrecht, *Phys. Rev. B* **77**, 214310 (2008).
- ³⁸J. P. C. Ruff, B. D. Gaulin, J. P. Castellan, K. C. Rule, J. P. Clancy, J. Rodriguez, and H. A. Dabkowska, *Phys. Rev. Lett.* **99**, 237202 (2007).
- ³⁹Y. Nakanishi, T. Kumagai, M. Yoshizawa, K. Matsuhira, S. Takagi, and Z. Hiroi, *Phys. Rev. B* **83**, 184434 (2011).
- ⁴⁰K. Goto, H. Takatsu, T. Taniguchi, and H. Kadowaki, *J. Phys. Soc. Jpn.* **81**, 015001 (2012).
- ⁴¹S. Onoda and Y. Tanaka, *Phys. Rev. Lett.* **105**, 047201 (2010).
- ⁴²S. Onoda and Y. Tanaka, *Phys. Rev. B* **83**, 094411 (2011).
- ⁴³S.-W. Han, J. S. Gardner, and C. H. Booth, *Phys. Rev. B* **69**, 024416 (2004).
- ⁴⁴J. A. Quilliam, C. G. A. Mugford, A. Gomez, S. W. Kycia, and J. B. Kycia, *Phys. Rev. Lett.* **98**, 037203 (2007).
- ⁴⁵R. Siddharthan, B. S. Shastry, A. P. Ramirez, A. Hayashi, R. J. Cava, and S. Rosenkranz, *Phys. Rev. Lett.* **83**, 1854 (1999).
- ⁴⁶A. Cornelius, B. Light, R. S. Kumar, M. Eichenfield, T. Dutton, R. Pepin, and J. Gardner, *Physica B* **359–361**, 1243 (2005).
- ⁴⁷X. Ke, D. V. West, R. J. Cava, and P. Schiffer, *Phys. Rev. B* **80**, 144426 (2009).
- ⁴⁸I. Mirebeau, P. Bonville, and M. Hennion, *Phys. Rev. B* **76**, 184436 (2007).
- ⁴⁹J. Ollivier and H. Mutka, *J. Phys. Soc. Jpn.* **80**, SB003 (2011).
- ⁵⁰J. Jensen and A. R. Mackintosh, *Rare Earth Magnetism* (Clarendon, Oxford, 1991).
- ⁵¹G. A. Gehring and K. A. Gehring, *Rep. Prog. Phys.* **38**, 1 (1975).