Decoherence of a qubit due to either a quantum fluctuator, or classical telegraph noise

Henry J. Wold,[1] Håkon Brox,[1] Yuri M. Galperin,[1,2,3] and Joakim Bergli[1]
[1]*Department of Physics, University of Oslo, P.O. Box 1048 Blindern, 0316 Oslo, Norway*
[2]*Centre for Advanced Study, Drammensveien 78, Oslo, Norway 0271, Oslo, Norway*
[3]*A. F. Ioffe Physico-Technical Institute of Russian Academy of Sciences, 194021 St. Petersburg, Russia*
(Received 11 June 2012; published 5 November 2012)

We study the domain of applicability of the classical telegraph noise model in the study of decoherence in qubits. We investigate the decoherence of a qubit coupled to either a quantum fluctuator, a quantum two-level system (TLS) again coupled to an environment, or to a classical fluctuator modeled by random telegraph noise. In order to do this, we construct a model for the quantum fluctuator where we can adjust the temperature of its environment, and the decoherence rate independently. The model has a well-defined classical limit at any temperature and this corresponds to the appropriate random telegraph process, which is symmetric at high temperatures and becomes asymmetric at low temperatures. We find that the difference in the qubit decoherence rates predicted by the two models depends on the ratio between the qubit-fluctuator coupling and the decoherence rate in the pointer basis of the fluctuator. This is then the relevant parameter, which determines whether the fluctuator, has to be treated quantum mechanically or can be replaced by a classical telegraph process. We also compare the mutual information between the qubit and the fluctuator in the classical and the quantum model.

DOI: 10.1103/PhysRevB.86.205404 PACS number(s): 03.65.Yz, 03.67.Lx, 03.67.Bg, 74.78.Na

I. INTRODUCTION

The interaction between a quantum system and its environments leads to loss of quantum coherence, or decoherence, in the system. Understanding decoherence is crucial for grasping the boundary between quantum and classical physics.[1–4] It is also essential for testing theories describing quantum measurements.[5–8]

From an engineering point of view, the decay of coherence in quantum bit devices (qubits) is the most important obstacle for constructing a working quantum computer. Solid state qubits are leading candidates in the projects of designing quantum circuits, where the coherence times of the qubits are required to be sufficiently long to allow for manipulations and transfer of information by logical gates. The most important source of decoherence in many realizations of solid state qubits are believed to be bistable fluctuators— two-level systems (TLSs), present as tunneling states in the amorphous substrate[9,10] used to fabricate the qubit, or in the tunneling junction in superconductor-based devices.[11–18]

These fluctuators are quantum-mechanical systems that are, in turn, coupled to their own environments, which are conventionally considered as uncorrelated thermal baths. Usually, one does not worry about the fine details of the environment of the fluctuators, but rather uses simplified models. The most popular is the Bloch-Redfield approach,[19] where the environment is taken into account by introduction of the relaxation and decoherence rates of the fluctuators. If the fluctuators couple more strongly to their own environment than to the qubit, they are usually treated classically. This means that the dynamical description of the quantum fluctuator is replaced by a classical dynamics of a fluctuating system, which switches randomly between its two metastable states according to a random telegraph process (RTP).[20,21] This approach is often referred to as the spin-fluctuator model.[11,18,22] In many cases, however, the decoherence of the qubit is determined by only a few fluctuators that are more strongly coupled to the qubit than others.[23–27] In such cases, one might question the validity of the classical model. From a practical point of view, it is therefore important to know when such a simplified classical description can replace the full quantum mechanical one. It is also of more fundamental interest in view of the decoherence approach to the quantum-classical transition.[1–4]

In this paper, we will develop a simple model allowing to show when a quantum system can in practice be replaced by a classical one, in the sense that interference effects can no longer be observed due to the entanglement with the environment. However, we believe that this is only a question of a system becoming *in practice* classical, i.e., when we can use a classical model to calculate a physical property of a quantum system. It does not directly shed any light on the fundamental limitations of quantum mechanics, in particular, the measurement problem, where one can discuss deviations from linear quantum mechanics, see Ref. 5 for a discussion.

Previously, the boundary between quantum and classical regime for the fluctuator has been explored in a model where the qubit is coupled to an impurity state, and an electron can tunnel between this state and an electron reservoir (metal).[28,29] The same model has also been used in order to study the effect of Coulomb interaction between the charged impurities and the reservoir electrons.[30]

The qubit dephasing rate calculated in the quantum model was found to converge to the classical result in the high-temperature limit. In the study by Abel and Marquardt,[29] a threshold for strong coupling between the qubit and the fluctuator was defined by the onset of visibility oscillations in the qubit as a function of the ratio between the coupling to the qubit and the reservoir. The threshold for visibility oscillations was found for higher values of the qubit coupling in the quantum model compared to the classical model, the thresholds finally converge at high T/γ, where γ is the fluctuator-reservoir coupling. Thus both in the decoherence rate and in the visibility oscillations the classical limit is recovered at high temperature. In this model, the temperature plays a dual role: it affects both the energy relaxation rate of the fluctuator, which maps to the switching rate of the RTP, and it affects the dephasing rate of the fluctuator. The usefulness of separation of the two effects is seen by the fact that it is perfectly possible to consider finite-temperature classical fluctuators by using an asymetric RTP.[31,32] This is never obtained in any limit of the model discussed in Refs. 28 and 29.

The subsequent considerations are based on the following qualitative picture: the dephasing of the qubit is caused by the generation of entanglement between the qubit and the environment. If the qubit and the fluctuator are strongly coupled, then they behave as a combined four-level quantum system and the quantum nature of the fluctuator will be important. In such a situation, one cannot replace it by a classical RTP. On the other hand, if the fluctuator is sufficiently strongly coupled to the environment, it means that the information about its state is continuously transferred to the environment and this prevents any quantum interference to take place. From this, we can guess that the relevant quantity determining whether the fluctuator can be considered either classical or quantum is the ratio of the qubit-fluctuator coupling (which determines the rate of entanglement generation between the qubit and the fluctuator) and the fluctuator dephasing rate.

The goal of this paper is to study the applicability of the classical model for qubit decoherence due to a quantum fluctuator. In order to achieve this, we study a model where the dephasing rate of the fluctuator can be varied independently of the temperature, so that the classical limit can be taken at any temperature and correspond to the proper assymetric RTP. By use of a model borrowed from the study of fluctuators in glasses, but where we allow for more freedom in the choise of parameters than we find in typical glasses, we compare the pure decoherence rate of the qubit subject to either a quantum fluctuator, in turn coupled to its environment, or a classical fluctuatur, modeled by random telegraph noise. Our model allows us to separate the effects of temperature, coupling to the bath, and decoherence rate of the fluctuator. We find that the difference in the qubit decoherence rate predicted by the quantum model and the classical one depends on the ratio, $\xi/\bar{\gamma}_2$, where ξ is the qubit-fluctuator coupling strength and $\bar{\gamma}_2$ is the decoherence rate of the fluctuator in the pointer basis.

II. MODEL

A. Quantum model for the fluctuator

We start by describing the quantum-mechanical model for the fluctuator. The model we use for the fluctuator originates in the study of tunneling states in glasses, i.e., a particle, or a group of particles that can be approximated by a single configurational coordinate in a double-well potential.[33] It gives rise to a potential on the qubit that depends on its position in the double well.

Following Refs. 9, 10, and 33, the Hamiltonian for the coupled qubit fluctuator is split into the Hamiltonians H_q for the qubit, H_f for the fluctuator, H_i for the qubit-fluctuator interaction, H_e for the environment and H_{fe} for the fluctuator-environment interaction:

$$H = H_q + H_f + H_i + H_e + H_{fe}, \quad H_q = E_q\tau_z,$$
$$H_f = (1/2)(\Delta\sigma_z + \Delta_0\sigma_x), \quad H_i = (1/2)\xi\tau_z\sigma_z, \qquad (1)$$

where the Pauli matrices τ_α and σ_α are operators in the Hilbert spaces of the qubit and the quantum fluctuator, respectively.

The energy splitting Δ and the tunnel amplitude Δ_0 can be calculated from the shape of the double-well potential.[33] The energy of the qubit depends on the position of the particle in the double well (we will in the following refer to the eigenstates of σ_z as the position basis) and the coupling strength is given by ξ. In this work, we will assume the simplified case where the qubit does not directly interact with the environment and therefore has no intrinsic dynamics in the absence of the fluctuator. Furthermore, we consider a model where the qubit is subject to pure dephasing, $[H_q, H_i] = 0$, there is no energy relaxation of the qubit in this model and the decoherence of the qubit is therefore insensitive to the qubit energy splitting E_q. When energy relaxation is present, coherent beatings between the qubit and resonant fluctuators are observed.[23,34] In this strong coupling regime, the fluctuator has to be treated as a quantum system. Our present work concentrates solely on nonresonant fluctuators, which are typically modeled classically.

The double-well potential is, in general, perturbed by electromagnetic and strain fields modifying the asymmetry energy Δ, while perturbations of the barrier height can usually be ignored.[35–37] In our model, we therefore assume that the environment couples to the fluctuator in the position basis, i.e., the eigenbasis of σ_z. Rather than formally specifying H_e and H_{fe} we take the freedom to consider two kinds of interaction between the fluctuator and the external environment, resonant and nonresonant. We will later in addition also use parameters for the fluctuator-environment coupling that are outside what we typically encounter in glasses. This is done in order to have more freedom to tune the parameters that are relevant to study the domain of applicability of the RTP model.

The resonant phonons creates a strain field u_{ik} that modifies the double-well potential of the TLS as follows:

$$\Delta = \Delta^{(0)} + \lambda_{ik}u_{ik}, \quad \Delta_0 = \text{const},$$

where $\Delta^{(0)}$ is the energy splitting in the absence of the strain field and λ_{ik} is the deformation potential of the fluctuator. In the energy basis of the fluctuator, this interaction creates two terms:

$$\left(\frac{\Delta}{\sqrt{\Delta^2+\Delta_0^2}}\tilde{\sigma}_z + \frac{\Delta_0}{\sqrt{\Delta^2+\Delta_0^2}}\tilde{\sigma}_x\right)\lambda_{ik}u_{ik},$$

where $\tilde{\sigma}_z$ and $\tilde{\sigma}_x$ act in the energy eigenbasis of the fluctuator.

The first term will give rise to pure dephasing of the fluctuator, while the second gives rise to relaxation. We will in the following assume that the rate of resonant phonons is small compared to the nonresonant ones, and that the contribution to pure dephasing given by the first term can be neglected. Resonant interaction, e.g., phonons with frequency close to the eigenfrequency of the fluctuator, are therefore responsible for direct transitions between the eigenstates of the fluctuator, $|\psi_g\rangle$ and $|\psi_e\rangle$.

We model this interaction by use of the generalized measurement operators defined for a small time step Δt as[38]

$$
\begin{aligned}
M_1(\Delta t) &= \sqrt{\gamma_{\mathrm{ab}}(T)\Delta t}\; I\otimes\sigma_x|\psi_g\rangle\langle\psi_g|,\\
M_2(\Delta t) &= \sqrt{\gamma_{\mathrm{em}}(T)\Delta t}\; I\otimes\sigma_x|\psi_e\rangle\langle\psi_e|,\\
M_3(\Delta t) &= \sqrt{1-M_1^\dagger M_1 - M_2^\dagger M_2}\,.
\end{aligned}
\tag{2}
$$

Here, I is the identity matrix in the Hilbert space of the qubit and the matrices $\sigma_x|\psi_{g(e)}\rangle\langle\psi_{g(e)}|$ "measures" whether the fluctuator is in the ground (excited) state, projects the fluctuator onto this state and flips it. The rates for absorbtion and emission are

$$
\begin{aligned}
\gamma_{\mathrm{ab}}(T) &= \gamma_1 N(E) = \frac{\gamma_1}{e^{E/T}-1},\\
\gamma_{\mathrm{em}}(T) &= \gamma_1[N(E)+1] = \frac{\gamma_1}{1-e^{-E/T}}.
\end{aligned}
\tag{3}
$$

Here, T is the temperature, $N(E)=(e^{E/T}-1)^{-1}$ is the Planck distribution, and $E=\sqrt{\Delta^2+\Delta_0^2}$ is the energy splitting of the fluctuator. The nonresonant interaction does not cause transitions between the eigenstates of the fluctuator. However, we might assume that, in general, the state of a phonon interacting with the fluctuator is perturbed by the interaction, and that the perturbation depends on the position of the system in the double well. These are assumed to be low-frequency phonons $\hbar\omega \ll E$, which does not significantly alter the level splitting of the fluctuator. Schematically, we can write

$$
|\psi_i\rangle|\phi_0^{\mathrm{ph}}\rangle \xrightarrow{t} |\psi_i\rangle|\phi_i^{\mathrm{ph}}\rangle, \tag{4}
$$

where $i\in\{0,1\}$ index the state of the fluctuator in the position basis, $|\phi_0^{\mathrm{ph}}\rangle$ is the initial state of the phonon and $|\phi_i^{\mathrm{ph}}\rangle$ is the state of the phonon after the interaction, conditioned upon that the fluctuator was initially in the state indexed by i. The interaction (4) results in entanglement between the phonon and the fluctuator, reducing the coherence of the latter. The rate of decoherence due to nonresonant phonons depends on the overlap element $\alpha=\langle\phi_0^{\mathrm{ph}}|\phi_1^{\mathrm{ph}}\rangle$ and on the rate of phonons interacting with the system. We model this interaction by the single parameter γ_2, which is responsible for the decay rate of the off-diagonal density matrix elements of the fluctuator in the position basis.

In this model, we effectively adjust the nature of H_{fe} by the ratio Δ_0/Δ. Therefore the equilibrium density matrix of the fluctuator will not necessarily lie along the z axis of the Bloch sphere. The equilibrium density matrix is determined by the rate γ_2 due to nonresonant phonons responsible for decay perpendicular to the z axis on the Bloch sphere and by relaxation to the thermal level along the z' axis in the eigenbasis of the fluctuator induced at the rate γ_1 by resonant phonons.

Note also that differences in the qubit decoherence between the quantum and the classical model is not observed when the z' axis is parallel with the z axis. The situation is illustrated in Fig. 1. We define the decoherence rate of the fluctuator, $\bar\gamma_2$, by the rate at which the off-diagonal density matrix elements decay in the basis where the density matrix is diagonal in equilibrium.

The time evolution in the quantum model is obtained by numerical integration of the von Neumann equation for the Hamiltonian given by Eq. (1), with two modifications. We add

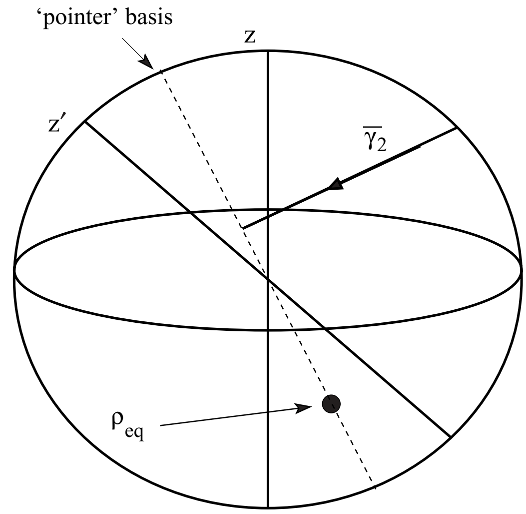

FIG. 1. The Bloch sphere for the fluctuator coupled to both nonresonant and resonant phonons. The nonresonant phonons are responsible for decay perpendicular to the z axis, the eigenbasis of σ_z, while the resonant phonons are responsible for relaxation parallell to the z' axis, which is the eigenbasis of the fluctuator. We define the pointer basis by the basis in which the equilibrium density matrix ρ_{eq} is diagonal. The rate of decay perpendicular to this axis is denoted by $\bar\gamma_2$.

a damping term γ_2 to our differential equation:

$$
\dot\rho_{\alpha\alpha'} = i\langle\alpha|[\rho,H]|\alpha'\rangle - \Lambda_{\alpha\alpha'}\rho_{\alpha\alpha'}, \tag{5}
$$

where ρ is the density matrix of the system composed of the qubit and the fluctuator and $\Lambda=\gamma_2 I\otimes\sigma_x$, which determines the decay of the off-diagonal density matrix elements of the fluctuator in the eigenbasis of σ_z. In addition, the fluctuator absorbs and emits phonons at the rates $\gamma_{\mathrm{ab}}(T)$ and $\gamma_{\mathrm{em}}(T)$. The absorption and emission of phonons is implemented as follows: for each time step Δt, we make a transformation to the eigenbasis of the fluctuator,

$$
\bar\rho = R(\theta)\rho R^\dagger(\theta), \tag{6}
$$

using the rotation matrix

$$
R(\theta) = I\otimes\begin{pmatrix}\cos\frac{\theta}{2} & \sin\frac{\theta}{2}\\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix},\quad \theta\equiv\arctan\left(\frac{\Delta_0}{\Delta}\right).
$$

The density matrix is then updated according to the rates of absorption and emission as

$$
\bar\rho' = M_1\bar\rho M_1^\dagger + M_2\bar\rho M_2^\dagger + M_3\bar\rho M_3^\dagger, \tag{7}
$$

before we make the inverse transform $\rho' = R^\dagger(\theta)\bar\rho' R(\theta)$, back to the position basis. Here, ρ' is the density matrix after the (potential) interaction with the resonant phonons.

B. Classical telegraph noise

Pure dephasing of the qubit by a classical telegraph noise can be described by the interaction Hamiltonian

$$
H_i = (1/2)\xi(t)\tau_z, \tag{8}
$$

where $\xi(t)=\pm\xi$ is the position of the fluctuator at time t. For details on this model see, e.g., Ref. 39 and references therein.

The probability for the fluctuator to switch from the state ξ_- to ξ_+, and from ξ_+ to ξ_- in the interval dt is given by $\Gamma_{-+}dt$ and $\Gamma_{+-}dt$, respectively. To describe finite temperature, we will consider the situation where the flipping rates Γ_{-+} and Γ_{+-} of the fluctuator are, in general, not identical, but the states are symmetric $\xi_- = -\xi_+$. The situation with asymmetric switching rates was previously studied in Refs. 31 and 32. The equilibrium average is given by

$$\langle \xi \rangle = \xi (p_+^{\mathrm{eq}} - p_-^{\mathrm{eq}}) = \xi(\Gamma_{-+} - \Gamma_{+-})/\Gamma, \tag{9}$$

where

$$\Gamma = \Gamma_{-+} + \Gamma_{+-}, \tag{10}$$

and $p_\pm(t)$ is the probability for the fluctuator to be found in the state $\xi_\pm$. The relaxation towards equilibrium is exponential with rate Γ.

The decoherence of the qubit is obtained by averaging over the realizations and initial conditions of the noise process $\xi(t)$. For a given realization of $\xi(t)$, the Schrödinger equation yields a superposition of the eigenstates of the qubit with a contribution to the relative phase $\phi(t) = \int_0^t \xi(t')dt'$. Averaged over the realizations of the stochastic process $\xi(t)$, we obtain the qubit coherence $D(t) = \langle e^{i\phi(t)} \rangle$. Here, we will use the transfer matrix method developed by Joynt *et al.*,[40] where we obtain directly the ensemble averaged Bloch vector of the qubit.

The state of the qubit-fluctuator system can be stored in the six-dimensional vector

$$\vec{q}(t) = \vec{m}_+(t) \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} p_+(t) + \vec{m}_-(t) \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} p_-(t), \tag{11}$$

where $\vec{m}_\pm$ is the Bloch vector of the qubit conditioned upon the state $\xi_\pm$ of the fluctuator. The propagator for $\vec{q}$ averaged over the individual realizations of the RTP can be expressed as $A(t) = e^{-Bt}$, where

$$B = I_3 \otimes V - i\frac{\xi}{2} L_z \otimes \upsilon_z, \quad V = \begin{pmatrix} \Gamma_{+-} & -\Gamma_{-+} \\ -\Gamma_{+-} & \Gamma_{-+} \end{pmatrix},$$

while I_3 and L_z are generators of the SO_3 group and υ_Z is the Pauli matrix. A direct advantage of this approach is that the qubit state conditioned upon whether the fluctuator is in the state $\xi_\pm$, $\rho_q^\pm$ follows directly from $\vec{q}$.

III. RESULTS

In order to compare the decoherence of the qubit subject to either the quantum fluctuator, or the classical telegraph noise, we calculate similar relaxation rates towards the equilibrium level in the two models. First, we choose a set of parameters, Δ, Δ_0, γ_1, γ_2, and T for the quantum model and prepare the fluctuator in the initial state $|\psi_1\rangle$. At this preliminary stage, we are not interested in the qubit and consider the fluctuator and its environment decoupled from the qubit. We compute numerically the equilibrium occupation probabilities p_0^{eq} and p_1^{eq} of the quantum fluctuator in the position basis as well as the relaxation rate Γ. Note that both the equilibrium occupations and the relaxation rate are, in general, complicated functions of all the parameters in our model. In this work, we always restrict ourselves to the regime where the fluctuator is overdamped $\Delta, \Delta_0 \ll \gamma_2$, i.e., the decoherence rate is sufficiently large such that coherent oscillations are not observed in the fluctuator. In this regime, the decay of the fluctuator towards its equilibrium value can be fitted to a simple exponential. Beyond this regime, the fluctuator behave as a quantum system, and can therefore not be modeled by the classical telegraph process. Note also that since the states $|\psi_i\rangle$ are not eigenstates of the Hamiltonian, the occupation numbers p_i^{eq} are not given by the Boltzmann weights at the bath temperature.

The decoherence rate is expressed through the rates $\Gamma_{\pm\mp}$ and the equilibrium occupancy $\langle \xi \rangle$ with the help of Eqs. (9) and (10). The qubit decoherence rate is, in general, a sum over multiple rates. For symmetric telegraph noise and pure dephasing, the decay of coherence in the qubit $D(t)$ is given by[39]

$$D(t) = \frac{e^{-\Gamma t/2}}{2\mu}[(\mu+1)e^{\Gamma\mu t/2} + (\mu-1)e^{-\Gamma\mu t/2}], \tag{12}$$

where $\mu \equiv \sqrt{1-(2\xi/\Gamma)^2}$. However, in the regime where the coupling to the qubit is weak compared to the damping of the fluctuator, $\Gamma > \xi$, the long-time behavior of the decoherence is strongly dominated by a single rate,

$$\Gamma_q^c = \Gamma(1-\mu)/2.$$

We finally compute the decoherence rate Γ_q^q of the qubit when it is coupled to the same quantum fluctuator from which we calculated the relaxation rate and equilibrium occupations previously, but this time the initial state of the fluctuator is the thermal equilibrium state. The decoherence rate of the qubit is calculated by numerical simulation of the coupled qubit-fluctuator density matrix $\rho(t)$ from which we can find the qubit density matrix by tracing out the degrees of freedom of the quantum fluctuator. From the qubit density matrix, $\rho^q(t) = \mathrm{Tr}_f[\rho(t)]$, we find the coherence $|\rho^q_{\uparrow\downarrow}(t)|$, where $\uparrow$ and $\downarrow$ denote the eigenstates of the qubit. Finally, the long-time behavior of $|\rho^q_{\uparrow\downarrow}(t)|$ is fitted to the exponential function $e^{-\Gamma_q^q t}$. Note that the initially $|\rho^q_{\uparrow\downarrow}(t)|$ might have contributions from several rates, like in the classical model (12). Note also that in the regime where the fluctuator is near resonant with the qubit, these two systems need to be treated as a four-level system, and the dynamics is characterized by four distinct rates. This regime was studied in Ref. 41 in order to characterize the effect of coherent impurities on the qubit.

The relative difference in the decoherence rate of the qubit due to classical telegraph noise and the quantum fluctuator is defined as

$$\delta\Gamma_q = \left(\Gamma_q^q - \Gamma_q^c\right)/\Gamma_q^c, \tag{13}$$

where Γ_q^q and Γ_q^c are the decoherence rate of the qubit subject to the quantum fluctuator and to the classical telegraph noise, respectively. This quantity is presented in Fig. 2 as a function of the dephasing rate of the fluctuator γ_2 and temperature T. We have restricted ourselves to a parameter range where the fluctuator does not undergo coherent oscillations. It is evident that the relative difference in the qubit decoherence rate is small for strong decoherence of the fluctuator, and for high temperatures. In this case, we can safely use the simple RTP model rather than the much more complicated quantum model. Superimposed on the contours for $\delta\Gamma_q$, we have plotted curves where the ratio $\xi/\bar{\gamma}_2$ is constant. We find that the

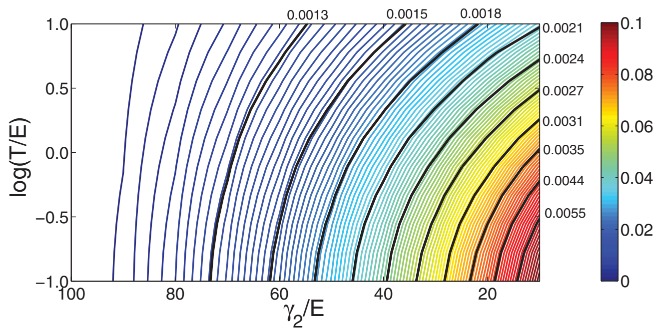

FIG. 2. (Color online) Contour plot of the relative difference $\delta\Gamma_q$ in the decoherence rate of the qubit subject to either classical telegraph noise, or a quantum fluctuator. In units of the fluctuator energy splitting E, the parameters of the quantum fluctuator are $\Delta = \Delta_0 = 1/\sqrt{2}$ and $\gamma_1 = 1.0$, the coupling to the qubit is $\xi = 0.1$. Color coding for $\delta\Gamma_q$ is shown on the right. The relaxation rate to equilibrium along the σ_z axis is the same for both the quantum and the classical fluctuator. Contours where the ratio $\xi/\bar{\gamma}_2$ is constant are plotted for comparison (black lines).

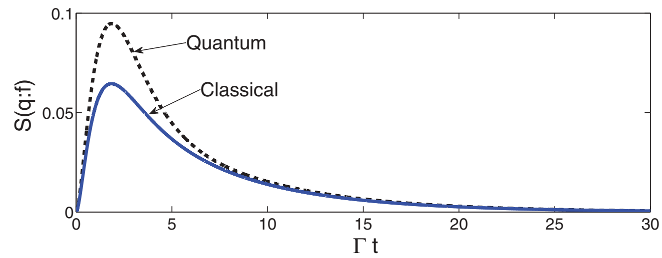

FIG. 3. (Color online) Mutual information $S(q : f)$ for the qubit coupled to the quantum fluctuator (black, dashed) and the qubit subject to the classical spin fluctuator (blue, solid). The mutual information is larger when both systems are treated as quantum objects, due to quantum entanglement between the two systems. In this simulation the parameters, in units of E, are $\xi = 0.1$, $\Delta = \Delta_0 = 1/\sqrt{2}$, $\gamma_1 = 1.0$, $\gamma_2 = 20$, and $E/T = 1.0$.

difference between the quantum and the classical fluctuator depends to a very good accuracy on the ratio $\xi/\bar{\gamma}_2$. Note that we have numerically checked that the dependence of the qubit decoherence on the parameter $\xi/\bar{\gamma}_2$ holds also in the regime where $\xi > \Gamma$ confirming that the RTP model can be applied in the strong-coupling regime also in the case when the qubit couples strongly to the fluctuator, as long as $\xi \ll \bar{\gamma}_2$. The requirement $\Gamma > \xi$ is only needed to ensure that the qubit decoherence follows a simple exponential law.

When the qubit is put in contact with the quantum fluctuator, the qubit and the fluctuator will in general entangle due to their coupling. The mutual information, the information about the state of one of the systems that can be inferred by measuring the other, will for the quantum fluctuator have an entanglement contribution in addition to the classical correlation.

The mutual information for the qubit-quantum fluctuator is defined straightforwardly by the von Neumann entropy[38]

$$S(q : f) = S(\rho_q) + S(\rho_f) - S(\rho_{qf}), \tag{14}$$

where ρ_q, ρ_q, and ρ_{qf} are the density matrices of the qubit, the fluctuator, and the composite system, respectively. When we treat the qubit subject to a classical telegraph noise, we introduce quantum states $|\pm\rangle$ corresponding to the states $\xi_\pm$ of the RTP and use the formula

$$\rho_{qf} = p_+\rho_q^+\rho_{f+} + p_-\rho_q^-\rho_{f-}. \tag{15}$$

Here, $p_\pm$ is the probability for the telegraph process to be found in the state $\xi_\pm$, $\rho_q^\pm$ is the density matrix of the qubit conditioned upon that the telegraph process is in the state $\xi_\pm$ and $\rho_{f\pm} = |\pm\rangle\langle\pm|$.

The time evolution of the mutual information for a qubit coupled either to the quantum or the classical fluctuators is shown in Fig. 3. The entanglement between the two systems builds up at a rate given by the coupling ξ but is lost to the environment at a rate given by the decoherence rate of the quantum fluctuator, $\bar{\gamma}_2$. The increased information about the qubit encoded in the quantum fluctuator, compared to the classical fluctuator, increases the transfer of entropy to the environment, thus increasing the decoherence rate of the qubit. This effect might explain the positive $\delta\Gamma_q$ found for low values of T and γ_2. It has been stated, see, e.g., Refs. 42 and 43, that there exist situations where increased information transfer decreases the decoherence rate of the qubit. However, we are not sure that the information transfer is reduced in the particular system discussed in Refs. 42 and 43.

Experimentally, since the composite density matrix ρ_{qf} is required, the mutual information can only be extracted in the case where one has access to measurement on both the qubit and the fluctuator simultaneously. Since the fluctuator by definition is a system of the environment outside our control, this cannot be achieved. However, the mutual information could potentially be studied in two coupled qubits, where one of the qubits are subject to controlled noise and takes the role of the fluctuator. Qubits subject to engineered noise under the control of the experimentalist has been realized in optically trapped $^9Be^+$ ions,[44] where also the required quantum gates has already been implemented in a similar systems.[45]

IV. DISCUSSION

In general, the dynamics of the quantum fluctuator in an environment depends on three parameters; the relaxation rate γ_1, the dephasing rate γ_2 and the temperature T determining the equilibrium occupations. In this paper, we use a model where the processes responsible for pure dephasing couple to the position basis, while the relaxation processes take place in the eigenbasis of the fluctuator. This model was used in order to study the relevance of the classical RTP model for description of decoherence of a qubit. If the interaction responsible for pure dephasing processes in the fluctuator (characterized by γ_2) commutes with the qubit-fluctuator Hamiltonian, i.e., $\Delta_0 = 0$ in our model, then the pure dephasing rate γ_2 will not have any effect on the decoherence rate of the qubit as long as the fluctuator is prepared in the thermal equilibrium state. The quantum fluctuator will in this case *always behave as a classical fluctuator* and can therefore straightforwardly be modeled by the classical telegraph noise.

In general, the difference in decoherence rate $\delta\Gamma$ depends on the ratio Δ_0/Δ in addition to the ratio $\xi/\bar{\gamma}_2$. We find that $\delta\Gamma$

increases monotonously as a function of the ratio Δ_0/Δ for $\Delta_0/\Delta \in [0,\pi/4]$ and that $\delta\Gamma = 0$ for $\Delta_0/\Delta = 0$. However, the contours of constant $\xi/\bar{\gamma}_2$ in the $\ln T$ versus γ_2 plot, match those of constant $\delta\Gamma$ for all values of Δ_0/Δ.

Furthermore, we note that our results do not tell us that it is, in principle, not possible in ad hoc fashion to construct a classical telegraph model, e.g., a classical model with feedback, providing the same decoherence rate for the qubit as the quantum fluctuator, even in the regime where the deviation $\delta\Gamma_q$ between the two models are large according to Fig. 2. We show that the decoherence rate of the qubit differs in the two models in the case where the relaxation rates of the classical and quantum fluctuator are identical. To the best of our knowledge, there exist no general relationship between the quantum fluctuator model and the classical spin-fluctuator model. Therefore one should be careful in applying the classical telegraph model unless one expects the decoherence rates of the fluctuators to be much larger than the qubit-fluctuator coupling. $\xi/\bar{\gamma}_2 \ll 1$. However, in systems such as glasses this inequality is usually expected to hold, and the quantum fluctuator can be treated effectively by random telegraph noise,[33] with an exception if the system is subject to an external ac field.[46]

The pointer states of a quantum system are defined as the pure states that are the least affected by environmental decoherence.[1,3] It is generally believed that when the dynamics of the system is dominated by the interaction with the environment, the pointer states are the eigenstates of the interaction Hamiltonaian.[1] On the other hand, when the system is weakly coupled to the environment, the pointer states are assumed to be the eigenstates of the isolated system.[2] Our model can be considered to interpolate between the two extremes. If we define the pointer basis as the basis where the Bloch vector of the system lies along the z axis in equilibrium, the decoherence rate $\bar{\gamma}_2$ of the system is the rate of decay of the off-diagonal elements of the density matrix in this basis.

As a final note we mention that our main result, that the difference in decoherence rate of the qubit between the quantum fluctuator model and the telegraph noise model, might be model specific. Further work is needed in order to settle whether or not this result is universal.

In conclusion, we have constructed a model for the quantum fluctuator where we can study its effect on the qubit as a function of both the temperature and its decoherence due to its interaction with the environment. We have compared the decoherence rate of the qubit found in this model, and in the widely used classical telegraph noise model. We find that the difference in the qubit decoherence rates depends on the ratio $\xi/\bar{\gamma}_2$ between the strength of the qubit-fluctuator coupling and decoherence rate of the fluctuator in the pointer basis. In the limit $\xi/\bar{\gamma}_2 \ll 1$, the fluctuator behaves essentially classically and the qubit decoherence rate can accurately be predicted by the telegraph noise model. Our results validate the application of the RTP model for the study of decoherence in qubits also when the coupling between the qubit and the fluctuator is strong as long as the fluctuator couples even more strongly to its own environment.

This work is part of the master project of one of the authors (H.J.W.) and more details can be found in his thesis.[47]

ACKNOWLEDGMENTS

We acknowledge useful comments by S. Gurvitz, A. Aharony, O. Entin-Wohlman, and E. Paladino.

[1] W. H. Zurek, Phys. Rev. D **24**, 1516 (1981).
[2] J. P. Paz and W. H. Zurek, Phys. Rev. Lett. **82**, 5181 (1999).
[3] W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).
[4] M. Schlosshauer, Rev. Mod. Phys. **76**, 1267 (2005).
[5] A. J. Leggett, J. Phys.: Condens. Matter **14**, R415 (2002).
[6] J. van Wezel and T. H. Oosterkamp, Proc. R. Soc. A **468**, 35 (2012).
[7] A. Bassi and G. Ghirardi, Phys. Rep. **379**, 257 (2003).
[8] S. L. Adler and A. Bassi, Science **325**, 275 (2009).
[9] P. W. Anderson, B. I. Halperin, and C. M. Varma, Philos. Mag. **25**, 1 (1972).
[10] W. A. Phillips, J. Low Temp. Phys. **7**, 351 (1972).
[11] E. Paladino, L. Faoro, G. Falci, and R. Fazio, Phys. Rev. Lett. **88**, 228304 (2002).
[12] R. W. Simmonds, K. M. Lang, D. A. Hite, S. Nam, D. P. Pappas, and J. M. Martinis, Phys. Rev. Lett. **93**, 077003 (2004).
[13] J. M. Martinis, K. B. Cooper, R. McDermott, M. Steffen, M. Ansmann, K. D. Osborn, K. Cicak, S. Oh, D. P. Pappas, R. W. Simmonds *et al.*, Phys. Rev. Lett. **95**, 210503 (2005).
[14] Y. Shalibo, Y. Rofe, D. Shwa, F. Zeides, M. Neeley, J. M. Martinis, and N. Katz, Phys. Rev. Lett. **105**, 177001 (2010).
[15] L. Tian and R. W. Simmonds, Phys. Rev. Lett. **99**, 137002 (2007).
[16] A. Shnirman, G. Schön, I. Martin, and Y. Makhlin, Phys. Rev. Lett. **94**, 127002 (2005).
[17] Y. M. Galperin, B. L. Altshuler, and D. V. Shantsev, in *Fundamental Problems of Mesoscopic Physics*, NATO Science Series, Vol. 154 (Springer Netherlands, 2004), pp. 141–165.
[18] Y. M. Galperin, B. L. Altshuler, J. Bergli, and D. V. Shantsev, Phys. Rev. Lett. **96**, 097009 (2006).
[19] C. P. Schlichter, *Principles of Magnetic Resonance* (Springer-Verlag, Berlin, 1990).
[20] S. Kogan, *Electronic Noise and Fluctuations in Solids* (Cambridge Univ. Press, Cambridge, 1996).
[21] M. Kirton and M. Uren, Adv. Phys. **38**, 367 (1989).
[22] J. Bergli, Y. M. Galperin, and B. L. Altshuler, Phys. Rev. B **74**, 024509 (2006).
[23] R. W. Simmonds, K. M. Lang, D. A. Hite, S. Nam, D. P. Pappas, and J. M. Martinis, Phys. Rev. Lett. **93**, 077003 (2004).
[24] O. Astafiev, Y. A. Pashkin, Y. Nakamura, T. Yamamoto, and J. S. Tsai, Phys. Rev. Lett. **93**, 267007 (2004).
[25] R. H. Koch, D. P. DiVincenzo, and J. Clarke, Phys. Rev. Lett. **98**, 267003 (2007).
[26] Y. M. Galperin, D. V. Shantsev, J. Bergli, and B. L. Altshuler, Europhys. Lett. **71**, 21 (2005).

[27]C. Müller, A. Shnirman, and Y. Makhlin, Phys. Rev. B **80**, 134517 (2009).

[28]A. Grishin, I. V. Yurkevich, and I. V. Lerner, Phys. Rev. B **72**, 060509 (2005).

[29]B. Abel and F. Marquardt, Phys. Rev. B **78**, 201302 (2008).

[30]I. V. Yurkevich, J. Baldwin, I. V. Lerner, and B. L. Altshuler, Phys. Rev. B **81**, 121305 (2010).

[31]G. Falci, E. Paladino, and R. Fazio, in *Quantum Phenomena in Mesoscopic Systems*, Proceedings of the International School of Physics "Enrico Fermi", Vol. 151 (IOS Press Amsterdam, 2003), pp. 173–198.

[32]Y. Jung, E. Barkai, and R. J. Silbey, Chem. Phys. **284**, 181 (2002).

[33]W. A. Phillips, Rep. Prog. Phys. **50**, 1657 (1987).

[34]J. Lisenfeld, C. Müller, J. H. Cole, P. Bushev, A. Lukashenko, A. Shnirman, and A. V. Ustinov, Phys. Rev. Lett. **105**, 230504 (2010).

[35]A. C. Anderson, Phys. Rev. B **34**, 1317 (1986).

[36]W. A. Phillips, *Amorphous Solids: Low-Temperature Properties*, Vol. 24 (Springer, Berlin, 1981).

[37]Y. M. Galperin, V. G. Karpov, and V. I. Kozub, Adv. Phys. **38**, 669 (1989).

[38]M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).

[39]J. Bergli, Y. M. Galperin, and B. L. Altshuler, New J. Phys. **11**, 025002 (2009).

[40]B. Cheng, Q.-H. Wang, and R. Joynt, Phys. Rev. A **78**, 022313 (2008).

[41]E. Paladino, M. Sassetti, G. Falci, and U. Weiss, Phys. Rev. B **77**, 041303 (2008).

[42]A. Aharony, S. Gurvitz, O. Entin-Wohlman, and S. Dattagupta, Phys. Rev. B **82**, 245417 (2010).

[43]Y. Ye, Y. Cao, X.-Q. Li, and S. Gurvitz, Phys. Rev. B **84**, 245311 (2011).

[44]M. J. Biercuk, H. Uys, A. P. VanDevender, N. Shiga, W. M. Itano, and J. J. Bollinger, Phys. Rev. A **79**, 062324 (2009).

[45]D. Leibfried, B. DeMarco, V. Meyer, D. Lucas, M. Barrett, J. Britton, W. M. Itano, B. Jelenkovic, C. Langer, T. Rosenband *et al.*, Nature (London) **422**, 412 (2003).

[46]H. Brox, J. Bergli, and Y. M. Galperin, Phys. Rev. B **84**, 245416 (2011).

[47]H. J. Wold, Master's thesis, University of Oslo, Norway, 2011, http://urn.nb.no/URN:NBN:no-28902.