Excitons in cores of exciton-polariton vortices

Nina S. Voronova[1,*] and Yurii E. Lozovik[2,3,†]

[1]*National Research Nuclear University MEPhI, 115409 Moscow, Russia*

[2]*Institute for Spectroscopy, RAS, 142190 Troitsk, Moscow Region, Russia*

[3]*Moscow Institute of Physics and Technology (State University), 141700 Dolgoprudny, Moscow Region, Russia*

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The inner structure of vortices in Bose-Einstein condensates of exciton polaritons is studied theoretically. We show analytically that the healing lengths for the exciton and photon components of exciton-polariton condensate are essentially different. Namely, the exciton healing length may be about two orders of magnitude smaller than the photon healing length. Experimentally, in near-field photoluminescence, the photon part of the exciton-polariton condensate is detected. The suggested theory shows that the cores of experimentally observed vortices are photon cores, and there could be thousands of unobserved exciton polaritons with a strongly reduced photonic fraction inside them.

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I. INTRODUCTION

Bose-Einstein condensation (BEC) of exciton polaritons in optical microcavities[1] remains in the focus of both theoretical and experimental studies. Polaritons are quasiparticles resulting from strong light-matter coupling between cavity photons and excitons in a quantum well embedded inside a cavity.[2] Owing to their extremely light effective mass, BEC in polariton systems can be achieved at very high temperatures compared to BEC in systems of cold atoms. In experiments, there are obstacles such as finite particle lifetime (due to photon escape from the cavity) that prevent achieving thermal equilibrium and therefore the implementation of real Bose condensation. Nevertheless, in recent years there has been great experimental success in the field[3–11] despite the fact that there has been a lot of discussion about clear experimental evidence for BEC in the system.[12] Stimulated scattering of polaritons due to exciton-exciton interactions provides arising of macroscopically large numbers of particles that occupy the lowest energy state, which may be treated as quasicondensation. Narrowing of the photoluminescence peak in such systems has been reported[3] as well as the nonlinear dependence of the emission intensity, although both give no information about statistics of the particles. In 2006, a significant increase in temporal and spatial coherence within the entire polariton system was experimentally observed[5] indicating that the polaritons are indeed Bose condensed. Later a number of experimental works appeared reporting observations of spontaneous quantized vortices[7,13,14] and soliton[11] formation, and methods to manipulate polaritons with different kinds of traps;[6,10] collective fluid dynamics and superfluidity of the polariton condensate were also evidenced and discussed.[8,9,11] A number of theoretical works followed, addressing half vortices,[15] vortices,[16,17] solitons,[18–20] polariton superfluidity,[21] and fluid dynamics,[22] as well as polarization features of the condensate[23,24] and spin-related phenomena.[25,26] Recently, there has been a lot of experimental activity on spontaneous oscillations between the polariton condensates created by multiple separate pump spots.[27,28]

Exciton polaritons in an optical microcavity represent a quasi-two-dimensional (2D) system of bosons of two types, photons and excitons, undergoing mutual transformations, which leads to arising of the new type of particles in the strong-coupling regime. These particles have a different dispersion law consisting of the two branches, corresponding to the lower and upper polariton states.[29] At low temperatures only the lower branch of the dispersion is macroscopically occupied. Transition toward a superfluid state expected for Bose particles in an extended 2D system is not BEC but a topological Berezinskii-Kosterlitz-Thouless (BKT) transition.[30,31] It is a result of vortex-antivortex pair creation in the condensate. A detailed analysis of creation and annihilation of polariton vortex-antivortex pairs in the optical parametric oscillator regime is presented in Refs. 32 and 33. Vortices are an important subject for investigation not only regarding the BKT transition; they also contribute to the phenomenon of turbulence in nonequilibrium systems.[34] So far theoretical works on vortices[15,23,32] have approached the problem by introducing the Gross-Pitaevskii (GP) equation for the wave function of lower polariton condensate, considering the structure of half vortices and vortices from the viewpoint of polariton polarization features.

In the present work we confine our consideration to equilibrium polariton condensates (without pumping and photon decay from the cavity), discussing mere stationary vortical solutions. Instead of using the GP equation for the wave function of lower polaritons we choose a more general two-component approach deriving a set of two separate equations of the GP type with sources for the exciton and photon components of the polariton condensate (in the strong-coupling regime).[21] This approach proves to be especially useful in cases when spatial profiles of the condensate components do not coincide (e.g., in strong traps where the localization radii of photon and exciton condensate wave functions differ essentially[35]).

The paper is organized as follows. In Sec. II we introduce all necessary notations, present general equations, and discuss characteristic spatial scales of the problem. Section III is devoted to deriving the stationary vortical solutions for the two components of the polariton condensate separately. The outcome of the analytical investigation is presented in Sec. IV, whereas the discussion of numerical calculation results for all values of parameters is given in Sec. V. Section VI contains our summary and concluding remarks.

II. GENERAL EQUATIONS

Within the framework of the mean-field approximation microcavity exciton polaritons in equilibrium at zero temperature can be described by a set of two coupled equations as employed in Refs. 21 and 35 which take into account particle transfer between the photon and exciton subsystems of the polariton condensate. While photons are described by the Schrödinger equation, we use the Gross-Pitaevskii equation for condensed excitons, with both equations containing source terms. Thus, the set of equations for photon and exciton condensate wave functions $\psi(\mathbf{r},t)$ and $\chi(\mathbf{r},t)$ reads

$$\begin{cases} i\hbar \dfrac{\partial}{\partial t}\psi(\mathbf{r},t) = -\dfrac{\hbar^2}{2m_{\rm ph}}\nabla^2\psi(\mathbf{r},t) + \dfrac{\hbar\Omega_R}{2}\chi(\mathbf{r},t), \\ i\hbar \dfrac{\partial}{\partial t}\chi(\mathbf{r},t) = -\dfrac{\hbar^2}{2m_{\rm ex}}\nabla^2\chi(\mathbf{r},t) + g|\chi(\mathbf{r},t)|^2\chi(\mathbf{r},t) + \dfrac{\hbar\Omega_R}{2}\psi(\mathbf{r},t), \end{cases} \tag{1}$$

where $m_{\rm ph} = \pi\hbar\sqrt{\varepsilon}/Lc$ is the cavity photon (longitudinal) effective mass, L is the microcavity width, ε is the dielectric constant of media, $\hbar\Omega_R$ is the energy of Rabi splitting between the photon and exciton modes, and $m_{\rm ex} = m_e + m_h$ is the exciton mass. In these equations we neglect the spin degree of freedom and consider the polariton condensate in the absence of external potential (for simplicity).

We introduce the current densities of the condensates:

$$\boldsymbol{j}_\psi(\mathbf{r},t) = -\frac{i\hbar}{2\,m_{\rm ph}}(\psi^*\nabla\psi - \psi\nabla\psi^*) = n_\psi\frac{\hbar}{m_{\rm ph}}\nabla S_\psi,$$

$$\boldsymbol{j}_\chi(\mathbf{r},t) = -\frac{i\hbar}{2\,m_{\rm ex}}(\chi^*\nabla\chi - \chi\nabla\chi^*) = n_\chi\frac{\hbar}{m_{\rm ex}}\nabla S_\chi,$$

where $n_{\psi,\chi}$ and $S_{\psi,\chi}$ are the densities and phases of the photon and exciton components of the condensate, respectively, and

$$\psi = |\psi|e^{iS_\psi} = \sqrt{n_\psi}e^{iS_\psi}, \quad \chi = |\chi|e^{iS_\chi} = \sqrt{n_\chi}e^{iS_\chi}. \tag{2}$$

One can easily derive the set of explicit equations for the densities and phases of the condensates:

$$\begin{cases} \dfrac{\partial n_\psi}{\partial t} + {\rm div}\boldsymbol{j}_\psi = \Omega_R\sqrt{n_\psi n_\chi}\sin(S_\psi - S_\chi), \\ \dfrac{\partial n_\chi}{\partial t} + {\rm div}\boldsymbol{j}_\chi = -\Omega_R\sqrt{n_\psi n_\chi}\sin(S_\psi - S_\chi), \\ \hbar\dfrac{\partial S_\psi}{\partial t} + \dfrac{m_{\rm ph}v_{\rm ph}^2}{2} - \dfrac{\hbar^2}{2m_{\rm ph}}\dfrac{\nabla^2\sqrt{n_\psi}}{\sqrt{n_\psi}} + \dfrac{\hbar\Omega_R}{2}\sqrt{\dfrac{n_\chi}{n_\psi}}\cos(S_\psi - S_\chi) = 0, \\ \hbar\dfrac{\partial S_\chi}{\partial t} + \dfrac{m_{\rm ex}v_{\rm ex}^2}{2} - \dfrac{\hbar^2}{2m_{\rm ex}}\dfrac{\nabla^2\sqrt{n_\chi}}{\sqrt{n_\chi}} + V + gn_\chi + \dfrac{\hbar\Omega_R}{2}\sqrt{\dfrac{n_\psi}{n_\chi}}\cos(S_\psi - S_\chi) = 0. \end{cases} \tag{3}$$

Here $v_{\rm ph}$ and $v_{\rm ex}$ represent the velocities of the condensate flows:

$$\boldsymbol{v}_{\rm ph} = \frac{\hbar}{m_{\rm ph}}\nabla S_\psi, \quad \boldsymbol{v}_{\rm ex} = \frac{\hbar}{m_{\rm ex}}\nabla S_\chi. \tag{4}$$

Introducing the coupled equations for the two condensates in the above form (analogous to the traditional Gross-Pitaevskii approach) we disregard the depletion of the condensates (both thermal and due to interactions) and assume the conservation of the total number of polaritons $N = \int(|\psi|^2 + |\chi|^2)d\mathbf{r}$.

The first two equations in (3) together yield the continuity equation for the polariton condensate from which $dN/dt = 0$ follows immediately. Those equations also reveal that in the stationary case the relative phase of the two components of the condensate $S_\psi - S_\chi$ should equal 0 or π. The equilibrium lower polariton state corresponds to the relative phase π, while the upper polariton state corresponds to $S_\psi - S_\chi = 0$.

In the search for topological defects such as vortices, one should first analyze typical distances that characterize the density variations taking place in the system. In the Thomas-Fermi (TF) limit, that is, at the distances where the density of the condensate changes slowly spacewise, the quantum pressure terms (containing the gradients of densities) in the third and fourth equations of the set (3) can be neglected compared to the other terms. This happens when the distances are much larger than the healing length, which for the "exciton" equation [the fourth equation in (3)] yields a known expression,[36]

$$\xi_{\rm ex} = \frac{\hbar}{\sqrt{2m_{\rm ex}gn_\chi}}, \tag{5}$$

while for the "photon" equation [the third equation in (3)] it cannot be derived in the same way due to the absence of the corresponding interaction term. The expression for photon condensate healing length $\xi_{\rm ph}$ will be obtained in Sec. III.

III. STATIONARY VORTICES IN THE TWO-COMPONENT POLARITON CONDENSATE

In the case of stationary solutions wave functions of the condensate components evolve in time according to

$$\psi(\mathbf{r},t)=\psi(\mathbf{r})e^{-(i/\hbar)(\mu-E_0)t}, \quad \chi(\mathbf{r},t)=\chi(\mathbf{r})e^{-(i/\hbar)(\mu-E_0)t}, \quad (6)$$

where we imply zero detuning between the exciton and cavity photon modes ($E_0=\pi\hbar c/L\sqrt{\varepsilon}$) and assume the value of chemical potential μ to be fixed by conservation of the full number of particles in the system. The set of equations (1) takes the simple form

$$\begin{cases} -\dfrac{\hbar^2}{2m_{\mathrm{ph}}}\nabla^2\psi(\mathbf{r})+(E_0-\mu)\psi(\mathbf{r})+\dfrac{\hbar\Omega_R}{2}\chi(\mathbf{r})=0, \\ -\dfrac{\hbar^2}{2m_{\mathrm{ex}}}\nabla^2\chi(\mathbf{r})+(E_0-\mu)\chi(\mathbf{r})+g|\chi(\mathbf{r})|^2\chi(\mathbf{r})+\dfrac{\hbar\Omega_R}{2}\psi(\mathbf{r})=0. \end{cases} \quad (7)$$

The above set of equations admits different solutions, including vortex states. For uniform condensates, the first equation in (7) yields a simple algebraic relation between the wave functions of the components which, being substituted to the second equation, gives

$$g|\chi|^2=gn_\chi=\frac{(\hbar\Omega_R/2)^2}{(E_0-\mu)}\left[1-\left(\frac{E_0-\mu}{\hbar\Omega_R/2}\right)^2\right]. \quad (8)$$

Therefore all possible values of the chemical potential μ (for lower polaritons) are enclosed within the limits $E_0-\hbar\Omega_R/2<\mu<E_0$.

We are looking for vortical solutions of the equations (7) which should be single valued and possess cylindrical symmetry relative to the rotation around the z axis. Hence, the wave functions in cylindrical coordinates can be written in the following form:

$$\psi(\mathbf{r})=\sqrt{n_{\psi0}}\,e^{is\varphi}f_\psi\left(\frac{r}{\xi_{\mathrm{ph}}}\right), \quad (9)$$

$$\chi(\mathbf{r})=\sqrt{n_{\chi0}}\,e^{i(s\varphi-\pi)}f_\chi\left(\frac{r}{\xi_{\mathrm{ex}}}\right), \quad (10)$$

where $n_{\psi0}$ and $n_{\chi0}$ are the unperturbed densities of the photon and exciton condensates respectively and s is an integer number.

The phases of the wave functions (9) and (10) correspond to the circulation of the velocity fields over a contour around the z axis $\oint \boldsymbol{v}_{\mathrm{ph,ex}}d\mathbf{l}=2\pi s\hbar/m_{\mathrm{ph,ex}}$ which turns out to be quantized, and it is also implied that the relative phase $S_\psi-S_\chi$ remains fixed and equal to π. At large distances from the vortex core (in the TF approximation) the densities of the condensates $|\psi|^2$, $|\chi|^2$ must approach their uniform values $n_{\psi0}$, $n_{\chi0}$ and the functions f_ψ and f_χ tend to 1 while $r\to\infty$. Since only those vortical solutions are thermodynamically stable which correspond to the lowest possible circulation value, hereafter we consider the case $s=1$ only:

$$\psi(\mathbf{r})=\sqrt{n_{\psi0}}\,e^{i\varphi}f_\psi(\tilde{\eta}), \quad \chi(\mathbf{r})=-\sqrt{n_{\chi0}}\,e^{i\varphi}f_\chi(\eta), \quad (11)$$

where $\tilde{\eta}=r/\xi_{\mathrm{ph}}$ and $\eta=r/\xi_{\mathrm{ex}}$ are the corresponding healing lengths which can now be obtained from (7), and they are given by [see also (5)]

$$\xi_{\mathrm{ph}}=\frac{\hbar}{\sqrt{2m_{\mathrm{ph}}(E_0-\mu)}}, \quad \xi_{\mathrm{ex}}=\frac{\hbar}{\sqrt{2m_{\mathrm{ex}}gn_{\chi0}}}. \quad (12)$$

One can see now that since $m_{\mathrm{ph}}/m_{\mathrm{ex}}\sim10^{-4}$, the healing lengths of the condensate components in the general case may drastically differ, depending on the number of particles in the system [the relation between $(E_0-\mu)$ and $gn_{\chi0}$ is given by (8)]. Thus, for most of the allowed values of chemical potential, $\xi_{\mathrm{ph}}\gg\xi_{\mathrm{ex}}$ (see Sec. IV) and the set of equations (7) has two characteristic spatial scales. In the region $r\lesssim\xi_{\mathrm{ex}}$, the substitution (11) yields

$$\begin{cases} \dfrac{1}{\eta}\dfrac{d}{d\eta}\left(\eta\dfrac{df_\psi}{d\eta}\right)-\dfrac{1}{\eta^2}f_\psi-\dfrac{m_{\mathrm{ph}}}{m_{\mathrm{ex}}}\dfrac{E_0-\mu}{gn_{\chi0}}f_\psi+\dfrac{m_{\mathrm{ph}}}{m_{\mathrm{ex}}}\dfrac{\hbar\Omega_R}{2gn_{\chi0}}\sqrt{\dfrac{n_{\chi0}}{n_{\psi0}}}f_\chi=0, \\ \dfrac{1}{\eta}\dfrac{d}{d\eta}\left(\eta\dfrac{df_\chi}{d\eta}\right)-\dfrac{1}{\eta^2}f_\chi-\dfrac{E_0-\mu}{gn_{\chi0}}f_\chi-f_\chi^3+\dfrac{\hbar\Omega_R}{2gn_{\chi0}}\sqrt{\dfrac{n_{\psi0}}{n_{\chi0}}}f_\psi=0, \end{cases} \quad (13)$$

while for the distances $r\gg\xi_{\mathrm{ex}}$ the equations for the functions $f_{\psi,\chi}$ have the form

$$\begin{cases} \dfrac{1}{\tilde{\eta}}\dfrac{d}{d\tilde{\eta}}\left(\tilde{\eta}\dfrac{df_\psi}{d\tilde{\eta}}\right)-\dfrac{1}{\tilde{\eta}^2}f_\psi-f_\psi+\dfrac{\hbar\Omega_R}{2(E_0-\mu)}\sqrt{\dfrac{n_{\chi0}}{n_{\psi0}}}f_\chi=0, \\ \dfrac{m_{\mathrm{ph}}}{m_{\mathrm{ex}}}\dfrac{1}{\tilde{\eta}}\dfrac{d}{d\tilde{\eta}}\left(\tilde{\eta}\dfrac{df_\chi}{d\tilde{\eta}}\right)-\dfrac{m_{\mathrm{ph}}}{m_{\mathrm{ex}}}\dfrac{1}{\tilde{\eta}^2}f_\chi-f_\chi-\dfrac{gn_{\chi0}}{E_0-\mu}f_\chi^3+\dfrac{\hbar\Omega_R}{2(E_0-\mu)}\sqrt{\dfrac{n_{\psi0}}{n_{\chi0}}}f_\psi=0, \end{cases} \quad (14)$$

with the constraints $f_{\psi,\chi}(0) = 0$ and $f_{\psi,\chi}(\infty) = 1$ in both cases. The set of equations (13) describes a vortex in the exciton component of the condensate, and the set of equations (14) treats that of the photon component.

Another effect of the difference between the photon and exciton effective masses is that the flow velocities (4) of the two components inside the polariton vortex are different. Since $v(r) \sim \hbar/mr$ (depending on the distance from the vortex center), the photon subsystem rotates much faster than the exciton one.

IV. ANALYTICAL SOLUTION

For simplicity of further analysis we introduce a dimensionless parameter $a = 2(E_0 - \mu)/\hbar\Omega_R$ which ranges from 0 to 1 depending on the value of μ. The ratio between the healing lengths (12) is given by

$$\frac{\xi_{\rm ph}}{\xi_{\rm ex}} = \sqrt{\frac{m_{\rm ex}}{m_{\rm ph}}}\sqrt{\frac{1-a^2}{a^2}}. \tag{15}$$

According to the definition of the parameter a, Fig. 1 shows that for all values of the chemical potential μ but the $\mu_{\rm min} = E_0 - \hbar\Omega_R/2$ the healing length of the exciton component is much smaller than that of the photon condensate.

It is important to note here that the healing lengths (12) were obtained analytically from the comparison of terms in Eqs. (7) in reasonable assumption that the polariton system contains enough particles for the interaction to play a role. In dilute cases the term containing χ^3 will be much smaller than the linear terms containing χ, and the healing length for the exciton component of the condensate may be expressed in a different way. Accordingly the relation (15) should be considered fully valid for systems with high pump power and less applicable for rarefied gases.

The boundary condition at $r \to \infty$ yields the relation between the unperturbed densities of the condensate components $\sqrt{n_{\chi 0}/n_{\psi 0}} = a$. Taking this into account along with (8), the set

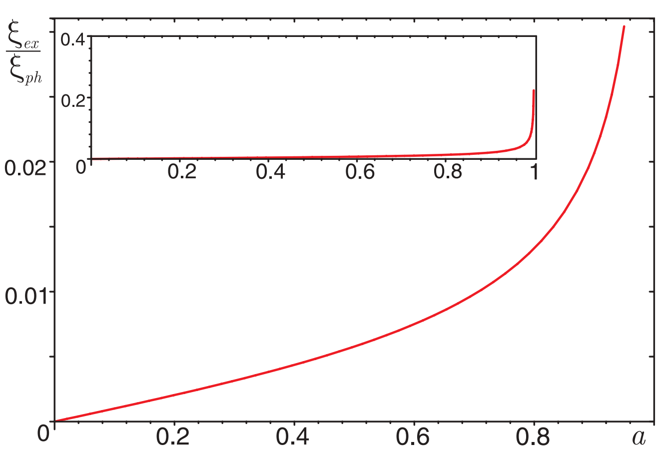

FIG. 1. (Color online) Ratio between the healing lengths for the exciton and photon components of the polariton condensate $\xi_{\rm ex}/\xi_{\rm ph}$ versus $a = 2(E_0 - \mu)/\hbar\Omega_R$. As a increases, the ratio slowly grows yet always remains much smaller than unity, and steeply rises to infinity only when a approaches its limit 1 (see inset), which is never physically realized (see the text).

of equations (13) can be rewritten for short distances $r \lesssim \xi_{\rm ex}$ as follows:

$$\begin{cases} f_\psi'' + \dfrac{f_\psi'}{\eta} - \dfrac{f_\psi}{\eta^2} - \dfrac{m_{\rm ph}}{m_{\rm ex}}\dfrac{a^2}{1-a^2}(f_\psi - f_\chi) = 0, \\ f_\chi'' + \dfrac{f_\chi'}{\eta} - \dfrac{f_\chi}{\eta^2} - \dfrac{a^2}{1-a^2}f_\chi - f_\chi^3 + \dfrac{1}{1-a^2}f_\psi = 0. \end{cases} \tag{16}$$

The factor $m_{\rm ph}/m_{\rm ex}$ is of the order of 10^{-4} which for most values of a but unity reduces the first equation in (16) to $f_\psi'' + f_\psi'/\eta - f_\psi/\eta^2 = 0$. At the scales $r \lesssim \xi_{\rm ex}$ the nontrivial solution for the photon condensate wave function tends to zero as $f_\psi(\eta \to 0) \sim \eta$. The second equation for the function f_χ describes a vortex in the exciton subsystem. At larger distances $\eta \gg \xi_{\rm ex}$, the spatial derivatives can be neglected which leads to a nonlinear algebraic equation connecting f_χ with f_ψ:

$$-\frac{a^2}{1-a^2}f_\chi - f_\chi^3 + \frac{1}{1-a^2}f_\psi = 0. \tag{17}$$

Besides, at these distances the function f_χ should approach its limiting value 1; therefore it may be supposed that $f_\chi = 1 - \delta, \delta \ll 1$, and an approximate relation between the exciton and photon functions may be obtained:

$$f_\chi = \frac{2 - 2a^2 + f_\psi}{3 - 2a^2}, \tag{18}$$

which should be valid for all distances much larger than $\xi_{\rm ex}$, which means everywhere far from the exciton vortex core.

In the region $\xi_{\rm ex} \ll r \lesssim \xi_{\rm ph}$ one ought to solve the set of equations (14) which reads

$$\begin{cases} f_\psi'' + \dfrac{f_\psi'}{\tilde\eta} - \dfrac{f_\psi}{\tilde\eta^2} - f_\psi + f_\chi = 0, \\ \dfrac{m_{\rm ph}}{m_{\rm ex}}\left(f_\chi'' + \dfrac{f_\chi'}{\tilde\eta} - \dfrac{f_\chi}{\tilde\eta^2}\right) - f_\chi - \dfrac{1-a^2}{a^2}f_\chi^3 + \dfrac{1}{a^2}f_\psi = 0. \end{cases} \tag{19}$$

It is easily seen that with the derivative terms neglected, the second equation of this set is equivalent to (17), while the first one with the substitution (18) yields the equation for the function f_ψ:

$$f_\psi'' + \frac{f_\psi'}{\tilde\eta} - \frac{f_\psi}{\tilde\eta^2} - \frac{2a(1-a^2)}{3-2a^2}f_\psi + \frac{2a(1-a^2)}{3-2a^2} = 0,$$

which with the given boundary condition has an analytical solution:

$$f_\psi(\tilde\eta) = \frac{\pi}{2}\left[I_1(\alpha\,\tilde\eta) - L_1(\alpha\,\tilde\eta)\right], \tag{20}$$

where $I_1(\alpha\tilde\eta)$ and $L_1(\alpha\tilde\eta)$ represent modified Bessel and Struve functions, and $\alpha = \sqrt{2a(1-a^2)/(3-2a^2)}$.

Equations (20) and (18) give accurate solutions of the initial set of equations (7) for everywhere in space except the region $r \lesssim \xi_{\rm ex}$ around zero where the photon function f_ψ tends linearly to zero while the exciton function f_χ is a solution of the second equation in (16). The results of analytical calculation are shown in Fig. 2.

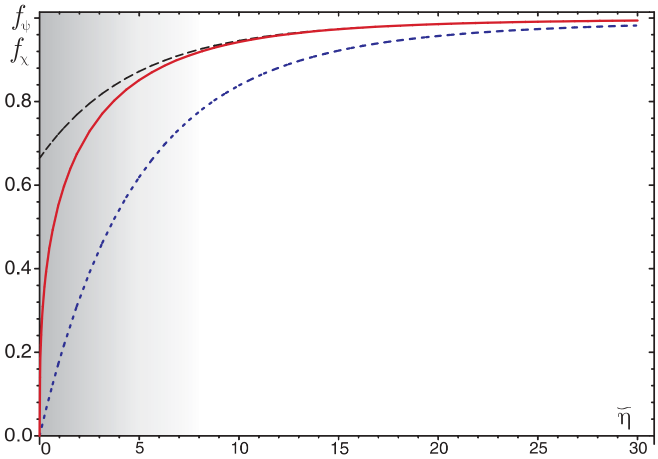

FIG. 2. (Color online) Analytical solution of the set of equations (19) for $a = 0.1$ (for this value, $\tilde{\eta} = 5$ corresponds to ≈ 15 μm). The function f_ψ (blue dotted line) is given by (20). The function f_χ (red solid line) is calculated as an exact solution of the cubic Eq. (17). Black dashed line represent the approximation for f_χ given by the formula (18) which appears to be invalid in the region of the exciton vortex core (grey field); therefore these dependencies can be considered correct only starting from the region of the photon vortex core characteristic size and further.

V. RESULTS OF NUMERICAL CALCULATION AND DISCUSSION

We solved the problem numerically on the exciton scale in order to obtain the solutions which are valid at all distances. Thus, we have solved the set of equations (16) for different values of the parameter a. The results of the calculations are shown in Fig. 3.

For small values of a which correspond to large values of the chemical potential and therefore to the large number of polaritons in the system, the effect is most pronounced (as was anticipated above in Fig. 1): The vortex in the exciton condensate heals about one hundred times faster than that of the photon subsystem [Fig. 3(a) and the relation (15)].

As the number of particles decreases (i.e., a is growing) we find that the healing lengths become closer to each other; therefore the components are less separated in space [Fig. 3(b)], although the difference between $\xi_{\rm ex}$ and $\xi_{\rm ph}$ remains about one order of magnitude [see (15)], and it becomes really small only in the case when the number of polaritons tends to zero, the chemical potential μ approaches its minimal value $E_0 - \hbar\Omega_R/2$, and a increases up to almost unity [see Fig. 3(c)].

It can be also seen from the results of the numerical calculation that, as was predicted in the formula (12), the size of the photon vortex core grows with increasing of the chemical potential (i.e., decreasing a). Therefore the polariton vortex should be expected to become larger with the increase of the pump power in the experimental setups, as it is the photon component of the polariton vortex that is detected on experiment.

There is one more peculiar feature of the two-component condensate considered. As was mentioned before, the unperturbed densities of the components are not equal but relate to each other as $n_{\chi 0}/n_{\psi 0} = a^2$. The condensate density profiles possess not only different healing lengths but also efficiently different uniform values at infinity (depending on the value of chemical potential). We plot n_ψ and n_χ in Fig. 4 for the intermediate value $a = 0.5$. For lower values of a (higher μ), the effect becomes even more dramatic: For $a = 0.1$ the difference between the densities is of two orders of magnitude. On the contrary, as a increases (smaller μ), the densities tend to heal at similar radius and to similar uniform values.

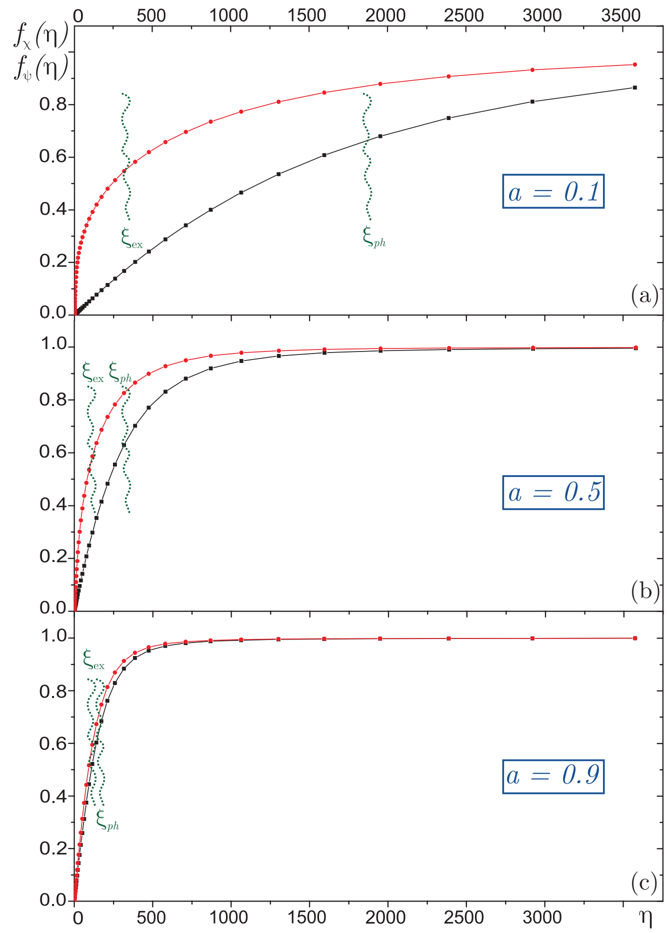

FIG. 3. (Color online) Vortical solutions of the set of GP-like equations for the two-component polariton condensate as functions of the radial coordinate measured in the units of the exciton condensate healing length $\eta = r/\xi_{\rm ex}$ for (a) $a = 0.1$, (b) $a = 0.5$, (c) $a = 0.9$. Red circles represent $f_\chi(\eta)$; black squares represent $f_\psi(\eta)$.

We would like to note that in the case of resonance the Hopfield coefficients[29] are equal and the equilibrium concentrations of photons and excitons in the system should seemingly be equal as well. The result demonstrated above can be explained by the presence of exciton-exciton interactions (in the case of no interactions $g \to 0$ in our framework one gets $n_{\psi 0} = n_{\chi 0}$). This remark suggests an experimental method to evidence the different spatial distribution of the photon and exciton components of the polariton condensate: In the case of resonant pumping, the increase in pump power should lead to the decrease of the exciton fraction in the system and therefore the nonlinear optical properties of the condensate should vanish.

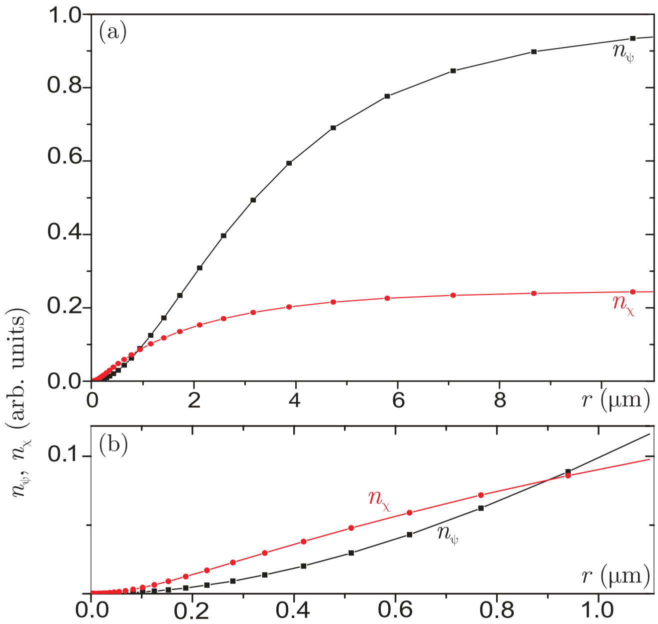

FIG. 4. (Color online) (a) The (normalized) densities of the photon and exciton components of the condensate as functions of distance from the vortex center for $a = 0.5$. The densities are given by $n_\chi(r) = n_{\chi 0} f_\chi^2$ and $n_\psi(r) = n_{\psi 0} f_\psi^2$, respectively, where $n_{\chi 0}$ and $n_{\psi 0}$ are the densities of the uniform condensates which ratio equals a^2 (see the text). The distance r is calculated from the dimensionless value $\eta = r/\xi_{\mathrm{ex}}$ for parameter values $\hbar\Omega_R = 5$ meV, $m_{\mathrm{ex}} = 0.153 m_e$. (b) The core region.

VI. CONCLUSIONS

In the presented study, we have demonstrated that the approach based on the two separate Gross-Pitaevskii-like equations for the photon and exciton components of the polariton condensate allows us to obtain the results concealed from the traditional one-component GP approach for lower polaritons. Here we analyzed the impact of the sufficiently high ratio between the effective masses of an exciton and cavity photon on the structure of the polariton vortex. We have shown that it leads to the distinctive difference between the healing lengths of the vortices in the exciton and photon subsystems of the condensate. The core size of an excitonic vortex appears to be much smaller than that for the photons in almost all regimes (at almost all values of the chemical potential); hence unobserved excitons exist inside the photon vortex core at almost all pump powers. The uniform values of the exciton and photon densities at infinity are also shown to be essentially different. Thus, while the exciton and photon components are coupled to each other in the momentum space, they may have essentially differently scaled distributions in the coordinate space. From the polaritons point of view, the particles with the small wave vectors (far from the vortex core) become more photonic as the density is increasing, while the particles with the growing excitonic fraction appear in the vortex center which corresponds to higher polaritons' wave vectors.

This being shown, we would like to emphasize that while the photonic component of the polariton condensate is detected in near-field photoluminescence, the effects demonstrated in the present paper could be experimentally observed by measuring local nonlinear optical properties of the exciton component of the condensate (see, e.g., Ref. 37), and the exciton decay could also be locally observed in the near field.

It is also important to note that the solutions for vortices found in this work should also be valid for half vortices,[7,15] with the difference that only one spin component of the polariton condensate will be perturbed while the other one will remain uniform in space.

In view of the demonstrated results we would like to make a link to the other systems with multiple coherence lengths such as multiband superconductors with different coherence lengths for different bands (see Ref. 38 and references therein). The relation between these systems and the two-component polariton condensate will be discussed elsewhere.

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*nsvoronova@mephi.ru
†lozovik@isan.troitsk.ru

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