Spin-orbit force due to Rashba coupling at the spin resonance condition

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We analyze the effect of Rashba type of spin-orbit (SO) coupling on the electron dynamics and the rf electrical conductivity. We show that in addition to the momentum current an additional SO current occurs which can be attributed to a SO contribution to the electric Lorentz force. This Rashba SO force is proportional to the time derivative of the electron magnetization. Therefore, in a static electromagnetic field SO interaction does not affect the electric or the spin current. Applying an rf electric current, however, an rf magnetization can be efficiently induced via the rf Rashba field. Thus, at the Larmor frequency a characteristic *current induced electron spin resonance* occurs. There the absorbed electric power is efficiently converted into magnetic energy.

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I. INTRODUCTION

Coupling of momentum and spin of an electron is a consequence of its moving charge which causes a magnetic field which in turn affects the moving electron, as it is known from atomic physics as spin-orbit (SO) coupling. In solids with lower than mirror symmetry, and/or in the presence of a built-in electric field, the Hamiltonian contains also an additional bilinear SO term, proportional to both the spin and the momentum.[1] This Rashba term manifests itself in various phenomena like Dyakonov-Perel spin relaxation,[2–4] spin dephasing,[3] a giant anisotropy of the electron spin resonance (ESR) linewidth,[3] a characteristic anisotropy of the g factor,[3] and others.[4] One of the most direct exemplifications of the Rashba field is the shift of the ESR resonance field when an electric current is applied.[5] Spin precession in the Rashba field has been proposed as the main control mechanism in the seminal concept of the Datta-Das transistor.[6] The intrinsic spin-Hall effect,[7] however, has been excluded after some discussion, specifically for the Rashba case where the spin splitting varies linearly with momentum.[8–10]

A particular SO effect appears when an rf current with a frequency corresponding to the Larmor frequency is applied:[11] In an ESR-like experiment, spin precession can be excited by a resonant microwave current as predicted by Rashba and Efros.[12] Excitation via this current-induced ESR[13] can be by orders of magnitude more efficient than the usual excitation of magnetic dipole transitions by a microwave magnetic field.[5,12] This may be of practical importance in the context of spin manipulation.[12,14] The dynamic spin-Hall effect is another effect resulting from the current induced spin precession[15] and closely related to the results of this paper.

One of the primary consequences of the Rashba type of SO coupling is the dependence of the electron velocity on its spin orientation. Imbalance of spin-up and spin-down populations, that is, a magnetization of conduction electrons, induces a "Rashba current" $j^{(R)}$, which occurs in addition to the usual "momentum" current $j^{(p)}$. The latter is also modified due to a finite Rashba spin-orbit parameter[1] $\alpha \neq 0$. We show that at thermal equilibrium and even for low frequency currents ($\omega\tau \ll 1$, where τ is the momentum relaxation time) the Rashba current is compensated by the change of the momentum current ($j_\alpha^{(p)} - j_{\alpha=0}^{(p)}$) due to SO interaction. As it turns out, this is no longer the case for rf fields: With increasing frequency, momentum relaxation is less and less able to compensate the Rashba current and a net SO effect on the current becomes visible, particularly at spin resonance. There the microwave electric field can effectively induce precession of the magnetization which results in an oscillating Rashba current and thus in a net SO-induced current. We show that the ESR electric power absorption due to this current can strongly exceed the classical magnetic dipole absorption. This resonantly absorbed electric power is effectively converted, via SO coupling, into magnetic energy. The rate for the latter turns out to exceed the total power absorbed at ESR conditions, which explains the experimentally observed[5,11] negative absorption signal.

In this paper we introduce a formalism to describe the influence of Rashba-SO interaction on the electric current and the magnetization. In contrast to Ref. 12, we use a semiclassical approach which allows us to take into account different rates of spin and momentum relaxation and the partial compensation of Rashba and momentum currents. Considering the electron and spin dynamics, we introduce the term "Rashba spin-orbit (RSO) force," and explore its effects. We show that the RSO force, acting on the electrons in spin-unpaired states, is proportional to the time derivative of the electronic magnetization. Therefore it can be applied to a wide class of spin-galvanic effects where the dynamic magnetization may have different origin. One can discuss, for example, electric properties originating not only from magnetic and electric rf fields but also from the magnetization induced by optical generation or by spin injection.

Spin-dependent forces have been introduced by Shen[16] and Chudnovsky[15,17] in general form for a time-independent Hamiltonian with SO interaction. Here we analyze the specific case of a two-dimensional (2D) sample placed in an rf electric field, where spin precession is induced by the Rashba field originating from the resulting rf current. We show that this current induced ESR (CI ESR) is characterized by a peculiar line shape in agreement with experiment.[5,11]

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II. HAMILTONIAN OF THE SYSTEM

In systems without mirror symmetry, SO interaction causes the second, the Rashba term, characterized by the unit vector $\mathbf{n}$ and the spin Pauli matrices $\boldsymbol{\sigma}$ in

$$\mathcal{H}=\frac{1}{2m^*}\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)^2+\upsilon_R\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)\times\mathbf{n}\cdot\boldsymbol{\sigma}-\frac{g}{2}\mu_B\mathbf{B}_0\cdot\boldsymbol{\sigma}. \tag{1}$$

Here $\mathbf{p}$ is the canonical momentum and m^* is the effective electron mass. The parameter $\upsilon_R=\alpha/\hbar$ in the second term is the material- and structure-dependent Rashba parameter[1] α divided by $\hbar$. It describes the magnitude of the SO Rashba coupling. In the case of asymmetric Si quantum wells it results from the built-in electric field. The last term stands for the Zeeman energy in an external magnetic field $\mathbf{B}_0$. In contrast to the usual ESR experiment where magnetic dipole transitions are excited by a microwave magnetic field, we analyze here the influence of an in-plane rf electric field $\mathbf{E}_1(t)=Re[\mathbf{E}_{1\omega}\exp(-i\omega t)]$. We consider a two-dimensional (2D) electron gas choosing the vector potential in the form of $\mathbf{A}=\frac{1}{2}\mathbf{B}_{0\perp}\times\mathbf{r}-c\int^t\mathbf{E}_1(t')dt'$, where $\mathbf{B}_{0\perp}$ is the transverse component of the static magnetic field applied. The configuration of the external fields relative to the 2D sample is shown in Fig. 1. Here the unit vector $\mathbf{n}$ is perpendicular to the 2D plane. For Si/SiGe, the Rashba parameter of $\upsilon_R=4\ \mathrm{ms}^{-1}$ corresponds to a built-in field[18] on the order of 2×10^{10} V/m.

Among the direct consequences of the Rashba SO coupling are the following. The velocity of an electron:

$$\mathbf{v}=\frac{\partial\mathcal{H}}{\partial\mathbf{p}}=\mathbf{v}^{(p)}+\mathbf{v}^{(R)}, \tag{2}$$

can be decomposed into the in-plane momentum velocity:

$$\mathbf{v}^{(p)}=\frac{1}{m^*}\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right), \tag{3}$$

and a spin-dependent Rashba component:

$$\mathbf{v}^{(R)}=\upsilon_R\mathbf{n}\times\boldsymbol{\sigma}, \tag{4}$$

which is oriented in-plane and perpendicular to the spin of an electron.

Another consequence of Rashba coupling is that the electron spin is affected by the SO Rashba field as well. The latter is proportional to the momentum velocity:

$$\mathbf{B}_R=\frac{\upsilon_R m^*}{\mu_B}\mathbf{n}\times\mathbf{v}^{(p)}. \tag{5}$$

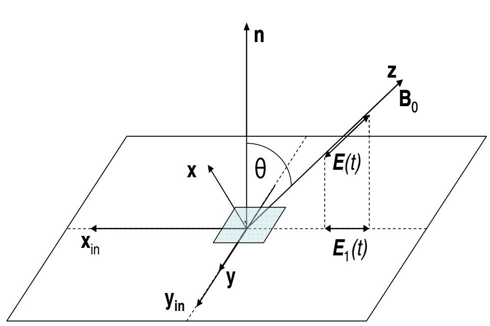

FIG. 1. (Color online) Considered geometry: The static magnetic field B_0 and the microwave electric field are parallel and tilted by θ with respect to the surface normal $\mathbf{n}$. This gives an in-plane field of $E_1(t)=-E(t)\sin\theta$.

In the new variables Eqs. (2)–(5) the Hamiltonian Eq. (1) has the simple form

$$\mathcal{H}=\frac{1}{2}m^*\mathbf{v}^{(p)2}-\left(\mathbf{B}_R+\frac{g}{2}\mathbf{B}_0\right)\boldsymbol{\mu}, \tag{6}$$

where $\boldsymbol{\mu}=\mu_B\boldsymbol{\sigma}$ is the magnetic moment of an electron.

III. OSCILLATORY SOLUTION OF THE ELECTRON EQUATION OF MOTION

To analyze the electron dynamics we consider the time derivative

$$\frac{d\mathbf{v}^{(p)}}{dt}=\frac{1}{i\hbar}[\mathbf{v}^{(p)},\mathcal{H}]+\frac{\partial\mathbf{v}^{(p)}}{\partial t}$$

which leads to the equation

$$\frac{d\mathbf{v}^{(p)}}{dt}=\frac{e}{m^*}\left(\mathbf{E}_1+\frac{1}{c}\mathbf{v}^{(p)}\times\mathbf{B}_{0\perp}+\frac{1}{c}\mathbf{v}^{(R)}\times\mathbf{B}_{0\perp}\right). \tag{7a}$$

The first two terms in Eq. (7a) are the classical Lorentz force. The third one ($c^{-1}\cdot\mathbf{v}^{(R)}\times\mathbf{B}_{0\perp}$) originates from Rashba coupling. One has to notice here that the one-electron Lorentz force acting on the momentum velocity is ruled by the total velocity $\mathbf{v}\times\mathbf{B}_{0\perp}/c$, defined in Eq. (2) and thus it depends on the spin orientation entering via Eq. (4). Considering that the momentum velocity of an electron relaxes much faster (by a few orders of magnitude) than the spin, we notice that $\mathbf{v}^{(p)}$ converges to the spin-dependent value $\mathbf{v}^{(p)}_{rel}=-\mathbf{v}^{(R)}(\boldsymbol{\sigma})$, which corresponds to the temporary energy minimum defined by $\partial\mathcal{H}/\partial\mathbf{v}^{(p)}=0$. Following the Drude approach, we include the relaxation term $(\mathbf{v}^{(p)}-\mathbf{v}^{(p)}_{\mathrm{rel}})/\tau$ in Eq. (7a):

$$\frac{d\mathbf{v}^{(p)}}{dt}=\frac{e}{m^*}\left(\mathbf{E}_1+\frac{1}{c}\mathbf{v}^{(p)}\times\mathbf{B}_{0\perp}+\frac{1}{c}\mathbf{v}^{(R)}\times\mathbf{B}_{0\perp}\right)-\frac{\mathbf{v}^{(p)}-\mathbf{v}^{(p)}_{\mathrm{rel}}}{\tau}. \tag{7b}$$

Subsequently we add to both sides of (7a) the derivative $d\mathbf{v}^{(R)}/dt=(\upsilon_R/\mu_B)\cdot\mathbf{n}\times d\boldsymbol{\mu}/dt$. Then we get the following equation for the total electron velocity:

$$\frac{d\mathbf{v}}{dt}=\frac{e}{m^*}\left(\mathbf{E}_1+\frac{1}{c}\mathbf{v}\times\mathbf{B}_{0\perp}+\frac{1}{e}\mathbf{F}^{(\mathrm{SO})}\right)-\frac{\mathbf{v}}{\tau}, \tag{8}$$

with the RSO force defined by

$$\mathbf{F}^{(\mathrm{SO})}=\frac{m^*\upsilon_R}{\mu_B}\left(\mathbf{n}\times\frac{d\boldsymbol{\mu}}{dt}\right). \tag{9}$$

Equations (8) and (9) allow us to analyze the electric field effect in spin dynamics, no matter whether the magnetization is induced by spin injection, by optic excitation, or by magnetization precession. In other words, the RSO force does not depend on the mechanism which rules the spin dynamics. Without the RSO force, Eq. (8) is the classical steady state equation with a well known solution defining the electric conductivity tensor[19] $\hat{\sigma}$. Equations (8) and (9) show that SO Rashba coupling causes an additional SO electric field

$\mathbf{F}^{(\mathrm{SO})}/e$, proportional to the derivative of the magnetization, which induces a SO contribution to the electric current $\mathbf{j}^{(\mathrm{SO})} = \mathbf{j}^{(p)}_{\alpha} - \mathbf{j}^{(p)}_{\alpha=0} + \mathbf{j}^{(R)}$, in addition to the classic current density $\mathbf{j}^{(p)}_{\alpha=0}$. The equations also show that there is no SO electric current for a static magnetization. The equations thus imply that an RSO force and the resulting current appear when momentum relaxation becomes too slow to fully compensate the Rashba current ruled by a fast spin dynamics.

The oscillating external electric field $\mathbf{E}_1(t) = \mathbf{E}_{1\omega}\exp(-i\omega t)$[20] causes oscillating currents $\mathbf{v} = \mathbf{v}_0^{(p)} + \mathbf{v}_\omega^{(p)}\exp(-i\omega t)$ and oscillating magnetic moments $\boldsymbol{\mu} = \boldsymbol{\mu}_0 + \boldsymbol{\mu}_\omega\exp(-i\omega t)$, where $\boldsymbol{\mu}_\omega$ is finite only for electrons in spin unpaired states. In the Drude model, $\mathbf{v}_0^{(p)}$ changes after each electron scattering event, while $\mathbf{v}_\omega^{(p)}$ remains constant. Also the amplitude of the electron magnetic moment $\boldsymbol{\mu}_\omega$, and in particular its phase, do not change in spite of many electron collisions. This assumption is justified by the fact that for Si/SiGe the momentum relaxation time (at the Fermi level) is $\tau \leqslant 10^{-11}$ s, while the transverse spin relaxation time T_2 is much longer (10^{-7}–10^{-6} s).

Following Lax *et al.*[19] we obtain from Eq. (8) the Fourier amplitudes:

$$\mathbf{v}_\omega = \frac{1}{n_e e}\hat{\sigma}(\omega)\left(\mathbf{E}_{1\omega} + \frac{1}{e}\mathbf{F}^{(\mathrm{SO})}_\omega\right), \tag{10}$$

where $\hat{\sigma}(\omega)$ is the familiar SO-independent dynamic electric conductivity tensor, depending on the external magnetic field, and n_e is the surface density of the free electron gas.

The amplitude of the RSO force,

$$\mathbf{F}^{(\mathrm{SO})}_\omega = -i\omega\frac{\alpha m^*}{\hbar\mu_B}\mathbf{n}\times\boldsymbol{\mu}_\omega, \tag{11}$$

is proportional to the frequency and the amplitude of the oscillating magnetic moment (in Si/SiGe for $\omega = 2\pi\cdot 9\times 10^{10}$ Hz the value of RSO force is of the order of $F^{(\mathrm{SO})}_\omega \approx 2.5\times 10^{-3}$ eV/cm).

In order to determine the Fourier amplitude $\boldsymbol{\mu}_\omega$ we have to analyze the influence of the Rashba interaction on the moving electron. The Rashba field Eq. (5) is a linear function of $\mathbf{v}^{(p)}$ and it is equal to $\mathbf{B}_R = \mathbf{B}_{R0} + \mathbf{B}_R(t)$, with $\mathbf{B}_{R0} = (v_R m^*/\mu_B)\cdot\mathbf{n}\times\mathbf{v}_0^{(p)}$ and $\mathbf{B}_R(t) = \mathbf{B}_{R\omega}\exp(-i\omega t)$, where

$$\mathbf{B}_{R\omega} = (v_R m^*/\mu_B)\cdot\mathbf{n}\times\mathbf{v}_\omega^{(p)}. \tag{12}$$

(The static component $\mathbf{B}_{R0}$ is small compared to the external constant field $\mathbf{B}_0$; for an rf electric field amplitude in a microwave cavity of the order of 10^2 V/cm and $B_0 \approx 0.3$ T we have $B_{R0} \approx 0.03 B_0$.) Because of the Rashba term in the Hamiltonian Eq. (1) the symmetry of the system in momentum space is broken. The electron magnetic moment $\boldsymbol{\mu}$ can be decomposed into an isotropic $\boldsymbol{\mu}(v_0^{(p)},t)$ and a small anisotropic part $\Delta\boldsymbol{\mu}(\mathbf{v}_0^{(p)},t)$. As it was shown by Duckheim and Loss[13] the equation of spin motion, averaged over all directions of $\mathbf{v}_0^{(p)}$, reads

$$\frac{d\boldsymbol{\mu}\left(v_0^{(p)},t\right)}{dt} = -\gamma\cdot\boldsymbol{\mu}\left(v_0^{(p)},t\right)\times[g\mathbf{B}_0/2 + \mathbf{B}_{R\perp}(t)], \tag{13}$$

where the gyroscopic factor is $\gamma = -2\mu_B/\hbar$ and $\mathbf{B}_{R\perp}$ denotes the Rashba field component perpendicular to $\mathbf{B}_0$. The constant component $\mathbf{B}_{R0}$ of the Rashba field is fixed once and again after each electron scattering event. This component has been neglected in Eq. (13) since the contribution of the correction $\Delta\boldsymbol{\mu}(\mathbf{v}_0^{(p)},t)\mathbf{B}_{R0}$ to the spin motion is of higher order in the SO Rashba coupling.

Adding to the right-hand side of Eq. (13) the incoherent terms we recognize the Bloch equations which in the linear limit lead to the familiar solution:

$$\mathbf{M}_\omega = \hat{\chi}(\omega)\mathbf{B}_{R\perp\omega}, \tag{14}$$

where $\mathbf{M}_\omega = \delta n_e\cdot\boldsymbol{\mu}_\omega$ is the electron gas magnetization and δn_e is the surface density of electrons in spin-unpaired states [the number of electron states with uncompensated spins per unit area is $\delta n_e \approx D(E)\mu_B B_0$, where $D(E) = m^*/\pi\hbar^2$ is the density of states for 2D electron gas]. The susceptibility tensor

$$\hat{\chi} = \frac{1}{2}\begin{bmatrix}(\chi_+ + \chi_-)\cos^2\theta & -i(\chi_+ - \chi_-)\cos\theta\\ i(\chi_+ - \chi_-)\cos\theta & \chi_+ + \chi_-\end{bmatrix} \tag{15}$$

connects the in-plane coordinates of the Rashba field with the in-plane coordinates of the magnetization $\mathbf{M}_\omega$. In the rotating coordinate system $(\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})/\sqrt{2}$ (with $\hat{\mathbf{z}}$ parallel to $\mathbf{B}_0$) $\hat{\chi}$ is diagonal with the components $\chi_\pm = \mp\gamma M_0/(\omega \mp \omega_L + i/T_2)$ and Eq. (14) has the simple form $\mu_{\omega\pm} = (\delta n_e)^{-1}\chi_\pm(\omega)B_{R\omega\pm}$, where for the vector $\boldsymbol{\mu}_\omega$ (and likewise for $\mathbf{B}_{R\omega}$) $\mu_{\omega\pm} = (\mu_{\omega x} \mp i\mu_{\omega y})/\sqrt{2}$.

The static magnetization $M_0 = \chi_0 B_0$ is defined by the Pauli paramagnetic susceptibility constant which for a 2D electron gas is equal to $\chi_0 = D(E)\cdot\mu_B^2$. The Larmor frequency is $\omega_L = \gamma g B_0/2$.

For the oscillatory solution of the problem of the electron motion we need the dependence of $\mathbf{B}_{R\omega}$ [Eq. (12)] and subsequently of $\mathbf{v}_\omega^{(p)}$ on the external electric field $\mathbf{E}_{1\omega}$ and then we express the RSO force [see Eq. (11)] in terms of $\mathbf{E}_{1\omega}$. We find this dependence from Eq. (10) by subtracting $\mathbf{v}_\omega^{(R)} = \frac{\alpha}{\hbar\mu_B}\mathbf{n}\times\boldsymbol{\mu}_\omega$ on both sides or from the oscillatory part of Eq. (7b) and we obtain

$$\mathbf{v}_\omega^{(p)} = \frac{1}{n_e e}\hat{\sigma}(\omega)\cdot\left[\mathbf{E}_{1\omega} - n_e e\hat{\rho}(0)\mathbf{v}_\omega^{(R)}\right]. \tag{16}$$

Here the resistivity tensor $\hat{\rho}(\omega)$ [defined as the inverse of the conductivity tensor $\hat{\rho}(\omega) = \hat{\sigma}^{-1}(\omega)$] for the 2D sample in the external fields (depicted in Fig. 1) has the form

$$\hat{\rho}(\omega) = \sigma_0^{-1}\begin{bmatrix}1 - i\omega\tau & \omega_c\tau\cos\theta\\ -\omega_c\tau\cos\theta & 1 - i\omega\tau\end{bmatrix}, \tag{17}$$

with the Drude conductivity $\sigma_0 = n_e e^2\tau/m*$ and the cyclotron frequency $\omega_c = -eB_0/m^*c$.

Taking the above into account, we obtain the equation

$$\mathbf{v}_\omega^{(p)} = (n_e e)^{-1}\hat{\sigma}(\omega)\left[\mathbf{E}_{1\omega} - n_e e\hat{\Delta}(\omega)\mathbf{v}_\omega^{(p)}\right]$$

with $\hat{\Delta}(\omega) = (\delta n_e)^{-1}(\alpha/\hbar\mu_B)^2 m^*\hat{\rho}(0)\hat{n}\hat{\chi}(\omega)\hat{n}$ ($\hat{n}$ is proportional to the Pauli spin matrix $\hat{n} = -i\sigma_y$). Then, using the iteration method we can express $\mathbf{v}_\omega^{(p)}$, at least in principle, as a function of $\mathbf{E}_{1\omega}$. In the linear approximation we get the solution for electrons in spin-unpaired states:

$$\mathbf{v}_\omega^{(p)} = \frac{1}{n_e e}\hat{\sigma}(\omega)\cdot[\hat{I} - \hat{\Delta}(\omega)\hat{\sigma}(\omega)]\mathbf{E}_{1\omega}. \tag{18}$$

The Rashba coupling thus contributes to the electron momentum current by the term $\hat{\Delta}$ which is proportional to α^2. For Si/SiGe the correction $\hat{\Delta}(\omega)\sigma_0$ is of the order of $m^*(v^{(R)})^2T_2/\hbar \approx 0.3 \times (10^{-2} - 10^{-1})$.

IV. ELECTRIC POWER ABSORPTION AND ENERGY TRANSFER VIA THE RASHBA FIELD

The momentary power absorbed by an electron is the time derivative of the one-electron Hamiltonian $P(t) = \partial\mathcal{H}/\partial t$. If the oscillating electric field is the only external time-dependent field, then $\partial\mathcal{H}/\partial t = e\mathbf{E}_1(t)\mathbf{v}(t)$ and the time averaged value of the electric power absorption (per unit area of the sample) is given by the oscillatory amplitudes by

$$P_E(\omega) = (1/2)\mathrm{Re}\{\mathbf{E}^*_{1\omega}\mathbf{j}_\omega\}, \tag{19}$$

with the electron current $\mathbf{j}_\omega = n_e e\langle\mathbf{v}_\omega\rangle$:

$$\mathbf{j}_\omega = \hat{\sigma}(\omega)\big(\mathbf{E}_{1\omega} + e^{-1}\big\langle\mathbf{F}^{(\mathrm{SO})}_\omega\big\rangle\big), \tag{20}$$

where $\langle X\rangle$ means the averaged value of X over all occupied electron states. In the Drude (spin-free) model we get $\langle\mathbf{v}_\omega\rangle = \mathbf{v}_\omega$, while in the presence of the RSO force a difference appears, $\langle\mathbf{v}_\omega\rangle \neq \mathbf{v}_\omega$, where $\mathbf{v}_\omega$ is defined by Eq. (10). This is a consequence of the fact that only some of the electrons, namely those in spin-unpaired states, contribute to the electron magnetization $\mathbf{M}_\omega = n_e\langle\boldsymbol{\mu}_\omega\rangle \approx \delta n_e\boldsymbol{\mu}_\omega$. The electric current Eq. (20) is the sum of two parts, the classical $\mathbf{j}^{(p)}_{\alpha=0,\omega} = \hat{\sigma}(\omega)\mathbf{E}_{1\omega}$ and the SO part $\mathbf{j}^{(\mathrm{SO})}_\omega = e^{-1}\hat{\sigma}(\omega)\langle\mathbf{F}^{(\mathrm{SO})}_\omega\rangle$, where $\langle\mathbf{F}^{(\mathrm{SO})}_\omega\rangle = -i\omega(\alpha m^*/\hbar\mu_B)\mathbf{n}\times\langle\boldsymbol{\mu}_\omega\rangle$.

The additional SO current $\mathbf{j}^{(\mathrm{SO})}_\omega$, described by the RSO force and the classical dynamic conductivity, is the only consequence of the Rashba coupling, in spite of the complex components of this current. It is proportional to the frequency and the amplitude of the precessing magnetization at the frequency ω. Thus, in the absorption power P_E [Eq. (19)] we can distinguish two components: The classical, nonresonant Joule heat $P^{(J)}_\omega = (1/2)\mathrm{Re}[\mathbf{E}^*_{1\omega}\hat{\sigma}(\omega)\mathbf{E}_{1\omega}]$ and the second one, which is observed in spin resonance experiments,

$$P^{(\mathrm{SO})}_\omega = \tfrac{1}{2}\mathrm{Re}\left[\mathbf{E}^*_{1\omega}\hat{\sigma}(\omega)e^{-1}\big\langle\mathbf{F}^{(\mathrm{SO})}_\omega\big\rangle\right]. \tag{21}$$

As for the nonresonant part, we consider a constant electric field, that is, the case $\omega = 0$, $\mathbf{E}_1(t) = \mathbf{E}_{10}$. According to Eq. (20) the total electric current is $\mathbf{j}_0 = ne\langle\mathbf{v}_0\rangle = \hat{\sigma}(0)\mathbf{E}_{10}$ (where $\mathbf{j}_0 = \mathbf{j}^{(p)}_{\alpha=0}$) and simultaneously $\mathbf{j}^{(\mathrm{SO})}_0 = 0$. The momentum current is then equal to $\mathbf{j}^{(p)}_{\alpha,0} = \mathbf{j}_0 - \mathbf{j}^{(R)}_0$, where $\mathbf{j}^{(R)}_0 = (n_e e\alpha/\hbar\mu_B)\mathbf{n}\times\langle\boldsymbol{\mu}_0\rangle$, with the static magnetization $n_e\langle\boldsymbol{\mu}_0\rangle = \mathbf{M}_0$. In the case of $\mathbf{E}_{10} = 0$ we have $\mathbf{j}_0 = 0$ and due to the broken symmetry of the Hamiltonian in momentum space [see Eqs. (5) and (6) and Fig. 2] the momentum current $\mathbf{j}^{(p)}_{\alpha,0} = -\mathbf{j}^{(R)}_0$ does not vanish.

Now let us analyze the expression (19): $P_E(\omega) = (n_e e/2)\mathrm{Re}[\mathbf{E}^*_{1\omega}\langle\mathbf{v}_\omega\rangle]$. Combining the oscillatory part of Eq. (7) with the equality $\mathbf{B}^*_{R\omega}\boldsymbol{\mu}_\omega = -m^*\mathbf{v}^{(p)*}_\omega\mathbf{v}^{(R)}_\omega$ we get

$$P_E(\omega) = \frac{E^{(\mathrm{kin})}_\omega}{\tau} + P^{(M)}_\omega, \tag{22}$$

where $E^{(\mathrm{kin})}_\omega = n_e(m^*/2)|\langle\mathbf{v}_\omega\rangle|^2$ is the kinetic energy of the electron gas, and $P^{(M)}_\omega$ is the Rashba magnetic power absorption, which describes the energy transfer via the Rashba field to the Zeeman energy. $P^{(M)}_\omega$ is the time averaged value of $P^{(M)}(t) = -\mathbf{M}(t)d\mathbf{B}_R/dt$. Expressed by the oscillatory amplitudes of the Rashba field and magnetization, it reads

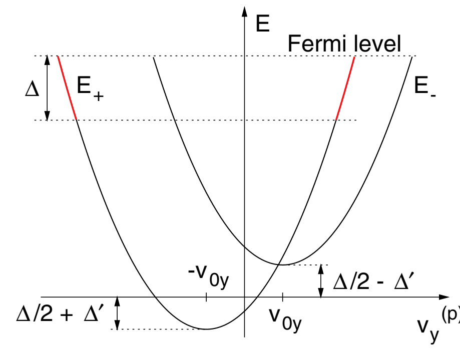

FIG. 2. (Color online) Conduction band minima for the two spin orientations in the presence of an external magnetic field. The two eigenvalues $E_\pm$ of the Hamiltonian (6), $E_\pm = \frac{1}{2}m^*[v_x^2 + (v_y \pm v_{0y})^2] - \Delta' \mp \frac{1}{2}\Delta$, correspond to the upper $+$ ($\boldsymbol{\mu} = \boldsymbol{\mu}_{||}$, parallel to $\mathbf{B}_0$) and lower $-$ ($\boldsymbol{\mu} = -\boldsymbol{\mu}_{||}$ antiparallel to $\mathbf{B}_0$) branches of the spectrum ($v_{0y} = |\frac{\alpha}{\hbar\mu_B}\mathbf{n}\times\boldsymbol{\mu}_{||}|\sin\theta$, $\Delta' = \frac{1}{2}m^*v_{0y}^2$, $\Delta = gB_0\mu_{||}$). The annulus indicated in red color in momentum space corresponds to spin-unpaired electron states.

$$P^{(M)}_\omega = (\omega/2)\mathrm{Im}(\mathbf{B}^*_{R\omega}\mathbf{M}_\omega), \tag{23a}$$

which has a simple form[21] in a rotating coordinate frame:

$$P^{(M)}_\omega = (\omega/2)[\chi''_+(\omega)\,|B_{R\omega+}|^2 + \chi''_-(\omega)\,|B_{R\omega-}|^2], \tag{23b}$$

where $\chi''_\pm$ denotes the imaginary part of $\chi_\pm$ ($+$ corresponds to the ordinary electron spin resonance).

In a static field, the mean values of the precession amplitudes of individual electron spins vanish. A finite precession of the magnetization results from an rf field with a frequency close to the Larmor frequency. Traditionally an rf magnetic field B_1 is used to excite spin precession. Rashba coupling causes, however, that the rf Rashba field can additionally induce magnetization precession, that is, a coherent precession of individual spins.

For weak or moderate Rashba coupling, and typical electric field strengths applicable in experiments, the momentum current $\mathbf{j}^{(p)}_\omega$ can be well approximated by the total current, as described by Eq. (20). The maximum value of the Rashba velocity is negligible in comparison to the drift velocity. In particular, for a 2DEG in Si/SiGe structures where $v_R = 4$ m/s and the electron mobility $\mu \cong 30$ m^2/Vs, the drift velocity exceeds the Rashba velocity already for $E_1 \geqslant 2$ V/m, while the microwave field in the microwave cavity is of the order of $E_1 \cong 10^4$ V/m at a microwave power of 1 μW.

On the other hand, for a high mobility, the Rashba field induced by the electric component of a microwave [Eq. (12)] is by orders of magnitude bigger than the amplitude of the microwave magnetic field. For the listed values of μ and v_R the Rashba field is $B_{R\omega} \cong 7\times10^3 B_1$. This comparison shows that even for the Si/SiGe case, where SO coupling is very weak as compared to GaAs or InSb, the current induced ESR excitation is much more effective than magnetic dipole excitation.[3,5,11] Therefore, for high mobility electrons, the most effective excitation of spin resonance takes place when the sample is placed in the maximum of E_1, that is, at the node of B_1 in an ESR microwave cavity.

Here $\mathbf{F}_{\omega}^{(\mathrm{SO})}$ [see Eq. (11)] is proportional to the precession amplitude $\boldsymbol{\mu}_{\omega}$. This amplitude of the transverse magnetization is induced by the transverse component of the rf magnetic field $\boldsymbol{\mu}_{\omega\perp} = (\delta n_e)^{-1}\hat{\chi}(\omega)\mathbf{B}_{\omega\perp}$. The real (= dispersive) component $\chi'(\omega)$ describes $\mu_{\omega\perp}$ which occurs in phase with $B_{\omega\perp}$ and the imaginary (= absorption) component $\chi''(\omega)$ corresponds to the magnetization shifted by $\pi/2$ in phase. The power transferred to the magnetic system, given by Eq. (23b), is described by the imaginary component only.

In contrast, the experimentally observed spectrum[5,11] of the electric absorption, as described by Eq. (21), is proportional to a sum of imaginary and real components of $\hat{\chi}(\omega)$. If the sample is placed at the node of B_1, the amplitude of B_ω is dominated by the current induced Rashba field, as described by Eq. (12). It is proportional to the local electric field, expressed by Eq. (18). The amplitudes of the real and imaginary contributions to the CI ESR spectrum, A'_{SO} and A''_{SO}, which stand for coefficients of the real and imaginary components of $\chi(\omega)$, are obtained from the set of Eqs. (11), (12), (14), (18), and (21). They scale with α^2 and the in-plane component of $E^2\sin^2\theta$ (θ is the angle between $\mathbf{n}$ and $\mathbf{E}$, $\theta = \pi/2$ corresponds to in-plane orientation of $\mathbf{E}$). The ratio of the two amplitudes results from a complex interplay of the phase shifts between $\mathbf{E}_1(t)$, $\mathbf{j}(t)$, $\mathbf{B}_R(t)$, $\boldsymbol{\mu}(t)$, and, finally, $\mathbf{F}^{(\mathrm{SO})}(t)$. The occurrence of a dispersive component in the observed absorption spectra was a long-standing puzzling problem.[21] Here it is shown to be a unique attribute of the CI ESR induced by a microwave electric field.

Both amplitudes of CI ESR spectra depend on the experimental geometry and on electron mobility in a complex way. An example of the dependence of the amplitudes on the sample orientation θ is shown in Fig. 3.

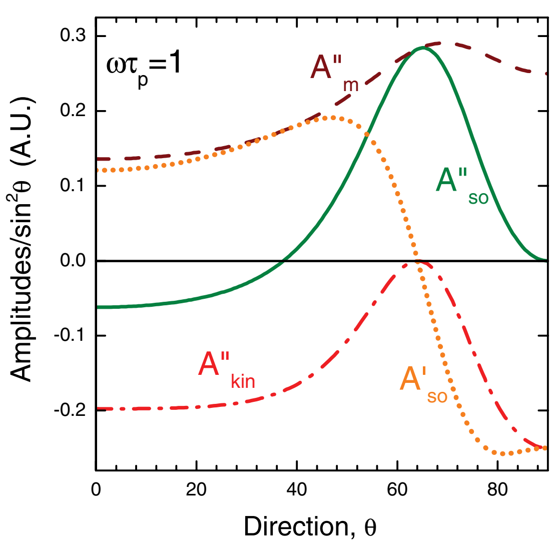

FIG. 3. (Color online) Angular dependence of the total dispersion amplitude A'_{SO} (dotted orange line), and the absorption parts $A''_{\mathrm{SO}} = A''_M + A''_{\mathrm{kin}}$ (solid green line) for $\omega\tau_p = 1$. (Dash-dotted line: A''_{kin}, dashed line: A''_M.) The amplitudes are normalized by $\sin^2\theta$.

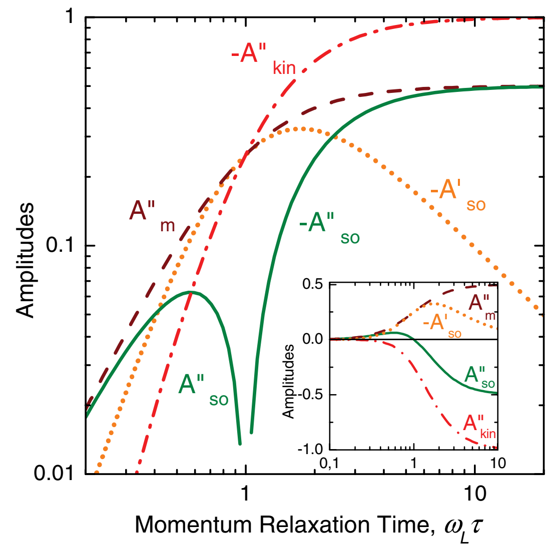

FIG. 4. (Color online) Dependence of the CI ESR amplitudes on momentum relaxation time, normalized by the Larmor frequency, for $\theta = \pi/2$. The solid (green) line stands for the total electric absorption amplitude and the dotted (orange) for the dispersion amplitude. The dashed (dark red) line describes the energy transfer to the magnetic energy and the dashed-dotted line (red) stands for the reduction of the Joule heat at spin resonance. Inset: dependencies in linear scale, demonstrating the signs of all amplitudes.

For $\omega\tau \cong 1$ both amplitudes are of the same order of magnitude. For such a high mobility they are expected to change sign with θ. A peculiarity occurs at $\theta \cong 64^\circ$, where the 2D cyclotron frequency agrees with the Larmor frequency.

The dashed line in Fig. 4 represents the transfer of resonance energy to the magnetic energy A''_M as described by Eqs. (23a) and (23b). Remarkably, the amplitude A''_M is bigger than the total amplitude A''_{SO}. Comparing Eqs. (19) and (22) we can express the gain in magnetic power in excess of $P_{\omega}^{(\mathrm{SO})}$ as follows: $P_{\omega}^{(\mathrm{SO})} - P_{\omega}^{(M)} = E_{\omega}^{(\mathrm{kin})}/\tau - P_{\omega}^{(J)} \leqslant 0$.

As it is shown in Figs. 3 and 4 (in the latter all amplitudes are plotted as a function of the momentum relaxation time, proportional to the electron mobility), A''_{kin} is negative in the whole range of τ and for all directions θ.

As seen in Fig. 4, in the low frequency limit all components of CI ESR absorption vanish. The dispersive component vanishes faster, with $\omega^2\tau^3$, than the absorption one, with $\omega\tau^2$. Consequently, the resonance line shape for low mobility is expected to be a classical absorption line $\chi''(\omega)$, while a dispersive component occurs for high mobility only. The difference $A''_{\mathrm{kin}} = A''_{\mathrm{SO}} - A''_M$ increases with $\omega^3\tau^4$, being thus negligible in the low mobility limit. It becomes effective only for $\omega\tau \geqslant 1$.

V. CONCLUSIONS

In conclusion, we have analyzed the complex nature of electric currents in the presence of Rashba type SO coupling. We consider the coherent interaction of an electron with an oscillating Rashba field induced by an electric rf perturbation. We show that the oscillatory parts of the electron velocity and the electron spin vary coherently with electric perturbation, independently of random electron collisions with crystal imperfections. Since the conductivity tensor is used in the description of the electron motion, the cyclotron motion and random scattering (described by the relaxation time τ) are taken into account. The spin oscillations at resonance frequency are described by the magnetic susceptibility tensor which depends on the transverse relaxation time T_2 (containing information of the spin interaction with crystal imperfections) which, in agreement with experiment, is assumed to be much longer than momentum relaxation time τ.

In spite of the fact that the electron velocity is a sum of two components of very different character and characterized by distinct relaxation mechanisms, the electric current can be fully described by a sum of a classical current and an additional component caused by the RSO force, both ruled by Ohm's law with a classic conductivity tensor. Since the RSO force is proportional to the time derivative of magnetization, the effect vanishes for static fields and a significant effect occurs for rf electric fields with a frequency close to the Larmor frequency. The formalism presented well describes details of current induced ESR, a powerful tool for effective spin manipulation.[11,14] In particular, it explains a specific resonance line shape and resonance amplitude and their complex dependencies on experimental geometry and electron mobility.[21]

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