

Nernst effect as a signature of quantum fluctuations in quasi-one-dimensional superconductors

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We study a model for the transverse thermoelectric response due to quantum superconducting fluctuations in a two-leg Josephson ladder, subject to a perpendicular magnetic field B and a transverse temperature gradient. Assuming a weak Josephson coupling on the rungs, the off-diagonal Peltier coefficient (α_{xy}) and the Nernst effect are evaluated as functions of B and the temperature T . In this regime, the Nernst effect is found to exhibit a prominent peak close to the superconductor–insulator transition, which becomes progressively enhanced at low T . In addition, we derive a relation to the diamagnetic response $\alpha_{xy} = -M/T_0$, where M is the equilibrium magnetization and T_0 a plasma energy in the superconducting legs.

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I. INTRODUCTION

In low-dimensional superconducting (SC) systems (ultrathin films, wires, and Josephson arrays), fluctuations of the SC order parameter field lead to a broadening of the transition to the SC state, and give rise to anomalous transport properties in the adjacent normal phase.¹ While close to or above the critical temperature T_c thermally excited fluctuations dominate these conduction anomalies, quantum fluctuations are expected to dominate at low temperatures $T \ll T_c$, where superconductivity is weakened due to, e.g., the effect of a magnetic field, disorder, or repulsive Coulomb interactions. Their most dramatic manifestation is the onset of a superconductor–insulator transition (SIT) when an external parameter such as magnetic field or thickness is tuned beyond a critical point.^{2,3}

A striking signature of the fluctuations' regime, which attracted much attention in the recent years, is the anomalous enhancement of transverse thermoelectric effects in the presence of a perpendicular magnetic field B . In particular, a substantial Nernst effect measured far above T_c , e.g., in the underdoped regime of high- T_c superconductors^{4,5} and disordered thin films,^{6,7} As the Nernst signal (a voltage developing in response to a temperature gradient in the perpendicular direction) is typically small in ordinary metals, its magnification in such systems has been attributed to the dynamics of thermally excited Gaussian SC fluctuations,^{8–11} or mobile vortices above a Kosterlitz-Thouless¹² transition.¹³ Theoretical studies were also extended to the quantum critical regime of SC fluctuations.¹⁴

Conceptually the above-mentioned theoretical models share a common intuitive idea: In the phase-disordered, vortex liquid state (which is qualitatively equivalent to a regime dominated by dynamical Gaussian fluctuations), vortex flow generated parallel to a thermal gradient ($\nabla_y T$) naturally induces an electric field (E_x) in the perpendicular direction. Consequently, the general expression for the Nernst coefficient

$$\nu \equiv \frac{E_x}{(\nabla_y T)B} = \frac{\rho_{xx}\alpha_{xy} - \rho_{xy}\alpha_{yy}}{B} \quad (1)$$

is overwhelmingly dominated by the first term in the numerator, dictated by the off-diagonal Peltier coefficient α_{xy} : SC fluctuations typically do not contribute to the second term due to particle-hole symmetry. This is in sharp contrast

to ordinary metals, where the two terms almost cancel. A measurement of the Nernst signal is therefore often regarded as a direct probe of α_{xy} , which is an interesting quantity: While being a transport coefficient, it is intimately related to thermodynamic properties. In particular, it was found (both experimentally and theoretically) to be proportional to the diamagnetic response:^{5,9,13} $\alpha_{xy} \sim -M/T$. In the clean limit (i.e., for Galilean invariant systems), it was shown to encode the entropy per carrier.^{15–17}

Note, however, that even in the case where Eq. (1) is dominated by the first term, the overall contribution to ν is not determined by α_{xy} alone, but rather by its product with the electric resistivity ρ_{xx} . The observation of a large Nernst signal therefore necessitates a reasonably resistive normal state. This indicates that a large Nernst signal is a subtle effect: On one hand it requires the presence of superconducting fluctuations, and on the other hand requires the superconducting fluctuations to be *dynamic* to produce a sizable voltage drop. A conjunction of these competing tendencies occurs in the fluctuations-dominated regime. Moreover, the Nernst effect is expected to serve as a sensitive probe of a SIT. It should be pointed out, however, that when the normal state adjacent to the SC transition is an insulator, α_{xy} *cannot* be directly deduced from ν : Unlike most cases studied in the literature thus far, ρ_{xx} cannot be assumed constant. Rather, it possesses a significant dependence of its own on the deviation from the critical point, and on T (in particular, a *divergence* in the $T \rightarrow 0$ limit). As a result, although α_{xy} is bound to vanish for $T \rightarrow 0$ by the third law of thermodynamics, ν may in principle approach a finite value in this limit.

Motivated by the above general observations, in the present paper we develop a theory for the transverse thermoelectric coefficients and their relation to diamagnetism in the quasi-one-dimensional (1D) superconducting device depicted in Fig. 1, in which the geometry dictates an appreciable Nernst effect in the fluctuations-dominated regime. The device considered is a two-leg Josephson ladder subject to a perpendicular magnetic field B , where a small temperature difference between the legs induces voltage along the ladder due to the transport of vortices across the junction.¹⁸ At low T , one expects vortex transport to be dominated by quantum tunneling. This system serves as a minimal setup for observing transverse thermoelectric effects.¹⁹ The relative simplicity of the model describing

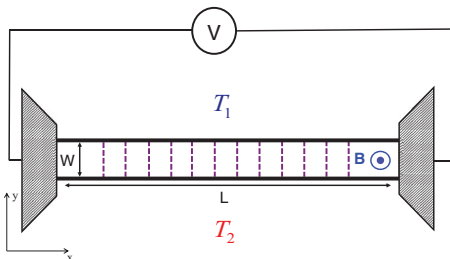


FIG. 1. (Color online) A scheme for the measurement of the Nernst effect in a Josephson ladder subject to a magnetic field B perpendicular to the plane, and a temperature difference between the top (T_1) and bottom (T_2) SC wires. Dashed lines represent Josephson coupling.

the quantum dynamics of SC fluctuations allows an explicit evaluation of α_{xy} , ν , and the magnetization density M in a wide range of parameters. In particular, we investigate their behavior when the wires' parameters are tuned through a SIT, and find a prominent peak in ν close to the transition, which becomes progressively enhanced²⁰ at low T . We further confirm the proportionality relation between α_{xy} and $-M$, however, the prefactor is $1/T_0$, with T_0 the plasma energy scale, rather than $1/T$ as found in the two-dimensional (2D) case.^{9,13}

The rest of the paper is organized as follows. In Sec. II we set up the model. In Sec. III we detail our derivation of the transverse thermoelectric coefficients α_{xy} (Sec. III A) and ν (Sec. III B). In Sec. IV we derive a relation between α_{xy} and the diamagnetic magnetization ($-M$). Finally, in Sec. V we summarize our main results and conclusions.

II. MODEL

We consider the system indicated in Fig. 1, which consists of two SC wires of length L parallel to the x direction separated by a thin insulating layer of width W , which allows a weak Josephson coupling J per unit length. In each of the separate wires ($n = 1, 2$), the 1D quantum dynamics of fluctuations in the phase of the SC condensate is governed by a Hamiltonian of the form (in units where $\hbar = 1$)

$$H_n = \frac{v}{2\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \left[K(\partial_x \theta_n)^2 + \frac{1}{K}(\partial_x \phi_n)^2 \right]; \quad (2)$$

here $\phi_n(x)$ is the collective phase field, and $\theta_n(x)$ its conjugate field (obeying $[\phi_n(x), \partial_x \theta_n(x')] = i\pi \delta(x' - x)$) which denotes Cooper pair number fluctuations. This model can be viewed as describing, e.g., the continuum limit of a Josephson chain,³ where the Josephson coupling (E_J) and charging energy (E_c) between adjacent SC grains are related to the parameters of H_n by $K = \sqrt{E_c/E_J}$ and $v = \sqrt{E_c E_J} \pi a$, with a the grain size. Note that Eq. (2) is a low-energy approximation of the quantum fluctuations. As will be shown later, to get nontrivial transverse thermoelectric effects we will need to keep corrections to H_n involving, e.g., higher derivatives of the fields. Such corrections take into account, for example, fluctuations in the current density within the finite width of the wires ($\sim a$) and the coupling of the collective modes to microscopic degrees of freedom.

The Josephson coupling between the wires is given by

$$H_J = -J \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \cos\{\phi_1 - \phi_2 - qx\}, \quad (3)$$

where q is the deviation of the vortex density in the junction area from a commensurate value

$$q = 2\pi \left(\frac{WB}{\Phi_0} \bmod \frac{1}{a} \right), \quad (4)$$

in which (for $\hbar = c = 1$) $\Phi_0 = \pi/e$ is the flux quantum. Assuming the hierarchy of scales $Ja \ll T \ll v/a$ (with T an average temperature of the system), H_J [Eq. (3)] can be treated perturbatively. Note that the first inequality justifies this approximation for an arbitrary value of the Luttinger parameter K in Eq. (2): For $K < 2$ and sufficiently small q , the Josephson term becomes relevant,²¹ and induces a SC phase where fluctuations in the relative phase between the wires are gapped^{22,23} in the $T \rightarrow 0$ limit. In turn, additional perturbations such as interwire charging energy^{22,23} and disorder²⁴ generate a transition to an insulating $T \rightarrow 0$ phase for sufficiently large K . Since, as shown below, in both extreme phases the Nernst effect is strongly suppressed, we focus our attention on the *intermediate* T regime, where temperature exceeds the energy scale associated with all these perturbations.

In addition to H_J , we introduce a weak backscattering term due to disorder of the form

$$H_D = \sum_{n=1,2} \int dx \zeta_n(x) \cos\{2\theta_n(x)\}, \quad (5)$$

$$\overline{\zeta_n(x)} = 0, \quad \overline{\zeta_n(x)\zeta_{n'}(x')} = D\delta(x - x')\delta_{n,n'},$$

where the overline denotes disorder averaging. This term generates the leading contribution to the resistivity ρ_{xx} , and thus to ν via Eq. (1).

III. TRANSVERSE THERMOELECTRIC COEFFICIENTS

We next consider a temperature difference $\Delta T = T_1 - T_2$ between the top and bottom wires (see Fig. 1), each assumed to be at equilibrium with a separate heat reservoir. As a result of the transverse Peltier effect, a current is induced along the legs of the ladder. Alternatively, if one maintains open boundary conditions, a voltage develops along the wires due to the Nernst effect. Below we derive the corresponding coefficients.

A. Off-diagonal thermoelectric coefficient α_{xy}

We first evaluate the electric current I_x induced along the ladder for $\Delta T \ll T \equiv \frac{T_1 + T_2}{2}$, which yields the transverse Peltier coefficient

$$\alpha_{xy} = \frac{I_x}{\Delta T}. \quad (6)$$

Alternatively, implementing the Onsager relations,⁹ one can deduce it from the coefficient $\tilde{\alpha}_{xy} = T\alpha_{xy}$, which dictates the heat current $I_x^{(h)}$ generated along the device in response to a voltage difference V_y between the two wires. We show

explicitly below that the result of both calculations is indeed the same.

The electric current is given by the expectation value

$$I_x = 2e\pi \langle \dot{\theta}_1 + \dot{\theta}_2 \rangle, \quad (7)$$

where the current operators $\dot{\theta}_n(x,t) = i[H, \theta_n]$ ($H = H_0 + H_J$ where $H_0 \equiv H_1 + H_2$) are evaluated perturbatively in H_J [Eq. (3)] using the interaction representation. The

leading contribution to I_x arises from the second order

$$\begin{aligned} \dot{\theta}_n(x,t) &= U(t)\dot{\theta}_n^{(0)}(x,t)U^\dagger(t), \quad \text{where} \\ \dot{\theta}_n^{(0)}(x,t) &= \frac{v}{K} \partial_x \phi_n(x,t), \end{aligned} \quad (8)$$

$$U(t) = 1 + i \int_{-\infty}^t dt_1 H_J(t_1) - \int_{-\infty}^t dt_1 \int_{-\infty}^{t_1} dt_2 H_J(t_1) H_J(t_2).$$

Employing Eq. (3) and inserting the resulting expressions for $\dot{\theta}_n$ in Eq. (7), we obtain

$$\begin{aligned} I_x &= \frac{\pi e v J^2}{2} \int_{-\infty}^t dt_1 \int_{-\infty}^{t_1} dt_2 \int_{-\frac{L}{2}}^{\frac{L}{2}} dx_1 \int_{-\frac{L}{2}}^{\frac{L}{2}} dx_2 \sin[q(x_1 - x_2)] i \text{Im} m \left[e^{-\frac{K}{2} F_{T_1}(x_1 - x_2; t_1 - t_2)} e^{-\frac{K}{2} F_{T_2}(x_1 - x_2; t_1 - t_2)} \right] \\ &\quad \times \left[\left\{ \partial_x F_{T_1}(x - x_1; t - t_1) + \partial_x F_{T_1}(x_2 - x; t_2 - t) \right\} - \left\{ \partial_x F_{T_2}(x - x_1; t - t_1) + \partial_x F_{T_2}(x_2 - x; t_2 - t) \right\} \right. \\ &\quad \left. - \left\{ \partial_x F_{T_1}(x - x_2; t - t_1) + \partial_x F_{T_1}(x_1 - x; t_2 - t) \right\} + \left\{ \partial_x F_{T_2}(x - x_2; t - t_1) + \partial_x F_{T_2}(x_1 - x; t_2 - t) \right\} \right], \end{aligned} \quad (9)$$

where we use the Boson correlation function $F_T(x;t) \equiv \frac{1}{K} \langle [\phi_n(x,t) - \phi_n(0,0)]^2 \rangle$ at fixed temperature T (Ref. 21)

$$F_T(x;t) = \frac{1}{2} \log \left[\frac{v^2}{\pi^2 a^2 T^2} \left\{ \sinh \left[\pi T \left(\frac{x}{v} - t + i\epsilon \text{sign}(t) \right) \right] \sinh \left[\pi T \left(\frac{x}{v} + t - i\epsilon \text{sign}(t) \right) \right] \right\} \right], \quad (10)$$

in which $\epsilon \sim a/v$ is associated with the short-distance cutoff.

Performing the integral first over the center-of-mass coordinate $x_c = \frac{x_1 + x_2}{2}$ in Eq. (9), and taking the limit $\epsilon \rightarrow 0$ in Eq. (10), it is easy to see that, since $\text{Im}\{F\} = \pi$ is independent of T , the resulting expression actually vanishes. This follows from the Lorentz invariance of the model for phase fluctuations, Eq. (2). As we discuss further in Sec. V, this behavior is in fact quite characteristic: Deviations from a linear energy-momentum dispersion are required to get a finite α_{xy} . In the present case, a nonvanishing result (of order ϵ) would emerge if ϵ in Eq. (10) is kept finite. This signifies that the leading contribution to α_{xy} arises from the physics on scales of the short-distance cutoff, which depends on microscopic details. We therefore need to include such corrections to Eq. (2), namely the terms which violate the Lorentz invariance: As will be elaborated in Sec. V, such terms are indeed necessary to provide the Josephson vortices in this system with entropy.

Tracing back to the underlying microscopic theory of SC devices, Eq. (2) is an effective Hamiltonian for the

collective fields ϕ_n, θ_n arising to leading order in a gradient expansion. A variety of higher-energy corrections, allowed by the symmetries of the problem, are present in any physical system. In particular, as a concrete example, corrections to the Josephson Hamiltonian which hybridize phase and charge fluctuations have been derived in earlier literature^{25,26} for a single junction. When incorporated in the continuum limit of a model for Josephson array, these yield higher order derivatives, e.g., a term of the form²⁶

$$H_n^{\text{corr}} = \frac{Cva}{2\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \left[(i \partial_x \phi_n) (\partial_x^2 \theta_n) + \text{H.c.} \right], \quad (11)$$

where C is a dimensionless constant. This adds a correction to the current operator in the wire n [Eq. (8)] of the form

$$\delta \dot{\theta}_n^{(0)} = iCva (\partial_x^2 \theta_n) \approx \frac{iCa}{K} \partial_t \partial_x \phi_n \quad (12)$$

where in the last approximation we have used the leading term in the equation of motion for ϕ_n . Inserting these corrections to $\dot{\theta}_1, \dot{\theta}_2$ into Eqs. (7) and (8) we obtain

$$\begin{aligned} I_x &\approx -\frac{\pi eCaJ^2}{2} \int_0^\infty dt \int_{-\frac{L}{2}}^{\frac{L}{2}} dx_c \int_{-L}^L dx \sin[qx] \text{Im} m \left[e^{-\frac{K}{2} F_{T_1}(x;t)} e^{-\frac{K}{2} F_{T_2}(x;t)} \right] \\ &\quad \times \left[\left\{ \partial_x F_{T_1} \left(-x_c - \frac{x}{2}; 0 \right) + \partial_x F_{T_1} \left(x_c - \frac{x}{2}; -t \right) \right\} - \left\{ \partial_x F_{T_2} \left(-x_c - \frac{x}{2}; 0 \right) + \partial_x F_{T_2} \left(x_c - \frac{x}{2}; -t \right) \right\} \right. \\ &\quad \left. - \left\{ \partial_x F_{T_1} \left(-x_c + \frac{x}{2}; 0 \right) + \partial_x F_{T_1} \left(x_c + \frac{x}{2}; -t \right) \right\} + \left\{ \partial_x F_{T_2} \left(-x_c + \frac{x}{2}; 0 \right) + \partial_x F_{T_2} \left(x_c + \frac{x}{2}; -t \right) \right\} \right], \\ &\approx -(\Delta T) 2\pi^2 eCaJ^2 \int_0^\infty dt \int_{-\infty}^\infty dx x \sin[qx] \text{Im} \{ \chi(x,t) \}, \\ \chi(x,t) &= \frac{(\pi a T / v)^K}{\left(\sinh \left[\pi T \left(\frac{x}{v} - t + i\epsilon \right) \right] \sinh \left[\pi T \left(\frac{x}{v} + t - i\epsilon \right) \right] \right)^{K/2}}, \end{aligned} \quad (13)$$

where in the last step we have assumed further $\Delta T \ll T$ and $L \rightarrow \infty$, and inserted the explicit expression for F_T Eq. (10). Employing the definition Eq. (6), we evaluate the remaining integrals and finally get

$$\alpha_{xy} = -\frac{e(\pi J)^2 a^3 \mathcal{C}}{4v^2} \sin\left(\frac{\pi K}{2}\right) \left(\frac{2\pi aT}{v}\right)^{K-2} \times \partial_q \left\{ \left| B\left(i\frac{vq}{4\pi T} + \frac{K}{4}, 1 - \frac{K}{2}\right) \right|^2 \right\}, \quad (14)$$

where $B(z, w)$ is the Beta function.²⁸ Note that the resulting $\alpha_{xy}(T)$ exhibits a power-law T dependence, which indicates an apparent divergence in the $T \rightarrow 0$ for sufficiently small K . Since α_{xy} is known to be proportional to the entropy of carriers, such behavior would violate the third law of thermodynamics. We emphasize, however, that this is an artifact of the approximation leading to Eq. (14), which assumes a finite temperature and in particular $J \ll T$. While the true $T \rightarrow 0$ is beyond the scope of the present theory, we speculate that due to an opening of a gap (either superconducting or insulating, depending on the value of K), the coefficient α_{xy} is suppressed as expected.

We now consider the alternative setup where an electric voltage V_y is imposed between the top and bottom wires (at uniform T), and evaluate the heat current $I_x^{(h)}$ induced along the ladder. For $J = 0$, but accounting for the corrections H_n^{corr} [Eq. (11)], the local heat current operator is given by²⁷

$$J_h^{(0)} = v^2 \sum_{n=1,2} \partial_x \theta_n \left(\partial_x \phi_n + i\mathcal{C} \frac{a}{v} \partial_t \partial_x \phi_n \right). \quad (15)$$

The voltage bias corresponds to a difference in chemical potentials in the two legs, $\mu_{1,2} = \pm eV_y$, which introduce constant shifts of $\partial_x \theta_{1,2}$ by $\pm \pi eV_y/vK$. The heat current $I_x^{(h)} = \langle U(t) J_h^{(0)} U^\dagger(t) \rangle$ [with $U(t)$ expanded to second order in H_J as in Eq. (8)] is hence given by

$$I_x^{(h)} = e\pi V_y \langle U(t) (\dot{\theta}_1^{(0)} - \dot{\theta}_2^{(0)}) U^\dagger(t) \rangle. \quad (16)$$

The resulting expression coincides with $(V_y T/\Delta T) I_x$. We thus confirm that $\tilde{\alpha}_{xy} = I_x^{(h)}/V_y = T\alpha_{xy}$, as required by Onsager's relation.

B. Nernst coefficient ν

To derive the Nernst coefficient, we next employ Eq. (1) noting that within our level of approximations α_{yy} and ρ_{xy} vanish due to particle-hole symmetry. The Nernst signal in the setup depicted in Fig. 1, defined as $\nu = |V/\Delta T B|$, is hence determined by the product of α_{xy} [Eq. (14)] and the longitudinal resistance of the ladder R_{xx} . To leading order in H_D [Eq. (5)]^{3,21,24}

$$R_{xx} = \frac{\pi^3 L D a^2}{2e^2 v^2} \cos\left(\frac{\pi}{K}\right) B\left(\frac{1}{K}, 1 - \frac{2}{K}\right) \left(\frac{2\pi aT}{v}\right)^{\frac{2}{K}-2}. \quad (17)$$

At low magnetic field such that $q = 2\pi W B/\Phi_0 \ll T/v$, this yields an expression for $\nu \approx \alpha_{xy} R_{xx}/B$ of the form

$$\nu \approx \nu_0 \mathcal{F}(K) \left(\frac{2\pi aT}{v}\right)^{K+\frac{2}{K}-6}, \quad (18)$$

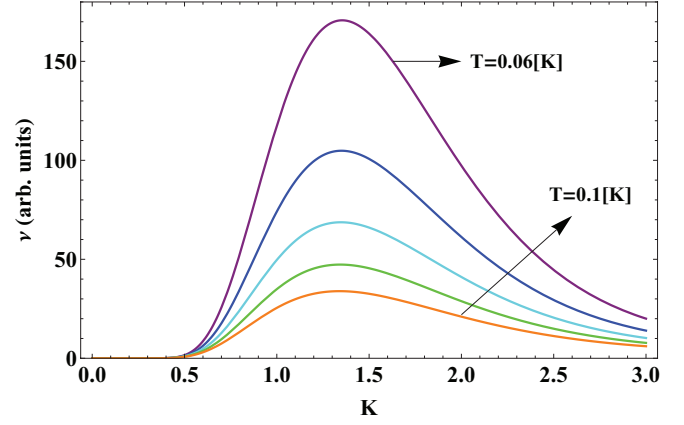


FIG. 2. (Color online) Isotherms of ν as a function of K for $v/2\pi a = 1$ K, $T = 0.06, 0.07, 0.08, 0.09, 0.1$ K.

where the constant prefactor $\nu_0 \propto L D J^2$ and

$$\mathcal{F}(K) \equiv \frac{\Gamma^2\left(\frac{K}{4}\right) \Gamma\left(1 - \frac{K}{2}\right) \Gamma\left(\frac{1}{K}\right) \left\{ \psi'\left(\frac{K}{4}\right) - \psi'\left(1 - \frac{K}{4}\right) \right\}}{2^{2/K} \Gamma^2\left(1 - \frac{K}{4}\right) \Gamma\left(\frac{K}{2}\right) \Gamma\left(\frac{1}{K} + \frac{1}{2}\right)} \quad (19)$$

[$\Gamma(z)$, $\psi'(z)$ are the Gamma and Trigamma functions, respectively²⁸].

The resulting dependence of ν on K , the parameter which tunes the SIT in the SC wires, is depicted in Fig. 2 for different values of $T \ll v/a$, and for low magnetic fields where $vq \ll T$. In this regime, ν exhibits a pronounced maximum at $K^*(T)$, slightly below the transition from SC to insulator^{3,29} ($K_c = 2$). As T is lowered, the peak becomes progressively enhanced and $K^* \sim \sqrt{2}$ as dictated by the rightmost exponential factor in Eq. (18). This nonmonotonous behavior can be traced back to the competition between electric resistance (which signifies the rate of phase slips), and α_{xy} (which signifies the strength of the diamagnetic response).

IV. MAGNETIZATION M

We next investigate the relation of α_{xy} to the diamagnetic magnetization density $M = -(1/LW)(\partial F/\partial B)$, assuming that the B dependence of the free energy F is primarily restricted to the flux in the junction area, i.e., the parameter q in Eq. (3). To leading order in H_J and at low magnetic field $B = (\Phi_0/2\pi W)q$,

$$M \cong \frac{2\pi}{\Phi_0} \frac{T}{2L} \partial_q \langle S_J^2 \rangle_0, \quad (20)$$

$$S_J = -J \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \int_0^{\frac{1}{T}} d\tau \cos\{\phi_1(x, \tau) - \phi_2(x, \tau) - qx\}.$$

The resulting expression for M is identical to Eq. (14) up to a constant prefactor. We thus obtain the relation

$$\alpha_{xy} = -\frac{M}{T_0}, \quad T_0 \equiv \frac{v}{\pi a \mathcal{C}} \sim \sqrt{E_c E_J}, \quad (21)$$

where the last proportionality relation associates the energy scale T_0 with the plasma energy in the Josephson chain forming the legs of the ladder. This resembles the linkage pointed out in earlier literature, except the thermal energy scale T in the

prefactor (as obtained for full-fledged 2D systems) is replaced here by the characteristic scale of quantum dynamical phase slips.

V. DISCUSSION

To summarize, we studied the transverse thermoelectric coefficients due to quantum SC fluctuations in a Josephson two-leg ladder, and their relation to diamagnetism. Most importantly, we predict a large Nernst signal, particularly at moderately low temperatures ($Ja \ll T \ll T_0$) where a pronounced peak is predicted close to the SIT. This behavior reflects a subtle interplay between diamagnetism (favored in the SC phase), and dynamical phase fluctuations (which proliferate in the insulator).

A crucial step in the derivation of the leading contribution to the transverse Peltier coefficient α_{xy} relies on a correction to the Hamiltonian which violates Lorentz invariance of the model for quantum phase fluctuations in the SC wires. Such terms are indeed necessary to couple the charge current to the thermal current: The correlation between them is at the heart of the Peltier effect. This point can be understood also from a different angle, implementing the language of vortex physics: It is in fact possible to view α_{xy} as a dual of the ordinary (longitudinal) thermopower $Q = \Delta V / \Delta T$, which is known to encode the specific heat (or entropy) of charge carriers. In a dual representation, the electric current in the transverse direction plays the role of a “voltage” applying a *longitudinal* (Magnus) force on the vortices, which balances the force imposed by the temperature gradient. The latter force is proportional to the entropy of the vortices. Hence, α_{xy} vanishes in the case where the vortices carry no entropy (see also Ref. 19).

We finally note that the remarkably simple relation to the entropy per carrier $\alpha_{xy} \sim -(s/B)$, derived for clean (Galilean invariant) systems,^{15,16} does not hold here. Indeed, this relation can be recovered, e.g., by employing the Boltzmann equation for an ordinary conductor in the limit $\omega_c \tau \rightarrow \infty$ (with ω_c the cyclotron frequency and τ a mean free time³⁰). In contrast, for $\omega_c \tau \ll 1$ the same calculation yields $\alpha_{xy} \sim B\tau^2$. In our case, the later limit is appropriate: While translational invariance holds in the x direction, charge conductance along the y direction is governed by weak tunneling between two discrete points $\sim J$. We hence expect $\alpha_{xy} \sim BJ^2$, in accord with Eq. (14).

As a concluding remark, we note that a qualitatively similar behavior of the Nernst effect is expected to hold in 2D SC films, or an infinite stack of such ladders (which is essentially equivalent to an anisotropic ultrathin SC film). Possibly, it can also explain some properties of the existing data: See, for example, Fig. 3 in the paper by Pourret *et al.*,⁷ where the Nernst signal measured in NbSi films exhibits an increase (and sharpening) of the peak for $T < T_c$. It is therefore suggestive that more elaborate Josephson arrays models can serve as a useful arena for studying transverse thermoelectric effects in disorder SC films.

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