

Electrical control of the Kondo effect in a helical edge liquid

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Magnetic impurities affect the transport properties of the helical edge states of quantum spin Hall insulators by causing single-electron backscattering. We study such a system in the presence of a Rashba spin-orbit interaction induced by an external electric field, showing that this can be used to control the Kondo temperature, as well as the correction to the conductance due to the impurity. Surprisingly, for a strongly anisotropic electron-impurity spin exchange, Kondo screening may get obstructed by the presence of a noncollinear spin interaction mediated by the Rashba coupling. This challenges the expectation that the Kondo effect is stable against time-reversal invariant perturbations.

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Introduction. The discovery that HgTe quantum wells support a quantum spin Hall (QSH) state¹ has set off an avalanche of studies addressing the properties of this novel phase of matter.² A key issue has been to determine the conditions for stability of the current-carrying states at the edge of the sample as this is the feature that most directly impacts prospects for future applications in electronics and spintronics. In the simplest picture of a QSH system the edge states are *helical*, with counterpropagating electrons carrying opposite spins. By time-reversal invariance electron transport then becomes ballistic, provided that the electron-electron (e - e) interaction is sufficiently well screened so that higher-order scattering processes do not come into play.^{3,4}

The picture gets an added twist when including effects from magnetic impurities, contributed by dopant ions or electrons trapped by potential inhomogeneities. Since an edge electron can backscatter from an impurity via spin exchange, time-reversal invariance no longer protects the helical states from mixing. In addition, correlated two-electron⁵ and inelastic single-electron processes^{6,7} must now also be accounted for. As a result, at high temperatures T electron scattering off the impurity leads to a $\ln(T)$ correction⁸ of the conductance at low frequencies ω , which, however, vanishes⁹ in the dc limit $\omega \rightarrow 0$. At low T , for weak e - e interactions, the quantized edge conductance $G_0 = e^2/h$ is restored as $T \rightarrow 0$ with power laws distinctive of a helical edge liquid. For strong interactions the edge liquid freezes into an insulator at $T = 0$, with thermally induced transport via tunneling of fractionalized charge excitations through the impurity.⁸

A more complete description of edge transport in a QSH system must include also the presence of a Rashba spin-orbit interaction. This interaction, which can be tuned by an external gate voltage, is a built-in feature of a quantum well.¹⁰ In fact, HgTe quantum wells exhibit some of the largest known Rashba couplings of any semiconductor heterostructures.¹¹ As a consequence, spin is no longer conserved, contrary to what is assumed in the minimal model of a QSH system.¹² However, since the Rashba interaction preserves time-reversal invariance, Kramers' theorem guarantees that the edge states are still connected via a time-reversal transformation ("Kramers pair").¹³ Provided that the Rashba interaction is spatially uniform and the e - e interaction is not too strong, this ensures the robustness of the helical edge liquid.¹⁴

What is the physics with both Kondo *and* Rashba interactions present? In this Rapid Communication we address this question with a renormalization group (RG) analysis as well as a linear-response and rate-equation approach. Specifically, we predict that the Kondo temperature T_K —which sets the scale below which the electrons screen the impurity—can be controlled by varying the strength of the Rashba interaction. Surprisingly, for a strongly anisotropic Kondo exchange, a noncollinear spin interaction mediated by the Rashba coupling becomes relevant (in the sense of RG) and competes with the Kondo screening. This challenges the expectation that the Kondo effect is stable against time-reversal invariant perturbations.¹⁵ Moreover, we show that the impurity contribution to the dc conductance at temperatures $T > T_K$ can be switched on and off by adjusting the Rashba coupling. With the Rashba coupling being tunable by a gate voltage, this suggests an inroad to control charge transport at the edge of a QSH device.

Model. To model the edge electrons, we introduce the two-spinors $\Psi^T = (\psi_\uparrow, \psi_\downarrow)$, where ψ_\uparrow (ψ_\downarrow) annihilates a right-moving (left-moving) electron with spin up (spin down) along the growth direction of the quantum well. Neglecting e - e interactions, the edge Hamiltonian can then be written as

$$H = v_F \int dx \Psi^\dagger(x) [-i\sigma^z \partial_x] \Psi(x) + \alpha \int dx \Psi^\dagger(x) [-i\sigma^y \partial_x] \Psi(x) + \Psi^\dagger(0) [J_x \sigma^x S^x + J_y \sigma^y S^y + J_z \sigma^z S^z] \Psi(0), \quad (1)$$

with v_F the Fermi velocity parametrizing the linear kinetic energy. The second term encodes the Rashba interaction of strength α , with the third term being an antiferromagnetic Kondo interaction between electrons (with Pauli matrices $\sigma^i, i = x, y, z$) and a spin-1/2 magnetic impurity (with Pauli matrices $S^i, i = x, y, z$) at $x = 0$. The spin-orbit induced magnetic anisotropy for an impurity at a quantum well interface¹⁶ implies that¹⁷ $J_x = J_y \neq J_z$. Unless otherwise stated, we use $\hbar = k_B \equiv 1$.

The Rashba term in Eq. (1) can be absorbed into the kinetic term by a spinor rotation¹⁸ $\Psi' = e^{-i\sigma^x \theta/2} \Psi$. By rotating also the impurity spin, $S' = e^{-iS^x \theta/2} S e^{iS^x \theta/2}$, one obtains

$H = H'_0 + H'_K$, with

$$H'_0 = v_\alpha \int dx \Psi^\dagger(x) [-i\sigma^z \partial_x] \Psi(x), \quad (2)$$

$$H'_K = \Psi^\dagger(0) [J_x \sigma^x S^x + J'_y \sigma^{y'} S^{y'} + J'_z \sigma^{z'} S^{z'} + J_E (\sigma^{y'} S^{z'} + \sigma^{z'} S^{y'})] \Psi(0), \quad (3)$$

where $J'_y = J_y \cos^2 \theta + J_z \sin^2 \theta$, $J'_z = J_z \cos^2 \theta + J_y \sin^2 \theta$, and $J_E = (J_y - J_z) \cos \theta \sin \theta$. The Rashba rotation angle θ is determined through $\cos \theta = v_F/v_\alpha$, $\sin \theta = \alpha/v_\alpha$, and $v_\alpha = \sqrt{v_F^2 + \alpha^2}$. Note that the spin in the rotated basis is quantized along the z' direction which forms an angle θ with the z axis. Also note that the Kondo interaction in this basis not only becomes spin nonconserving, but also picks up a noncollinear term for $J_y \neq J_z$.

Including e - e interactions, and assuming a band filling incommensurate with the lattice,² time-reversal invariance constrains the possible scattering channels in the rotated basis to dispersive ($\sim g_d$) and forward ($\sim g_f$) scattering, in addition to correlated two-particle backscattering^{3,4} ($\sim g_{bs}$) and inelastic single-particle backscattering^{6,7,19} ($\sim g_{ie}$) at the impurity site. Adding the corresponding interaction terms to H'_0 and H'_K in (2) and (3), and employing standard bosonization,²⁰ the full Hamiltonian for the edge liquid can now be expressed as a free boson model, $(v/2) \int dx [(\partial_x \varphi)^2 + (\partial_x \vartheta)^2]$, with three local terms added at $x = 0$:

$$H'_K = \frac{A}{\kappa} \cos(\sqrt{4\pi K} \varphi) + \frac{B}{\kappa} \sin(\sqrt{4\pi K} \varphi) + \frac{C}{\sqrt{K}} \partial_x \vartheta, \quad (4)$$

$$H'_{bs} = \frac{g_{bs}}{2(\pi\kappa)^2} \cos(\sqrt{16\pi K} \varphi), \quad (5)$$

$$H'_{ie} = \frac{g_{ie}}{2\pi^2 \sqrt{K}} : (\partial_x^2 \vartheta) \cos(\sqrt{4\pi K} \varphi) : . \quad (6)$$

Here φ is a nonchiral Bose field with ϑ its dual, $v\partial_x \vartheta = \partial_t \varphi$ with $v = [(v_\alpha + g_f/\pi)^2 - (g_d/\pi)^2]^{1/2}$, $K = [(\pi v_\alpha + g_f - g_d)/(\pi v_\alpha + g_f + g_d)]^{1/2}$, and $\kappa \approx v_F/D$ is the edge state penetration depth acting as short-distance cutoff with D the bandwidth, and $: \dots :$ denotes normal ordering. In H'_K we have defined $A = J_x S^x/\pi$, $B = (J'_y S^{y'} + J_E S^{z'})/\pi$, and $C = (J'_z S^{z'} + J_E S^{y'})/\pi$. The presence of H'_{ie} requires breaking of the $U(1)$ spin symmetry, as brought about by the Rashba interaction.

Kondo temperature. The bosonized theory is tailor made for a perturbative RG analysis, allowing us to determine the temperature scale below which the edge electrons couple strongly to the impurity. We first note that the backscattering term in (5) is that of the well-known boundary-sine Gordon model. For $K < 2/3$ it dominates over the inelastic backscattering in (6), and turns relevant for $K < 1/4$ with a weak to strong-coupling crossover at a temperature²¹ $T_{bs} \approx D g_{bs}^{1/(1-4K)}$. As a consequence, the enhancement of backscattering as $T \rightarrow 0$ results in a zero-temperature insulating state when $K < 1/4$.

Turning to the Rashba-rotated Kondo interaction H'_K in Eq. (4), we obtain for its one-loop RG equations

$$\begin{aligned} \partial_l \tilde{J}_x &= (1 - K) \tilde{J}_x + \nu K (\tilde{J}'_y \tilde{J}'_z - \tilde{J}_{E1} \tilde{J}_{E2}), \\ \partial_l \tilde{J}'_y &= (1 - K) \tilde{J}'_y + \nu K \tilde{J}_x \tilde{J}'_z, \quad \partial_l \tilde{J}'_z = \nu K \tilde{J}_x \tilde{J}'_y, \\ \partial_l \tilde{J}_{E1} &= (1 - K) \tilde{J}_{E1} - \nu K \tilde{J}_x \tilde{J}_{E2}, \quad \partial_l \tilde{J}_{E2} = -\nu K \tilde{J}_x \tilde{J}_{E1}, \end{aligned} \quad (7)$$

with the “tilde” indicating that the couplings depend on the renormalization length l , and where $\nu \equiv 1/(\pi v)$. The two terms proportional to J_E in Eq. (4) flow individually under RG, with the corresponding renormalized coupling constants here denoted \tilde{J}_{E1} and \tilde{J}_{E2} . In deriving Eqs. (7) we have used that higher-order contributions involving an intermediate process governed by H'_{bs} or H'_{ie} are suppressed, since these transfer spin or energy incompatible with H'_K . In a recent work,²² Kondo scattering without Rashba interaction was studied, and different physics in the regime $\nu J_z \geq 2K$ was found, not accessible perturbatively in νJ_z . Since its realization in an HgTe quantum well requires anomalously weak screening of the e - e interaction we do not consider this regime here.

The role of the Rashba rotation in Eqs. (7) is both to determine the bare values $\tilde{J}'_{y,z}(l=0) \equiv J'_{y,z}$ and to introduce the noncollinear couplings $\tilde{J}_{E1,E2}$. To explore the outcome, we first examine the case of a strongly screened e - e interaction, $K \approx 1$. For this case, the first-order terms of Eq. (7) can be neglected and $\tilde{J}_{E1} = \tilde{J}_{E2} = \tilde{J}_E$, since their scaling equations will be identical. In this limit, \tilde{J}_E quickly flows to zero. We take the Kondo temperature T_K to be the value of $T = D \exp(-l)$, where one of the couplings in Eq. (7) first grows past $1/(\nu K)$, making the renormalized H'_K in Eq. (4) dominate the free theory. For $K \approx 1$ we then see that

$$T_K \approx D \exp\left(-\frac{1}{\nu J_x} \frac{\text{arcsinh}(\zeta)}{\zeta}\right), \quad (8)$$

where $\zeta = \sqrt{(J_z/J_x)^2 - 1}$ is an anisotropy parameter.⁸ Here the θ dependence lies predominantly in ν . Note that Kondo temperatures modified by spin-orbit couplings, as in (8), or by spin-dependent hopping, have been proposed recently also for ordinary conduction electrons.^{23–27}

In the opposite limit of a strong e - e interaction, the second-order terms of the scaling equations can be neglected, as long as $1 - K \gg \tilde{J} \nu K$, for all $\tilde{J} = \tilde{J}_x, \tilde{J}'_y, \tilde{J}_{E1}$. The scaling equations in this limit reduce to $\partial_l \tilde{J} = (1 - K) \tilde{J}$, with solutions $\tilde{J} = J e^{(1-K)l}$. With $l = \ln(D/T)$, one can now use the $\tilde{J} = 1/(\nu K)$ criterion to find the Kondo temperature

$$T_K \approx D (J_{\max} \nu K)^{1/(1-K)}, \quad (9)$$

where $J_{\max} = \max[J_x, J'_y, J_E]$.

In Fig. 1 we exhibit T_K for both “easy-plane” and “easy-axis” Kondo interaction. To isolate the effect of the Rashba interaction from that of the e - e interaction we choose to plot T_K as a function of θ and K_0 , with $K_0 \equiv K(\theta=0)$ the ordinary Luttinger parameter. For $|J_E| > |J_x|, |J'_y|$, the noncollinear term $\sim \sigma^{y'} S^{z'}$ in Eq. (3) dominates the RG flow for values of K in the shaded “dome” (the size of which is set by the ratio $J_z/J_{x,y}$). As this term disfavors a spin singlet, Kondo screening will be obstructed in the corresponding interval of Rashba couplings.²⁸ This runs contrary to the

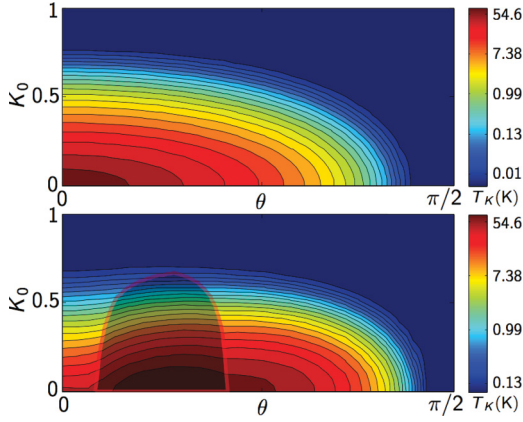


FIG. 1. (Color online) The Kondo temperature T_K as a function of the Rashba angle θ and the ordinary Luttinger parameter K_0 . The T_K scale is logarithmic and red and blue colors indicate high and low T_K , respectively. Top: $J_x = J_y \geq J_z$ (here, $J_x/a_0 = J_z/a_0 = 10$ meV); bottom: $J_x = J_y < J_z$ (here, $J_x/a_0 = 5$ meV and $J_z/a_0 = 50$ meV). In the shaded area, \tilde{J}_{E1} dominates the perturbative RG flow, hence obstructing singlet formation.

expectation that a spin-orbit interaction does not impair the Kondo effect.^{15,29} However, this expectation is rooted in a noninteracting quasiparticle picture which breaks down in one dimension. Instead a Luttinger liquid is formed, with strongly correlated electron scattering.²⁰ As suggested by our RG analysis, when this scattering gets enhanced with lower values of K , it boosts the effect of the noncollinear spin interaction that works against the Kondo screening.

Conductance at low temperatures. Away from the “dome” in Fig. 1, the Rashba-rotated Kondo interaction easily sustains a Kondo temperature T_K below which the impurity gets screened. When $K > 1/4$ and two-particle backscattering is RG irrelevant, there is no correction δG to the conductance at zero temperature: As explained by Maciejko *et al.*,⁸ the topological nature of the QSH state implies a “healing” of the edge after the impurity has been effectively removed by the Kondo screening. For finite $T \ll T_K$, the leading operator δG is generated by either H'_{bs} or H'_{ie} , whatever operator has the lowest scaling dimension: For $1/4 < K < 2/3$ ($K > 2/3$) H'_{bs} (H'_{ie}) dominates, with^{6–8} $\delta G_{bs} \sim (T/T_K)^{8K-2}$ [$\delta G_{ie} \sim (T/T_K)^{2K+2}$]. Thus, in the noninteracting limit $K \approx 1$, the presence of a Rashba interaction is revealed by a T^4 scaling of the conductance, rather than the expected T^6 scaling as in the minimal model with $U(1)$ spin symmetry.⁸ The picture changes dramatically for $K < 1/4$. Now H_{bs} turns RG relevant, with g_{bs} entering a strong-coupling regime below the crossover temperature T_{bs} , implying zero conductance at $T = 0$. At finite $T \ll T_{bs}$, instanton processes restore its finite value, with⁸ $G \sim (T/T_{bs})^{2(1/4K-1)}$. To leading order this regime is blind to the Rashba interaction.

Conductance and currents at high temperatures. When $T > \max(T_K, T_{bs})$, scattering from H'_K as well as from H'_{bs} and H'_{ie} remains weak, and transport properties can be obtained perturbatively. We here focus on the correction δG to the conductance due to H'_K , noting that the contributions from H'_{bs} and H'_{ie} decouple and are insensitive to the strength of the Rashba interaction.

The current operator \hat{I} takes the form $\hat{I} = (e/2)\partial_t$ ($\cos\theta\Psi^\dagger\sigma^z\Psi' - \sin\theta\Psi^\dagger\sigma^y\Psi'$) in the rotated basis. After the unitary transformation $U = e^{i\sqrt{\pi}\lambda\varphi(0)S^z}$, which removes the J'_z term when $\lambda = J'_z/\pi v\sqrt{K}$, the bosonized part $\delta\hat{I}$ of the current operator due to H'_K is

$$\delta\hat{I} = \frac{ie}{2\pi\kappa} \left[\sum_{j=\pm} A_j e^{i\sqrt{\pi}(2\sqrt{K}-j\lambda)\varphi} S^j + iA_0 e^{i\sqrt{4\pi}K\varphi} S^z - \frac{iJ_x}{\sqrt{\pi}K} \sin\theta : \partial_x \vartheta e^{-i\sqrt{\pi}\lambda\varphi} : S^+ \right] + \text{H.c.}, \quad (10)$$

where $A_\pm = (1/2)(J_x \pm J_y) \cos\theta$, $A_0 = (J_E/2) \cos\theta$. Using the Kubo formula to calculate the conductance correction $\delta G(\omega)$ at a frequency ω in the limit $J^2 \ll \omega \ll T$, with $J^2 = J_x^2, J_y^2, J_E^2$, we then find to $O(J^2)$

$$\delta G = -\frac{e^2}{\hbar} \sum_{j=-1}^{+1} F_j \cdot [A_j(T)]^2 - \frac{e^2}{\hbar} \mu J_x^2 \sin^2\theta, \quad (11)$$

which, in this limit, is independent of ω . Here $A_j(T) = A_j \cdot (2\pi T/D)^{(\sqrt{K}-j\lambda/2)^2-1}$. The constant $F_j \equiv F(2\sqrt{K}-j\lambda)$ with $F(x) = [\Gamma(x^2/4)]^2/[4\pi(\hbar v)^2\Gamma(x^2/2)]$, while $\mu = (1 + \lambda^2/2) \sin(\pi\lambda^2/4)/(\pi\hbar v\sqrt{K})^2$. At zero Rashba coupling, $\theta = 0$, Eq. (11) reproduces the finding in Refs. 8 and 9. By replacing the bare couplings with renormalized ones, the result in Eq. (11) can be RG improved to numerically obtain δG to all orders in perturbation theory in a leading-log approximation. At $\theta = 0$ this gives $\delta G \sim \ln(T)$, in agreement with Ref. 8.

As stressed in Ref. 9, the use of the Kubo formula rests on a perturbation expansion (in our case assuming that $J^2 \ll \omega$) which breaks down as $\omega \rightarrow 0$. To study the scaling of δG in the dc limit we will instead fall back on a semiclassical rate equation approximation where we assign classical probabilities to the states. The details of this calculation are provided in the Supplemental Material,³⁰ and we here only give the main results. In the dc limit, i.e., $\omega \ll J^2 \ll T$, the conductance correction becomes

$$\delta G = -\frac{e^2 \cos^2\theta}{2T} \left[\frac{4\gamma_0\gamma'_0 + (\gamma_0 + \gamma'_0)(\gamma_0^E + \gamma'_0{}^E) + \tilde{\gamma}_0^E\gamma_0^E}{\gamma_0 + \gamma'_0 + \tilde{\gamma}_0^E} \right],$$

with $\gamma_0 \sim (J_x + J_y)^2 T^{2(\sqrt{K}-\lambda/2)^2-1}$, $\gamma'_0 \sim (J_x - J_y)^2 T^{2(\sqrt{K}+\lambda/2)^2-1}$, $\gamma_0^E \sim J_E^2 T^{2K-1}$, and $\tilde{\gamma}_0^E \sim J_E^2 T$, where, for brevity, we have omitted various K -dependent prefactors. Note that with $J_x = J_y$, the vanishing δG becomes nonzero when turning on the Rashba interaction by an electric field. This suggests a means to manipulate the edge current by varying the bias of an external gate.

To explore this possibility we have calculated the δI - V characteristics, exploiting that in the rotated basis H'_K can be treated as a tunneling Hamiltonian³¹ and the part of δI corresponding to this tunneling current is then obtained as in Ref. 32. When $J^2 \ll \omega \ll T$, eV we find

$$\delta I \approx -e \sum_{j=-1}^{+1} \text{Im}\{B(K_j + ieV'/2\pi T, K_j - ieV'/2\pi T) \times C_j(T/D)^{2K_j-1} \sin[\pi(K_j - ieV'/2\pi T)]/\cos(\pi K_j)\} \quad (12)$$

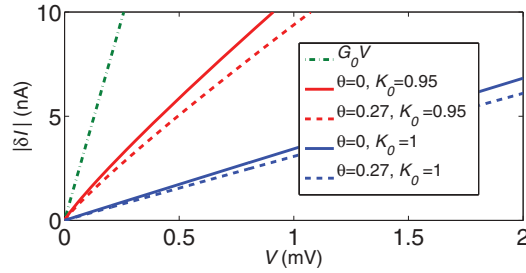


FIG. 2. (Color online) The RG-improved current correction (12) at $T = 30$ mK as a function of applied voltage, for different values of K_0 and θ . The dashed lines represent $\theta \approx 0.27$, corresponding to $\hbar\alpha = 10^{-10}$ eV m. Other parameters are defined in the text. The QSH edge current G_0V is plotted as a reference.

for $\delta I \equiv I - G_0V$, with $K_j \equiv (\sqrt{K} - j\lambda/2)^2$, B the beta function, and $V' \equiv V \cos \theta$. Here $C_{\pm 1} = c_{\pm}(J_x \pm J'_y)^2$ and $C_0 = c_0 J_E^2$, with $c_{\pm,0}$ constants depending on K , λ , and θ . In Fig. 2 we plot this for parameter values given below.

Experimental realization. Given our result in Eq. (12), is the Rashba dependence of δI large enough to be seen in an experiment? As a case study, let us consider an Mn^{2+} ion implanted close to the edge of an HgTe quantum well.³³ Mn^{2+} has spin $S = 5/2$, but, due to the large and positive single-ion anisotropy $\propto(S^z)^2$ at the quantum well interface, the higher spin components freeze out in the sub-Kelvin range, leaving behind a spin-1/2 doublet.¹⁶ Moreover, the single-ion anisotropy implies that the Kondo interaction with this effective spin-1/2 impurity is anisotropic, with $J_x = J_y = 3J_z = 3J_I$, where J_I is the isotropic bulk spin-exchange coupling.¹⁷ Its value can be assessed from the $sp-d$ exchange integrals for the bulk conduction electrons³⁴ in $\text{Hg}_{1-x}\text{Mn}_x\text{Te}$. Close to the Γ point of the Brillouin zone these integrals produce an antiferromagnetic exchange, $J_I > 0$. With the Mn^{2+} ion located within the penetration depth κ from the edge, a rough estimate yields an expected value of $J_I/a_0 \approx 10$ meV, with a_0 the lattice constant. Turning to the Rashba coupling α , gate controls have been demonstrated³⁵ in the laboratory with

$\hbar\alpha$ for an HgTe quantum well device running from 5×10^{-11} to 1×10^{-10} eV m as the bias of a top gate is varied from 2 to -2 V. As for the value of the interaction parameter K_0 in an HgTe quantum well, estimates^{8,14,36–38} range between 0.5 and 1, and depend on the geometry and composition of the heterostructure. Collecting the numbers, and putting^{1,39} $a_0 \approx 0.5$ nm, $v_F \approx 5.0 \times 10^5$ m/s, and $D \approx 300$ meV, we can use Eq. (12) to numerically plot the δI - V characteristics for different values⁴⁰ of α and K_0 , choosing $T = 30$ mK ($> T_K$) (see Fig. 2). As revealed by the graphs, the Rashba dependence of δI is appreciable, and should allow for an experimental test.⁴¹

Concluding remarks. We have studied the combined effect of a Kondo and a Rashba interaction at the edge of a quantum spin Hall system. The interplay between an anisotropic Kondo exchange and the Rashba interaction is found to result in a noncollinear electron-impurity spin interaction. A perturbative RG analysis indicates that this interaction may block the Kondo effect when the electron-electron interaction is weakly screened. We conjecture that this surprising result—challenging a time-honored expectation that the Kondo effect is blind to time-reversal invariant perturbations¹⁵—is due to the breakdown of single-particle physics in one dimension. It remains a challenge to unravel the microscopic scenario behind this intriguing phenomenon. In the second part of our work we derived expressions showing how charge transport at the edge is influenced by the simultaneous presence of a magnetic impurity and a Rashba interaction. A case study suggests that the predicted current-voltage characteristics should indeed be accessible in an experiment. Most interestingly, its manifest dependence on the gate-controllable Rashba coupling breaks a path for charge control in a helical electron system.

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¹M. König *et al.*, *Science* **318**, 766 (2007).

²For a review, see X.-L. Qi and S.-C. Zhang, *Rev. Mod. Phys.* **83**, 1057 (2011).

³C. Wu, B. A. Bernevig, and S. C. Zhang, *Phys. Rev. Lett.* **96**, 106401 (2006).

⁴C. Xu and J. E. Moore, *Phys. Rev. B* **73**, 045322 (2006).

⁵D. Meidan and Y. Oreg, *Phys. Rev. B* **72**, 121312(R) (2005).

⁶T. L. Schmidt, S. Rachel, F. von Oppen, and L. I. Glazman, *Phys. Rev. Lett.* **108**, 156402 (2012).

⁷N. Lezmy *et al.*, *Phys. Rev. B* **85**, 235304 (2012).

⁸J. Maciejko, C. Liu, Y. Oreg, X. L. Qi, C. Wu, and S. C. Zhang, *Phys. Rev. Lett.* **102**, 256803 (2009).

⁹Y. Tanaka, A. Furusaki, and K. A. Matveev, *Phys. Rev. Lett.* **106**, 236402 (2011).

¹⁰R. Winkler, *Spin-Orbit Interaction Effects in Two-Dimensional Electron and Hole Systems* (Springer, Berlin, 2003).

¹¹H. Buhmann, *J. Appl. Phys.* **109**, 102409 (2011).

¹²B. A. Bernevig *et al.*, *Science* **314**, 1757 (2006).

¹³C. L. Kane and E. J. Mele, *Phys. Rev. Lett.* **95**, 226801 (2005).

¹⁴A. Ström, H. Johannesson, and G. I. Japaridze, *Phys. Rev. Lett.* **104**, 256804 (2010).

¹⁵Y. Meir and N. S. Wingreen, *Phys. Rev. B* **50**, 4947 (1994).

¹⁶O. Újsághy and A. Zawadowski, *Phys. Rev. B* **57**, 11598 (1998).

¹⁷R. Zitko, R. Peters, and T. Pruschke, *Phys. Rev. B* **78**, 224404 (2008).

¹⁸J. I. Väyrynen and T. Ojanen, *Phys. Rev. Lett.* **106**, 076803 (2011).

¹⁹J. C. Budich, F. Dolcini, P. Recher, and B. Trauzettel, *Phys. Rev. Lett.* **108**, 086602 (2012).

²⁰T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, UK, 2003).

²¹C. L. Kane and M. P. A. Fisher, *Phys. Rev. B* **46**, 15233 (1992).

²²J. Maciejko, *Phys. Rev. B* **85**, 245108 (2012).

²³M. Pletyukhov and D. Schuricht, *Phys. Rev. B* **84**, 041309(R) (2011).

- ²⁴X.-Y. Feng and F.-C. Zhang, *J. Phys.: Condens. Matter* **23**, 105602 (2011).
- ²⁵R. Žitko and J. Bonča, *Phys. Rev. B* **84**, 193411 (2011).
- ²⁶M. Zarea, S. E. Ulloa, and N. Sandler, *Phys. Rev. Lett.* **108**, 046601 (2012).
- ²⁷L. Isaev, D. F. Agterberg, and I. Vekhter, *Phys. Rev. B* **85**, 081107 (2012).
- ²⁸The position of the “dome” in Fig. 1 is determined by the condition that the magnitude $|\tilde{J}_{E1}|$ of the scale-dependent amplitude of the noncollinear term $\sim \sigma^y S^z$ outgrows $|\tilde{J}_x|$, $|\tilde{J}_y|$, and $|\tilde{J}_z|$ under RG. Note that when $\theta > \pi/4$, this does not happen since now $J'_y = J_y \cos^2 \theta + J_z \sin^2 \theta$ will be large for large J_z , making $|\tilde{J}'_y|$ dominate the RG flow.
- ²⁹J. Malecki, *J. Stat. Phys.* **129**, 741 (2007).
- ³⁰See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevB.86.161103> for a detailed account of the rate-equation calculations.
- ³¹G. Mahan, *Many-Particle Physics* (Kluwer Academic/Plenum, New York, 2000).
- ³²X.-G. Wen, *Phys. Rev. B* **44**, 5708 (1991).
- ³³S. Datta *et al.*, *Superlattices Microstruct.* **1**, 327 (1985).
- ³⁴J. K. Furdyna, *J. Appl. Phys.* **64**, R29 (1988).
- ³⁵J. Hinz *et al.*, *Semicond. Sci. Technol.* **21**, 501 (2006).
- ³⁶C. Y. Hou, E. A. Kim, and C. Chamon, *Phys. Rev. Lett.* **102**, 076602 (2009).
- ³⁷A. Ström and H. Johannesson, *Phys. Rev. Lett.* **102**, 096806 (2009).
- ³⁸J. C. Y. Teo and C. L. Kane, *Phys. Rev. B* **79**, 235321 (2009).
- ³⁹A. L. Efros and B. I. Shklovskii, *Electronic Properties of Doped Semiconductors* (Springer, Heidelberg, 1989).
- ⁴⁰Since H'_K in Eq. (3) is RG relevant (marginally relevant) for $K < 1$ ($K = 1$), the condition $\delta I \ll G_0 V$ for perturbation theory to be valid restrains K to values close to unity in the chosen voltage interval.
- ⁴¹Note that a test requires the concurrent tunability of a back gate, so as to keep the electron density fixed (Ref. 42).
- ⁴²D. Grundler, *Phys. Rev. Lett.* **84**, 6074 (2000).