Calculation of the plasma frequency of a stack of coupled Josephson junctions irradiated with electromagnetic waves

Yu. M. Shukrinov,[1,2] I. R. Rahmonov,[1,3] and M. A. Gaafar[1,4]

[1]*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Moscow Region, Dubna, Russia*
[2]*Max Planck Institute for the Physics of Complex Systems, 01187 Dresden, Germany*
[3]*Umarov Physical Technical Institute, TAS, Dushanbe, 734063 Tajikistan*
[4]*Department of Physics, Faculty of Science, Menoufiya University, Egypt*

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We perform a precise numerical study of phase dynamics in high-temperature superconductors under electromagnetic radiation. We observe the charging of superconducting layers in the bias current interval corresponding to the Shapiro step. A remarkable change in the longitudinal plasma wavelength at parametric resonance is shown. Double resonance of the Josephson oscillations with radiation and plasma frequencies leads to additional parametric resonances and the non-Bessel Shapiro step.

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I. INTRODUCTION

Can superconducting layers in high-temperature superconductors (HTSCs) be charged? And if so, how would this be reflected in the properties of these materials? How strong is the influence of nonequilibrium effects on the physics of the intrinsic Josephson junctions (JJs) in HTSCs? These questions are very important for understanding the fundamental properties of superconductors[1–7] and radiation from the intrinsic Josephson junctions, naturally formed by a system of superconducting layers in HTSCs such as $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi-2122).[8] There is no consensus about the mechanism of the terahertz radiation from these materials, which makes their investigation of great relevance today.[9] To make this situation clear, one needs a detailed and precise numerical study of their phase dynamics because the electrical and magnetic properties of intrinsic JJs in HTSCs are strongly nonlinear.[9–14]

One of the most spectacular indications of the Josephson effect in HTSCs is a locking of the Josephson oscillations of each junction to the frequency of external electromagnetic radiation. This locking leads to the appearance of steps in the current voltage characteristics (IV characteristics) at quantized voltages, called the Shapiro steps.[15,16] Devices based on this effect are widely used as voltage standards. Therefore, a detailed study of the Shapiro steps in the intrinsic Josephson junctions at different resonance conditions would open an interesting field of research with potential for different applications.

Another interesting feature of the intrinsic JJ is a longitudinal plasma wave propagating along the c axis.[17,18] A system of superconducting layers in an anisotropic HTSC, which is characterized by the order parameter $\Delta_l(t) = |\Delta| \exp[i\theta_l(t)]$ with the time dependent phase $\theta_l(t)$, comprises N Josephson junctions.[8] The thickness of superconducting layers (about 3 Å) in an HTSC is comparable with the Debye length r_D of electric charge screening. Therefore, there is no complete screening of the charge in the separate layers, and the electric field induced in each JJ penetrates into the adjacent junctions. Thus, the electric neutrality of superconducting layers is dynamically broken and, in the case of the ac Josephson effect, a capacitive coupling appears between the adjacent junctions.[17] The absence of complete screening of charge in the superconducting layer leads to the formation of a generalized scalar potential Φ_l of the layer, which is related to the charge density Q_l in the superconducting layer as follows:[17,19] $Q_l = -\frac{1}{4\pi r_D^2}\Phi_l$. The existence of a relationship between the electric charge Q_l of the lth layer and the generalized scalar potential Φ_l of this layer reflects a nonequilibrium nature of the ac Josephson effect in layered HTSCs.[19] In this case, the diffusion contribution to the quasiparticle current arises due to the generalized scalar potential difference, which is taken into account in the capacitively coupled Josephson-junction model with diffusion current (CCJJ + DC model[20]). At $\omega_J = 2\omega_{\rm LPW}$ (ω_J and $\omega_{\rm LPW}$ are the Josephson and longitudinal plasma-wave frequencies, respectively) the parametric resonance is realized: the Josephson oscillations excite the longitudinal plasma wave. The charge in the superconducting layer at parametric resonance can have a complex oscillation depending on the number of junctions in the stack, coupling and dissipation parameters, and boundary conditions. Fourier analysis[21] of the temporal dependence of the charge in a superconducting layer shows in the spectrum different frequencies, in particular, $\omega_{\rm LPW}$, ω_J, and their combinations. The IV characteristics of the intrinsic JJ display a multiple branch structure[22–25] and have a breakpoint related to the parametric resonance and a parametric resonance region in the outermost branch before transition to the inner branch. External radiation essentially changes the physical picture of the coupled Josephson junctions. In particular, the conditions for double resonance $\omega = \omega_J = 2\omega_{\rm LPW}$ can be realized, where ω is the radiation frequency.

We would like to stress that the one-dimensional models with coupling between junctions capture the main features of the real intrinsic JJs, like hysteresis and branching of the IV characteristics, and help us to understand their physics. An interesting and important fact is that the one-dimensional models can also be used to describe the properties of a parallel array of the Josephson junctions, which is often considered as a model for long Josephson junctions. References 26 and 27 showed that the experimental data demonstrate a series of resonances in the IV characteristics of the array. These data were analyzed using the discrete sine-Gordon model and the extension of this model that includes a capacitive interaction between the neighboring Josephson junctions. The parametric instabilities of a one-dimensional parallel array of N identical Josephson junctions were predicted by theoretical analysis of the discrete sine-Gordon equation (also known as the Frenkel-Kontorova model) and observed experimentally in Ref. 28. In particular, the novel resonant steps related to the parametric instability were experimentally found in the IV characteristics of the discrete Josephson ring even when there were no vortices in the ring. It is therefore clear that the problem we consider is very general.

In this paper, we present the results of the investigation of the effects of electromagnetic radiation on the phase dynamics of the intrinsic JJs and the temporal oscillations of the electric charge in superconducting layers in HTSCs. We demonstrate the "charging" of the Shapiro step, i.e., the charging of superconducting layers in HTSCs in the bias current interval corresponding to the Shapiro step. An increase in the amplitude of radiation changes the plasma wavelength along the stack of junctions. An additional parametric resonance and "non-Bessel" Shapiro steps appear at double-resonance conditions when the Josephson frequency coincides with the radiation and plasma frequencies. We show that the radiation drastically changes the IV characteristics in the parametric resonance region and even leads to some irregular structures.

II. MODEL AND METHOD

To investigate the phase dynamics of the intrinsic JJ we use the one-dimensional CCJJ + DC model with the gauge-invariant phase differences $\varphi_l(t)$ between S layers l and $l+1$ in the presence of electromagnetic irradiation described by the following system of equations:

$$\frac{\partial \varphi_l}{\partial t} = V_l - \alpha(V_{l+1} + V_{l-1} - 2V_l)$$
$$\frac{\partial V_l}{\partial t} = I - \sin\varphi_l - \beta\frac{\partial \varphi_l}{\partial t} + A\sin\omega t + I_{\text{noise}}, \qquad (1)$$

where t is the dimensionless time normalized to the inverse plasma frequency ω_p^{-1}, $\omega_p = \sqrt{2eI_c/\hbar C}$, C is the capacitance of the junctions, $\beta = 1/\sqrt{\beta_c}$, β_c is the McCumber parameter, α gives the coupling between junctions,[17] and A is the amplitude of the radiation. To find the IV characteristics of the stack of the intrinsic JJ, we solve this system of nonlinear second-order differential equations in Eq. (1) using the fourth-order Runge-Kutta method. In our simulations we measure the voltage in units of $V_0 = \hbar\omega_p/(2e)$, the frequency in units of ω_p, the bias current I, and the amplitude of radiation A in units of I_c.

To calculate the voltages $V_l(I)$ at each I, we simulate the dynamics of the phases $\varphi_l(t)$ by solving the system of equations in Eq. (1) using the fourth-order Runge-Kutta method with a step in time T_p (the scheme of the numerical procedure and parameters of simulation was presented in Fig. 1 of Ref. 22). As a result, we find the temporal dependence of the voltages in each junction at a fixed value of bias current. So we can calculate the temporal dependence of the charge in each layer as well through the voltage difference in the neighbor junctions (see below). After completing the calculations for bias current value I the current value is increased or decreased by a small amount of δI (bias current step) to calculate the voltages in all junctions at the next point of the IV characteristics. So actually time dependence of voltage in each junction or charge in each layer consists of intervals at each fixed current value. We use the distribution of phases and voltages achieved at the previous point of the IV characteristics as the initial distribution for the current point. The average of the voltage $\bar{V}_l$ is given by

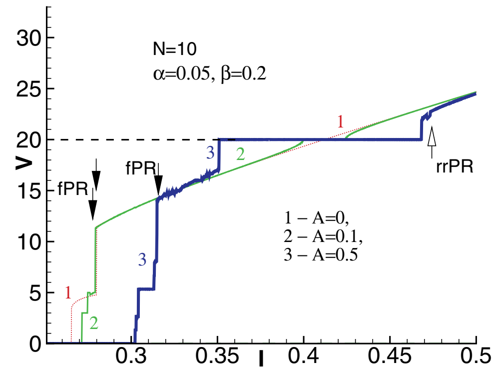

FIG. 1. (Color online) The IV characteristics of a stack with 10 coupled JJs without irradiation (curve 1) and under radiation with frequency $\omega = 2$ and amplitude $A = 0.1$ (curve 2) and amplitude $A = 0.5$ (curve 3). The filled arrows indicate the positions of the fundamental parametric resonance (fPR), while the hollow arrow indicates the radiation related parametric resonance (rrPR).

$$\bar{V}_l = \frac{1}{T_f - T_i}\int_{T_i}^{T_f} V_l dt, \qquad (2)$$

where T_i and T_f determine the interval for the temporal averaging.

To study time dependence of the electric charge in the S layers, we use the Maxwell equation $div(\varepsilon\varepsilon_0\vec{E}) = Q$, where ε and ε_0 are relative dielectric and electric constants, respectively. The charge density Q_l (in the following text referred to as *charge*) in the S layer l is proportional to the difference between the voltages V_l and V_{l+1} in the neighbor insulating layers $Q_l = Q_0\alpha(V_{l+1} - V_l)$, where $Q_0 = \varepsilon\varepsilon_0 V_0/r_D^2$. For $r_D = 3\times10^{-10}$ m, $\varepsilon = 25$, $\omega_p = 10^{12}$ s^{-1} we get $V_0 = 3\times10^{-4}$ V and $Q_0 = 8\times10^5$ C/m^3. So, at $Q = Q_0$ for a superconducting layer with area $S = 1\ \mu$m^2 and thickness $d_s = 3\times10^{-10}$ m the charge value is about 2.4×10^{-16} C. This value of charge is not high, but it creates an interesting physics. We have taken an even number of junctions in the stack ($N = 10$) to avoid additional modulations in the electric charge in the parametric resonance region which appear when N is odd, and to concentrate on the effect of radiation. This choice allows us to see the effect of the radiation more clearly. Numerical calculations have been done for a stack with the coupling parameter $\alpha = 0.05$, dissipation parameter $\beta = 0.2$, and periodic boundary conditions. We note that the qualitative results are not very sensitive to these parameter values and boundary conditions. The details of the model and simulation procedure are presented in Ref. 25.

III. RESULTS AND DISCUSSIONS

It is known that in the case of a single Josephson junction with an increase in the radiation amplitude A a hysteresis region decreases, i.e., it leads to the decrease of the critical current value and the increase of the return current I_R.[29] For a stack of coupled JJs the external radiation leads additionally to a series of novel effects related to the parametric resonance and the longitudinal plasma wave propagating along the c axis.[17,18] We demonstrate below three effects with an increase in the amplitude of radiation A: (i) the changing of longitudinal plasma wavelength, (ii) the additional resonances around the Shapiro step, and (iii) the double resonance $\omega_J = \omega = 2\omega_{\rm LPW}$.

A. Variation of longitudinal plasma wavelength

The equation for the Fourier component of the difference $\delta\varphi_l = \varphi_{l+1,l} - \varphi_{l,l-1}$ between neighbor junctions can be written in linear approximation in the form[23] $\ddot{\delta_k} + \beta(k)\dot{\delta_k} + \cos[\Omega(k)\tau]\delta_k = 0$, where $\tau = \omega_p(k)t$, $\omega_p(k) = \omega_p C$, $\beta(k) = \beta C$, $\Omega(k) = \Omega/C$, and $C = \sqrt{1 + 2\alpha[1 - \cos(kd)]}$, $d = d_s + d_i$, d_i is the thickness of insulating layer. This equation demonstrates the parametric resonance with a change in the parameters $\beta(k)$ and $\Omega(k)$ leading to the excitation of a longitudinal plasma wave with $\omega_p(k) = \omega_p\sqrt{1 + 2\alpha[1 - \cos(kd)]}$.

First, we consider the case $\omega > 2\omega_{\rm LPW}$, when the Shapiro step is above the parametric resonance region in IV characteristics. Irradiation leads to the decrease of the hysteresis in the IV characteristics,[21,29] so it is expected that the parametric resonance point ($\omega_J = 2\omega_{\rm LPW}$) would be shifted as well, and the longitudinal plasma-wave frequency would increase.

We investigate the influence of the external radiation on the parametric resonance by increasing the amplitude of the radiation at fixed frequency. Figure 1 shows three IV characteristics of a stack with 10 coupled JJs: without irradiation (curve 1) and under radiation with $\omega = 2$, $A = 0.1$ (curve 2) and $A = 0.5$ (curve 3). At $\omega = 0$ the parametric resonance is characterized by the breakpoint current $I_{bp} \simeq 0.28$ and breakpoint voltage $V_{\rm bp} \simeq 11.51$ corresponding to the Josephson frequency $\omega_J \simeq 1.151$.[23] The parametric resonance region in the IV characteristics is shifted up along the voltage axis with increase in the amplitude of radiation. As we can see, the first Shapiro step is developed on the outermost branch of the IV characteristics in the hysteresis region at $V = \omega_J N = 20$. The dashed line stresses this fact. The filled arrows indicate the positions corresponding to the appearance of a fundamental parametric resonance in the stack, which is realized without radiation too. The hollow arrow indicates an additional parametric resonance before the Shapiro step that is caused by irradiation. We call this resonance a radiation related parametric resonance to distinguish it from the fundamental parametric resonance. We will discuss it below.

Figure 2(a) shows the time dependence of the electric charge in two superconducting layers at $A = 0.1$ (charge in the first layer is shown by thick curve L1 and in the second one by thin curve L2). Instead of monotonic exponential increase of the charge observed in the case without radiation, we see exponential but modulated growth of the charge. The charges on the neighbor layers are equal in magnitude and opposite in sign. This is true for all adjacent layers and corresponds to the π mode, so the π mode survived under radiation with the amplitude $A = 0.1$. Fast Fourier transform (FFT) analysis of the time dependence of voltage $V(t)$ in each JJ and charge $Q(t)$ in each S layer (not presented here) show that this modulation is due to beating between the external and Josephson oscillations.

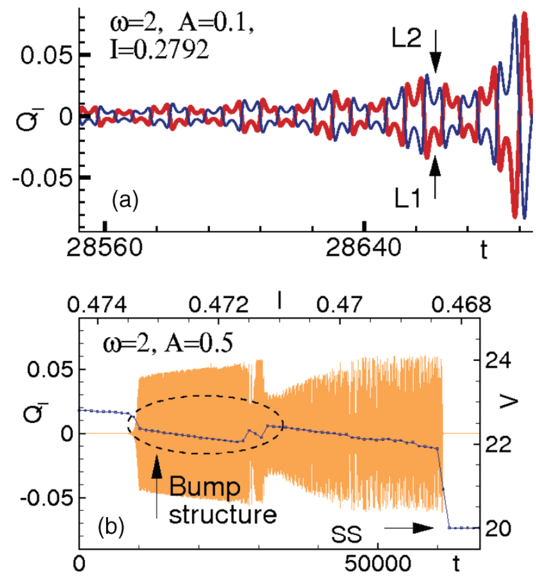

FIG. 2. (Color online) (a) The illustration of the modulation of the charge oscillations in the fundamental parametric resonance region ($I = 0.2792$) at $A = 0.1$. $L1$ and $L2$ show charges in the first and second superconducting layers, respectively. (b) $Q(t)$ dependence together with the IV characteristics (black curve with the symbols related to the right and upper axes) in the radiation related parametric resonance region at $A = 0.5$. The ellipse part of the IV characteristics shows the "bump" structure. The arrow indicates the position of the Shapiro step (SS) in the IV characteristics as well.

One of the interesting results we obtained is that the irradiation can change the character of the charge-time dependence essentially and bring about a "bump" structure on the outermost branch of the IV characteristics, as shown in the Fig. 2(b), which demonstrates the $Q(t)$ dependence together with the IV characteristics (black curve with the symbols associated with the right and upper axes). The "bump" structure in the IV characteristics is marked by an ellipse. At used parameters of simulation $\omega = 2$ and $A = 0.5$, as was mentioned above, the additional radiation related parametric resonance appears in the stack before the Shapiro step (shown by the hollow arrow in Fig. 1). It would be interesting if the charging of the S layers can appear with other types of resonances in coupled JJs.[11,30,31] This question has not been fully investigated.

We will now discuss the effect of the amplitude increase on the wavelength of the longitudinal plasma wave at fundamental parametric resonance. Figure 3 demonstrates this effect at $\omega = 2$. We see that, before the resonance region [Fig. 3(a)], the charge in the layers is zero (to within the noise level). In the growing region of the resonance [Fig. 3(b)] the amplitude of the charge oscillations increases exponentially, forming the longitudinal plasma wave with the wave number $k = \pi/d$ ($\lambda = 2d$). At $A = 0.14$ *the wavelength of the created longitudinal plasma wave at the fundamental parametric resonance is changed by the external radiation.* The charge distribution along the stack, presented in Fig. 3(c), illustrates the wave with $\lambda = 10d$. At $A = 0.23$ we found that the wavelength of the LPW changed from $\lambda = 10d$ to $5d$, as shown in Fig. 2(d).

The results of detailed investigations of the irradiation effects at $\omega = 2$ in the amplitude range (0,0.35) are summarized

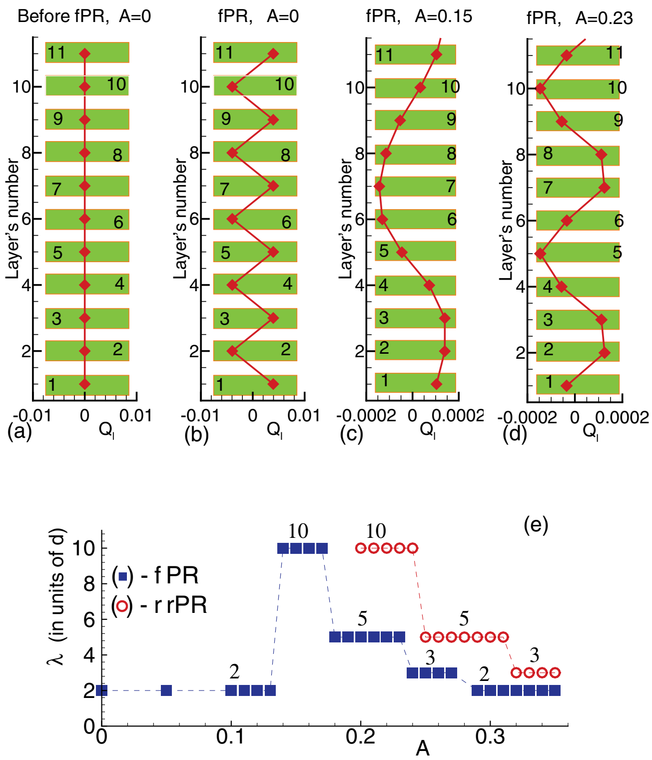

FIG. 3. (Color online) Demonstration of the change in the wavelength of the longitudinal plasma mode at the fundamental parametric resonance with increase of the amplitude of radiation. The numbers count the layers in the stack. (a) $A = 0$, before resonance. (b) $A = 0$, at resonance. (c) $A = 0.15$. (d) $A = 0.23$. (e) The longitudinal plasma wavelength at the fundamental parametric resonance (fPR) (filled squares) and radiation related parametric resonance (rrPR) (empty circles) in the amplitude interval (0,0.35) at $\omega = 2$.

in Fig. 3(e), which shows the variation of the longitudinal plasma wavelength with A. In the case of fundamental parametric resonance we register the following transitions of longitudinal plasma wave with increase in A: $\lambda = 2d \Rrightarrow \lambda = 10d \Rrightarrow \lambda = 5d \Rrightarrow \lambda = 3d \Rrightarrow \lambda = 2d$. An increase in A also changes the wavelength of the radiation related parametric resonance. In the case of the radiation related parametric resonance, as is demonstrated in Fig. 1, we observe the following transitions: $\lambda = 10d \Rrightarrow \lambda = 5d \Rrightarrow \lambda = 3d$ as A increases from 0 to 0.35.

B. Double resonance in a system of coupled Josephson junctions

The double-resonance condition $\omega_J = \omega = 2\omega_{\mathrm{LPW}}$ can be approached by decreasing the radiation frequency: it produces the Shapiro step in the parametric resonance region. In Fig. 4(a) we show the IV characteristics of a stack with 10 coupled JJs under radiation with the amplitude $A = 0.005$ and different frequencies. The numbers near the corresponding curves indicate the value of external radiation frequency. The thick curve (black online) shows the IV characteristics without irradiation, while the inset stresses the coincidence of all curves before the Shapiro step. The Shapiro step does not appear at a frequency smaller than $\omega = 1.151$, because before it a jump to another branch occurs.

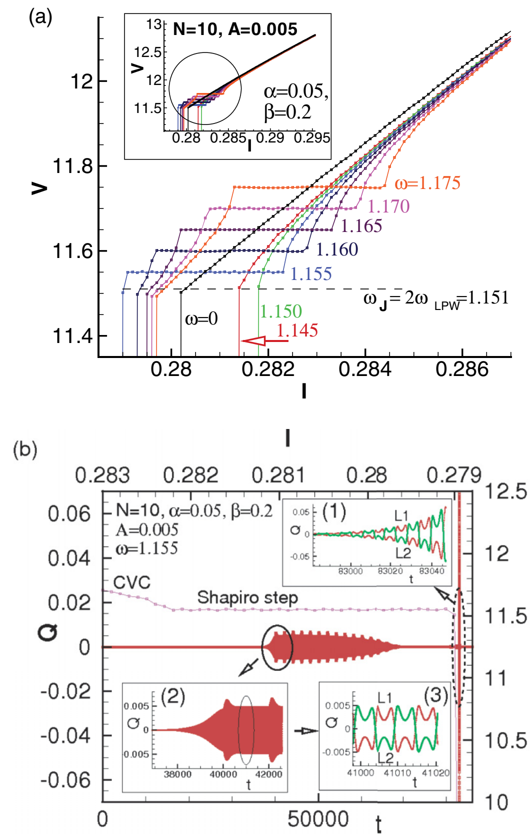

FIG. 4. (Color online) (a) IV characteristics of a stack with 10 coupled JJs under radiation with amplitude $A = 0.005$ and different frequencies. The thick curve (black online) shows IV characteristics without irradiation. Inset: Coincidence of the curves before the Shapiro step. (b) Demonstration of the Shapiro step "charging." We show the IV characteristic with the Shapiro step (CVC, right and upper axes) together with a time dependence of the charge in the superconducting layer. Inset 1: Enlargement of the charge oscillations in the parametric resonance region. Insets 2 and 3: Enlargements in consecutive order of the charge oscillations in the Shapiro step region.

The double resonance demonstrates an interesting feature of coupled JJs which is absent in the case of a single JJ: when the external frequency is close enough to the parametric resonance condition $\omega_J = 2\omega_{\mathrm{LPW}}$, *charge oscillations appear in the S layers in the current interval corresponding to the Shapiro step ("charging" of the Shapiro step)*. In our case, such charging appears starting from $\omega \simeq 1.1555$, while for the fundamental parametric resonance without radiation it is realized at a Josephson frequency of $\omega_J = 1.151$. The amplitude of oscillations and the current interval of charging ("width of charging") grow as the double-resonance condition is approached. Figure 4(b) demonstrates the charging of the Shapiro step at $\omega = 1.155$. It shows IV characteristics with the Shapiro step (the curve with symbols denoted as CVC, related to the right and upper axes) together with a time dependence of the charge in the first superconducting layer. The enlarged parts of the charge-time dependence are shown in consecutive order in insets 2 and 3. In inset 3 we clearly see that the charge oscillations in the S layers correspond to the π mode of a clearly longitudinal plasma wave. Inset 1 enlarges the charge oscillations in the time interval, corresponding to the bias current close to the transition to the inner branch and demonstrates that the fundamental parametric resonance survives at these radiation parameters. It also corresponds to the creation of the π mode of the longitudinal plasma wave. However, of course, there are no restrictions on the creation of longitudinal plasma waves with other wave numbers at different parameters of the system and different radiation.

C. Non-Bessel behavior of Shapiro steps at double resonance

How does the double resonance affect the width of the Shapiro step? Figure 5(a) shows the effect of the amplitude increase at $\omega = 1.151$, i.e., at double-resonance conditions. Inset 1 enlarges the part of the figure with small A. In this case, even small amplitude radiation leads to the "charging" of the Shapiro step. We see it in inset 2, where the charge-time dependence at $A = 0.06$ together with the IV characteristics (dark curve with the symbols associated with right axis for the voltage and upper axis for the current) are presented. At double resonance the transition to the inner branch occurs directly from the Shapiro step. In this case, *the Bessel function dependence of the Shapiro step width*[29] *on A for the coupled JJ is broken and we observe the "non-Bessel" Shapiro step*. To stress this effect, we show in Fig. 5(b) the A dependence of the bias current at the beginning and at the end of the Shapiro step for a single JJ (hollow diamond) and the stack of 10 JJs (filled squares) at $\omega = 1.151$ [Fig. 5(b)] and $\omega = 2$ [Fig. 5(c)]. The double arrows indicate the corresponding Shapiro step width. We see that at $\omega = 1.151$ the value of the Shapiro step width is cut off ("non-Bessel") in comparison with the cases of a single JJ and a stack at $\omega = 2$, when the Shapiro step is far from the fundamental parametric resonance in the voltage (bias current) value.

Finally, we show that an increase of the radiation amplitude at double-resonance conditions also leads to the appearance of an additional parametric resonance before the Shapiro step (radiation related parametric resonance, $I > I_{SS}$). This situation is demonstrated in inset 2 of Fig. 5(a) which shows the charge-time dependence together with the IV characteristics at $A = 0.06$. We see two charged regions: the radiation related parametric resonance region and the Shapiro step region. Analysis of the charge-time dependence in this case shows the creation of the longitudinal plasma wave with the wavelength $\lambda = 5d$. The appearance of the radiation related parametric resonance in the system before the Shapiro step is reflected in the IV characteristics by its deformation (the appearance of the breakpoint). We indicate such deformation by arrows at $A = 0.06$ and 0.08. Further increase of the radiation amplitude leads to the disappearance of the main Shapiro step. We note that we have observed a "charging" of some Shapiro step harmonics as well. The detailed description of this phenomenon and various manifestations of the double resonance in the coupled JJ will be considered elsewhere.

IV. SUMMARY

In summary, the nonequilibrium situation in the thin superconducting layers of HTSCs plays an important role in the variety of novel phenomena in the system of Josephson junctions naturally formed in these materials, related to their interaction with external radiation. Our detailed numerical study of the phase dynamics in the presence of radiation revealed a series of effects specific to the coupled Josephson junctions and absent in the case of a single junction. We observed the charging of the superconducting layers in the bias current interval corresponding to the Shapiro step ("charging" Shapiro step). A remarkable change in the wavelength of the longitudinal plasma mode at the parametric resonance is important for understanding the fundamental properties of superconductors and radiation emitted from the intrinsic Josephson junctions. We expect that the double resonance of the Josephson oscillations with radiation and plasma frequencies demonstrating additional parametric resonances and a "non-Bessel" Shapiro step will be an object of intensive experimental investigations. We stress the importance and necessity of further theoretical and experimental studies of the observed structural changes in the current voltage characteristics in the parametric resonance region and under electromagnetic radiation.

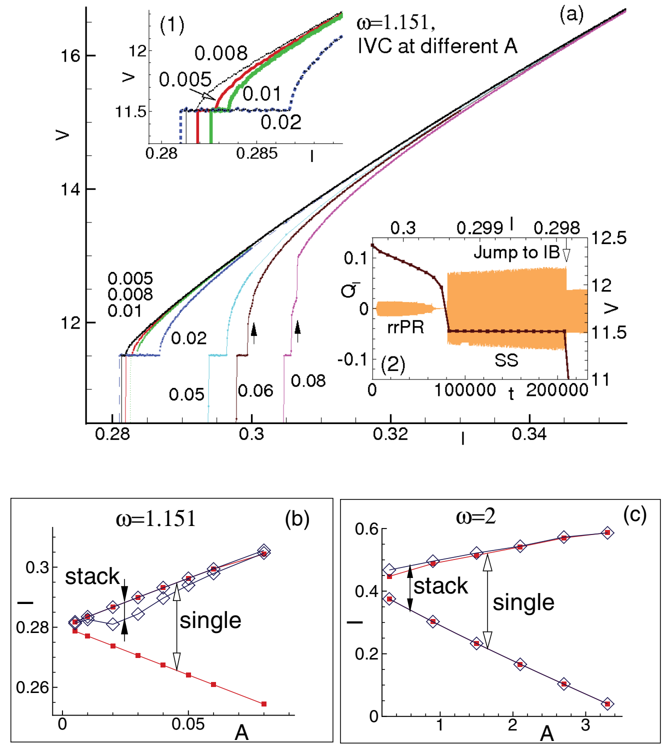

FIG. 5. (Color online) (a) IV characteristics of a stack with 10 JJs at $\omega = 1.151$ and different amplitudes of radiation. Inset 1: Enlargement of the part with small A. Inset 2: Charge-time dependence at $A = 0.06$ together with IV characteristic (dark curve with the symbols with right axis for the voltage and upper axis for the current). (b) Demonstration of Shapiro step width changing for single JJ (filled squares) and stack of intrinsic JJ (empty squares) at $\omega = 1.151$. (c) The same at $\omega = 2$.

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