Spin transistors based on spatial control of electron spins in double quantum wells

Pawel Pfeffer* and Wlodek Zawadzki
Institute of Physics, Polish Academy of Sciences, Aleja Lotnikow 32/46, 02-668 Warsaw, Poland
(Received 16 June 2012; revised manuscript received 24 October 2012; published 5 November 2012)

A system of two quantum wells (QWs), one made of HgCdTe and the other of HgCdMnTe, subjected to electric and magnetic fields F and B parallel to the growth direction, is proposed and described theoretically. It is shown that in such a system the spin g factor of mobile electrons strongly depends on the sign and magnitude of the electric field. By adjusting F at a constant B one can transfer almost all the electrons into one or the other QW and polarize their spins along the desired orientation. A change in B at a constant F can produce a similar transfer and polarization effect. It is indicated that the above system can be used to construct electric and magnetic spin transistors in the parallel-transport configuration.

DOI: 10.1103/PhysRevB.86.195303 PACS number(s): 73.21.Fg, 73.61.Ga, 75.25.−j, 75.50.Pp

I. INTRODUCTION

Spin systems in semiconductors have always been an important part of solid state investigations but in recent years they have attracted great interest due to possible spintronic applications. In order to process quantum information one needs to manipulate the local spins quickly and coherently, which could be done by applying external magnetic fields. This, however, requires their precise control at small length scales, which is not easy. Salis *et al.*[1] demonstrated that one can control the spin splitting and spin coherence using an external electric field, employing parabolic $Ga_{1-x}Al_xAs$ quantum wells (QWs) and manipulating the electron wave functions within such wells. The experiments of Salis *et al.* were described by the present authors[2] using the $\mathbf{k}\cdot\mathbf{p}$ theory for nonparabolic bands. The $\mathbf{k}\cdot\mathbf{p}$ theory was adapted to heterojunctions in which the effective mass and the spin g factor can depend not only on the energy but also on the spatial variable along the growth direction.

In the present work we propose and describe a system that will manipulate electron spins in more sophisticated ways that those realized in Ref. 1. Three additional possibilities are envisaged. First, the electron spin will respond not only to the intensity but also to the sign of the electric field F. Second, the spins will respond in a nonconventional way to the magnitude of the magnetic field B. Third, it will be possible to create reservoirs of spin-polarized electrons in specific spatial regions. The proposed system consists of two rectangular semiconductor quantum wells separated by a barrier. The left well is made of HgCdMnTe alloy, the right one of HgCdTe alloy, and the barriers are made of CdTe. The chemical compositions are chosen to make the well depths exactly or almost equal to each other. The alloy HgCdMnTe is a dilute magnetic semiconductor (DMS) in which an external magnetic field creates an additional magnetization by ordering the Mn ions. This additional magnetization results in a higher spin splitting of electron energies characterized by a higher effective spin-splitting factor g. By applying a positive or negative external electric field parallel to the growth direction, one can transfer free electrons to either the magnetic or nonmagnetic well, which will strongly affect their overall spin g factor. In this way, g will depend not only on the electric field intensity but on its sign as well. The other properties will become clear once the system is physically and mathematically described below.

In the last part we consider the possibility of using the spatial control of spins to construct an electric or a magnetic spin transistor in the parallel-transport configuration. It is shown that one should be able to change spin currents using electric and magnetic fields.

II. THEORY

Since the proposed alloys are relatively narrow-gap materials, we use the three-level $\mathbf{P}\cdot\mathbf{p}$ model (3LM) for their band structures. In our description the energy gap and other band parameters are functions of z. The model explicitly takes into account eight bands arising from the Γ_7^v, Γ_8^v (doubly degenerate), and Γ_6^c levels at the center of the Brillouin zone and treats the distant (upper and lower) levels as a perturbation. The resulting bands are spherical but nonparabolic. Our formulation includes external magnetic and electric fields parallel to the growth direction. The multiband $\mathbf{P}\cdot\mathbf{p}$ theory, which is the $\mathbf{k}\cdot\mathbf{p}$ theory generalized for the presence of external magnetic $\mathbf{B}$ and electric $\mathbf{F}$ fields, has the form[2]

$$\sum_l \left[\left(\frac{P^2}{2m_0}+E^l+H_M^l+V(z)+eFz-\mathcal{E}\right)\delta_{l'l} + \frac{1}{m_0}\mathbf{p}_{l'l}\cdot\mathbf{P}+\mu_B\mathbf{B}\cdot\boldsymbol{\sigma}_{l'l}\right] f_l = 0, \quad (1)$$

where $\mathcal{E}$ is the energy, $\mathbf{P}=\mathbf{p}+e\mathbf{A}$ is the kinetic momentum, $\mathbf{A}$ is the vector potential of the magnetic field $\mathbf{B}$, and $\boldsymbol{\sigma}_{l'l}=(1/\Omega)\langle u_{l'}|\boldsymbol{\sigma}|u_l\rangle$. Here $\boldsymbol{\sigma}$ are the Pauli spin matrices, Ω is the volume of the unit cell, u_l are periodic amplitudes of the Luttinger-Kohn functions, $\mu_B=e\hbar/2m_0$ is the Bohr magneton, $\mathbf{p}_{l'l}$ are the interband matrix elements of momentum, and H_M^l is the exchange interaction between mobile electrons and those localized around the Mn ions. The sum in Eq. (1) runs over all bands $l=1,2,\ldots,8$ included in the model, $l'=1,2,\ldots,8$ runs over the same bands, and E^l are the band-edge energies (see below). Within the 3LM there exist the interband matrix element of momentum P_0 and that of the spin-orbit interaction Δ_0 (see Ref. 3), as well as two matrix elements related to the magnetic Mn ions: α and β (see Refs. 4,5). The latter represent constants of the s-d and

p-d exchange integrals related to the conduction band $|S\rangle$ and valence bands $|P\rangle$, respectively.

For rectangular wells the potential $V(z)$ describes conduction-band offsets and barriers. The valence offsets are automatically determined by the corresponding energy gaps.

Equation (1) represents an 8×8 system of equations for eight envelope functions $f_l(\mathbf{r})$. Since we are interested in the eigenenergies and eigenfunctions for the conduction band, we express the valence functions $f_3, \ldots, f_8$ via the conduction functions f_1 and f_2 for the spin-up and spin-down states, respectively. The magnetic field $\mathbf{B} \parallel \mathbf{z}$ is described by the asymmetric gauge $\mathbf{A} = [-By,0,0]$ and the resulting envelope functions have the general form $f_l = \exp(ik_x x)\Phi_n[(y - y_0)/L]\chi_l(z)$, where Φ_n are the harmonic oscillator functions, and $y_0 = k_x L^2$, in which $L = (\hbar/eB)^{1/2}$ is the magnetic radius. After some manipulation the effective Hamiltonian for the f_1 and f_2 functions is

$$\hat{H} = \begin{bmatrix} \hat{A}^+ & \hat{K} \\ \hat{K}^\dagger & \hat{A}^- \end{bmatrix}, \tag{2}$$

where

$$\hat{A}^\pm = V(z) + eFz \pm \frac{A_M}{2} - \frac{\hbar^2}{2}\frac{\partial}{\partial z}\frac{1}{m^*(\mathcal{E},z)}\frac{\partial}{\partial z} + \frac{P_x^2 + P_y^2}{2m_l^*(\mathcal{E},z)} \pm \frac{\mu_B B}{2} g^*(\mathcal{E},z), \tag{3}$$

in which the effective mass is

$$\frac{m_0}{m^*(\mathcal{E},z)} = 1 + C - \frac{1}{3}E_{P_0}\left(\frac{2}{\tilde{E}_0} + \frac{1}{\tilde{G}_0}\right), \tag{4}$$

and the g factor is

$$g^*(\mathcal{E},z) = 2 + 2C' + \frac{2}{3}E_{P_0}\left(\frac{1}{\tilde{E}_0} - \frac{1}{\tilde{G}_0}\right), \tag{5}$$

where $\tilde{E}_0 = E_0 - \mathcal{E} + V(z) + eFz$ and $\tilde{G}_0 = G_0 - \mathcal{E} + V(z) + eFz$. Here $G_0 = E_0 + \Delta_0$. The spin splitting is related to the band structure [the last terms in Eq. (3)] and the DMS interaction. Explicitly, $g_M^*(B,z) = A_M/\mu_B B$, where $A_M = x\alpha\langle S_z\rangle$ and x is the mole fraction of magnetic ions. The average spin component parallel to the applied magnetic field is $\langle S_z\rangle = -S_0 B_S(y)$, the total Mn spin is $S = 5/2$, and $B_S(y)$ is the Brillouin function. We take $x = 0.08$, $S_0 = 1.02$, $\alpha = 0.61$ eV, and $\beta = -0.62$ eV.[6] One can combine the band structure and DMS contributions to the spin splitting in one effective $g_{\rm tot}^*$ factor. The small off-diagonal matrix elements $\hat{K}$ and $\hat{K}^\dagger$ in Eq. (2) appear due to inversion asymmetry of the system along the growth direction z, which results in an additional Bychkov-Rashba spin splitting (see below). The effective mass and the g factor in Eqs. (4) and (5), respectively, are written neglecting small contributions of the semimagnetic β terms in the valence bands. The equations $\hat{A}^+ f_n^+ = \mathcal{E}_n^+ f_n^+$ and $\hat{A}^- f_n^- = \mathcal{E}_n^- f_n^-$ are first solved separately for the system of two QWs at fixed values of the fields B and F using simple boundary conditions at the interfaces for the functions and their derivatives $\chi|_+ = \chi|_-$ and $[(1/m^*)\partial\chi/\partial z)]|_+ = [(1/m^*)\partial\chi/\partial z)]|_-$. The transverse motion in the x-y plane is quantized into the Landau levels $\hbar\omega_c(n + 1/2)$. The electric field term eFz is small but not negligible compared to the offsets of the QWs.

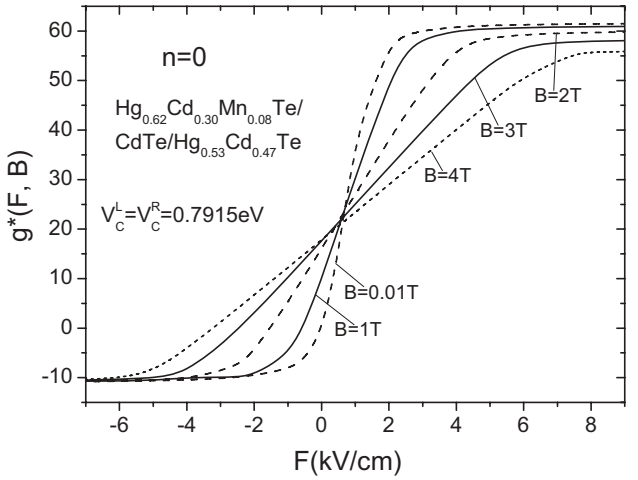

FIG. 1. Calculated spin g factor for the $n = 0$ Landau level in two quantum wells of equal depths versus external electric field F parallel to the growth direction for fixed magnetic fields B. It is seen that the g factor sharply depends on the magnitude and sign of F. The widths of both wells are 9 nm; the barrier width is 4 nm.

III. SPATIAL SPIN CONTROL

We will consider two structures. In the first, the chemical compositions of the QWs are such that without external fields their depths (conduction-band offsets V_C) are identical and equal to 0.7915 eV. In the second, the the depth of the left HgCdMnTe QW is 0.7915 eV, while the depth of the right HgCdTe QW is 0.7855 eV. The zero of energy $\mathcal{E}$ is taken at the bottom of the left QW. In Figs. 1 and 2 we show the calculated electron spin g values for the two structures versus an external electric field F at fixed magnetic field intensities B. It is seen that, indeed, the g values strongly depend not only on the value of F but also on its sign. Generally speaking, if the positive electric field pushes the electrons into the DMS QW they experience the additional magnetization of the Mn ions and the overall g value is higher. It is also seen that, by changing the chemical compositions of the alloy (see Table I)

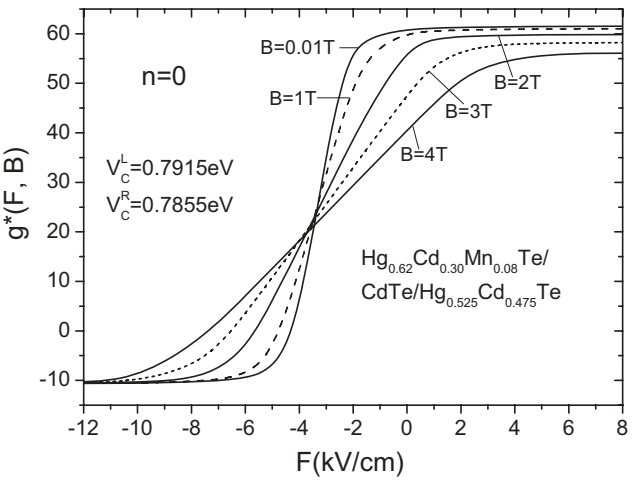

FIG. 2. As Fig. 1 for two QWs of slightly different depths. The overall picture is similar to that in Fig. 1 but the scale of the electric fields is considerably shifted.

TABLE I. Band parameters of alloys for different chemical compositions x and y, as used in the calculations; C and C' are far-band contributions to the band-edge values. The interband matrix element of momentum P_0 is taken to be independent of x and y: $E_{P_0} = 2\hbar^2 P_0^2/m_0 = 18$ eV.

	CdTe	$Hg_{.62}Cd_{.3}Mn_{.08}Te$	$Hg_{.53}Cd_{.47}Te$	$Hg_{.525}Cd_{.475}Te$
E_0 (eV)	−1.606	−0.495	−0.5125	−0.5211
G_0 (eV)	−2.516	−1.510	−1.5126	−1.5203
C	−0.104	−0.104	−0.104	−0.104
C'	−0.479	−0.479	−0.479	−0.479
V_C (eV)		0.7915	0.7915	0.7855
m_0^*/m_0	0.093	0.0343	0.0353	0.0359
g_0^*	−1.66	−15.25	−14.437	−14.093

and the corresponding depth of the right QW (without Mn ions), one strongly shifts the scale of the electric fields, while the overall pattern of the g behavior and its dependence on B remain similar. Since we assume that the number of mobile electrons is small and the temperature is low, so the electrons will occupy the lowest Landau and spin levels, one can use Figs. 1 and 2 to decide which spin level will be occupied for given values of F and B. As far as the wave functions in the system of two QWs are concerned, there are two factors that determine where the wave function (WF) is predominantly located. First, if the electron masses are the same for both wells, the WF is located mostly in the deeper QW. Second, if both QWs have the same depth, the WF is located mostly in the QW with the higher electron mass. Qualitatively, a positive electric field lowers the left well and raises the right one, which tends to shift the wave functions to the left QW.

We consider first the case of two QWs of equal depths. Taking $F = 0$ and $2 \leqslant B \leqslant 4$ T, it follows from Fig. 1 that the spin factor g is positive, so the electrons will occupy the lowest level $|0-\rangle$. The calculated WF for this case is shown in Fig. 3, and it is seen that the WF is almost entirely concentrated in

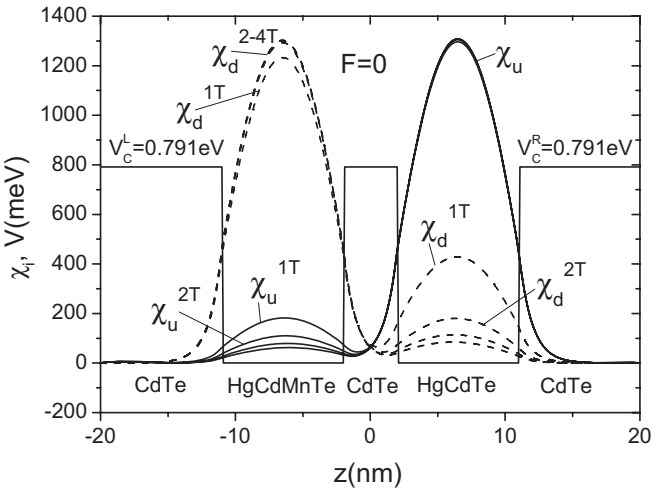

FIG. 3. Calculated wave functions of $|0+\rangle$ and $|0-\rangle$ states for two QWs of equal depths at vanishing electric field F and fixed magnetic fields B (indicated by superscripts). Spin-up functions, solid lines; spin-down functions, dashed lines. Electrons in the $|0-\rangle$ state are located mostly in the left QW.

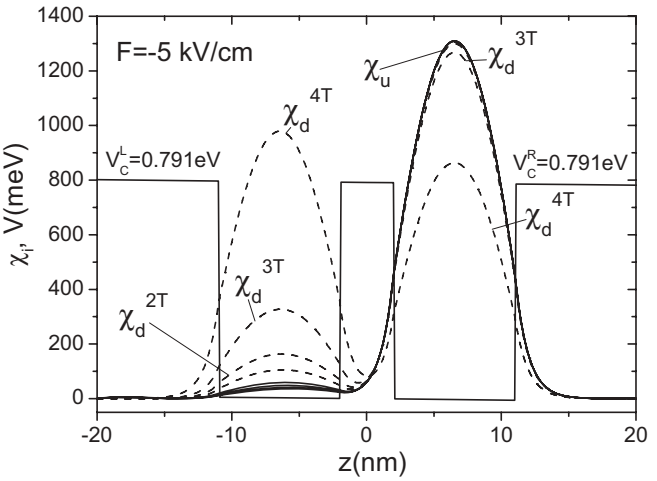

FIG. 4. As Fig. 3 but for electric field $F = -5$ kV/cm. Electrons in the lowest $|0+\rangle$ state are located in the right QW.

the left QW, which means that the electrons will be located there and they will have the spin-down orientation. If now one applies $F = -5$ kV/cm and $1 \leqslant B \leqslant 4$ T, it is seen from Fig. 1 that at these fields g is negative and the electrons will occupy the $|0+\rangle$ state. The calculation for this situation is shown in Fig. 4 and it is seen that the WF for the $|0+\rangle$ state is strongly concentrated in the right QW. Thus, the application of an electric field F at constant B transfers electrons from the left to the right QW and changes their spin orientation from spin down to spin up.

A similar effect can be obtained in a system of QWs having different depths. Taking $F = 0$ and $1 \leqslant B \leqslant 4$ T, it is seen in Fig. 2 that in this regime the spin g value is positive, so the occupied state is $|0-\rangle$. It follows from Fig. 5 that the calculated WF for the spin-down state is in the left QW and the electrons will be located there. If now one applies the fields $F = -5$ kV/cm and$B = 1$ T, the g value is negative (see Fig. 2) and the occupied state is $|0+\rangle$. The calculated WF for this situation is shown in Fig. 6; one can again see that the WF for this state is almost completely concentrated in the

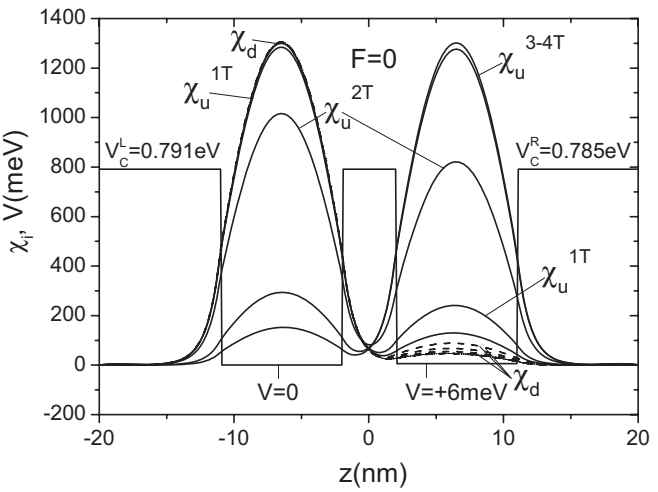

FIG. 5. As Fig. 3 but for two QWs of different depths at $F = 0$. Electrons in the lowest $|0-\rangle$ state are located in the left QW.

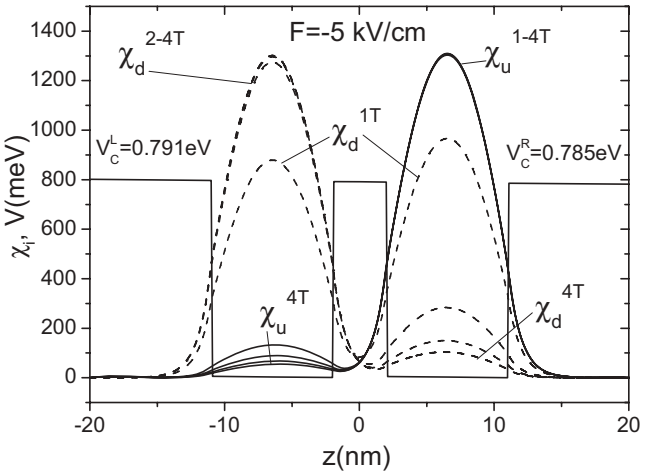

FIG. 6. As Fig. 5, but for $F = -5$ kV/cm. At small magnetic fields B the electrons are in the $|0+\rangle$ state and they are located in the right QW, while for high B they are in the $|0-\rangle$ state and are located in the left QW (see the text).

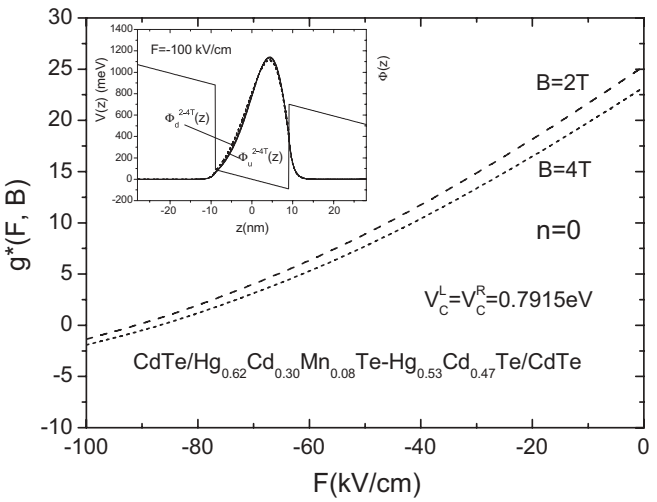

FIG. 7. Spin g factor for the $n = 0$ Landau level in one quantum well versus the external electric field **F** parallel to the growth direction, as calculated for two magnetic fields. The inhomogeneous well corresponds to the structure shown in Fig. 3 with the middle barrier reduced to zero width. Note the difference in scale for the electric field, as compared to Fig. 1. The inset shows the well profile and calculated wave functions. The two spin states almost coincide.

right QW. Thus, by changing the electric field one can move electrons from the left to the right QW and change their spin orientation from spin down to spin up.

Finally, we consider the case of unequal QWs, taking $F = -5$ kV/cm at different magnetic fields. For $B = 1$ T one reads from Fig. 2 that $g \approx -4.6$, so the electrons will be in the $|0+\rangle$ state and from Fig. 6 one concludes that they will be concentrated in the right QW. If now for the same $F = -5$ kV/cm the magnetic field is changed to $2 \leqslant B \leqslant 4$ T, it follows from Fig. 2 that g becomes positive, so the occupied state is $|0-\rangle$. The wave function for this state at the above fields is strongly concentrated in the left QW; see Fig. 6. Thus by changing the intensity of B alone at a constant electric field F one will reverse the electron spin and transfer electrons from the right to the left QW. In other words, electrons will move *along* the magnetic field, which is an unusual effect.

An asymmetric change of the spin g value by an external electric field can be realized with one quantum well also, but a one-well structure is less effective than the ones considered above. In Fig. 7 we report our calculations for the structure shown in Fig. 3, but with the middle barrier reduced to zero width. The resulting well is asymmetric, and the electric field, by displacing the wave function, makes the influence of the DMS (left) part stronger or weaker. As follows from Fig. 7, the g value depends in fact on the electric field, but not so strongly as in Fig. 1. A physical reason is seen in the inset: in the one-well structure the wave function is not as precisely concentrated as in the two-well arrangement. Moreover, in the one-well structure the spin-up and spin-down wave functions are almost not separated spatially.

As mentioned above, an asymmetric structure of two QWs with a nonvanishing electric field applied along the growth direction z will result in an additional Bychkov-Rashba spin splitting and spin mixing due to structure inversion asymmetry (SIA).[7,8] We considered this effect using Eq. (2). The SIA terms couple $|0+\rangle$ and $|1-\rangle$ states. It turns out that the additional spin splitting is quite small and an admixture of opposite spin constitutes not more than 7.6% of the the total wave function squared at electric fields of our interest. It should be mentioned that the same mechanism was used by Nowack *et al.*[9] to control a single-electron spin in a quantum dot by an electric field.

Spin-polarized electrons were created in the past by optical orientation (OO), exciting III-V semiconducting compounds (mostly GaAs) with circularly polarized light. Fabricated sources were subsequently used for experiments in various domains of physics: electron collisions with atoms, surface magnetism, parity nonconservation in inelastic scattering at high energies, etc.[10] The system we propose offers important advantages in comparison with OO. First, it creates an almost completely spin-polarized electron gas, while OO gives 40%–50% efficiency. Second, due to spin splitting by the magnetic field, electrons stay a long time in their spin state, while in the OO regime they quickly recombine. Third, the spin-polarized electrons are in spatial reservoirs which should facilitate applications. Also, electrons with polarized spins were spatially confined in so-called spin superlattices (SSs) made of periodic layers of nonmagnetic and DMS materials.[11–13] In an external magnetic field the DMS layers, characterized by higher spin g factors, have different energies from the nonmagnetic layers, and the electrons with opposite spins spatially separate, occupying lower energies for their spin orientation. Compared to SSs, our system is considerably more flexible due to the electric field. The latter allows one (1) to reverse the spin g factor, and (2) to put the spin-polarized electrons into the chosen QW. In addition, in SSs the spin separation is not stable since the electrons will eventually relax their spins and go to the contiguous layers in order to arrive at the lowest possible energy. In our case the system is stable because the electric field makes one QW lower than the other.

Finally, our system offers the following interesting possibility. If the temperature is raised, so that both $|0-\rangle$ and $|0+\rangle$

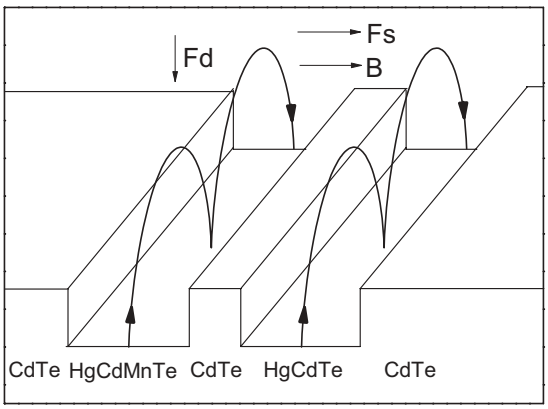

FIG. 8. A scheme for electric and magnetic spin transistors in the parallel-transport configuration based on the double-well heterostructures shown in Fig. 3 or 5. Electron trajectories in the ballistic regime are indicated. If the left and right electrodes in the back can drain only spin-down or spin-up electrons, respectively, the spin currents in each well can be controlled by the intensities of the electric field $\mathbf{F}s$ and magnetic field $\mathbf{B}$.

levels are populated, it follows from Figs. 3 and 6 that, for $3 \leqslant B \leqslant 4$ T, the spin-down electrons will be in the left QW while the spin-up electrons will be in the right QW. Thus the system spatially separates electrons with opposite spins, which amounts to an effective Stern-Gerlach experiment. The latter is, as is well known, not possible with electrons in a vacuum; see Ref. 14.

IV. SPIN TRANSISTORS

The two-well structures proposed above may be used to control spin currents via electric and magnetic fields in the parallel-transport configuration. The proposed scheme is shown in Fig. 8. In addition to the electric field $\mathbf{F}s$ parallel to the growth direction, as considered above, one needs an electric field along the interfaces which will drive the parallel currents. Suppose the left electrode in the back of the structure can drain only the spin-down electrons, while the right electrode can drain only the spin-up electrons. For the small number of electrons only the lowest spin state is occupied. Thus for a positive g value we are interested in the spin-down current. If now, for example, $\mathbf{B}$ and $\mathbf{F}s$ are the field intensities indicated in Fig. 3 (in particular $\mathbf{F}s = \mathbf{0}$), the left well will conduct the spin-down parallel current. If one applies a small negative field $\mathbf{F}s$, the probability of finding the spin-down electrons in the left QW diminishes. As a result the spin-down current in the left QW becomes smaller. The same reasoning applies to the spin-up current in the right QW for different values of $\mathbf{F}s$. If one raises the temperature or sends enough electrons to populate both the spin-up and spin-down lowest states, one can have a spin-up parallel current in one well and a spin-down current in the other. As follows from the above reasoning in Sec. III, one can also control the probability of finding the electron wave function in a given QW using a magnetic field alone at a constant electric field. Consider as an example the case shown in Fig. 4 at higher temperatures, when both spin states are populated. It can be seen that the spin-down wave function in the left QW strongly diminishes with increasing magnetic field. This means that one can change the corresponding spin-down parallel current in the left QW using the magnetic field intensity. Thus the above configuration can lead to both electric and magnetic spin transistors.

If the driving parallel field $\mathbf{F}d$ is much stronger than the $\mathbf{F}s$ field, one has the standard crossed-field $\mathbf{F}d \perp \mathbf{B}$ configuration. The electron orbit in this case represents a cycloid parallel to the interfaces, as indicated in Fig. 8. One then deals with the standard transverse magnetoresistance situation. If the field intensities $\mathbf{F}s$ and $\mathbf{F}d$ are comparable, one deals with a resultant field which is not perpendicular to the magnetic field and the electron trajectory will be more complicated. After a scattering event, the electron begins to move along the electric field. If the relaxation time is very short, it could be more advantageous to direct $\mathbf{F}d$ along the direction of the wells. As to electron mobilities in the parallel transport, it has been demonstrated that, in good samples, they are comparable to the bulk values.[15] For short devices one will then deal with the ballistic regime.

V. SUMMARY

A heterostructure of two interacting quantum wells is devised that can spatially contain spin-polarized electrons by means of electric and magnetic fields parallel to the growth direction. It is proposed to make one QW of the dilute magnetic semiconductor HgCdMnTe and the other of the nonmagnetic material HgCdTe, which will allow one to strongly influence the overall spin g factor, including its change of sign. It is indicated how in such a system one can put spin-polarized electrons into one or the other QW and control their spin orientation. By employing the proposed scheme one can construct an electric or magnetic spin transistor by driving spin-polarized electron currents in each QW parallel to the interfaces.

*Corresponding author: pfeff@ifpan.edu.pl

[1]G. Salis, Y. Kato, K. Ensslin, D. C. Driscoll, A. C. Gossard, and D. D. Awschalom, Nature (London) **414**, 619 (2001).
[2]P. Pfeffer and W. Zawadzki, Phys. Rev. B **72**, 035325 (2005).
[3]E. O. Kane, J. Phys. Chem. Solids **1**, 249 (1957).
[4]R. R. Galazka, in *Proceedings of the 14th International Conference on the Physics of Semiconductors*, edited by B. L. H. Wilson, International Conference Series, Vol. 43 (Institute of Physics, Edinburgh, 1978), p. 133.
[5]J. K. Furdyna, J. Appl. Phys. **64**, R29 (1988).
[6]J. Wrobel *et al.*, in *Proceedings of the 18th International Conference on the Physics of Semiconductors*, International Conference on the Physics of Semiconductors, edited by O. Engstrom (World Scientific, Singapore 1987), p. 1795.
[7]Y. A. Bychkov and E. I. Rashba, J. Phys. C **17**, 6039 (1984).
[8]W. Zawadzki and P. Pfeffer, Semicond. Sci. Technol. **19**, R1 (2004).
[9]K. C. Nowack, F. H. L. Koppens, Yu. V. Nazarov, and L. M. K. Vandersypen, Science **318**, 1430 (2007).

[10]D. T. Pierce and R. J. Celotta, in *Optical Orientation*, edited by F. Maier and B. P. Zakharchenya (North-Holland, Amsterdam, 1984).
[11]M. von Ortenberg, Phys. Rev. Lett. **49**, 1041 (1982).
[12]W. C. Chou, A. Petrou, J. Warnock, and B. T. Jonker, Phys. Rev. Lett. **67**, 3820 (1991).
[13]N. Dai, H. Luo, F. C. Zhang, N. Samarth, M. Dobrowolska, and J. K. Furdyna, Phys. Rev. Lett. **67**, 3824 (1991).
[14]J. Wrobel, T. Dietl, A. Lusakowski, G. Grabecki, K. Fronc, R. Hey, K.H. Ploog, and H. Shtrikman, Phys. Rev. Lett. **93**, 246601 (2004).
[15]A. Madhukar, Final Report No. AD-A232-341, Army Research Office (unpublished).