Creating double negative index materials using the Babinet principle with one metasurface

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Metamaterials are patterned metallic structures which permit access to a novel electromagnetic response, negative index of refraction, impossible to achieve with naturally occurring materials. Using the Babinet principle, the complementary split ring resonator (SRR) is etched in a metallic plate to provide negative ϵ, with perpendicular direction. Here we propose a new design, etched in a metallic plate to provide negative magnetic permeability μ, with perpendicular direction. The combined electromagnetic response of this planar metamaterial, where the negative μ comes from the aperture and the negative ϵ from the remainder of the continuous metallic plate, allows achievement of a double negative index metamaterial (NIM) with only one metasurface and strong transmission. These designs can be used to fabricate NIMs at microwave and optical wavelengths and three-dimensional metamaterials.

I. INTRODUCTION

Metamaterials are artificial materials that can be engineered to exhibit fascinating electromagnetic properties that do not occur in nature, such as negative refractive index, perfect imaging, and electromagnetic cloaking,[1–9] Chiral metamaterials have shown giant optical activity,[10] circular dichroism,[11] negative refraction,[12–15] and possible reversal of the Casimir force.[16] Most of the applications critically required a negative index of refraction in three dimensions (3D). It is an open question whether there is any 3D metamaterial design working for optical frequencies still feasible to fabricate. Here, we theoretically proposed a new design that provides negative index of refraction with one surface and can be used to fabricate 3D metamaterials. This is an important step towards the realization of bulk 3D double negative index metamaterials (NIMs) at optical wavelengths. Previous theoretical designs[6] obtain NIM in 3D at GHz and THz. Two groups independently proposed fully isotropic bulk magnetic metamaterial designs, based on SRRs arranged in a cubic lattice by employing spatial symmetries.[17,18] On the other hand, some designs of 3D isotropic NIMs exist, but fabricating them has remained a challenging task and is virtually impossible at optical frequencies. For example, Koschny *et al.*,[19] designed an early example of an isotropic NIM. However, high constant dielectric assumed across the gaps of the SRRs rendered the experimental realization impractical. Alternative approaches have also been investigated, including direct lasing fabrication,[6,20,21] and high refractive index spheres.[6,22]

Double negative index materials exhibit simultaneously negative magnetic permeability μ and electric permittivity ϵ over a common frequency range.[23] Negative permeabilities are the result of a strong resonant response to an external magnetic field; negative permittivity can appear by either a plasmonic or a resonant response (or both) to an external electric field. Pendry *et al.*[24] suggested a double metallic SRR design for negative μ and a parallel metallic wire periodic structure to give negative ϵ. Several variations of the initial design have been studied.[1,2,6,25] For example, a single ring resonator with several cuts has been proven capable of reaching negative μ at a

higher frequency,[25] and cut wire pairs and fishnet structures[1,2,6] allow for a negative magnetic response at optical frequencies. In these old designs, one needs two particles: one particle gives negative μ and the other gives negative ϵ at the same frequency range. Later, Rockstuhl and Lederer proposed a new design,[26] nanoapertures embedded in a continuous metal film, that gives $\epsilon < 0$ from the metallic surface and a resonant μ, which, however, did not reach $\mu < 0$ because of the weak resonance. Since the imaginary part of μ is large, $\mu > 0$ and $\epsilon < 0$ will still give negative index of refraction.[26,27] This design suffers from large losses, partially due to the weak magnetic dipole of the resonance but also—like other resonant structures at optical wavelengths—due to the generally high Ohmic losses in metals at high frequencies.[4] It is very interesting to develop a new design that will use only one "particle" on one surface that will provide negative index of refraction and also $\mu < 0$ and $\epsilon < 0$, with parallel direction on the surface. We will use Babinet's principle[28–31] to obtain a complementary structure in a metallic surface that will give $\mu < 0$ for the first time, instead of $\epsilon < 0$ resulting from Babinet-complementary resonators studied in the previous literature, with perpendicular direction on the surface. In addition, the remaining continuous metallic surface will give $\epsilon < 0$.

Babinet's principle has been used[28–34] to design artificial planar metamaterials, which can provide only negative electric permittivity. Inverse elements such as complementary SRRs (CSRRs) have been proposed as an alternative to conventional metallic wires for the design of planar metamaterials, which give $\epsilon < 0$. Complementary metamaterials show[31,34] properties similar to their inverse structures. The transmission coefficient t^c for the CSRR is related to the transmission coefficient t for the SRR by $t + t^c = 1$. In addition, according to Babinet's principle, their transmission and reflection behaviors, as well as their scattered electric and magnetic fields, are interchanged.[28–34] These complementary designs provide an effective negative permittivity,[28–31] with perpendicular direction. In this manuscript, we will use Babinet's principle to design complementary structures that will provide an effective negative permeability and the metallic surface to give negative permittivity. Therefore, we will obtain negative n with only

one planar metamaterial, with parallel direction of the surface of planar metamaterial.

II. BABINET's PRINCIPLE

Babinet's principle relates the fields scattered by two complementary planar structures of arbitrary shape made of infinitely thin, perfectly conducting sheets. If we consider an incident electromagnetic (EM) field, E_0, B_0, then its complementary EM field, $E_0{}^c$, $B_0{}^c$, is defined[28,35] as $E_0{}^c = -cB_0{}^c$ and $B_0{}^c = E_0{}^c/c$, which corresponds to a $90°$ rotation around the propagation axis. If we have a thin metallic surface, the reflected (E_r, H_r) and the transmitted (E_t, H_t) electric and magnetic fields satisfy the following conditions. If we have charge on the thin metallic surface, the perpendicular reflected electric field is opposite to the transmitted electric field, i.e., $\vec{n} \cdot \vec{E}_r = -\vec{n} \cdot \vec{E}_t$, where $\vec{n}$ is the axis vector perpendicular to the metallic surface. The parallel electric field is continuous to both sides of the thin metallic surface, i.e., $\vec{n} \times \vec{E}_r = \vec{n} \times \vec{E}_t$. If the incident EM wave generates current in the thin metallic surface, the perpendicular magnetic field is continuous, i.e., $\vec{n} \cdot \vec{H}_r = \vec{n} \cdot \vec{H}_t$, and the parallel magnetic field is opposite to the two sides of the metallic surface, i.e., $\vec{n} \times \vec{H}_r = -\vec{n} \times \vec{H}_t$. If we have a perfectly conducting surface, no magnetic dipole occurs in the plane due to this condition $\vec{n} \times \vec{H}_r = -\vec{n} \times \vec{H}_t$. In addition, no electric dipole occurs perpendicular to the plane, due to the condition $\vec{n} \cdot \vec{E}_r = -\vec{n} \cdot \vec{E}_t$. However, in-plane electric dipoles and magnetic dipoles perpendicular to the plane are possible. Due to the Babinet principle, we have two related complementary plane structures. The surface charge in one structure is related to the magnetic flux in the complement and the opposite occurs. The magnetic flux is related to the surface charge. Also, the parallel electric field in the plane can be generated, with surface electric current perpendicular to the electric field (see examples below). In particular, this means a resonance in one structure is related to a corresponding resonance in the dual fields in the complementary structure. In Fig. 1(a) we have a thin metallic rectangular patch. The propagation direction of the EM field is perpendicular to the metallic patch, and the electric field $\vec{E}_0$ is parallel to the long side of the rectangular patch. We have created an electric dipole in the y direction of Fig. 1(a). The electric current, $\vec{J}$, is along the direction of $\vec{E}_0$ and the magnetic flux is shown in Fig. 1(a). In Fig. 1(b) we presente the Babinet-complementary structure, where we have a thin metallic surface and a rectangular hole. As discussed above, the surface charge shown in Fig. 1(a) can be transformed to the magnetic flux. The positive (negative) charge, shown in Fig. 1(a), can be related to the magnetic flux going inside (outside) the metallic surface [see Fig. 1(b)] . The magnetic flux of Fig. 1(a) can be related with the charge of Fig. 1(b). Another example, shown in Fig. 2(a), is the metallic SRR. We present the current, the charge, and the magnetic flux. In Fig. 2(a), we have a strong magnetic dipole along the propagation direction of the EM wave. There is also a minor electric dipole in the SRR gap. This dipole moment is responsible for the bianisotropy of the SRR. In Fig. 2(b), we present the complementary structure, and used the idea of exchanging surface charge and magnetic flux. We do not have a magnetic dipole in the complementary SRR (CSRR),
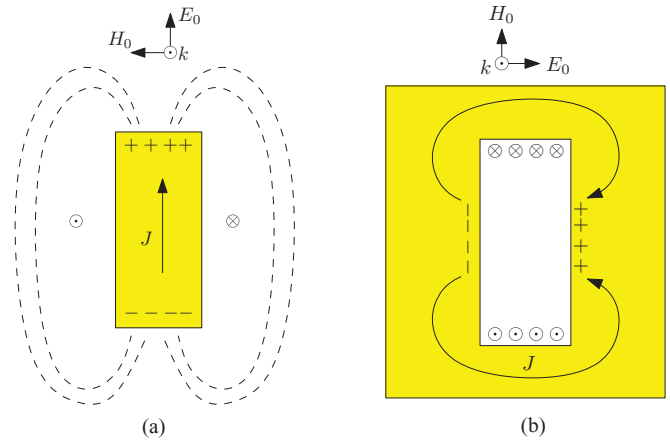

FIG. 1. (Color online) (a) Rectangular thin metallic element with the distribution of surface charge and magnetic flux. This design has a strong electric dipole in the y direction. (b) The complementary structure of (a), with the distribution of the surface charge and the magnetic flux. This complementary design has a strong electric dipole in the x direction.

but there is an electric dipole, due to the neck of the CSRR, parallel to the neck.

In Fig. 3(a), we have a new metallic design that gives an electric dipole and the complementary design gives a residual magnetic dipole shown in Fig. 3(b), for perpendicular direction on the metallic surface.

III. NEW DESIGNS FOR THE BABINET PRINCIPLE

In the previous section we reviewed the Babinet principle for different designs (Figs. 1–3) and the complementary designs. These designs (Figs. 1–3) can help us understand how the electric dipole on the metallic film is related to he magnetic dipole of the "complementary" design. We can use these ideas of Babinet's principle to use a metallic surface to give $\epsilon < 0$ and $\mu < 0$ and will have negative index of refraction, $n < 0$.
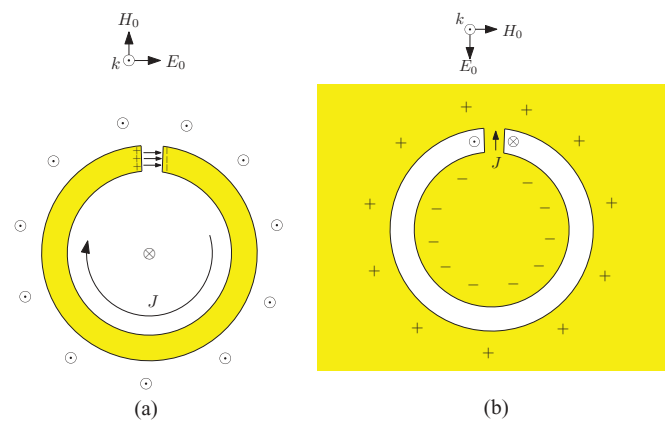

FIG. 2. (Color online) Thin metallic SRR structure (a) and the complementary SRR structure (b) with the distribution of the surface charges and the magnetic flux. The SRR structure has a strong magnetic dipole perpendicular to the metallic surface and a minor electric dipole from the SRR gap. The CSRR structure (b) has a minor electric dipole from the neck of the CSRR.
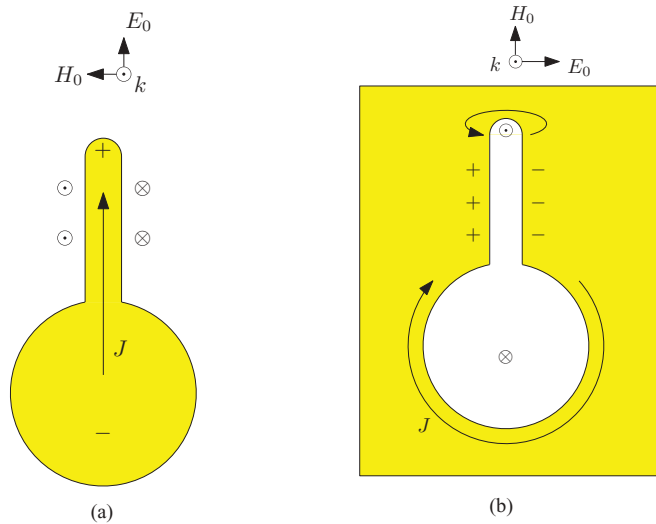
FIG. 3. (Color online) Thin metallic design (a) and the complementary design (b) with the distribution of the surface charge and the magnetic flux. The design (a) has an electric dipole and the complementary design (b) has a residual magnetic dipole.

In Fig. 4(a), we present a new metallic design that has no electric dipole but a resonant electric quadrupole moment. Considering at first normal incidence, the complementary structure show in Fig. 4(b) will possess a resonant mode with a residual magnetic dipole moment perpendicular to the plane as consequence of Babinet's principle. We have thus established the correspondence of a resonant electric quadrupole mode in the "octopus" structure to a resonant magnetic dipole mode in the complementary structure.

Strictly, Babinet's principle relates the field distributions between the direct structure and its complement when the wave propagates perpendicular to the screen. However, here we only use Babinet's principle to establish the mapping of the resonant modes from the direct structure to its complement: If the direct structure has some resonant eigenmode, e.g., the electric cut-wire resonance of the finite-length wire, then we can establish using Babinet's principle that also the complementary structure has a corresponding resonant eigenmode at this frequency, but with all fields replaced by their dual fields. The resonant modes are a property of the geometry; it does not matter how the particular mode is excited. So, the Babinet principle allows us to map fields between the corresponding resonances of mutually complementary structures. Therefore, after indentifying the resonances of the Babinet complement, we can elect to excite these resonances in a different geometry of incidence.

Now leaving the original Babinet setup behind, we realize that the correspondence of the two resonant modes in the direct and complementary structure, i.e., the derived "translation rule" of local charges, currents, and flux distributions on the surface, has to hold for any external excitation. In order to couple to the magnetic dipole of the complementary structure [Fig. 4(b)] we thus change the propagation direction to be parallel with the surface such that the magnetic field of the incident wave can couple to the ring current (i.e., the magnetic dipole) and we can obtain a negative μ. Notice the charges are shown in the "legs" of this design. The current goes around
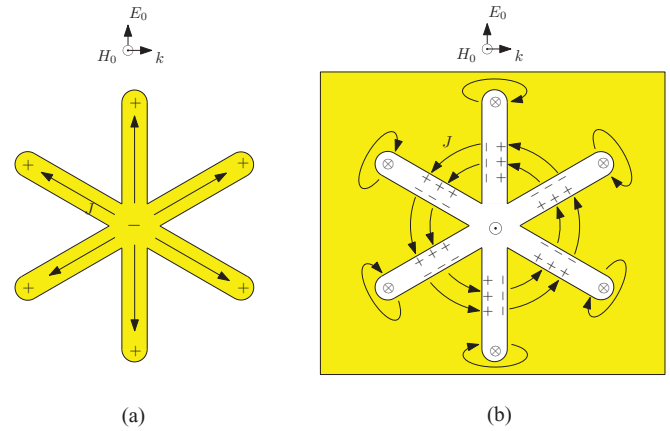

FIG. 4. (Color online) Thin metallic "octopus" design (a) and the complementary design (b) with the distribution of the surface charge and the magnetic flux. The design (a) does not have an electric dipole, but an electric quadrupole, and the complementary design (b) has a magnetic dipole perpendicular to the plane. Here the propagation direction is parallel to the surface.

and provides a magnetic dipole, which can give a negative permeability. In addition the metallic surface [Fig. 4(b)] will give negative permittivity, ϵ. This complementary design can help us to understand how to obtain negative μ and negative ϵ. The design shown in Fig. 4 overlaps with $\epsilon < 0$ and $\mu < 0$ and the impedance $z = \sqrt{\mu/\epsilon}$ is very large, and we have a lot of reflection and low transmission. Deforming the structure while maintaining the same principle topology of Fig. 4, we did a lot of simulations for different designs to have very low impedance ($z = 1$ for air) and to have very high transmission. We find a new design (see Fig. 5) which satisfies the criteria $\epsilon < 0$ and $\mu < 0$, and it has $|\epsilon|$ and $|\mu|$ of the same magnitude.

We will use an alternative design to obtain negative μ and the metallic surface will give negative ϵ at the same frequency region, and the magnitude of ϵ and μ will be comparable, and the impedance will be close to 1. The design operates at 10 GHz to keep sample fabrication and experimental verification simple but can be scaled to THz frequencies. In Fig. 5(a), we present a complementary design, which has a
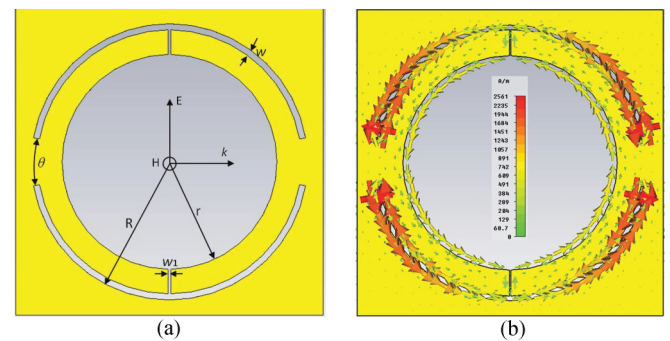

FIG. 5. (Color online) (a) The complementary design in a thin metallic surface; the dimensions are $R = 4.5$ mm, $r = 3.5$ mm, $w = 0.15$ mm, $w_1 = 0.05$ mm, and $\theta = 20°$. (b) The current density of the complementary design at 9.9 GHz gives a magnetic dipole perpendicular to the metallic surface. The propagation direction is parallel to the metallic surface.
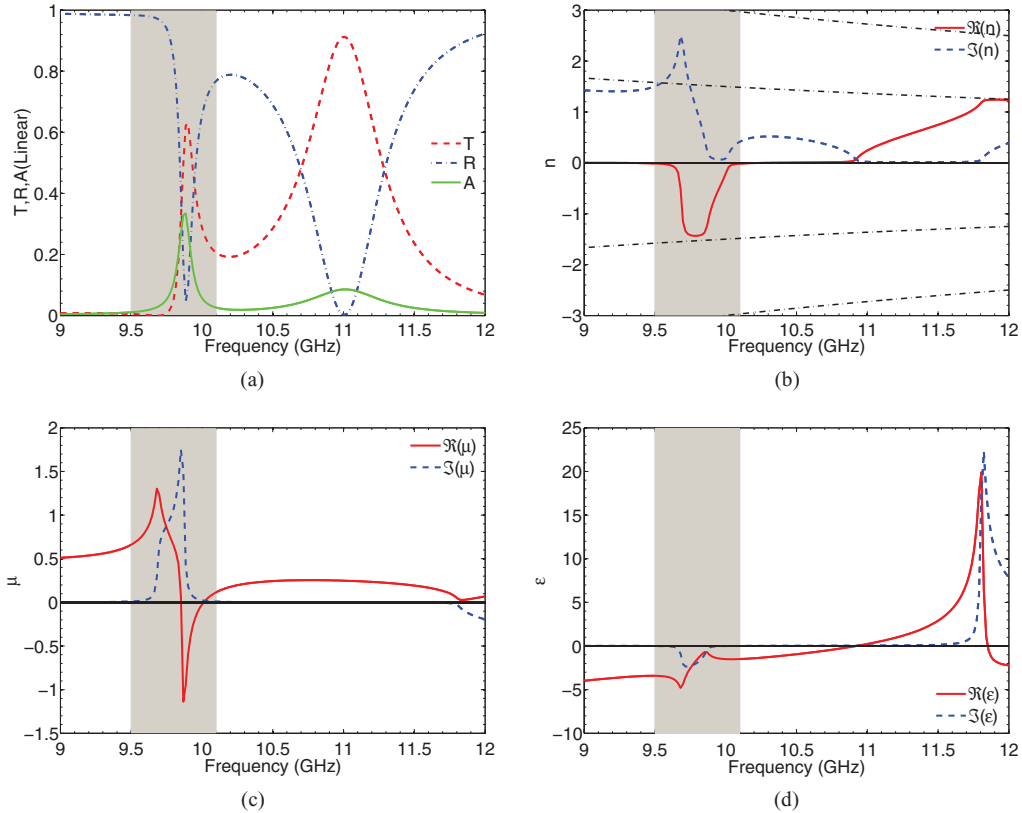
FIG. 6. (Color online) (a) T, R, and A vs frequencies for the complementary design. (b) Retrieval results for the index of refraction n; (c) the magnetic permeability μ; and (d) the electric permittivity ϵ. Negative ϵ is obtained by the thin metallic surface and negative μ is obtained by the complementary design. With the single thin metallic surface design, one obtains negative n.

big hole in the metallic surface that will give a negative μ. This structure is on a 35 μm thick copper sheet with lattice constant $a_x = a_y = 10$ mm and $a_z = 6$ mm. In Fig. 5(b), we plot the current density, which gives a circular current and provides magnetic dipole momentum. We have used the design of Fig. 5(a) to obtain transmission, reflection, and absorption $A = 1 - T - R$ with the CST. We simulated a periodic array of the proposed structures. With propagation direction parallel to the metallic surface, we have used the amplitude and the phase of transmission and reflection to obtain the frequency dependence of ϵ, μ, and n through the retrieval procedure.[36]

In Fig. 6, we plot the transmission T, reflection R, and absorption A versus the frequency. At the frequency 9.9 GHz, there is dip in the reflection. In the retrieval results, we obtain negative n with only one metallic surface with the appropriate hole design. The hole design gives negative μ and the metallic surface gives negative ϵ, the impedance is close to 1, and the transmission is close to 65%. The figure of merit (FOM = $|Re(n)/Im(n)|$) is 9 with $n = -1.0$ with frequency 9.9 GHz. The FOM $\simeq 8$ with $n = -0.5$ with frequency 9.95 GHz. This is a unique design that gives negative ϵ and μ, with only one metallic surface.

IV. CONCLUSION

In conclusion, we have shown it is possible to construct Babinet-complement structures (holes in planar metamaterials) that provide a resonant magnetic moment perpendicular to the screen. We use such a magnetic resonance providing $\mu < 0$ together with the $\epsilon < 0$ coming from the electric background provided by the continuous regions of the screen, to obtain a double negative index metamaterial for propagation direction parallel to the screen. In this material, both $\epsilon < 0$ and $\mu < 0$ are provided by the same single component. These designs can be used to fabricate three-dimensional metamaterials and obtain negative index of refraction at microwave and infrared wavelengths.

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[1]V. M. Shalaev, Nat. Photon. **1**, 41 (2007).

[2]C. M. Soukoulis, S. Linden, and M. Wegener, Science **315**, 47 (2007).

[3]N. I. Zheludev, Science **328**, 582 (2010).

[4]C. M. Soukoulis and M. Wegener, Science **330**, 1633 (2010).

[5]A. Boltasseva and H. A. Atwater, Science **331**, 290 (2011).

[6]C. M. Soukoulis and M. Wegener, Nat. Photon. **5**, 523 (2011).

[7]Y. Liu and X. Zhang, Chem. Soc. Rev. **40**, 2494 (2011).

[8]P. Tassin, T. Koschny, M. Kafesaki, and C. M. Soukoulis, Nat. Photon. **6**, 259 (2012).

[9]D. Schurig et al., Science **314**, 977 (2006).

[10]A. V. Rogacheva, V. A. Fedotov, A. S. Schwanecke, and N. I. Zheludev, Phys. Rev. Lett. **97**, 177401 (2006).

[11]J. K. Gansel et al., Science **325**, 1513 (2009).

[12]J. B. Pendry, Science. **306**, 1353 (2004).

[13]E. Plum et al., Phys. Rev. B **79**, 035407 (2009).

[14]J. Zhou et al., Phys. Rev. B **79**, 121104 (2009).

[15]S. Zhang et al., Phys. Rev. Lett. **102**, 023901 (2009).

[16]R. Zhao, J. Zhou, T. Koschny, E. N. Economou, and C. M. Soukoulis, Phys. Rev. Lett. **103**, 103602 (2009).

[17]W. J. Padilla, Opt. Express **15**, 1639 (2007).

[18]J. D. Baena, L. Jelinek, and R. Marqués, Phys. Rev. B **76**, 245115 (2007).

[19]T. Koschny, L. Zhang, and C. M. Soukoulis, Phys. Rev. B **71**, 121103 (2005).

[20]M. S. Rill et al., Nat. Mater. **7**, 543 (2008).

[21]D. O. Güney, T. Koschny, and C. M. Soukoulis, Opt. Express **18**, 12348 (2010).

[22]Q. Zhao, J. Zhou, F. Zhang, and D. Lippens, Mater. Today **12**, 60 (2009).

[23]V. Veselago, Sov. Phys. Usp. **10**, 509 (1968).

[24]J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, IEEE Trans. Microwave Theory Technol. **47**, 2075 (1999).

[25]J. Zhou et al., Phys. Rev. Lett. **95**, 223902 (2005).

[26]C. Rockstuhl and F. Lederer, Phys. Rev. B **76**, 125426 (2007).

[27]J. Zhou, L. Zhang, G. Tuttle, T. Koschny, and C. M. Soukoulis, Phys. Rev. B **73**, 041101 (2006).

[28]F. Falcone et al., Phys. Rev. Lett. **93**, 197401 (2004).

[29]T. Zentgraf et al., Phys. Rev. B **76**, 033407 (2007).

[30]C. Rockstuhl, T. Zentgraf, T. P. Meyrath, H. Giessen, and F. Lederer, Opt. Express **16**, 2080 (2008).

[31]H.-T. Chen et al., Opt. Express **15**, 1084 (2007).

[32]I. A. I. Al-Naid, C. Jansen, and M. Koch, Electron. Lett. **44**, 1228 (2008).

[33]R. Singh, E. Plum, W. Zhang, and N. I. Zheludev, Opt. Express **18**, 13425 (2010).

[34]A. Bitzer, A. Ortner, H. Merbold, T. Feurer, and M. Walther, Opt. Express **19**, 2537 (2011).

[35]J. D. Jackson, Classical Electrodynamics, 3rd ed. (Wiley, New York, 1999).

[36]D. R. Smith, S. Schultz, P. Markos, and C. M. Soukoulis, Phys. Rev. B **65**, 195104 (2002).