

Triplet supercurrent in ferromagnetic Josephson junctions by spin injectionA. G. Mal'shukov¹ and Arne Brataas²¹*Institute of Spectroscopy, Russian Academy of Sciences, 142190, Troitsk, Moscow oblast, Russia*²*Department of Physics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway*

(Received 8 June 2012; revised manuscript received 28 August 2012; published 17 September 2012)

We show that injecting nonequilibrium spins into the superconducting leads strongly enhances the stationary Josephson current through a superconductor-ferromagnet-superconductor junction. The resulting long-range supercurrent through a ferromagnet is carried by triplet Cooper pairs that are formed in *s*-wave superconductors by the combined effects of spin injection and exchange interaction. We quantify the exchange interaction in terms of Landau Fermi-liquid factors. The magnitude and direction of the long-range Josephson current can be manipulated by varying the angles of the injected polarizations with respect to the magnetization in the ferromagnet.

DOI: [10.1103/PhysRevB.86.094517](https://doi.org/10.1103/PhysRevB.86.094517)

PACS number(s): 72.25.Dc, 71.70.Ej, 73.40.Lq

I. INTRODUCTION

Studies of hybrid structures combining superconducting and ferromagnetic components attract much attention due to their unique, rich, and complex physical properties that are promising in a number of potential applications.¹ The interface of an *s*-wave superconductor with a ferromagnet is characterized by an unusual proximity effect that is spatially oscillating and can lead to a sign reversal of the critical current through superconductor-ferromagnet-superconductor (SFS) Josephson junctions. Such a reversal is equivalent to a π shift in the current-phase relation for the Josephson current. This interesting property is a motivation for using the so-called π junctions as elements of superconducting quantum circuits for potential application in quantum computing.² However, the proximity effect in ferromagnets does not reach far. Two critical tasks are to extend its range and to find a way to manipulate the π junction in order to switch the device between its various phase states. In contrast, Cooper pairs can be transferred over relatively long distances even in ferromagnets, if they are in a triplet state with ± 1 projections of their total spin onto the spin-quantization axis. Various mechanisms have been proposed that convert a singlet pair into a triplet pair, such as a spatially dependent magnetization,³ spin-flip scattering at ferromagnetic-superconductor (FS) interfaces,⁴ and precessing magnetization.⁵ A number of works in this direction have been reviewed in Ref. 1.

In this work, we will show that these tasks can be fulfilled via the production and manipulation of a long-range proximity effect by injecting spins into superconducting leads. The novelty of our idea is based on the important, and so far unaddressed, role played by the electron-electron interaction in SFS. Our insight is that the combined effects of spin-injection and electron-electron interaction generate a long-range proximity effect despite the strong exchange field in the ferromagnet. The conventional wisdom is that spin-polarized electrons can only exist as excitations in *s*-wave superconductors because the Cooper pairs do not carry a spin. However, we will demonstrate that this simple picture, which is based on the neglect of electron-electron interactions beyond superconducting pairing correlations, misses qualitatively

important effects. Quantitatively, in simple metals, the exchange interaction of itinerant carriers is noticeable and can be described in terms of Landau Fermi-liquid factors. Although the exchange interaction does not cause ferromagnetism in *s*-wave superconductors, it causes a transfer of spin polarization from the quasiparticle excitations to the condensate, in the form of polarized triplet Cooper pairs. When such a triplet pairing is generated by the combined effects of spin injection and exchange interaction, these pairs subsequently tunnel through the ferromagnetic layer via the long-range proximity effect, if the spin polarizations in the leads and the layer are not collinear. Only at this stage, which includes the so far unaddressed important electron-electron interaction, does the situation become similar to proposals of Refs. 1 and 3, where an inhomogeneous magnetization gives rise to the long-range effect provided by ± 1 triplets. The relative angles between the spin polarizations in the superconducting leads and in the ferromagnet can be varied by controlling the injected spin polarizations, making it possible to vary the magnitude and sign of the Josephson current. This enables manipulations of π junctions. In addition to the Josephson supercurrent, which is driven by the difference in the condensate phases, there is also a dissipative dc current. The latter is induced by the spin-polarization flow through the ferromagnetic layer with spin-dependent conductivity. This dissipative current also can be manipulated by varying the injected polarization angles. As it will be shown, at some angles it vanishes, so that the dissipative and supercurrents can be measured independently.

Various effects of an injected spin polarization and spin current on the electric transport in SFS junctions^{6,7} and other superconducting systems⁸⁻¹¹ have recently been considered. Despite this interest, the fact that the exchange interaction transfers the spin polarization from the quasiparticles to the condensate has not been addressed so far.

The article is organized by the following way: In Sec. II, an expression is derived connecting the triplet components of the anomalous Green function to the nonequilibrium spin polarization in superconducting leads. In Sec. III, the Josephson and dissipative currents are calculated. Finally, our results are discussed in Sec. IV.

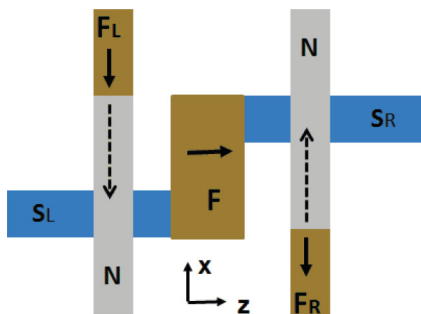


FIG. 1. (Color online) A sketch of the system. The electric current flows in normal leads N through contacts with ferromagnetic leads F_L and F_R . Spin density is injected from F_L and F_R into N and further penetrates across tunneling barriers into superconductors S_L and S_R . The Josephson current flows between these leads through a ferromagnetic layer F. Arrows show possible magnetizations of the ferromagnets.

II. TRIPLET ELECTRON PAIRING FUNCTION INDUCED BY SPIN INJECTION

How to efficiently inject a spin polarization into paramagnetic metals is well known.¹² A nonequilibrium spin accumulation is induced by the electric current through a paramagnetic-ferromagnetic interface. We consider the scenario in which the spin polarization further diffuses from a paramagnet through a resistive barrier into a superconducting lead, so that the electric circuit where the spin injection takes place is effectively separated from the superconducting circuit. We assume that the steady-state spin polarizations are generated in both superconducting leads, in the vicinity of the F layer. The sketch of the system is shown in Fig. 1. For clarity, we simplify the problem by assuming that the FS contacts contain a barrier, so that the proximity effect is weak. We also assume that the spin-relaxation time τ_{spin} in the leads is long, so that the spin diffusion length l_{spin} is large compared to the SN contact sizes and the coherence length. Consequently, the spin densities $s_{L(R)}$ and the order parameters $\Delta_{L(R)}$ only vary slowly in space near the left (L) and right (R) contacts.

The electronic transport through an SFS system, whose characteristic dimensions are larger than the elastic mean free path, can be described in terms of Usadel equations for angular averaged Green functions g (for a review see Ref. 13). These functions are matrices in the Keldysh, spin, and Nambu spaces. We choose the spin and Nambu spaces so that the one-particle destruction operators are $c_{1\mathbf{k}\uparrow} = c_{\mathbf{k}\uparrow}$, $c_{1\mathbf{k}\downarrow} = c_{\mathbf{k}\downarrow}$, $c_{2\mathbf{k}\uparrow} = c_{-\mathbf{k}\downarrow}^\dagger$, and $c_{2\mathbf{k}\downarrow} = -c_{-\mathbf{k}\uparrow}^\dagger$, where the labels 1 and 2 denote the Nambu spinor components, while \uparrow and \downarrow are the spin indices. The Keldysh component g^K of the Green function can be represented as¹³

$$g^K = g^r h - h g^a, \quad (1)$$

where g^r and g^a are the retarded and advanced functions, respectively, and the distribution function h is a diagonal matrix in the Nambu space.

In order to determine the distribution h in the superconducting leads, the interfaces between these leads and the spin-polarized normal metals must be considered. We use standard boundary conditions relating fluxes through S-N (S-F) inter-

faces to Green functions in superconductors and normal metals (ferromagnets). It is assumed that the spin-relaxation rates in the superconducting leads are slow enough ($l_{\text{spin}} \gg r_{sn}\sigma_s$) and the leakage of the spin polarization through the SF boundary is sufficiently slow $r_{sn}/A_{sn} \ll r_{sf}/A_{sf}$, where $1/r_{sn}$ and $1/r_{sf}$ are the interface conductances (per unit square) of SN and SF interfaces, A_{sn} and A_{sf} are the SN and SF contact areas, and σ_s is the normal-state conductivity of the superconductor's lead. With these assumptions, the distribution functions in the superconductor $h^{(s)}$ and normal metal $h^{(n)}$ are equal to each other, $h^{(s)} = h^{(n)}$. We further assume that nonequilibrium spins in N leads are thermalized with chemical potentials μ_\uparrow and μ_\downarrow for the two spin directions. Therefore, denoting by the subscripts 11 and 22 the corresponding matrix elements in the Nambu space, we get for $h_{\uparrow(\downarrow)} \equiv h_{11\uparrow(\downarrow)}^{(s)} = h_{11\uparrow(\downarrow)}^{(n)}$ and $\bar{h}_{\uparrow(\downarrow)} \equiv h_{22\uparrow(\downarrow)}^{(s)} = h_{22\uparrow(\downarrow)}^{(n)}$,

$$h_{\uparrow(\downarrow)} = \bar{h}_{\uparrow(\downarrow)} = \tanh \frac{\omega - \mu_{\uparrow(\downarrow)}}{2k_B T}. \quad (2)$$

At the same time, the retarded (g^r) and advanced (g^a) Green functions have the same forms as in an equilibrium superconductor.

Our calculation so far reiterates the conventional wisdom of spin injection in superconductors: The effects are limited to a spin-dependent statistical distribution function, while the retarded and advanced Green functions do not change. In this picture, spin injection does not lead to the appearance of triplet correlations in the condensate wave function, which would cause long-range Josephson tunneling through a ferromagnetic layer. Fortunately, there is a mechanism to generate triplet correlations in spin-polarized superconducting leads, which others have so far overlooked. The electron-electron exchange interaction provides a coupling between a spin accumulation and the spectral properties of superconductors, in that spin-polarized quasiparticles produce an effective Zeeman field. The latter, in its turn, gives rise to *triplet* correlations that are described via the corresponding spin components of the anomalous functions g_{12}^r and g_{12}^a . In Fermi-liquid theory, the effective Zeeman energy is $\epsilon_{xc}(\sigma\mathbf{N})$, where \mathbf{N} is a unit vector parallel to the injected spin polarization $\mathbf{S} = \mathbf{N}\mathbf{S}$ and

$$\epsilon_{xc} = GS/2N_F. \quad (3)$$

The spin-accumulation magnitude is

$$S = -\frac{N_F}{4(1+G)} \int d\omega \text{Tr} \left(\frac{(1+\tau_3)}{2} \sigma_z g^K \right), \quad (4)$$

where τ_3 and σ_z are the Pauli matrices acting in the Nambu and spin spaces, respectively, and N_F is the density of states at the Fermi level. The renormalization factor $1/(1+G)$, where G is the exchange Landau-Fermi liquid parameter, appears when the spin density of Eq. (4) is expressed in terms of a semiclassical Green function integrated over energy.¹⁴ This factor is not qualitatively important in our case, since G is not too close to the paramagnet instability $G = -1$.¹⁵ The exchange Coulomb interaction in metals gives rise to a negative G . For example, the calculated value is -0.17 in Al.¹⁶ The spin density (4) strongly depends on temperature, mostly via the temperature dependence of the superconducting gap in the energy spectrum. In order to determine S and Δ in both

leads, Eq. (4) has to be solved together with the S -dependent self-consistency equation for Δ .⁹

Via the effective Zeeman energy of Eq. (3), the retarded and advanced Green functions become spin-dependent.¹ Indeed, choosing the quantization axis along \mathbf{N} , the anomalous functions $f_{\uparrow\downarrow}^r = g_{12\uparrow\uparrow}^r$ and $f_{\downarrow\uparrow}^r = -g_{12\downarrow\downarrow}^r$ become

$$f_{\uparrow\downarrow(\downarrow\uparrow)}^r = \pm \frac{|\Delta| \exp(i\phi)}{\sqrt{(\omega \mp \epsilon_{xc} + i\delta)^2 - |\Delta|^2}}, \quad (5)$$

where the phase ϕ of the order parameter Δ equals ϕ_L and ϕ_R at the left and right contacts, respectively. The triplet component of this function with 0 spin projection onto the z axis is $f_0^r = (f_{\uparrow\downarrow}^r + f_{\downarrow\uparrow}^r)/\sqrt{2}$, while the triplet components with ± 1 projections vanish, $f_{\pm 1}^r = f_{\uparrow\downarrow(\downarrow\uparrow)}^r = 0$. The advanced function, as well as the conjugated functions f^\dagger , are determined from symmetry relations.

It is more transparent to discuss the Green functions in a basis where the spin-quantization axis is parallel to the magnetization in the ferromagnetic layer, which is along z , as shown in Fig. 1. We assume that the spin polarizations in the left and right leads are rotated with respect to this axis by the angles θ_L and θ_R , respectively. We follow the convention that the three components of the triplet f_0, f_1, f_{-1} are related to a 3D vector $\mathbf{a} = (a_x, a_y, a_z)$ with $a_z = f_0, a_x = (f_{-1} - f_1)/\sqrt{2}$ and $a_y = i(f_{-1} + f_1)/\sqrt{2}$.¹⁷ Hence, in the geometry shown in Fig. 1, after a rotation of \mathbf{a} around the y axis, we get in the new basis $f'_0 = f_0 \cos \theta$ and $f'_1 = -f'_{-1} = -f_0 \sin \theta/\sqrt{2}$. So, by using Eq. (5) the triplet components in the left and right superconducting leads are

$$f_{\pm 1R(L)} = -\frac{\sin \theta_{R(L)}}{2} (f_{\uparrow\downarrow} + f_{\downarrow\uparrow}), \quad (6)$$

where the labels r and a have been omitted from here and the same magnitudes of ϵ_{xc} are assumed in both leads. In the new basis, the distribution function (2) is

$$h_{L(R)} = h_{\uparrow} \frac{(1 + \sigma_z \cos \theta_{L(R)})}{2} + h_{\downarrow} \frac{(1 - \sigma_z \cos \theta_{L(R)})}{2} + \sigma_y \sin(\theta_{L(R)}) \frac{h_{\uparrow} - h_{\downarrow}}{2}. \quad (7)$$

III. THE JOSEPHSON AND DISSIPATIVE CURRENTS

What we have established is that the superconducting leads acquire triplet pairing correlations determined by nonequilibrium spin polarizations whose directions are tilted with respect to the ferromagnet's magnetization in the SFS junction. We will show that the current through such a triplet pairing-ferromagnet-triplet pairing system consists of two parts: a dissipative contribution controlled by the nonequilibrium distribution of spins in the device, and a supercurrent driven by the phase difference between superconductors and provided by the triplet components of the superconducting condensates in the left and right leads.

Let us first consider the dissipative current. It can be expressed in terms of the distribution function h_f inside the ferromagnet. Due to precession in the exchange field B_{ex} , the spins that are not parallel to it decay quickly on the length scale $\sqrt{D_f/B_{\text{ex}}}$, where D_f is the diffusion constant.

Therefore, only the components of h_f that are parallel and antiparallel to z , denoted as $h_{f\uparrow}$ and $h_{f\downarrow}$, remain finite inside the ferromagnet, if the junction length $L \gg \sqrt{D_f/B_{\text{ex}}}$. When the spin-relaxation length is larger than L , in the linear approximation these collinear components satisfy the spin-conserving diffusion equation $D_{f\sigma} \nabla_x^2 h_{f\sigma} = 0$, where $\sigma = \uparrow, \downarrow$, that takes into account spin-dependent diffusion coefficient in a strong ferromagnet. The solution of this equation is a linear function of x whose slope is obtained from the boundary conditions $\mp r_{sf\sigma} \sigma_{f\sigma} \nabla_x h_{f\sigma}|_{x=x_{L(R)}} = h_{L(R)\sigma} - h_{f\sigma}|_{x=x_{L(R)}}$, where $h_{L(R)\sigma}$ are given by the first two terms of Eq. (7). Taking into account that $r_{sf\sigma}$ and $\sigma_{f\sigma}$ can depend on the electron spin and assuming equal barrier transmittances at L and R contacts, we obtain

$$h_{f\sigma} = \frac{h_{R\sigma} + h_{L\sigma}}{2} + \frac{h_{R\sigma} - h_{L\sigma}}{1 + 2\gamma_{\sigma}} \frac{x}{L}, \quad (8)$$

where $x_{R(L)} = \pm L/2$ and $\gamma_{\sigma} = (r_{sf\sigma} \sigma_{f\sigma}/L) \gg 1$. Using Eqs. (8) and (7) we compute the dissipative part of the current through the junction:

$$j_d = \sum_{\sigma} \int d\omega \sigma_{f\sigma} \nabla_x h_{f\sigma} = \frac{\delta\mu}{eL} \left(\frac{\sigma_{f\uparrow}}{1 + 2\gamma_{\uparrow}} - \frac{\sigma_{f\downarrow}}{1 + 2\gamma_{\downarrow}} \right) (\cos \theta_R - \cos \theta_L). \quad (9)$$

This current is proportional to the difference in the spin-up and spin-down conductances $(2r_{sf\sigma} + L/\sigma_{f\sigma})^{-1}$ of the total ferromagnetic layer, including the interfaces; this is the well-known¹⁹ connection between spin and electric transport in ferromagnets. The electric current attains its maximum when $\cos \theta_R = -\cos \theta_L = \pm 1$ and vanishes at $\theta_R = \theta_L$, as well as at $\theta_R, \theta_L = \pm \pi/2$. Such an angular dependence has a simple physical explanation. The electric current (9) is proportional to the spin current through the junction. The latter attains its maximum when the nonequilibrium spin polarizations in the ferromagnetic layers are oppositely directed, and it vanishes if these polarizations are collinear and have equal magnitudes. The spin current obviously also vanishes if these polarizations are perpendicular to the ferromagnetic magnetization axis, since perpendicular components do not penetrate deep into ferromagnet. The spin flow through the junction is accompanied by energy dissipation. It is determined by the Ohmic losses in the ferromagnet during transport of spin-polarized electrons in the leads having spin-dependent electrochemical potentials. The dissipative current of Eq. (9) is independent of the superconducting phases ϕ_L and ϕ_R . We assume that the electric potentials of both contacts are equal. If the load is present in the circuit, the spin current will induce a voltage difference. The latter, in its turn, can cause periodic oscillations of the Josephson current.⁶

When $\sqrt{D_f/B_{\text{ex}}}$ is much shorter than the junction length L and the coherence length, the up- and down-spin Fermi surfaces become decoupled. In this regime, the supercurrent j_s through the junction is determined by the decoupled tunneling of ± 1 triplet Cooper pairs at their respective ferromagnet's Fermi surfaces. Unlike the dissipative current, the spin dependence of the electron diffusion coefficients and conductivities is not so important, at least in the case when $B_{\text{ex}} \ll E_F$. Therefore, in the leading approximation we set $D_{f\uparrow} = D_{f\downarrow} = D_f$, and a similar relation for the

conductivities. Furthermore, in the linear approximation, only the first term of Eq. (8) has to be taken into account. Moreover, since the Josephson current is determined by part of the distribution function that is odd in frequency, from Eqs. (8), (7), and (2) only the spin-independent part $h_{f\uparrow} + h_{f\downarrow}$ contributes to the current. It is given by

$$j_s = \frac{\sigma_f}{16e} \sum_{m=\pm 1} \int d\omega [(f_m^r \nabla_x f_m^{r\dagger} - \nabla_x f_m^r f_m^{r\dagger}) - (f^r \rightarrow f^a)] \frac{(h_{f\uparrow} + h_{f\downarrow})}{2}, \quad (10)$$

where f_m^r ($m = \pm 1$) are the retarded triplet components of the anomalous function in ferromagnet and $f_m^a(\omega) = f_m^r(-\omega)$, while $f_m^{r\dagger}(\omega) = f_m^{r*}(-\omega)$. When the spin-relaxation length is much larger than L , and within the linearized approximation, $f_{\pm 1}^r$ obey¹

$$D_f \nabla_x^2 f_m + 2i\omega f_m = 0, \quad (11)$$

with the boundary conditions¹⁸ $r_{sf} \sigma_f \nabla_x f_m|_{x=x_{R(L)}} = \pm f_{mR(L)}$, where $f_{mR(L)}$ are given by Eqs. (5) and (6). After transforming the integral in Eq. (10) into a sum over the frequencies $\omega_n = \pi k_B T (2n + 1)$, j_s can be finally represented in the form

$$j_s = \sin(\phi_L - \phi_R) \frac{L \sin \theta_R \sin \theta_L}{e \sigma_f R_{sf}^2} K, \quad (12)$$

where

$$K = |\Delta|^2 k_B T \sum_{\omega_n > 0, v=\pm 1} \frac{1}{k_v L \sinh(k_v L)} \times \left[\frac{1}{\sqrt{(\omega_n + iH_v)^2 + |\Delta|^2}} - \frac{1}{\sqrt{(\omega_n - iH_{-v})^2 + |\Delta|^2}} \right]^2 \quad (13)$$

and $k_v = \sqrt{2(\omega_n + i\nu\delta\mu)/D_f}$, with $H_v = \epsilon_{xc} + \nu\delta\mu$ and $2\delta\mu = \mu_{\uparrow} - \mu_{\downarrow}$.

IV. DISCUSSION

As follows from Eq. (12), the Josephson current depends on the directions of the nonequilibrium spin polarizations in the superconducting leads. The current reaches its maximum when the spin accumulations in the leads are perpendicular to the magnetization in the ferromagnet, $\theta_R = \theta_L = \pi/2$. It reverses its sign when the spin polarization in one of the leads flips its direction. Therefore, in the setup shown in Fig. 1, the junction can be switched into the π state by simply reversing the electric current through one of the FN contacts. It should be noted that the dissipative current given by Eq. (9) vanishes when the relative angles are such that the supercurrent reaches its maximum. Hence, the dissipative transport can be turned off, a feature that can be important for practical purposes. Equations (12) and (13) also imply that the long-range proximity effect, described via j_s , vanishes when the exchange interaction $\epsilon_{xc} = 0$. The dependence of j_s on the spin-potential $\delta\mu$ is shown in Fig. 2. A finite spin potential causes variations of the order parameter Δ and spin density entering in Eq. (12), which have been found from a pair of self-consistent equations. In our

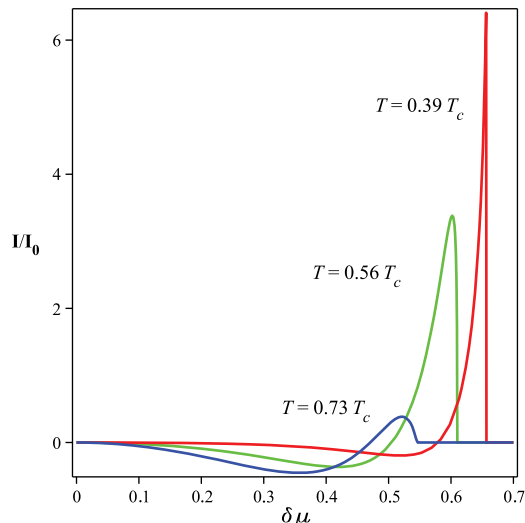


FIG. 2. (Color online) The Josephson current as a function of the spin potential $\delta\mu$ measured in units of the unperturbed superconducting gap Δ_0 , at $L = \zeta$, where $\zeta = \sqrt{D_f/2|\Delta_0|}$ and $I_0 = (2\zeta\pi k_B T/e r_{sf}^2 \sigma_f) \times 10^{-2}$.

calculation of Δ and S , we neglected the exchange field ϵ_{xc} , assuming that $\epsilon_{xc} \ll \delta\mu$. This is a realistic assumption, taking into account that $S < \delta\mu N_F$ in Eq. (3) and $|G|$ is considerably less than 1 in some superconducting metals (e.g., Al). In this limit, the dependence of Δ on $\delta\mu$ is formally the same as in a thermally equilibrium superconductor subject to a Zeeman splitting equal to $\delta\mu$. Such a scenario is well studied in the literature (see, e.g., Ref. 20). In Fig. 2, we see that the critical current changes sign at some values of $\delta\mu$. This is caused by injection of nonequilibrium spins into the ferromagnetic layer. As a result, the distribution function in Eq. (10) is different from the equilibrium distribution. There is some similarity of this effect with a current reversal observed in Josephson transistors.²¹ At the lower temperature, the supercurrent vs spin potential is more peaked in the range of higher $\delta\mu$, since the spin density increases sharply together with ϵ_{xc} in this range. At even higher $\delta\mu$, the superconductivity is destroyed by spin injection. That causes a sudden drop of the current. We believe that this narrow range can be easily observed experimentally in the set up shown in Fig. 1, because $\delta\mu$ can be fine tuned by varying the current through the normal leads.

Figure 2 is calculated at $L = \sqrt{D_f/2|\Delta_0|}$, which is the characteristic length of the ± 1 triplet proximity effect in the range of temperatures considered. This length is obviously much larger than the s -wave Cooper pair penetration depth $\sqrt{D_f/B_{ex}}$ and therefore clearly demonstrates how the range of the proximity effect becomes much longer by spin injection into the superconducting leads.

In conclusion, spin injection into s -wave superconductors can dramatically increase the stationary Josephson current in a SFS system. This enhancement is provided by ± 1 triplet components of the electron pairing function. They are generated in superconducting leads by exchange fields that are noncollinear with the ferromagnet magnetization. These fields, in turn, are induced by an injected spin polarization. Besides a strong effect on the Josephson current, spin injection also gives rise to a dissipative current that at zero bias potential is induced

due to spin dependence of the ferromagnet conductivity. Both Josephson and dissipative currents can be manipulated by varying the injected spin directions in the leads enabling control of π junctions.

ACKNOWLEDGMENT

Anatoly Mal'shukov gratefully acknowledges the hospitality of NTNU.

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