

**Enhanced energy relaxation process of a quantum memory coupled to a superconducting qubit**

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For quantum information processing, each physical system has a different advantage as regards implementation, so hybrid systems that benefit from the advantage of several systems would provide a promising approach. One common hybrid approach involves combining a superconducting qubit as a controllable qubit and another quantum system with a long coherence time as a memory qubit. The use of a superconducting qubit gives us excellent controllability of the quantum states, and the memory qubit is capable of storing information for a long time. It has been believed that selective coupling can be realized between a superconducting qubit and a memory qubit by tuning the energy splitting between them. However, we have shown that this detuning approach has a fundamental drawback as regards energy leakage from the memory qubit. Even if the superconducting qubit is effectively separated by reasonable detuning, a non-negligible incoherent energy relaxation in the memory qubit occurs via residual weak coupling when the superconducting qubit is affected by severe dephasing. This energy transport from the memory qubit to the control qubit can be interpreted as the appearance of the anti-quantum Zeno effect induced by the fluctuation in the superconducting qubit. We also discuss possible ways to avoid this energy relaxation process, which is feasible with existing technology.

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**I. INTRODUCTION**

A superconducting qubit provides us with excellent controllability of the system for quantum information processing. Coherent manipulations of superconducting qubits have already been demonstrated experimentally, and actually it is possible to perform a single-qubit rotation within a few nanoseconds by using a resonant microwave field.<sup>1</sup> Also, a high-fidelity single-qubit measurement has already been achieved with existing technology.<sup>2</sup> Specifically, a method using a Josephson bifurcation amplifier<sup>3,4</sup> (JBA) has been used experimentally to perform a nondestructive measurement on the superconducting qubit.

However, the coherence time of the superconducting qubit is usually relatively short, where the typical dephasing time is of the order of a microsecond at the optimal point and becomes tens of nanoseconds far from the optimal point.<sup>5-7</sup> Recently, to overcome the problem of the short coherence time, a hybrid approach has been suggested that one can use another physical system as a quantum memory. One promising system for quantum memory is an atomic ensemble of electronic spins such as *P*-doped Si and nitrogen atoms in fullerene cages C<sub>60</sub> where the spin ensemble is coupled with the superconducting qubit through a microwave cavity.<sup>8-10</sup> Magnetic coupling between a superconducting flux qubit and a spin ensemble such as nitrogen-vacancy centers in diamond can also provide such a hybrid system.<sup>11,12</sup> Spin-ensemble qubits typically have a long coherence time of, for example, tens of milliseconds, which is a million times longer than that of a superconducting qubit.<sup>8-11</sup> Moreover, for the electron spins bound to donors in silicon, coherence times can be as long as several seconds.<sup>13</sup>

It is known that a superconducting qubit could potentially provide a memory qubit if the lifetime could be increased. The control and the measurement setup used for the superconducting qubit, however, induces decoherence, and so there is a tradeoff relationship between efficient control and a long coherence time.<sup>14</sup> This means that, by sacrificing controllability, it would be possible to have a much longer coherence time

for a memory superconducting qubit.<sup>14,15</sup> By combining a superconducting qubit with excellent controllability and another superconducting qubit with a long coherence time, we can construct a hybrid system to take advantage of both characteristics. For example, a recent experiment coupling two superconducting phase qubits with a resonant cavity<sup>16</sup> showed the possibility of utilizing one of the qubits for control and the other for memory where two phase qubits are entangled through the common quantum bus, namely, the resonant cavity.<sup>17</sup> Another example is a hybrid system consisting of a superconducting qubit and a microwave cavity. Strong coupling has been realized between the superconducting qubit and microwave cavity,<sup>18,19</sup> which shows a possible application of a high-*Q* cavity as a quantum memory for storing the information.

However, in this paper, we point out quantitatively that such hybrid systems composed of a superconducting qubit and a memory qubit could have a potential error caused by unwanted energy leakage from the memory qubit. When we transfer the quantum information to the memory, we have to tune the energy of the superconducting qubit on resonance with the memory qubit, and then it becomes possible to swap the information from the controllable qubit to the memory qubit. Subsequently, by changing external magnetic field, we can detune the energy of the superconducting qubit to decouple from the memory qubit. Importantly, the superconducting qubit is usually affected by severe dephasing,<sup>5,6</sup> and this induces an incoherent energy leakage from the memory qubit to the superconducting qubit during information storage. This energy relaxation caused by dephasing violates the energy conservation, and this phenomenon can be understood as an occurrence of anti-Zeno effect for quantum transport.<sup>20-23</sup>

With reasonable experimental parameters, we evaluate the actual lifetime of the memory qubit, and this turns out to be much shorter than the previously expected lifetime of the memory qubits.<sup>8-11</sup> We will suggest possible ways to avoid such an energy relaxation process, which is feasible with current technology.

The remainder of this paper is organized as follows. In Sec. II, we review the concept of quantum Zeno and anti-Zeno effects. Sections III and IV present the details of our calculations to show how unwanted relaxation occurs in the memory qubit due to the instability of the detuned but weakly coupled superconducting qubit. In Sec. V, we suggest some ways to avoid such relaxation by using the idea of the decoherence-free subspace. Section VI concludes our discussion.

## II. QUANTUM ZENO AND ANTI-ZENO EFFECTS

Quantum Zeno and anti-Zeno effects are fascinating phenomena predicted by quantum mechanics.<sup>24–26</sup> Let us summarize the quantum Zeno and anti-Zeno effects. When an unstable excited state decays to a ground state, we can define the survival probability  $P_e(t)$  as the population remaining in the excited state at time  $t$ . If this survival probability exhibits a quadratic behavior in the initial stage of the decay such as  $P_e(t) \simeq 1 - \Gamma^2 t^2$  for  $\Gamma t \ll 1$ , it is possible to confine the state to the excited level via frequent projective measurements. When we perform  $N$  projective measurements with a time interval  $\tau = \frac{t}{N}$  to determine whether or not the state is still in the excited state, the probability of projecting the state in the excited level for all  $N$  measurements is calculated as  $P(t, N) \simeq (1 - \Gamma^2 \tau^2)^N \simeq 1 - \Gamma^2 \frac{t^2}{N}$ . This success probability approaches unity as the number of measurements increases. So, the system is frozen and decay can be completely suppressed, which is called the quantum Zeno effect. On the other hand, if the survival probability exhibits an exponential decay such as  $P_e(t) = e^{-\Gamma t}$ , the probability of confining the state to the excited level by performing  $N$  measurements is calculated as  $P(t, N) = (e^{-\Gamma \tau})^N = e^{-\Gamma t}$ . Here, the projective measurements do not change the success probability, which means that we can not observe quantum Zeno effect for such an exponential decay system. Interestingly, it is known that unstable systems show quadratic decay initially and exponential decay later.<sup>27</sup> The temporal scale used to denote the crossover from quadratic to exponential decay is known as the jump time.<sup>28</sup> Therefore, to observe the quantum Zeno effect, it is necessary to perform projective measurements on a time scale shorter than the jump time. Moreover, it was predicted that, when projective measurements are performed on a time scale comparable to the jump time, the decay is effectively accelerated, and this is called the anti-quantum Zeno effect.<sup>25,29</sup> Recently, it was also predicted that the anti-quantum Zeno effect can be induced when we perform false measurements,<sup>30</sup> namely, the decay dynamics of the unstable state can be enhanced by frequent measurements with an erroneous apparatus where the energy band of the measurement apparatus is significantly different from the energy of the signal emitted from the unstable state.

## III. ANALYTICAL STUDY OF ENHANCED ENERGY RELAXATION PROCESS

Let us study the unexpected relaxation of the memory qubit in a hybrid system shown in Fig. 1 quantitatively. Although such relaxation behavior has been studied by Refs. 20–23 in an anti-Zeno context for quantum energy transport, we introduce

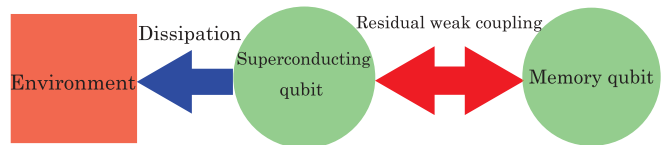


FIG. 1. (Color online) Schematic of the enhanced relaxation of the memory qubit via imperfect decoupling from a superconducting qubit. We assume that the superconducting qubit is affected by decoherence, while there is no direct coupling between the environment and the memory qubit. Since we can tune the energy of the superconducting qubit, it is possible to make the energy of the superconducting qubit on resonance with the energy of the memory qubit to transfer the information. It is also possible to tune the energy of the superconducting qubit far from the resonance when keeping the stored information. However, this type of selective coupling has a significant drawback during the information storage in the memory qubit, as explained in the main text.

a simpler solvable model and we derive an analytical form of the energy relaxation time of the memory qubit. To describe the coupling between the superconducting qubit and the memory qubit, we use the following Hamiltonian called the Jaynes-Cummings model or the Tavis-Cummings model:

$$H = \frac{\omega_{sc}}{2} \hat{\sigma}_z^{(sc)} + \frac{\omega_m}{2} \hat{\sigma}_z^{(m)} + g(\hat{\sigma}_+^{(sc)} \hat{\sigma}_-^{(m)} + \hat{\sigma}_-^{(sc)} \hat{\sigma}_+^{(m)}), \quad (1)$$

where  $\omega^{(sc)}$  ( $\omega^{(m)}$ ) denotes the energy of the superconducting qubit (memory qubit) and  $g$  denotes the coupling strength of the interaction. Note that, although we refer to a superconducting qubit as a control system coupled with the memory in this paper, the analysis here can be applied to any system as long as the interaction with the memory device is described by the Jaynes-Cummings model or the Tavis-Cummings model. These models are of fundamental importance not only for the present setup, but also for many variations, including coupling between superconducting qubits,<sup>31,32</sup> a superconducting qubit interacting with a microwave cavity,<sup>16,17</sup> a superconducting resonator coupled with a spin ensemble,<sup>33–36</sup> or a superconducting flux qubit magnetically coupled with nitrogen-vacancy centers in diamond.<sup>11</sup> Since we can change the energy of the superconducting qubit, it is possible to detune the energy between qubits when keeping the information stored in the memory. In this paper,  $\Delta = \omega_{sc} - \omega_m$  denotes detuning during such a storage. We choose the initial state as  $|0\rangle_{sc}|1\rangle_m$  to represent the storage of the excitation in the memory qubit. Note that, since the Hamiltonian conserves the total number of the excitation, the bases taken into account are  $|0\rangle_{sc}|1\rangle_m$  and  $|1\rangle_{sc}|0\rangle_m$ , as long as we consider only the dephasing of the superconducting qubit as the decoherence source. In other words, the state of the coupled system is always in the Hilbert subspace spanned by these two bases. Also, it is worth mentioning that, throughout this paper, we assume that the memory qubit is coupled only with the superconducting qubit and has no direct interaction with the environment. The assumption here is valid as long as the lifetime of the memory qubit is much longer than that of a superconducting qubit. This is actually true for typical memory qubits because the coherence time of memory qubits can be tens of milliseconds, which is a million times longer than that of superconducting qubits.<sup>8–11</sup>

To obtain an analytical solution of the dynamics under the effect of the dephasing, we adopt a simple model where the system is affected by the unitary operation and the decoherence alternatively, so that we can obtain a recursion equation as  $\rho_{(n+1)\tau} = \hat{\mathcal{E}}(e^{-iH\tau}\rho_{n\tau}e^{iH\tau})$ . Here,  $\rho_{n\tau}$  denotes the density matrix of the system at time  $n\tau$ ,  $\hat{\mathcal{E}}$  denotes the dephasing process, and  $\tau$  denotes a period during the unitary operation. In the limit for a small  $\tau$ , this simplification can be justified by the Trotter expansion.<sup>37</sup> Moreover, the effect of dephasing can be considered a process of measurements without postselection, which we refer to as a ‘‘nonselective measurement.’’ For example, if a pure state  $\alpha|0\rangle + \beta|1\rangle$  decoheres due to the dephasing, we finally obtain a mixed state  $|\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$ , which is the same state as that obtained after performing a projective measurement with respect to  $\hat{\sigma}_z = |0\rangle\langle 0| - |1\rangle\langle 1|$  on the pure state and discarding the measurement results.<sup>38,39</sup> Therefore, our model can be interpreted as one where the environment ‘‘sees’’ the system frequently to degrade the quantum coherence, which provides us with an intuitive connection between our calculation and the quantum Zeno effect. When the time  $\tau$  is comparable to the dephasing time  $T_2^{(\text{sc})}$  of the superconducting qubit such as  $\tau = \alpha T_2^{(\text{sc})}$ , where  $\alpha$  is a fitting parameter of the order of unity, the off-diagonal terms of the density matrix become small. So, we consider the superoperator  $\hat{\mathcal{E}}$  to be a nonselective measurement process for removing out the off-diagonal terms as follows:

$$\begin{aligned} \hat{\mathcal{E}}(\rho_{n\tau}) &\simeq (|0\rangle_{\text{sc}}\langle 0| \otimes \hat{\mathbb{1}}_m)\rho_{n\tau}(|0\rangle_{\text{sc}}\langle 0| \otimes \hat{\mathbb{1}}_m) \\ &\quad + (|1\rangle_{\text{sc}}\langle 1| \otimes \hat{\mathbb{1}}_m)\rho_{n\tau}(|1\rangle_{\text{sc}}\langle 1| \otimes \hat{\mathbb{1}}_m) \\ &= \hat{\mathcal{P}}_0^{(\text{sc})}\rho_{n\tau}\hat{\mathcal{P}}_0^{(\text{sc})} + \hat{\mathcal{P}}_1^{(\text{sc})}\rho_{n\tau}\hat{\mathcal{P}}_1^{(\text{sc})}, \end{aligned} \quad (2)$$

where  $\hat{\mathcal{P}}_0^{(\text{sc})}$  ( $\hat{\mathcal{P}}_1^{(\text{sc})}$ ) is the projection operator to a state  $|0\rangle_{\text{sc}}$  ( $|1\rangle_{\text{sc}}$ ). Under this approximation, the mixed state after performing this superoperator  $\hat{\mathcal{E}}$  should be described as  $\rho_{n\tau} = p_{a,n\tau}|10\rangle_{\text{sc},m}\langle 10| + p_{b,n\tau}|01\rangle_{\text{sc},m}\langle 01|$  where  $p_{a,n\tau}$  and  $p_{b,n\tau}$  denote the population of each state. So, we obtain the recursion equations as follows:

$$\begin{aligned} p_{a,(n+1)\tau} &= \frac{1}{4g^2 + \Delta^2}(2g^2 + \Delta^2 p_{a,n\tau} \\ &\quad + 2g^2(p_{a,n\tau} - p_{b,n\tau})\cos t\sqrt{4g^2 + \Delta^2}), \\ p_{b,(n+1)\tau} &= \frac{1}{4g^2 + \Delta^2}(2g^2 + \Delta^2 p_{b,n\tau} \\ &\quad - 2g^2(p_{a,n\tau} - p_{b,n\tau})\cos t\sqrt{4g^2 + \Delta^2}). \end{aligned}$$

By solving these equations with the initial condition of  $p_{a,0} = 0$  and  $p_{b,0} = 1$ , we obtain

$$\begin{aligned} p_{a,n} &\simeq \frac{1}{2} \left[ 1 - \left( \frac{\Delta^2}{4g^2 + \Delta^2} \right)^n \right], \\ p_{b,n} &\simeq \frac{1}{2} \left[ 1 + \left( \frac{\Delta^2}{4g^2 + \Delta^2} \right)^n \right], \end{aligned} \quad (3)$$

where we use a rotating-wave approximation such that  $\cos\sqrt{4g^2 + \Delta^2}t$  should vanish due to the high-frequency oscillation. We define the effective relaxation time induced by this anti-Zeno effect as the time at which the population of

the excitation of the memory becomes  $\frac{p_{b,0} - p_{b,\infty}}{2}$ . So, we can calculate this effective relaxation time of the memory qubit as

$$\tilde{T}_1^{(\text{m})} = \frac{\alpha \ln 2}{\ln(1 + \frac{4g^2}{\Delta^2})} T_2^{(\text{sc})} \simeq \alpha \ln 2 \frac{\Delta^2}{4g^2} T_2^{(\text{sc})}. \quad (4)$$

Here, we assumed  $\frac{g}{\Delta} \ll 1$ , namely, the coupling is much smaller than the detuning, which is appropriate for actual experiments in order to decouple the system. Although the energy relaxation might be considered to be exponentially small for a large detuning, the effective relaxation time of the memory is only quadratically dependent on detuning. Moreover,  $\tilde{T}_1^{(\text{m})}$  is limited by the dephasing time of the superconducting qubit. Since a detuned superconducting qubit is strongly affected by the environment,<sup>5–7</sup> the dephasing time of the superconducting qubit becomes as small as tens of nanoseconds, which could lead to a severe energy leakage from the memory qubits. It is also worth mentioning that, even if we couple a microwave cavity with the memory device instead of the superconducting qubit,<sup>33–36,40,41</sup> any imperfection of the cavity in such a coupled system will also cause similar energy leakage from the memory qubits.

This kind of acceleration of the energy relaxation can be understood in terms of the violation of energy conservation caused by the dephasing process, which has been discussed for biology systems.<sup>20–23</sup> Also, if we consider the superconducting qubit as a measurement apparatus for the memory qubit, it would be also possible to interpret this enhanced relaxation as the appearance of the anti-Zeno effect induced by false measurements of erroneous apparatus.<sup>30</sup> The decay is accelerated by the difference between the energy band of the measurement apparatus and the energy of the signal emitted from the unstable state.<sup>30</sup> In our case, the detuned superconducting qubit would be interpreted as the erroneous apparatus to measure the signal, namely, to determine which energy eigenstate the memory qubit is in, so that the energy transport from the memory qubit could be accelerated due to the imperfection of the apparatus. In addition, a similar expression has also been used in quantum optics, and is called the scattering rate.<sup>42</sup> For example, when we drive the Rabi oscillations of an unstable two-level system with a detuned light, the total scattering rate of light from the laser field can be also suppressed only quadratically against the detuning.<sup>42</sup> So, it would be possible to interpret the enhanced relaxation rate in our calculation as the scattering rate of the excited population, although in our case the scattering is caused by the dephasing of the superconducting qubit.

Regardless of the interpretation of the enhanced relaxation of the memory qubit, our results are of significant importance from a practical point of view. Since the threshold of the acceptable error rate for achieving fault-tolerant quantum computation is quite small, typically of the order of 1%,<sup>43</sup> it is essential to find ways to store quantum states in reliable memory devices isolated from the environment. Although the observed lifetime of the spin ensemble coherently coupled with a superconducting qubit is typically as short as tens of nanoseconds in the current technology,<sup>12,34</sup> it is theoretically predicted that one would be able to prolong the lifetime by improving the properties of the spin ensemble.<sup>11</sup> It is known that the lifetime of electron spins can be tens of milliseconds

or more for some systems,<sup>8–11</sup> and therefore a hybrid system with a long-lived quantum memory would be realized by these systems in the future. However, our result suggests that, even with a quantum memory having improved properties, the superconducting qubit itself ultimately may limit the memory time of such hybrid systems. We have shown that the standard way to decouple the control qubit from the memory qubit by detuning may not sufficiently suppress the noise in the stored quantum states, which casts a doubt on the feasibility of using memory qubit strategies for scalable quantum computation. Therefore, our result motivates us to find another decoupling method to protect the memory qubits from the noise induced by a superconducting qubit for the sake of future realization of such hybrid systems, which we discuss later in our paper.

It is worth mentioning that this incoherent energy relaxation is much more severe than the well-known errors caused by the dispersive Hamiltonian. Without decoherence, the Hamiltonian between the superconducting qubit and the memory qubit can be represented as a dispersive form  $H = \frac{g^2}{\Delta} \hat{\sigma}_z^{(sc)} \hat{\sigma}_z^{(m)}$  for a large detuning.<sup>44</sup> Therefore, if we tune the superconducting qubit so that it is on resonance with a third-party system such as another qubit for information operations, the superconducting qubit in a superposition state induces phase errors in the detuned memory qubit. The error rate is  $\epsilon \simeq \frac{g^2 t_1}{\Delta}$  where  $t_1$  is the time required for information operations on the third system. Fortunately, such information operations can be performed in tens of nanoseconds and so this kind of phase error can be small. Moreover, as long as the superconducting qubit is detuned from any other qubits, the effect of the dispersive Hamiltonian on the memory qubit can be negligible by polarizing the superconducting qubit into the ground state. These results seem to show the suitability of this scheme for the long-term storage of information. However, as we have shown above, this naive illustration is no longer valid when we take into account the effect of the dephasing from the environment. In fact, incoherent energy relaxation via the anti-Zeno effect occurs whenever the information is stored in the memory qubit. In spite of the fact that the memory qubit is assumed to retain the information for a long period, the information continues to leak during the storage, so the total error accumulation will be significant when we try to access the information in the memory after such a storage.

#### IV. NUMERICAL STUDY OF ENHANCED ENERGY RELAXATION PROCESS

Although we have obtained an analytical formula for the effective relaxation time of the memory qubit with some approximation, it is also possible to obtain a more rigorous result by solving a Lindblad master equation numerically. So, we adopt  $\frac{d\rho}{dt} = -i[H, \rho] - \frac{1}{2T_2^{(sc)}} [\hat{\sigma}_z^{(sc)}, [\hat{\sigma}_z^{(sc)}, \rho]] - \frac{1}{2T_1^{(sc)}} (\hat{\sigma}_+^{(sc)} \hat{\sigma}_-^{(sc)} \rho + \rho \hat{\sigma}_+^{(sc)} \hat{\sigma}_-^{(sc)} - \hat{\sigma}_-^{(sc)} \rho \hat{\sigma}_+^{(sc)})$ , where  $T_1^{(sc)}$  denotes the relaxation time of a superconducting qubit. By solving this master equation numerically, we are able to plot the population decay behavior of the memory qubit as shown in Fig. 3. In the absence of the relaxation in the superconducting qubit, the population of the memory qubit decays to half of the initial population, while the population decays to zero under the effect of the relaxation on the superconducting qubit. From the numerical solution, we plot  $\tilde{T}_1^{(m)}$  as a function of  $T_2^{(sc)}$  in Fig. 4. Even for a long dephasing time and huge detuning such as  $T_2^{(sc)} = 35$  ns and  $\frac{\Delta}{g} = 44$ , the effective relaxation time  $\tilde{T}_1^{(m)}$  is around 14  $\mu$ s, which is much shorter than the typical lifetime of the memory qubit.<sup>8–11</sup> Therefore, our results show that the standard strategy to detune the superconducting qubit with the memory qubit actually fails due to the energy leakage from the memory qubit during the storage of the information. It should be noted that, since our model is quite general, the result here is significant for every hybrid system where a device having a short coherence time couples with a memory device,

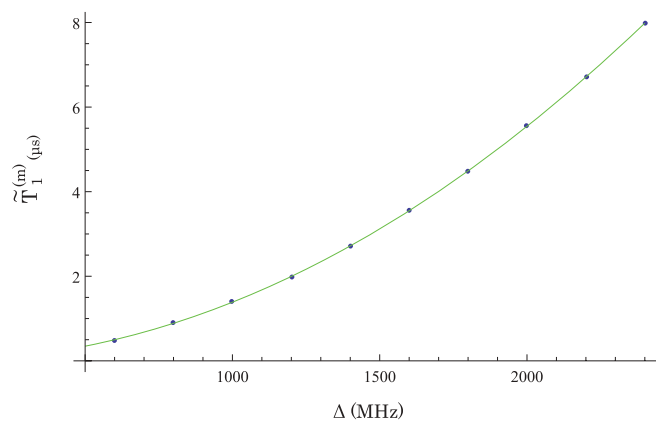


FIG. 2. (Color online) A plot of the relaxation time of the memory qubit induced by the anti-Zeno effect of as a function of the detuning. The discrete plot denotes a numerical result and the continuous line denotes an analytical result. We set parameters as  $T_2^{(sc)} = 10$  ns for the dephasing time of the superconducting qubit and at  $g/2\pi = 25$  MHz for the coupling strength between the qubits. These results show that the relaxation time is quadratically dependent on the energy detuning.

Fig. 2. The behavior of our analytical solution matches the numerical solutions, and this shows the validity of our approximation. This justifies our interpretation that the dephasing corresponds to nonselective measurements and the energy relaxation in the memory qubit is caused by the anti-Zeno effect. Surprisingly, even for a large detuning such as  $\frac{\Delta}{g} \simeq 50$ , the relaxation time is just a few microseconds. Since a typical memory qubit is considered to have a long lifetime of, for example, tens of milliseconds,<sup>8–11</sup> this result shows that the actual relaxation time induced by imperfect decoupling is much shorter than previously expected.

A superconducting qubit can be affected by both dephasing and relaxation. To model both the dephasing and relaxation on the superconducting qubit, we add a relaxation term to the Lindblad master equation and we adopt the following master equation:  $\frac{d\rho}{dt} = -i[H, \rho] - \frac{1}{2T_2^{(sc)}} [\hat{\sigma}_z^{(sc)}, [\hat{\sigma}_z^{(sc)}, \rho]] - \frac{1}{2T_1^{(sc)}} (\hat{\sigma}_+^{(sc)} \hat{\sigma}_-^{(sc)} \rho + \rho \hat{\sigma}_+^{(sc)} \hat{\sigma}_-^{(sc)} - \hat{\sigma}_-^{(sc)} \rho \hat{\sigma}_+^{(sc)})$ , where  $T_1^{(sc)}$  denotes the relaxation time of a superconducting qubit. By solving this master equation numerically, we are able to plot the population decay behavior of the memory qubit as shown in Fig. 3. In the absence of the relaxation in the superconducting qubit, the population of the memory qubit decays to half of the initial population, while the population decays to zero under the effect of the relaxation on the superconducting qubit. From the numerical solution, we plot  $\tilde{T}_1^{(m)}$  as a function of  $T_2^{(sc)}$  in Fig. 4. Even for a long dephasing time and huge detuning such as  $T_2^{(sc)} = 35$  ns and  $\frac{\Delta}{g} = 44$ , the effective relaxation time  $\tilde{T}_1^{(m)}$  is around 14  $\mu$ s, which is much shorter than the typical lifetime of the memory qubit.<sup>8–11</sup> Therefore, our results show that the standard strategy to detune the superconducting qubit with the memory qubit actually fails due to the energy leakage from the memory qubit during the storage of the information. It should be noted that, since our model is quite general, the result here is significant for every hybrid system where a device having a short coherence time couples with a memory device,

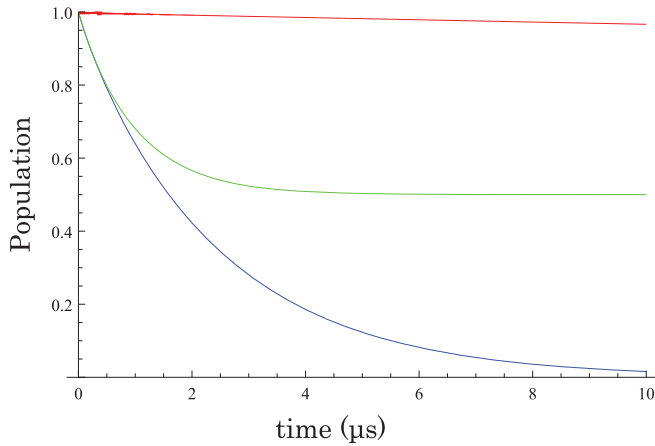


FIG. 3. (Color online) The population decay of the memory qubit induced by the anti-Zeno effect as a function of a time. The coupling constant between qubits is set at  $2\pi \times 25$  MHz. From the bottom, we plot a numerical result with  $T_1^{(\text{sc})} = 400$  ns and  $T_2^{(\text{sc})} = 10$  ns, a numerical result with  $T_2^{(\text{sc})} = 10$  and  $T_1^{(\text{sc})} = \infty$ , and a numerical result with  $T_2^{(\text{sc})} = \infty$  and  $T_1^{(\text{sc})} = 400$  ns.

as long as the coupling is represented by the Jaynes-Cummings model or the Tavis-Cummings model.

## V. OVERCOMING ENERGY RELAXATION PROCESS BY USING A DECOHERENCE-FREE SUBSPACE

Finally, we discuss possible solutions to the problem of the energy relaxation caused by the anti-Zeno effect. As discussed in the previous section, significant dephasing of a superconducting qubit can nullify the advantage of the long lifetime of the memory qubit due to the indirect relaxation, and therefore if we are to realize a hybrid system, it is crucial

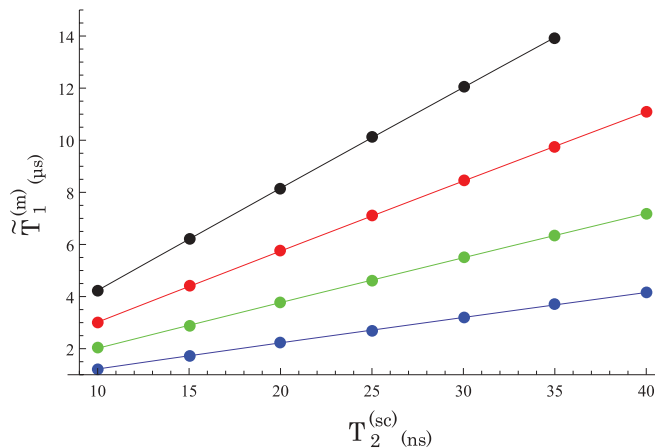


FIG. 4. (Color online) The relaxation time of the memory qubit induced by the anti-Zeno effect. The horizontal line denotes the dephasing time of the superconducting qubit. Dots correspond to numerically obtained data. Also, continuous lines are drawn through the points as a guide to the eye. We set the relaxation time of the superconducting qubit at  $T_1^{(\text{sc})} = 400$  ns and the coupling constant between qubits at  $g/2\pi = 25$  MHz. The lowest line is that where the detuning  $\Delta/2\pi$  is 600 MHz, and the other lines are where  $\Delta/2\pi = 800, 1000$ , and 1200 MHz, respectively.

that we overcome such enhanced relaxation problems. As an example, we discuss how to avoid this enhanced relaxation when the memory qubit consists of an ensemble of microscopic spins with a long life time. When one uses an ensemble composed of  $N \frac{1}{2}$  spins as a memory qubit, the excitation of the superconducting qubit is transferred to the ensemble and stored as a collective mode. A state with a single collective mode in the ensemble is represented as  $|W\rangle = \frac{1}{\sqrt{N}} \sum_{l=1}^L \hat{\sigma}_+^{(l)} |\downarrow \downarrow \dots \downarrow\rangle$ , where  $\hat{\sigma}_+^{(l)}$  denotes the raising operator of a spin and  $|\downarrow\rangle$  denotes the ground state of a single spin. This strategy of utilizing the spin ensemble directly coupled with the superconducting qubit as a memory is suggested theoretically in Ref. 11. However, if we adopt their strategy straightforwardly, the memory ensemble will suffer from the relaxation induced by the anti-quantum Zeno effect as we have discussed. So, our purpose here is to decouple this excitation in the ensemble from the superconducting qubit.

To achieve this, we can apply a spatial magnetic field gradient  $\frac{dB}{dx}$  (T/m) with some time duration to the state  $|W\rangle$  of the ensemble so that we obtain the state  $|W_\theta\rangle = \frac{1}{\sqrt{N}} \sum_{l=1}^N e^{i\theta l} \hat{\sigma}_+^{(l)} |\downarrow \downarrow \dots \downarrow\rangle$ . Here, we have  $\theta = \tau \mu \frac{dB}{dx} \Delta x$  where  $\mu$  denotes the magnetic moment of the spin,  $\tau$  denotes application time of such a field gradient, and  $\Delta x$  denotes the distance between the spins. Since we have  $\langle W|W_\theta\rangle = \frac{1}{N} \sum_{l=1}^N e^{i\theta l}$ , the state  $|W_\theta\rangle$  becomes orthogonal with the state  $|W\rangle$  for  $\theta N = 2\pi$ , and therefore we can decouple the ensemble from the superconducting qubit. For reasonable parameters such as  $2\pi \times 28$  GHz/T for the Zeeman splitting and  $N \Delta x = 20 \mu\text{m}$  for the ensemble length, we need a field gradient 10 T/m to achieve the orthogonal state in hundreds of nanoseconds. This idea of applying a field gradient was developed in the field of holographic quantum computation for different purposes.<sup>40,41</sup> In holographic quantum computation, we utilize a hybrid system composed of a superconducting qubit, a spin ensemble, and a microwave cavity. A field gradient will be applied to transfer the collective excitation of the ensemble to another spin-wave mode so that we can store many qubits in one ensemble. However, the effect of relaxation enhanced by the anti-Zeno effect, in other words, indirect relaxation caused by the imperfect decoupling, has not been discussed in previous research. We use the technique to apply a field gradient for the efficient decoupling of the superconducting qubit from the spin ensemble so that a long time storage of quantum states should be possible in the memory.

Although we have mentioned a solution for the spin-ensemble memory here, the basic idea could be applied to many other systems. We discuss this point briefly, although the details will be published elsewhere. As the quantum memory, one can use multiple memory qubits collectively coupled with the control superconducting qubit. It becomes possible to make a decoherence-free subspace<sup>45</sup> against the interaction from the control qubit as follows: The Hamiltonian between the control superconducting qubit and memory qubits would be  $H = H_S^{(\text{sc})} + H_S^{(\text{m}1)} + H_S^{(\text{m}2)} + H_I^{(\text{sc},\text{m}1)} + H_I^{(\text{sc},\text{m}2)}$  where we have  $H_S^{(j)} = \frac{\omega_j}{2} \hat{\sigma}_z^{(j)}$  and  $H_I^{(j,k)} = g(\hat{\sigma}_+^{(j)} \hat{\sigma}_-^{(k)} + \hat{\sigma}_-^{(j)} \hat{\sigma}_+^{(k)})$ . An excitation of the control superconducting qubit can be transferred into the collective excitation on the memory qubits such as  $\frac{1}{\sqrt{2}}(|01\rangle_{\text{m}1,\text{m}2} + |10\rangle_{\text{m}1,\text{m}2})$ . By performing a

local phase flip on one of the memory qubits, we have  $\frac{1}{\sqrt{2}}(|01\rangle_{m1,m2} - |10\rangle_{m1,m2})$ , which is completely orthogonal to the previous state and decoupled from the controllable qubit. In other words, this state is in the decoherence-free subspace for the residual coupling from the superconducting qubit. Therefore, the memory qubit is immune to the energy leakage induced by the quantum anti-Zeno effect. So, this decoupling using a decoherence-free subspace is much more robust than the traditional way of decoupling the system by detuning. Recently, the coherent control of three qubits and the generation of an entanglement between them have been demonstrated by using superconducting qubits,<sup>16,31,46</sup> and therefore our suggestion here is feasible even with current technology. It is worth mentioning that, while a decoherence-free subspace has been usually utilized to protect qubits from decoherence induced by the environment,<sup>45</sup> we here suggest another use of a decoherence-free subspace, namely, an efficient decoupling method in a hybrid system composed of a superconducting qubit and a quantum memory, which has not been discussed previously.

## VI. CONCLUSION

In conclusion, we have studied the indirect energy relaxation in a hybrid system consisting of a superconducting qubit

and a memory qubit. Even if we employ the detuning between a memory qubit and a superconducting qubit to decouple them, the dephasing of the superconducting qubit could significantly affect the coherence of the memory qubit. The violation of the energy conservation law due to dephasing of the superconducting qubit induces an incoherent energy leakage from the memory, which leads to a significant degradation in the lifetime of the quantum memory. This can be interpreted as a manifestation of the quantum anti-Zeno effect. If this indirect relaxation is inevitable, a hybrid scheme with a superconducting qubit would not be promising unless we can succeed in making the coherence time of the superconducting qubit longer than the present value. However, we can find a possible solution to this problem, for example, via decoupling from the superconducting qubit using a decoherence-free subspace for the memory qubits. Our model is quite general, and therefore the results reported here can be applied to many systems.

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