Chiral $p_x + ip_y$ superconducting nanowire coupled to two metallic rings pierced by a flux

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(Received 27 March 2012; revised manuscript received 17 August 2012; published 18 September 2012)

We consider a p-wave superconducting nanowire weakly coupled to two metallic rings. At the two interfaces between the nanowire and the metallic rings the pairing order parameter vanishes, as a result two zero-mode Majorana fermions appear. The two metallic rings are pierced by external magnetic fluxes. By determining the correlation between the persistent currents in the two rings, we identify the special features of Majorana fermions, such as nonlocality, which can be experimentally observed in this setup.

DOI: 10.1103/PhysRevB.86.094519 PACS number(s): 74.45.+c, 71.10.Pm, 73.23.Ra

I. INTRODUCTION

Topological superconductors are characterized by invariance under charge conjugation symmetry. As a result of this invariance zero-mode neutral fermions called Majorana fermions appear at the interfaces between a superconductor and a metal. Majorana fermions have been proposed to exist in a variety of semiconductor devices based on the proximity effect of a superconductor and metals.[1] Due to the robustness against noise the Majorana fermions can be used as qubits for quantum computing.[2] However, the quest to observe Majorana fermions in a physical system continues although they may have been observed in a recent tunneling experiment[3] and possibly other experiments.[4,5] One of the materials where Majorana fermions might be observed is Sr_2RuO_4 where the pairing order parameter has p-wave symmetry[6] and the excitations are given by half vortices, which are neutral zero mode fermions.[2,7–14]

Here we propose a specific setup where Majorana fermions can be observed by measuring persistent current correlations. We consider a geometry of two metallic nanorings coupled to a p-wave superconducting nanowire (Fig. 1). For the p-wave wire zero-mode Majorana fermions appear at the edges of the wire. The metallic rings are pierced by two magnetic fluxes φ_1 and φ_2. The coupling between the electrons in the rings occurs through the p-wave Majorana zero modes. For independent rings the current in each ring will depend only on the flux in the same ring. When the two rings are coupled through the p-wave wire we expect that the current will be affected by the flux in the other ring. The two rings are attached to a charging voltage[15] such that the Fermi momentum will be the same in the rings and in the wire. Therefore resonant tunneling current can be observed under this situation. When the energy corresponding to the flux difference of the rings is comparable to the zero-mode excitation energy in the wire a tunneling current between the rings will be observed. The current in each ring is nonlocal and depends on the two fluxes.

The explicit model consists of the p-wave wire coupled to two metallic rings.[16] Using the left and right movers (fermions), we obtain a continuum representation of the wire. At the two edges of the wire we obtain two neutral zero modes. The coupling between the wire and the metallic rings is dominated by the zero modes and given by $\epsilon \approx |\Delta_0| e^{-L|\Delta_0|}$, $L|\Delta_0| \gg 1$ where Δ_0 is the p-wave pairing field and L is the length of the wire. We compute the current in each ring as a function of the external flux and find that when the flux difference (energy) is comparable to the zero-mode excitation energy the tunneling current gives rise to a nonlocal flux dependence. When $L \to \infty$ the quasiparticle energy scales to zero and the spectrum of the quasiparticles overlaps with the zero modes resulting in a more elaborate picture. In the next section we present the explicit model. Our results on persistent currents and nonlocality are given in Sec. III. The main conclusions are summarized in Sec. IV.

II. MODEL

The p-wave wire of length L is given by the Hamiltonian $H_{\rm pw}$,

$$H_{\rm pw} = -\sum_x [tC^+(x)C(x+a) + \hat{\Delta}C(x)C(x+a) + \text{H.c.}] - \mu_F \sum_x C^+(x)C(x). \quad (1)$$

The pairing gap is given by $\hat{\Delta}$ and the polarized fermion operator is given by $C(x) \equiv C_{\sigma=\uparrow}(x)$. The pairing gap obeys the vanishing pairing boundary conditions $\hat{\Delta}(x=0) = \hat{\Delta}(x = L) = 0$ and $\hat{\Delta}(x) = \hat{\Delta}_0$ for $0 < x < L$.

We restrict ourselves to the weak pairing phase $|\mu_F| < t$ and introduce the right and left fermions for the wire, $C(x = na) = C_R(x) + C_L(x) \to C(x) = e^{ik_F x}\hat{C}_R(x) + e^{-ik_F x}\hat{C}_L(x)$, where k_F is the Fermi momentum. Then Eq. (1) is replaced by the Hamiltonian

$$H_{\rm pw} = \int dx[\Psi^\dagger(x) v_F \sigma^z(-i\partial_x)\Psi(x) + \Delta(x)\Psi^\dagger(x)\sigma^y\Psi(x)] \equiv \int dx[\Psi^\dagger(x)h(x)\Psi(x)], \quad (2)$$

where $\Psi^\dagger(x)$ is the two-component spinor operator given by $\Psi^\dagger(x) = [C_R^\dagger(x), C_L(x)]$. The Hamiltonian $h(x)$ in the first quantized form obeys the eigenvalue equation $h(x)\Phi_E(x) = E\Phi_E(x)$. The eigenspinor $\Phi_E(x)$ is constrained by the Majorana pseudoreality condition of charge conjugation symmetry $[\Phi_E(x)]_c \equiv \mathbf{C}\Phi_E^*(x) \equiv \Phi_{-E}(-x)$ where $\mathbf{C} = \sigma^z$ represents the charge conjugation operator. As a result the Hamiltonian $h(x)$ possesses the conjugation $\mathbf{C}[h(x)]^*\mathbf{C}^{-1} = h(-x)$. The one-dimensional Majorana chain possesses a $\mathbf{Z}_2$ invariant, which counts the number of electrons (modulo-two) in the ground state.

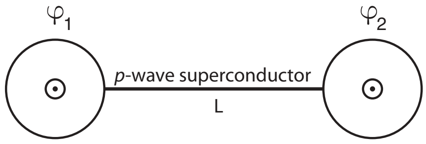

FIG. 1. Two metallic nanorings connected by a p-wave superconducting nanowire pierced by fluxes φ_1 and φ_2. The length of the wire is L whereas the circumference of each nanoring is $l_{\rm ring}$.

The Hamiltonian in Eq. (1) breaks time-reversal symmetry. The time-reversal operator for our problem is given by $\hat{T}\Phi(x,t)$ where $\hat{T}=\sigma^x\mathbf{K}$ and $\mathbf{K}$ is the anti-unitary conjugation operator. The Hamiltonian in the momentum space reveals that the time-reversal symmetry is broken $\hat{T}^{-1}h(\vec{k})\hat{T}=-h(-\vec{k})$ [time-reversal invariance demands that the Hamiltonian transform according to $\hat{T}^{-1}h(\vec{k})\hat{T}=h(-\vec{k})$]. Since time-reversal symmetry is broken the topological invariant is an integer $\mathbf{Z}$[17] although the even-odd effect might be important.

The pairing field $\Delta(x)\equiv 4\hat{\Delta}(x)\sin(k_F a)$ can be written as $\Delta(x)=M_L(x)+M_R(x-L)$ where $M_L(x)=\frac{\Delta(0)}{2}\mathrm{sgn}(x)$ and $M_R(x-L)=\frac{\Delta(0)}{2}\mathrm{sgn}(x-L)$ obey the domain wall property: $M_L(-x)=-M_L(x)$ (at $x=0$), $M_R[-(x-L)]=-M_R(x-L)$ (at $x=L$). The zero-mode eigenfunctions are given by $\eta_\lambda(x)=[\eta_1(x),\eta_2(x)]^T$ and are eigenstates of the operator $\sigma^x\eta_\lambda(x)=\lambda\eta_\lambda(x)$ with $\lambda=\pm1$. The left zero-mode spinor, localized around $x=0$, is identified with $\lambda=-1$, and the right one, localized around $x=L$, is identified with $\lambda=1$,

$$\begin{aligned}\eta_{\rm Left}(x)&\equiv\eta_{\lambda=-1}(x)=e^{\frac{-1}{v_F}\int_0^x\Delta(x')\,dx'}\frac{1}{\sqrt{2}}[1,-1]^T,\\ \eta_{\rm Right}(x)&\equiv\eta_{\lambda=1}(x)=e^{\frac{1}{v_F}\int_L^x\Delta(x')\,dx'}\frac{1}{\sqrt{2}}[1,1]^T.\end{aligned}\tag{3}$$

The spinor operator $\Psi(x)$ with the two zero-mode Majorana operators α_l (at the left edge) and α_r (at the right edge), $(\alpha_r)^2=(\alpha_l)^2=\frac{1}{2}$, takes the form

$$\Psi(x)\to\Psi(x)+\alpha_r\eta_{\lambda=1}(x)+\alpha_l\eta_{\lambda=-1}(x).\tag{4}$$

As a result the low-energy Hamiltonian of the p-wave wire is given by

$$\begin{aligned}H_{\rm pw}&=\int dx[v_F\Psi^\dagger(x)\sigma^z(-i\partial_x)\Psi(x)+\Delta(x)\Psi^\dagger(x)\sigma^y\Psi(x)]\\ &\approx\frac{i}{2}\epsilon\alpha_l\alpha_r,\end{aligned}\tag{5}$$

where $\epsilon\approx|\Delta_0|e^{-L|\Delta_0|}$ and $L|\Delta_0|\gg1$. In Eq. (5) we have ignored the non-zero-mode excitations, which have energies larger than ϵ.

In the experimental situation we may think of a nanowire, which is placed on top of a p-wave superconductor. Due to the proximity effect the nanowire can be treated as a p-wave nanowire.[2,3] For N conducting channels we will have N zero Majorana modes at each boundary. Since time-reversal symmetry is broken, the topological invariant is an integer $\mathbf{Z}$ (for $d>1$)[17] although the physics of a paired versus unpaired Majorana mode might be crucial.

At this stage we include the Hamiltonians of the two rings pierced by the fluxes $\hat{\varphi}_i$, $i=1,2$ and length (i.e., circumference) $l_{\rm ring}\ll L$. The left ring is restricted to the region $-l_{\rm ring}\leqslant x\leqslant 0$, and the right ring is restricted to $L\leqslant x\leqslant L+l_{\rm ring}$. Since only the wire fields at $x=0$ and $x=L$ are involved, we fold the space of the right ring $i=2$ such that both rings are restricted to the region $-l_{\rm ring}\leqslant x\leqslant 0$. Consequently, the external fluxes obey $\hat{\varphi}_1\to\hat{\varphi}_1$ and $\hat{\varphi}_2\to-\hat{\varphi}_2$. In addition, we replace the fermion operator for each ring $\psi_i(x)\equiv\psi_{i\,\sigma=\uparrow}(x)$, $i=1,2$ (because of the Zeeman splitting in each ring we restrict ourselves to a one-component fermion) by the right $R_i(x)$ and left $L_i(x)$ fermions

$$\psi_i(x)=R_i(x)e^{ik_Fx}+L_i(x)e^{-ik_Fx}.\tag{6}$$

The presence of a magnetic field on the wire and rings removes the spin degeneracy. For a magnetic field H in the z direction perpendicular to a vector $\vec{d}$, which characterizes the p-wave order parameter Δ given by $\Delta\propto\vec{\sigma}\cdot\vec{d}$, we have two sets of solutions for the two spin states. For this case the Fermi momentum in the wire and rings will depend on the magnetic field H.[18] We thus have $k^{\rm wire}_{F,\sigma\uparrow}\equiv k^{\rm wire}_F+\mu_BH/v_F$ equal to the Fermi momentum for each ring. We observe that the coupling between the wire and the rings depends on the matching condition between the Fermi momentum in the rings and in the wire. Therefore for N channels the matching condition between the Fermi momentum will allow us to select the strong coupling zero mode. For the remaining part we will consider only a single polarization for the wire and rings. For each ring we replace the right and left movers by four Majorana operators,

$$r_i\equiv-i[R_i(0)-R_i^\dagger(0)],\quad l_i\equiv L_i(0)+L_i^\dagger(0).\tag{7}$$

The matrix element between the wire and rings is denoted by $-g$. Therefore, the low-energy Hamiltonian is given by

$$H_T=\frac{-ig}{\sqrt{2}}[\alpha_l(r_1+l_1)+\alpha_r(r_2-l_2)].\tag{8}$$

Given the fact that we have two Majorana zero modes α_l and α_r, we can replace them by a single fermion, which has a definite parity.[19] Thus $q=\alpha_l+i\alpha_r$, $q^\dagger=\alpha_l-i\alpha_r$, which obey $[q,q^\dagger]_+=1$, $q^\dagger|0\rangle=|1\rangle$ and $q|1\rangle=|0\rangle$. Here $|0\rangle$ is the ground state of wire and rings: $R_{i,p}(x)|0\rangle=L_{i,p}(x)|0\rangle=R^\dagger_{i,h}(x)|0\rangle=L^\dagger_{i,h}(x)|0\rangle=q|0\rangle=0$, where $R_{i,p}(x)$, $L_{i,p}(x)$ are the particle operators and $R^\dagger_{i,h}(x)$, $L^\dagger_{i,h}(x)$ are the hole operators. The right and left movers are given as a linear combination of particle and hole operators. Further, $R_{i,p}(x)$ $L_{i,p}(x)$ represent the annihilation operators of particles and $R^\dagger_{i,h}(x)$, $L^\dagger_{i,h}(x)$ are the creation operators for holes

$$R_i(x)=R_{i,p}(x)+R^\dagger_{i,h}(x);\quad L_i(x)=L_{i,p}(x)+L^\dagger_{i,h}(x).\tag{9}$$

Using the fermionic representation, we replace $H_{\rm pw}$ given in Eq. (5) and H_T given in Eq. (8) by

$$\begin{aligned}H_{\rm pw}+H_T\equiv\epsilon q^\dagger q-\frac{ig}{2\sqrt{2}}&[(q+q^\dagger)(r_1+l_1)\\ &-i(q-q^\dagger)(r_2-l_2)].\end{aligned}\tag{10}$$

The value of the wire energy ϵ in Eqs. (5) and (10) is based on the projection of the spinor [in Eq. (4)] on the zero modes $\eta_{\lambda=1}(x)$ and $\eta_{\lambda=-1}(x)$. Since the modes in the rings couple to the zero modes as well as excitations in the wire, we expect the modes in the rings to attain a finite width Γ. The Hamiltonian in Eqs. (8) and (10) indicates that when an electron is injected into the wire from one lead a hole excitation occurs at the second lead. Therefore instead of the Andreev reflection[1] where the electron and hole are excited at the same lead, here we have a *crossed* Andreev reflection, which splits the Cooper pair over the two leads. The Hamiltonian in Eq. (10) shows that the two Majorana fermions combine into a single fermion, which couples to the two leads, prohibiting the Andreev reflection in favor of the crossed Andreev reflection.

We perform an exact integration over the fermion operators $q, q^\dagger$ and find the time-dependent effective interaction $H_{\text{eff}}(t)$,

$$H_{\text{eff}}(t) = \frac{-ig^2}{2}\int_{-\infty}^{\infty} dt' \mu[t-t'] e^{-i\frac{\epsilon}{\hbar}(t-t')}[r_2(t) - l_2(t) + i(r_1(t) + l_1(t))][r_2(t') - l_2(t') - i(r_1(t') + l_1(t'))], \tag{11}$$

where $\mu[t-t']$ is the step function, which is one for $t > t'$ and zero otherwise. When $\epsilon \to 0$ the term $e^{-i\frac{\epsilon}{\hbar}(t-t')}$ in Eq. (11) is replaced by one.

Using the scaling analysis given in Ref. 20, we observe that the effective interaction flows to the strong coupling limit. The coupling g is replaced by $g(b)$ where $b > 1$ represents the scaling parameter. We find $g^2(b) = g^2 b^{2-\alpha}$, $\alpha \approx 1$ flows to the strong coupling $g\ (b \gg 1) \to \infty$. The only way a solution will exist is if the effective interaction annihilates the ground state,

$$H_{\text{eff}}(t)|0\rangle = 0; \qquad [r_2(t) - l_2(t) - i(r_1(t) + l_1(t))]|0\rangle = 0. \tag{12}$$

Therefore, the physical solution is given by the *constraint* condition.[21] Since $R_{i,p}(x)|0\rangle = L_{i,p}(x)|0\rangle = R^\dagger_{i,h}(x)|0\rangle = L^\dagger_{i,h}(x)|0\rangle = 0$ the constraint condition implies for particles the equation $[i(R^\dagger_{2,p} - L^\dagger_{1,p}) - (L^\dagger_{2,p} - R^\dagger_{1,p})]|0\rangle = 0$, and for holes $[i(R_{2,h} - L_{1,h}) - (L_{2,h} - R_{1,h})]|0\rangle = 0$. We obtain the constraint equation $\psi_1(x=0) = e^{-i\frac{\pi}{2}}\psi_2(x=0) \equiv \widetilde{\psi}_2(x=0) = 0$. In Ref. 21 we have constructed the many particles wave function, which is sensitive to the even/odd number of electrons in the two rings. We find that for an odd number of electrons the current is drastically reduced. This result implies that only the even parity is relevant, an even parity in the p-wave wire and an even parity in the metallic rings. It is consistent with the fact that contrary to a single Majorana fermion, two Majorana fermions have a definite parity.[19] Therefore in our problem the even/odd degeneracy is removed by the rings which favor an even number of electrons.

Next we consider the weak coupling case, which corresponds to the finite energy limit $\epsilon \neq 0$. This limit will be studied perturbatively. We observe that in addition to the current produced by the flux in each ring, the effect of the Majorana fermions is to induce a correlation current. This current is revealed through the appearance of a tunneling current when the flux difference is equal to ϵ of the zero-mode excitations.

We will use the zero-mode Bosonization method.[20,22] The right $R_i(x)$ and left $L_i(x)$ fermions for each ring $i = 1,2$ are given by

$$\begin{aligned} R_i(x) &= \sqrt{\frac{\Lambda}{2\pi}} Z_i e^{i\alpha_{R,i}} e^{i\frac{2\pi}{l_{\text{ring}}}(N_{R,i}-\frac{1}{2})x} e^{i\sqrt{4\pi}\vartheta_{R,i}(x)}, \\ L_i(x) &= \sqrt{\frac{\Lambda}{2\pi}} Z_i e^{i\alpha_{L,i}} e^{i\frac{2\pi}{l_{\text{ring}}}(N_{L,i}-\frac{1}{2})x} e^{i\sqrt{4\pi}\vartheta_{L,i}(x)}. \end{aligned} \tag{13}$$

Here $Z_1 Z_2 = -Z_2 Z_1$ are the new Majorana fields, which ensure the anticommutation between the two rings in the Bosonic representation, $\vartheta_{R,i}(x)$ and $\vartheta_{L,i}(x)$ are the Bosonic fields which describe the particle-hole excitations, $\alpha_{R,i}$ $\alpha_{L,i}$ are the Bosonic zero modes, and $N_{R,j}$, $N_{L,j}$ are the conjugate variables. The Bosonic zero modes obey the commutation rules $[-\alpha_{L,i}, N_{L,j}] = i\delta_{i,j}$ and $[\alpha_{R,i}, N_{R,j}] = i\delta_{i,j}$. As a result we obtain the zero-mode representation for the uncoupled rings in terms of the number operators $N_{L,i}$, $N_{R,i}$ and fluxes $\hat{\varphi}_i$ in each ring

$$\begin{aligned} H_0 = &\frac{\pi v_F \hbar}{2l_{\text{ring}}}[(N_{L,1} - N_{R,1} + 2\hat{\varphi}_1)^2 + (N_{L,1} + N_{R,1})^2] \\ &+ \frac{\pi v_F \hbar}{2l_{\text{ring}}}[(N_{L,2} - N_{R,2} + 2\hat{\varphi}_2)^2 + (N_{L,2} + N_{R,2})^2]. \end{aligned} \tag{14}$$

We will use the *interaction picture* for $\alpha^I_{R,i}(t)$ and $\alpha^I_{L,i}(t)$ in order to evaluate $H_{\text{eff}}(t)$ in Eq. (11). We find that $H_{\text{eff}}(t)$ is given in terms of the zero-mode functions $F(t)$ and $G(t+\tau)$

$$H_{\text{eff}}(t) = \frac{-i\hat{g}^2}{2}\int_{-\infty}^{\infty} d\tau \mu[\tau] F(t) e^{-i\frac{\epsilon}{\hbar}\tau} G(t+\tau), \tag{15}$$

where $\hat{g}^2 = g^2\frac{k_F}{2\pi}$. The functions $F(t) \equiv [r_2(t) - l_2(t) + i(r_1(t) + l_1(t))]$ and $G(t+\tau) \equiv (r_2(t+\tau) - l_2(t+\tau) - i(r_1(t+\tau) + l_1(t+\tau))$ are given in terms of the zero-mode fields $\alpha^I_{R,i}(t)$ and $\alpha^I_{L,i}(t)$.

We perform the integration with respect to τ and find from Eq. (15)

$$H_{\text{eff}} \approx H^{\text{real}}_{\text{eff}} + i H^{\text{Im}}_{\text{eff}}, \tag{16}$$

where $H^{\text{real}}_{\text{eff}}$ is the real part and $H^{\text{Im}}_{\text{eff}}$ is the imaginary part of the effective action. The real part $H^{\text{real}}_{\text{eff}}$ is given in terms of the dimensionless width $\hat{\Gamma} = \frac{\Gamma}{\hbar}(\frac{2\pi v_F}{l_{\text{ring}}})^{-1}$, which is the width of the levels in the rings, and dimensionless zero-mode energy $\nu_0 \equiv \frac{\epsilon}{\hbar}(\frac{2\pi v_F}{l_{\text{ring}}})^{-1}$

$$\begin{aligned} H^{\text{real}}_{\text{eff}} = &\frac{\hbar\hat{g}^2\nu_0}{\nu_0^2 + \hat{\Gamma}^2}[\cos(2\alpha_{R,2}) - \cos(2\alpha_{L,2}) - \cos(2\alpha_{L,1}) + \cos(2\alpha_{R,1}) + 2(\sin(\alpha_{R,2} + \alpha_{L,2}) - \sin(\alpha_{R,1} + \alpha_{L,1}))] \\ &- 2\hbar\hat{g}^2\left(\frac{\nu_0 - \hat{\varphi}_1}{(\nu_0 - \hat{\varphi}_1)^2 + \hat{\Gamma}^2} + \frac{\nu_0 + \hat{\varphi}_1}{(\nu_0 + \hat{\varphi}_1)^2 + \hat{\Gamma}^2}\right)[1 + \sin(\alpha_{R,1} - \alpha_{L,1})] \\ &- 2\hbar\hat{g}^2\left(\frac{\nu_0 - \hat{\varphi}_2}{(\nu_0 - \hat{\varphi}_2)^2 + \hat{\Gamma}^2} + \frac{\nu_0 + \hat{\varphi}_2}{(\nu_0 + \hat{\varphi}_2)^2 + \hat{\Gamma}^2}\right)[1 + \sin(\alpha_{R,2} - \alpha_{L,2})]. \end{aligned} \tag{17}$$

III. PERSISTENT CURRENTS

The imaginary part $H^{\mathrm{Im}}_{\mathrm{eff}}$ of Eq. (16) causes the current to vanish. Finite current solutions will be obtained for the ground states $|0\rangle$, which obey $H^{\mathrm{Im}}_{\mathrm{eff}}|0\rangle = 0$. In particular when $\nu_0 \approx |\frac{\hat{\varphi}_2-\hat{\varphi}_1}{2}|$ the imaginary part of the effective Hamiltonian $H^{\mathrm{Im}}_{\mathrm{eff}}$ is maximal and the existence of a finite current demands the condition $H^{\mathrm{Im}}_{\mathrm{eff}}|0\rangle = 0$. This condition is equivalent to the relation between the zero-mode fields $\alpha_{R,2} = \alpha_{R,1} + u$; $\alpha_{L,2} = \alpha_{L,1} - u$ for an arbitrary field u, which is integrated out. The equation $\nu_0 \approx |\frac{\hat{\varphi}_2-\hat{\varphi}_1}{2}|$ represents the new constraint condition, which replaces the constraint $\frac{\hat{\varphi}_2-\hat{\varphi}_1}{2} = 0$ obtained from Eq. (12) for the strong coupling case, $\epsilon \to 0$.

We enforce the condition $\nu_0 \approx |\frac{\hat{\varphi}_2-\hat{\varphi}_1}{2}|$ and obtain

$$\begin{aligned}\frac{H}{\hbar} \approx{}& \frac{\pi v_F}{2l_{\mathrm{ring}}}\left[\left(-i\frac{d}{d\alpha}+2\hat{\varphi}_1\right)^2+\left(-i\frac{d}{d\beta}\right)^2\right]+\frac{\pi v_F}{2l_{\mathrm{ring}}}\left[\left(-i\frac{d}{d\alpha}+2\hat{\varphi}_2\right)^2+\left(-i\frac{d}{d\beta}\right)^2\right]\\&+\left\{\frac{2\hat{g}^2\nu_0}{\nu_0^2+\hat{\Gamma}^2}\sin(\alpha)\sin(\beta)-2\hat{g}^2\left[\frac{\nu_0-\hat{\varphi}_1}{(\nu_0-\hat{\varphi}_1)^2+\hat{\Gamma}^2}+\frac{\nu_0+\hat{\varphi}_1}{(\nu_0+\hat{\varphi}_1)^2+\hat{\Gamma}^2}\right]\sin(\beta)\right.\\&\left.-2\hat{g}^2\left[\frac{\nu_0-\hat{\varphi}_1}{(\nu_0-\hat{\varphi}_1)^2+\hat{\Gamma}^2}+\frac{\nu_0+\hat{\varphi}_1}{(\nu_0+\hat{\varphi}_1)^2+\hat{\Gamma}^2}+\frac{\nu_0-\hat{\varphi}_2}{(\nu_0-\hat{\varphi}_2)^2+\hat{\Gamma}^2}+\frac{\nu_0+\hat{\varphi}_2}{(\nu_0+\hat{\varphi}_2)^2+\hat{\Gamma}^2}\right]\right\}\delta_{|\frac{\hat{\varphi}_1-\hat{\varphi}_2}{2}|,\nu_0}.\end{aligned} \tag{18}$$

The first line of Eq. (18) represents the Hamiltonian for the two metallic rings pierced by the external fluxes expressed in terms of the zero modes of the metallic rings. The second part of Eq. (18) represents the coupling between the wire and the two rings. We observe that this part is restricted by the constraint condition $|\frac{\hat{\varphi}_1-\hat{\varphi}_2}{2}| = \nu_0$. This constraint represents the effect of Majorana fermions on the p-wave wire.

In order to investigate the Hamiltonian in Eq. (18), we will use the algebra of the zero modes[22–24] where the charge and the current are given by $\hat{Q} = N_R + N_L \equiv -2i\frac{d}{d\beta}$, $\hat{J} = N_R - N_L \equiv -2i\frac{d}{d\alpha}$, which obey the conditions: $\hat{J}|J,Q\rangle = J|J,Q\rangle$; $J = 0,\pm1,\pm2,\ldots$, $\hat{Q}|J,Q\rangle = Q|J,Q\rangle$; $Q = 0,\pm1,\pm2,\ldots$; $e^{i\alpha}|J,Q\rangle = |J+1,Q\rangle$; $e^{-i\alpha}|J,Q\rangle = |J-1,Q\rangle$, $e^{i\beta}|J,Q\rangle = |J,Q+1\rangle$; and $e^{-i\beta}|J,Q\rangle = |J,Q-1\rangle$.

Using the algebra of the zero modes, we compute to lowest order the energy of the ground state as a function of the coupling constant $\lambda \equiv 2\hat{g}^2(\frac{\pi v_F}{l_{\mathrm{ring}}})^{-1} = 2g^2\frac{k_F l_{\mathrm{ring}}}{\pi v_F} < 1$, the electronic bandwidth and the external fluxes. We find for the ground-state energy $E(\hat{\varphi}_1,\hat{\varphi}_2)$

$$E(\hat{\varphi}_1,\hat{\varphi}_2) = \frac{2\hbar\pi v_F}{l_{\mathrm{ring}}}\left[\left(\hat{\varphi}_1{}^2+\hat{\varphi}_2{}^2\right)-2\lambda\left(\frac{\nu_0-\hat{\varphi}_1}{(\nu_0-\hat{\varphi}_1)^2+\hat{\Gamma}^2}+\frac{\nu_0+\hat{\varphi}_1}{(\nu_0+\hat{\varphi}_1)^2+\hat{\Gamma}^2}+\frac{\nu_0-\hat{\varphi}_2}{(\nu_0-\hat{\varphi}_2)^2+\hat{\Gamma}^2}+\frac{\nu_0+\hat{\varphi}_2}{(\nu_0+\hat{\varphi}_2)^2+\hat{\Gamma}^2}\right)\right]\delta_{|\frac{\hat{\varphi}_1-\hat{\varphi}_2}{2}|,\nu_0}. \tag{19}$$

Using Eq. (19) we compute the currents $I_i = \frac{\partial E(\hat{\varphi}_1,\hat{\varphi}_2)}{\partial\hat{\varphi}_i}$ for the two rings in units of $I_0 = \frac{\pi v_F}{l_{\mathrm{ring}}}$. In the absence of the constraint condition $\delta_{|\frac{\hat{\varphi}_1-\hat{\varphi}_2}{2}|,\nu_0}$ the current in each ring $\frac{I_i}{I_0} = i_i[\hat{\varphi}_i]$, $i = 1,2$ is given by

$$i_i[\hat{\varphi}_i] = \hat{\varphi}_i - 2\lambda\left[\frac{3(\nu_0-\hat{\varphi}_i)^2+\hat{\Gamma}^2}{((\nu_0-\hat{\varphi}_i)^2+\hat{\Gamma}^2)^2}+\frac{(\nu_0+\hat{\varphi}_i)^2-\hat{\Gamma}^2}{((\nu_0+\hat{\varphi}_i)^2+\hat{\Gamma}^2)^2}\right]. \tag{20}$$

We observe that the current in each ring depends on both the external flux applied and the zero-mode excitation energy ν_0.

Next we consider the effect of the zero mode. The condition for stable solutions imposes the constraint $\nu_0 \approx |\frac{\hat{\varphi}_1-\hat{\varphi}_2}{2}|$, which relates the flux difference to the zero-mode excitation energy ν_0. As a result we find that the current in the two rings is given by

$$\begin{aligned}i_1[\hat{\varphi}_1,\hat{\varphi}_2] &\approx \hat{\varphi}_1 - 2\lambda\left[\frac{3(\nu_0-\hat{\varphi}_1)^2+\hat{\Gamma}^2}{((\nu_0-\hat{\varphi}_1)^2+\hat{\Gamma}^2)^2}+\frac{(\nu_0+\hat{\varphi}_1)^2-\hat{\Gamma}^2}{((\nu_0+\hat{\varphi}_1)^2+\hat{\Gamma}^2)^2}\right]\left(\frac{1}{\pi}\right)\frac{\hat{\Gamma}}{\left(\nu_0-\frac{|(\hat{\varphi}_1-\hat{\varphi}_2)|}{2}\right)^2+\hat{\Gamma}^2},\\ i_2[\hat{\varphi}_1,\hat{\varphi}_2] &\approx \hat{\varphi}_2 - 2\lambda\left[\frac{3(\nu_0-\hat{\varphi}_2)^2+\hat{\Gamma}^2}{((\nu_0-\hat{\varphi}_2)^2+\hat{\Gamma}^2)^2}+\frac{(\nu_0+\hat{\varphi}_2)^2-\hat{\Gamma}^2}{((\nu_0+\hat{\varphi}_2)^2+\hat{\Gamma}^2)^2}\right]\left(\frac{1}{\pi}\right)\frac{\hat{\Gamma}}{\left(\nu_0-\frac{|(\hat{\varphi}_1-\hat{\varphi}_2)|}{2}\right)^2+\hat{\Gamma}^2}.\end{aligned} \tag{21}$$

In Fig. 2 we plot the current in ring one (left ring) as a function of the flux $\hat{\varphi}_1$ in ring one for a fixed $\hat{\varphi}_2 = 0.3$ in the second ring. For the case $\nu_0 = 0.002$ we observe that the current varies linearly with $\hat{\varphi}_1$ with resonances at $\hat{\varphi}_1 = 0$ and when $\hat{\varphi}_1 = \hat{\varphi}_2$. Figure 3 depicts the current in ring two (right ring) as a function of flux in ring one (left ring) for a fixed flux $\hat{\varphi}_2 = 0.3$ in ring

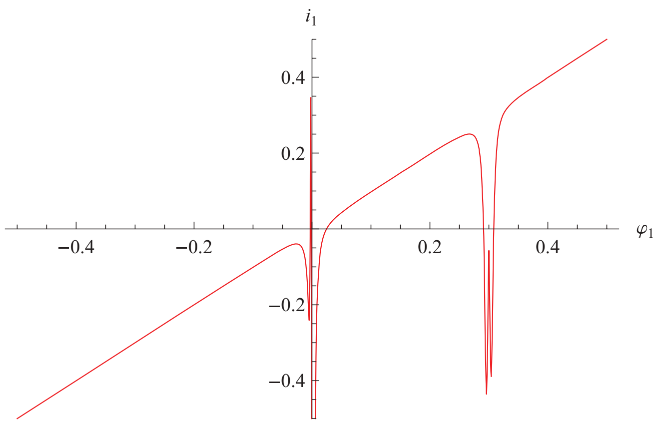

FIG. 2. (Color online) The current in ring one (left ring) as a function of the flux $\hat{\varphi}_1$ in ring one for a fixed flux in ring two $\hat{\varphi}_2 = 0.3$ for $\nu_0 = 0.002$ and $g = 0.01$. The structure of resonance peaks at $\hat{\varphi}_1 = 0$ and $\hat{\varphi}_1 = \hat{\varphi}_2$ follows from Eq. (21).

two for $\nu_0 = 0.002$. The current i_2 is constant except for a resonance when $\hat{\varphi}_1 = \hat{\varphi}_2$, clearly indicating the correlation induced tunneling current.

Next we increase the value of the zero-mode energy to $\nu = 0.01$. In Fig. 4 we show the current in ring one (left ring) as a function of the flux $\hat{\varphi}_1$ in ring one (left ring) for a fixed flux $\hat{\varphi}_2 = 0.3$ (right ring). We observe two peaks, which correspond to the condition $\nu_0 = \pm\frac{\hat{\varphi}_1 - \hat{\varphi}_2}{2}$. In addition we observe a peak when $\nu_0 = \pm\hat{\varphi}_1$ (here we show the graph only for the positive flux values). Finally, in Fig. 5 we depict the current in ring two as a function of flux in ring one with fixed flux $\hat{\varphi}_2 = 0.3$ and $\nu = 0.01$. The current is constant except when the condition $\nu_0 = \pm\frac{\hat{\varphi}_1 - \hat{\varphi}_2}{2}$ is satisfied again indicating the correlation. Due to the fact that we considered the weak coupling limit we observe, in addition to the correlation effects, the zero-order result $i_1 = \hat{\varphi}_1$ and $i_2 = \hat{\varphi}_2$.

IV. CONCLUSION

To summarize, we have investigated the dependence of the currents in the p-wave superconducting nanowire coupled to

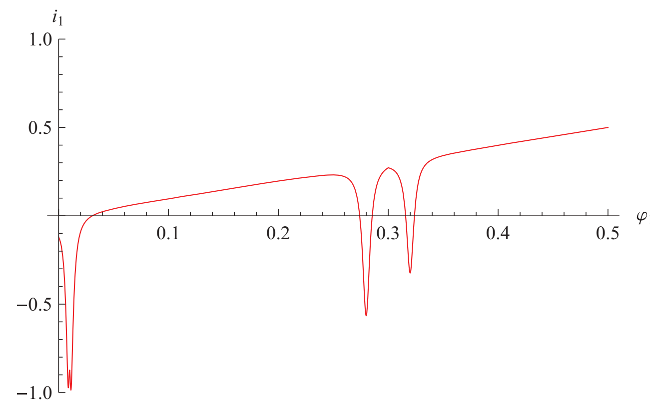

FIG. 4. (Color online) The current in ring one (left ring) as a function of the flux $\hat{\varphi}_1$ in ring one for a fixed flux in ring two $\hat{\varphi}_2 = 0.3$ for $\nu_0 = 0.01$ and $g = 0.01$.

two metallic nanorings on the fluxes for the entire regime of parameters.[20,22] In the limit of large L and $\epsilon \to 0$ the current vanishes in both rings when the two fluxes are different. We observe that for a finite energy ϵ and different fluxes the current dependence is more complex. The current in each ring varies linearly except when the two fluxes are equal, there is a resonance and a finite tunneling current. We can interpret this effect as a crossed Andreev reflection, instead of the usual Andreev reflection, which represents the fingerprint of the Majorana fermions in this setup. Based on this observation such a two-ring p-wave superconducting nanowire system can be used to realize qubits for quantum computing.[2] In other words, due to the nonlocality associated with the spatially separated Majorana zero modes (bound to the rings), as evident by the transformation of Eq. (8) into Eq. (10), quantum information can be encoded in this setup for topological quantum computing.[1,11,12,15] Our predictions can be verified by measuring correlation between the two magnetizations in the rings thus providing us with a distinct possibility of observing Majorana fermions.

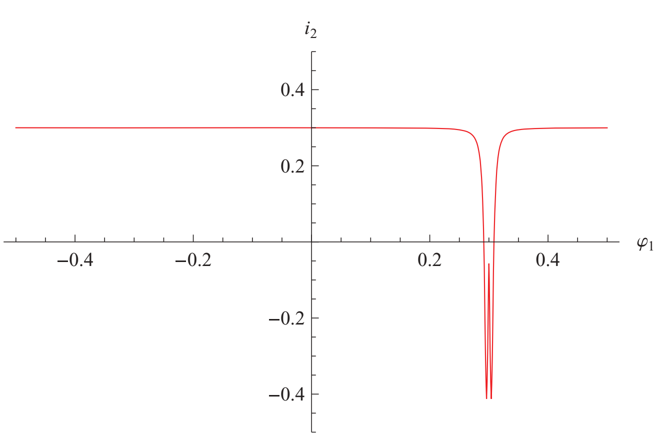

FIG. 3. (Color online) The current in ring two (right ring) as a function of the flux in ring one $\hat{\varphi}_1$ for a fixed flux $\hat{\varphi}_2 = 0.3$ in ring two for $\nu_0 = 0.002$ and $g = 0.01$.

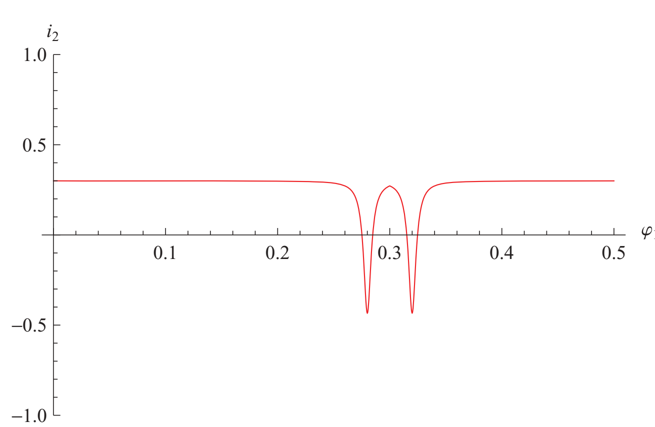

FIG. 5. (Color online) The current in ring two (right ring) as a function of the flux in ring one $\hat{\varphi}_1$ for a fixed flux $\hat{\varphi}_2 = 0.3$ in ring two for $\nu_0 = 0.01$ and $g = 0.01$.

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[1] C. W. J. Beenakker, arXiv:1112.1950v1.
[2] Alexei Yu. Kitaev, arXiv:cond-mat/0010440; Ann. Phys. (NY) **303**, 2 (2003).
[3] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, Science **336**, 1003 (2012).
[4] M. Ichioka, J. Phys. Soc. Jpn. **80**, 013602 (2011).
[5] M. T. Deng, C. L. Yu, G. Y. Huang, M. Larsson, P. Caroff, and H. Q. Xu, arXiv:1204.4130.
[6] C. Kallin and A. J. Belinsky, J. Phys.: Condens. Matter **21**, 164210 (2009).
[7] S. Tewari, S. Das Sarma, and D.-H. Lee, Phys. Rev. Lett. **99**, 037001 (2007).
[8] R. M. Lutchyn, J. D. Sau, and S. Das Sarma, Phys. Rev. Lett. **105**, 077001 (2010).
[9] Y. Oreg, G. Refael, and F. von Oppen, Phys. Rev. Lett. **105**, 177002 (2010).
[10] L. Jiang, D. Pekker, J. Alicea, G. Refael, Y. Oreg, and F. von Oppen, Phys. Rev. Lett. **107**, 236401 (2011).
[11] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. **82**, 3045 (2010).
[12] J. Alicea, Rep. Prog. Phys. **75**, 076501 (2012).
[13] I. Martin and A. F. Morpurgo, Phys. Rev. B **85**, 144505 (2012).
[14] Y. Tsutsumi and K. Machida, J. Phys. Soc. Jpn. **81**, 074607 (2012).
[15] L. Fu, Phys. Rev. Lett. **104**, 056402 (2010).
[16] *Advances in Condensed Matter and Material Research*, edited by H. Geelvinck and S. Reynst, Vol. 10 (Nova Publishers, Hauppauge, NY, 2011), pp. 379–402.
[17] S. Ryu, A. P. Schneider, A. Furusaki, and A. W. W. Ludwig, New J. Phys. **12**, 065010 (2010).
[18] K. Sengupta, I. Žutić, H.-J. Kwon, V. M. Yakovenko, and S. Das Sarma, Phys. Rev. B **63**, 144531 (2001).
[19] C. Chamon, R. Jackiw, Y. Nishida, S.-Y. Pi, and L. Santos, Phys. Rev. B **81**, 224515 (2010).
[20] D. Schmeltzer, A. R. Bishop, A. Saxena, and D. L. Smith, Phys. Rev. Lett. **90**, 116802 (2003).
[21] D. Schmeltzer and A. Saxena, Phys. Rev. B **81**, 195310 (2010).
[22] D. Schmeltzer and R. Berkovits, Phys. Lett. A **253**, 341 (1999).
[23] D. Schmeltzer, A. Kuklov, and M. Malard, J. Phys.: Condens. Matter **22**, 095301 (2010).
[24] D. Schmeltzer, J. Phys.: Condens. Matter **23**, 155601 (2011).