Error accounting algorithm for electron counting experiments

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Electron counting experiments attempt to provide a current of a known number of electrons per unit time. We propose architectures utilizing a few readily available electron pumps or turnstiles with modest error rates of 1 part per 10^4 with common sensitive electrometers to achieve the desirable accuracy of 1 part in 10^8. This is achieved not by counting all transferred electrons, but by counting only the errors of individual devices; these are less frequent and therefore readily recognized and accounted for. Our proposal thereby eases the route towards quantum-based standards for current and capacitance.

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I. INTRODUCTION

The long pursued goal of closing the quantum metrological triangle[1–3] involves the comparison of a current given by Josephson and quantum Hall effects, with one based on counting electrons flowing through a constraint. An equivalent alternative is to charge a capacitor with a counted number of electrons.[4]

For metrologically precise measurements of current about 1 nA is required.[5] This can be provided either by pumping at frequencies higher than those that have shown satisfactory accuracy or by parallelization. Higher operation frequencies increase the error rates, while parallelization is hindered by conflicting requirements on the devices, including yield and accuracy. We propose here a concept to alleviate both concerns; this is made possible by the relative simplicity of nonadiabatic single gate devices providing a quantized current, which have been realized in several technologies[6–9] and even parallelized.[10,11]

On the other hand, error rates akin to those of the shuttling experiment conducted for adiabatic pumps[4,5] have not been demonstrated, while direct current measurements indicate that error rates near $\Gamma = 1$ ppm can be achieved.[12]

To improve upon these results, we propose a circuit that utilizes the high precision and accuracy provided by electron pumps, but permits the correction of the remaining errors: We connect several pumps in a series and observe the charges on memory nodes between them. An example of such a circuit is shown in Fig. 1.

Each of the aforementioned pumps will provide a current very close to $e\Omega$, when its gate is driven with periodicity $1/\Omega$, $-e$ being the electron charge, and will barely pass electrons between the terminals in a suitable off state.

When all turnstiles move electrons with the same rate, then only each node-charge averaged over a pump period is constant. Whenever an error, a surplus (or deficit) electron being shifted by pump j, occurs then the charge on node $j-1$ will drop by one electron, while the charge on node j will rise by it, labeling the outside reservoirs as nodes 0 and N. Accordingly, th errors committed by different pumps result in different charge signals on the nodes. Such different signatures can be recognized and attributed. If this were to be done flawlessly, it would be possible to tell with certainty by how many electrons the charge transported by each pump deviates from the intended value.

But when the characteristic time to identify the charge state τ_M and the time between errors $1/\Gamma\Omega$ are comparable then, if all observed errors are attributed to the pump (or pumps) whose error would most likely explain the observed signature, some multipump errors will mistakenly be misattributed. Events, where the first two pumps of a three pump circuit both pump one electron more than intended, cannot be distinguished from the—more likely—case of one too few electrons pumped by the third pump. Only a four stage circuit would be able to distinguish these two scenarios. Circuits with an odd number of pumps N exhibit $2N!/[(N-1)/2]![(N+1)/2]!$ different scenarios of errors of $(N+1)/2$ pumps that cannot be distinguished from the errors of all the other pumps. If these happen within a time interval τ_M then they will be misattributed. The relative rate of such misattributions is then approximately

$$\Gamma^N_{\text{corrected}} \simeq 2\frac{N!}{\frac{N-1}{2}!\frac{N+1}{2}!}(\Gamma\Omega\tau_M)^{\frac{N+1}{2}}\frac{1}{\Omega\tau_M}. \tag{1}$$

So the corrected error-rate scales as the bare error rate Γ risen to the power of half the number of stages employed. This allows devices with a sufficiently low bare error rate to reach any desired corrected error rate with additional stages. This assertion is demonstrated numerically in Sec. III of this paper after the required analytical framework is provided in Sec. II.

II. MASTER EQUATION TREATMENT OF THE ARBITRARY ERROR-CORRECTION CIRCUIT

We identify the net errors the N pumps have committed in time interval $[0,t)$ with the stochastic process $\mathfrak{K}$ with outcome space $\mathbb{Z}^N$ over time and denote the accompanying probability distribution over $\mathbb{Z}^N$ with $\vec{\phi}(t)$. With $\vec{i} = (i_1, i_2, \ldots, i_N)$ the entries of vector $\vec{\phi}$ are labeled $\phi(\vec{i})(t)$ and are the joint probabilities of i_j errors having been being committed by pump j for any j in the time interval $[0,t)$. As the error events we intend to observe are assumed to be rare, we can assume $\mathfrak{K}$ to be a Markov process. For the discrete time series at integer multiples of a small time δt process $\mathfrak{K}$ is described by the Pauli-Master equation

$$\vec{\phi}(t+\delta t) - \vec{\phi}(t) = K\vec{\phi}(t)\delta t, \tag{2}$$

where K is the stochastic matrix describing $\mathfrak{K}$ and constant if $\mathfrak{K}$ is stationary. K is composed of the probabilities of an error

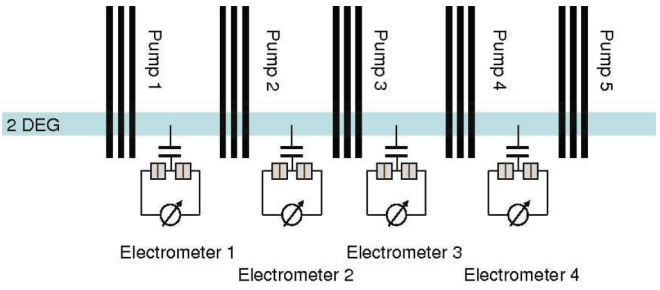

FIG. 1. (Color online) Schematic of a circuit for an error-correction scheme in a typical electron counting experiment composed of metallic single-electron transistors and electron pumps utilizing dynamic quantum dots in a two-dimensional electron gas.

$\vec{F}$ occurring during a time interval $[k\delta t,(k+1)\delta t)$ given initial state of the errors $\vec{i}$ as $P^{\delta t}_{\vec{F}}(k\delta t,\vec{i}) = \delta t\, K_{\vec{i}+\vec{F},\vec{i}}$. The probability for the realization of a particular trajectory $\mathcal{F}$, with $\vec{F}(k\delta t)$ errors committed in each time interval $[k\delta t,(k+1)\delta t)$, is then

$$\pi_{\mathcal{F}} = \prod_{k=0}^{T/\delta t} P^{\delta t}_{\vec{F}(k\delta t)}\left(k\delta t, \sum_{k'=0}^{k-1} \vec{F}(k'\delta t)\right). \tag{3}$$

In addition to the random walk of Eq. (3) we utilize a number s of the noisy measurements in the circuits of interest. For each time the outcome of the s measurements can be written as a vector $\vec{S}$ in $\mathbb{R}^s$. We assume that, given state $\vec{i}$ of the random walk, the measurement outcomes follow the form

$$\vec{S}_{[\vec{i}]}(k\delta t) = \vec{f}(\vec{i}(k\delta t),k\delta t) + \vec{n}(k\delta t), \tag{4}$$

where $\vec{n}(t)$ is stochastic additive noise and $\vec{f}$ the deterministic transfer-function-relating observed system to measurement outcomes. We assume the noise to be independent of and uncorrelated to the random walk, to be stationary, ergodic and markov and, crucially, to be uncorrelated in time, which is a valid approximation, when the errors are rare compared to the noise's autocorrelation time. For the noise averaged over an interval of length δt we denote the noise probability density over $\mathbb{R}^s$ with $\vec{\nu}_{\delta t}(t,\vec{n})$. Equivalently to the time trajectories of the error scenarios $\mathcal{F}$, we define $\mathcal{S}$ to be time trajectories of the measurement outcomes and note that the probability that the circuit commits a particular random walk $\mathcal{F}$ and generates electrometer outputs $\mathcal{S}$ over interval $[0,T)$ is

$$\Pi^{\mathcal{F}}_{\mathcal{S}} = \pi_{\mathcal{F}} \prod_{k=0}^{T/\delta t} \vec{\nu}_{\delta t}\left[k\delta t, \vec{S}(k\delta t) - \vec{f}\left(\sum_{k'=0}^{k} \vec{F}(k\delta t),k\delta t\right)\right]. \tag{5}$$

By Bayes' theorem the conditional probability $\Pi_{F|S}$ of a given random walk $\mathcal{F}$ having occurred when output $\mathcal{S}$ is exhibited is

$$\Pi_{\mathcal{F}|\mathcal{S}} = \Pi^{\mathcal{F}}_{\mathcal{S}} \Big/ \sum_{\mathcal{F}'} \Pi^{\mathcal{F}'}_{\mathcal{S}}, \tag{6}$$

whenever the denominator is nonzero. Accordingly, given $\mathcal{S}$ (over the entire time interval $[0,T)$) the probability that net errors $\vec{E}$ have occurred in the time interval $[0,t)$ is the sum of these conditional probabilities over all possible paths $\mathcal{F}$ satisfying said condition

$$\Pi(\vec{E},t|\mathcal{S}) = \sum_{\mathcal{F}|\vec{E},t} \Pi^{\mathcal{F}}_{\mathcal{S}} \Big/ \sum_{\mathcal{F}'} \Pi^{\mathcal{F}'}_{\mathcal{S}}. \tag{7}$$

While the probability of $\vec{E}$ errors having occurred in time interval $[0,t)$, given measurement $\mathcal{S}$ over that time interval and any measurement outside of it, is

$$\Pi(\vec{E},t|\mathcal{S},t) = \frac{\sum_{\mathcal{S}'(t')=\mathcal{S}(t')\forall t'\leqslant t} \sum_{\mathcal{F}'|\vec{E},t} \Pi^{\mathcal{F}'}_{\mathcal{S}'}}{\sum_{\mathcal{S}'(t')=\mathcal{S}(t')\forall t'\leqslant t} \sum_{\mathcal{F}} \Pi^{\mathcal{F}}_{\mathcal{S}'}}, \tag{8}$$

where the interest is not in identifying one particular random walk per se, but to identify the probabilities that the final result of the underlying random walk has been $\vec{E}$ net errors, provided measurements $\mathcal{S}$. In particular, if there is no need (or possibility) to identify in what precise time interval an error occurred then indeed $\Pi(\vec{E},T|\mathcal{S},T)$ is going to be the variable of interest. Obviously no more can be desired once δt is chosen sufficiently small.

More convenient to compute is the probability that up to time $t\vec{E}$ net errors have occurred and the measurement outcome $\mathcal{S}$ has been observed over the same interval, but irrespective of $\mathcal{S}$ for times $t' > t$

$$\psi(\vec{E},t|\mathcal{S},t) = \sum_{\vec{\mathcal{S}}'(t')=\vec{\mathcal{S}}(t')\forall t'\leqslant t} \sum_{\mathcal{F}'|\vec{E},t} \Pi^{\mathcal{F}'}_{\mathcal{S}'}. \tag{9}$$

When stepping forward in time by δt, the change of $\psi(\vec{E},t|S,t)$ then includes four terms: random walks leaving the second sum because of an error-event $\vec{F}(t+\delta t) \neq 0$; random walks entering the second sum for the same reason, $\vec{F}(t+\delta t) \neq 0$ and $\sum_{t'\leqslant t+\delta t} \vec{F}(t) = \vec{E}$ and $\vec{S}'(t') = \vec{S}(t')\forall t' \leqslant t+dt$; and paths $\mathcal{S}'$ leaving the first sum because $\vec{S}'(t+\delta t) \neq \vec{S}(t+\delta t)$, but satisfying $\vec{F}(t+\delta t) = 0$, to avoid double counting with respect to the first term. Of course there are also paths that stay within both sums, with $\sum_{t'\leqslant t} \vec{F}(t) = \sum_{t'\leqslant t+dt} \vec{F}(t) = \vec{E}$, and $\vec{S}'(t') = \vec{S}(t')\forall t' \leqslant t+\delta t$ while no paths can enter the first sum by its definition.

Accordingly

$$\begin{aligned}
&\psi(\vec{E},(k+1)\delta t|S,(k+1)\delta t) - \psi(\vec{E},k\delta t|S,k\delta t)\\
&\quad = -\sum_{\vec{F}(k\delta t)\neq\vec{0}} P_{\vec{F}}(k\delta t,\vec{E})\psi(\vec{E},k\delta t|S,k\delta t) + \vec{\nu}\{\vec{S}[(k+1)\delta t] - \vec{f}[\vec{E},(k+1)\delta t]\} \sum_{\vec{F}(k\delta t)\neq\vec{0}} P_{\vec{F}}(k\delta t,\vec{E}-\vec{F})\psi(\vec{E}-\vec{F},k\delta t|S,k\delta t)\\
&\qquad -(1-\vec{\nu}\{\vec{S}[(k+1)\delta t] - \vec{f}[\vec{E},(k+1)\delta t]\})P_{\vec{0}}\psi(\vec{E},t|S,t),
\end{aligned} \tag{10}$$

where $\vec{0}$ is the null vector. For each time ψ contains for each trajectory $\mathcal{S}$ a number of elements equal to the number of possible net error scenarios $\vec{E}$. For a given trajectory with N pumps under consideration, at each time the set of all $\psi(\vec{E},t,\mathcal{S},t)$ will be an element of $[0,1]^N$. Analogous to $\vec{\phi}$ it is convenient to summarize all $\psi(\vec{E},t,\mathcal{S},t)$ into $\vec{\psi}(t)$. Dropping the reference to $\mathcal{S}$. Eq. (10) simplifies to

$$\vec{\psi}(t+\delta t) = (1+U\delta t)(1+K\delta t)\vec{\psi}(t) \simeq e^{U\delta t}(1+K\delta t)\vec{\psi}(t). \tag{11}$$

Here $U\delta t$ includes the dependence on measurements $\vec{\nu}$ of Eq. (10) and is a diagonal matrix; while K does not depend on S and is constant if the error rates do not depend on time, that is, if $\mathfrak{K}$ is stationary. Remarkably the evolution of $\vec{\psi}$ is governed by a linear difference equation akin to Eq. (2). While the definitions both $P_{\vec{F}}$ and $\vec{\nu}$ depend on the choice of δt, we assume that it can be chosen sufficiently small, such that neither K nor U do.

For all times $t > 0$, it is obvious that

$$\|\vec{\psi}(t)\|_1 = \sum_{\vec{E}} \psi(\vec{E},t|\mathcal{S},t) = \sum_{S'(t')=S(t')\forall t'\leqslant t} \sum_{F} \Pi^F_{\mathcal{S}'}, \tag{12}$$

so it is convenient to define that

$$\psi(\vec{0},0,\mathcal{S},0) = \sum_{\vec{E}}\sum_{\mathcal{S}} \psi(\vec{E},0,\mathcal{S},0) = 1, \tag{13}$$

which implies that the set of all possible measurement outcomes S prior to the first measurement has one element of probability 1. With the initial condition Eq. (13) and the time development of Eq. (10) or Eq. (11) the final probability of $\vec{E}$ errors having occurred in interval $[0,T)$ and trajectories S having been measured can readily be computed as $\psi(\vec{E},T,\mathcal{S},T)$. The conditional probability of $\vec{E}$ errors having occurred in interval $[0,T)$ given trajectories S having been measured is then $\psi(\vec{E},T,\mathcal{S},T)/\sum_{\vec{E}'}\psi(\vec{E}',T,\mathcal{S},T)$, where the denominator is just the 1-norm of vector $\psi(T)$. Equation (11) is the master equation governing the net-error probabilities of the shrinking set satisfying trajectories S.

The expectation value and variance of the errors given trajectory S is then computed as

$$\begin{aligned}\langle\vec{E}|S\rangle &= \sum_{\vec{E}} \frac{\vec{E}\psi(\vec{E},T,\mathcal{S},T)}{\sum_{\vec{E}'}\psi(\vec{E}',T,\mathcal{S},T)},\\ \sigma_{\vec{E}|S} &= \sum_{\vec{E}} (\vec{E}-\langle\vec{E}|\mathcal{S}\rangle)^2 \frac{\psi(\vec{E},T,\mathcal{S},T)}{\sum_{\vec{E}'}\psi(\vec{E}',T,\mathcal{S},T)}.\end{aligned} \tag{14}$$

Given measurement outcome $\mathcal{S}$ this computes that (on average) $\langle\vec{E}|S\rangle$ net errors have occurred in time interval $[0,T]$; that quantity has an uncertainty of $\sigma_{\vec{E}|\mathcal{S}}$, however, the entire probability distribution $\vec{\psi}(T)$ is known. The mean of that later quantity weighted over all possible trajectories will be of particular interest below, $\langle\sigma_{\vec{E}|S}\rangle_{\mathcal{S}} = \sum_{\mathcal{S}}\Pi_{\mathcal{S}}\sigma_{\vec{E}\mathcal{S}}$, the expectation value of the uncertainty of the error-accounting scheme. For computer-generated $\mathcal{S}$ and $\mathcal{F}$ the average error of the error-accounting scheme can be computed as well as the difference $\delta^{\mathcal{S}}_{\vec{E}}$ of the computed $\langle\vec{E}|S\rangle$ and *ex ante* known underlying error of the random walk. However, $\langle\sigma_{\vec{E}|\mathcal{S}}\rangle_{\mathcal{S}}$ is a sufficient proxy that is accessible when only $\mathcal{S}$ is known, as in an experiment.

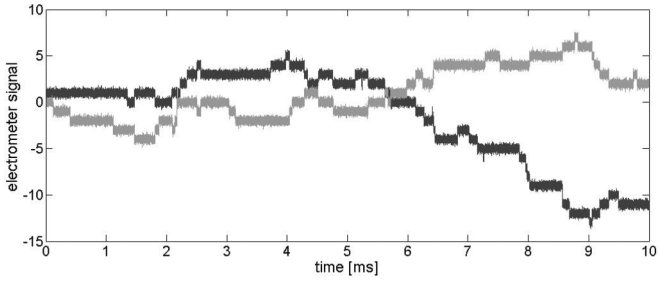

FIG. 2. Signal traces of first (black) and second (gray) electrometer. Pump error rates are 1000/s, per pump and sign, electrometer noise $10^{-5}e/\sqrt{\text{Hz}}$, and coupling capacitance ratio $C_C/C_{\text{node}} = 0.04$.

For any random walk governed by a known stochastic matrix K and additive totally random noise described by U, Eq. (11) is readily integrated for any one set of measurement outcomes. The expectation value of the occurred error and its uncertainty according to Eq. (14) follows. The average over all possible outcomes, however, can only be approximated by summing over a large set of trajectories.

III. NUMERICAL EVALUATION OF THE ONE-DIMENSIONAL PUMP CHAIN

The analytical treatment above is valid for an arbitrary circuit, while for numerical studies we confine ourselves to the simplest case, which is of particular experimental interest, the one-dimensional chain of electron pumps. In Eq. (4) this corresponds to $f_j(\vec{i}(t),t) = i_j - i_{j+1}$, where subindex j indicates the jth component of the vector of length $s = N-1$. The electrometer noise we assume to be ergodic, stationary, Gaussian, and white. The random walk we assume to be stationary—though the pulsed operation of the pumps may be of interest—and independent of the charges accumulated on the nodes. This case is understood even using only the arguments yielding Eq. (1) and could experimentally be realized by applying feedback voltages to the nodes. Then the net errors will diffuse freely for each pump and will accordingly diverge with $\sqrt{T}$. As the electrometers cannot distinguish states that differ by the same number of errors for all pumps, we combine charge equivalent states when computing $\vec{\psi}$. For free error diffusion we need to compute $\vec{\psi}$ only over a few states around the most likely error scenario, typically all states that deviate by up to three errors from the most likely error scenario. The integration of Eq. (10) is simplified as K is both constant and invariant under translation by any possible error. This last is no longer the case when the voltage dependence of the error rates is included; the constant K is here only invariant under uniform translations of errors on all pumps. Accordingly, $d\vec{\psi}/dt$ needs to be calculated over a larger configuration space. This is the case of mesoscopic feedback;[13,14] however, such a charge dependence of the error rates does not add conceptual complication to our treatment.

Figure 2 shows a typical set of signal traces[15] that can be expected from the combination of the diffusion dynamics and electrometer noise realistic for the radio frequency single electron transistor (RF-SET)[16–18] and similar devices.[19–21] These traces are used to numerically integrate Eq. (11) yielding

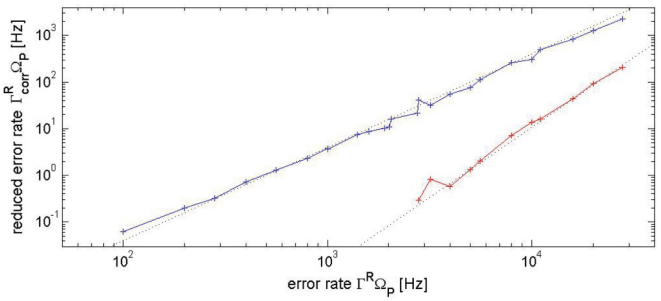

FIG. 3. (Color online) Corrected error rates with respect to time for three (upper curve) and five (lower curve) pump circuits as a function of the uncorrected error rates. Dashed lines are a fit to Eq. (1). Other parameters as in Fig. 2.

a perceived diffusion. The result of this perceived diffusion is subtracted from the underlying diffusion. This difference is the error that our algorithm misattributes. Nonetheless, the rate of such errors after correction is much smaller than the rate of errors of each single device. The numerical results for rates of these reduced errors are shown for various parameters in Fig. 3 in excellent agreement with Eq. (1).[15] Even for a rate of 200 errors per second a 1000-fold improvement of the error rate is achieved with only three pumps and two intermediate electrometers, so that even devices exhibiting a relative error rate of 10 ppm can reach the metrologically desirable error threshold of 10 ppb due to the proposed architecture. The five pump version achieves a 10 000-fold improvement even at a rate of 3000 errors per second. Pumps with intrinsic error rates of 100 ppm at pump rates of 30 MHz could thus be corrected to the required 10 ppb accuracy. If devices with 10 ppm were to be used, a 1000-fold improvement were to suffice, so that more than 8000 errors per second could be accounted for. This would correspond to a pump rate of 800 MHz, so that both the requirements of current magnitude and accuracy are met. Recall that the lowest reported error rate for single parameter pumps is about 1 ppm.[12]

IV. CONCLUSION

In summary, we have proposed a family of architectures utilizing either conventional electron pumps of modest quality or one of the single-gate devices that are capable of producing quantized current steps with the now common RF-SET or a similar device. The algorithm constructed in this work determines the charge transported by such a structure to a much greater precision than would be feasible by any single of the devices used. The relaxation on the requirements for the quality of the individual devices used simplifies the experimentation, even at a higher operation frequency, and increases the sample yield. It thereby opens the prospects towards the parallelization needed to increase the provided current. We note that the proposed architecture allows to monitor the error rates of individual pumps on the level of single electrons,[22] similar to those stated for adiabatic electron pumps in their shuttle mode.[4] This has not been possible for the single gate devices[6,8,9] in currently employed circuits. Accordingly, the architecture proposed here allows further research on these devices and thereby a determination regarding their suitability for metrological applications, while easing the required thresholds. During the preparation of this paper we participated in work demonstrating the integrated operation of the components required for the experimental realization of this concept.[23,24]

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