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Fermi edge singularity and finite-frequency spectral features in a semi-infinite one-dimensional wire

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We theoretically study a charge qubit interacting with electrons in a semi-infinite one-dimensional wire. The system displays the physics of the Fermi edge singularity. Our results generalize known results for the Fermi edge system to the regime where excitations induced by the qubit can resolve the spatial structure of the scattering region. We find resonant features in the qubit tunneling rate as a function of the qubit level splitting. They occur at integer multiples of hv_F/l. Here v_F is the Fermi velocity of the electrons in the wire, and l is the distance from the tip of the wire to the point where it interacts with the qubit. These features are due to the constructive interference of the amplitudes for creating single coherent left- or right-moving charge fluctuation (plasmon) in the electron gas. As the coupling between the qubit and the wire is increased, the resonances are washed out. This is a clear signature of the increasingly violent Fermi sea shake-up, associated with the creation of many plasmons whose individual energies are too low to meet the resonance condition.

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I. INTRODUCTION

Systems in which a localized impurity, with an internal quantum mechanical degree of freedom, interacts with an electron gas, play an important role in many-body theory. On the one hand, they allow theorists to investigate interaction effects and many-body correlations beyond the perturbative regime. On the other hand, they explain observed phenomena such as the resistance minimum (as a function of temperature) in dilute magnetic alloys, i.e., the Kondo effect.[1] Another impurity phenomenon that has been studied extensively is the so-called Fermi edge singularity.[2] In its original incarnation,[3,4] the effect refers to power-law singularities in the soft-x-ray absorption, emission, and photoemission spectra of metallic samples. As with the Kondo effect,[5] the phenomenon has received renewed attention due to technological breakthroughs in nanophysics and quantum transport.[6] In this context, an oft-considered system consists of an electron that tunnels inelastically from a localized state into a degenerate Fermi gas, losing energy ε in the process. The electrons in the Fermi gas absorb this energy by scattering off the localized hole left behind by the tunneling electron. The interaction leads to a tunneling rate with a power-law singularity at small ε. This is in contrast to a system without interactions, in which only elastic tunneling would have been possible. A detailed introductory discussion of the Fermi edge physics in this basic system can be found in Chap. 6 of Ref. 7.

Recent studies of the Fermi edge singularity have considered nonequilibrium,[8–15] nonstationary,[16] and band structure[17] phenomena. Most studies have focused on the behavior of transition rates in the vicinity of the singularity. As we show in this paper, the transition rate may also have distinct many-body signatures away from the singularity. We confine our attention to the equilibrium situation. (Equilibrium here refers to the initial state of the Fermi gas. The Fermi edge singularity always involves the relaxation of the localized impurity. This means that initially the impurity is in an excited state.)

The system we study consists of an electron gas interacting with a two-level system (charge qubit). A concrete realization of the qubit could be an electron tunneling between the lowest two states of a double quantum dot.[18,19] The qubit state space is spanned by the vectors $|+\rangle$ and $|-\rangle$. The total Hamiltonian for the system is $H = H_0 + H_T$, where

$$\begin{aligned} H_0 &= (H_+ + \varepsilon)|+\rangle\langle+| + H_-|-\rangle\langle-|, \\ H_T &= \gamma|+\rangle\langle-| + \gamma^*|-\rangle\langle+|. \end{aligned} \tag{1.1}$$

The energy ε represents a gate voltage that controls the qubit level splitting and γ is a small tunneling amplitude between the two qubit states. Both these parameters are typically under experimental control. The Hamiltonians $H_\pm = T + V_\pm$ describe the electron gas. The kinetic term T is the same for both Hamiltonians. The finite-range potentials $V_\pm$ represent the electrostatic potential produced by the qubit. This potential depends on the internal state of the qubit, so that $V_+ \neq V_-$. We denote the ground-state energies associated with the Fermi sea ground states $|F_\pm\rangle$ of $H_\pm$ as $E_0^{(\pm)}$. Without loss of generality we assume that $E_0^{(+)} + \varepsilon > E_0^{(-)}$, so that the ground state of H_0 has the qubit in the state $|-\rangle$.

From the point of view of the electron gas, the qubit acts as a dynamic localized impurity, while from the point of view of the qubit, the electron gas acts as a dissipative environment. The system can be mapped onto a spin-boson model,[20] i.e., a qubit coupled to a bosonic bath. (We explicitly perform this mapping in Sec. III.) The bosonic quanta associated with excitations in the electron gas are charge density fluctuations, also known as plasmons. The bath spectral function turns out to be Ohmic at small energies. The dynamics of the spin-boson model is a well-studied problem. It is known (see Sec. VII of Ref. 20) that for sufficiently large qubit level splittings, and with the bath at zero-temperature equilibrium, the qubit, when initialized in the excited state, undergoes exponential decay to the ground state at a rate that is accurately given by Fermi's golden rule. Through the mapping back to our model, this implies the following. If, at $t = 0$, our system is initialized in state $|F+\rangle \otimes |+\rangle$, then at later times t, the probability $n_+(t)$ of finding the qubit in state $|+\rangle$ is given by

$$n_+(t) = e^{-Wt}, \tag{1.2}$$

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provided that $\omega \gg W$, where W is the qubit tunneling rate calculated from Fermi's golden rule, and

$$\omega = \varepsilon + E_0^{(+)} - E_0^{(-)}. \quad (1.3)$$

This condition can be understood intuitively as follows. Equation (1.2) can only hold if the qubit levels $\pm$ remain sufficiently well-defined despite being broadened by the coupling of the qubit to the environment. Thus the qubit level broadening must be far smaller than the energy difference between the $+$ and the $-$ state. The qubit level broadening equals the tunneling rate W, while the effective qubit level splitting is ω.

The quantity we study in this work is the qubit transition rate W. Based on the above argument, our starting point is Fermi's golden rule, which gives

$$W = |\gamma|^2 \int_{-\infty}^{\infty} dt\, e^{i\varepsilon t} \langle F+|e^{iH_+ t} e^{-iH_- t}|F+\rangle. \quad (1.4)$$

(See Appendix A for details.) We obtain an explicit expression for W in terms of the energy ε and the potentials $V_\pm$.

How is this quantity connected to Fermi edge physics? First, we note that the tunneling rate W of Eq. (1.4) is equal to the photoemmision spectrum, i.e., the intensity of electrons ejected from the metal, at a fixed x-ray frequency ε, in the original incarnation of the problem. (See, for instance, Sec. IV in Ref. 2.) Furthermore, in the language of the Fermi edge singularity, the quantity $\langle F+|e^{iH_+ t} e^{-iH_- t}|F+\rangle$, which appears in Eq. (1.4) is known as the closed loop factor. (See, for instance, Sec. III D in Ref. 2.) For the fermion Hamiltonians $H_\pm$ given in Sec. II, we calculate the closed loop factor and hence the rate W of Eq. (1.4) exactly.

Before proceeding to the calculation, we briefly review relevant results from the literature. We stress that the derivations we present from Sec. II onward do not invoke or rely on the results from the literature quoted here. It is known that, for t much larger than the time an individual electron spends in the scattering region, the closed loop factor is given by

$$\langle F+|e^{iH_+ t} e^{-iH_- t}|F+\rangle \simeq e^{-i\Delta E t}(i\Lambda t)^{-\alpha}, \quad (1.5)$$

where $\Delta E = E_0^{(-)} - E_0^{(+)}$ is the difference between the ground-state energies of H_- and H_+ and Λ is an ultraviolet energy scale. [The branch with $\arg(i\Lambda t) = \pm\pi/2$ is implied.] The power-law exponent α is determined by the single-particle scattering matrices $S_\pm$ associated with the fermion Hamiltonians. Explicitly,[6,9]

$$\alpha = \mathrm{Tr}\left[\left(\frac{\ln S_+ S_-^\dagger}{2\pi}\right)^2\right]. \quad (1.6)$$

For sufficiently small ω, the asymptotic form of Eq. (1.5) gives rise to a tunneling rate

$$W = \frac{2\pi|\gamma|^2}{\Gamma(\alpha)}\left(\frac{\omega}{\Lambda}\right)^{\alpha}\frac{1}{\omega}\theta(\omega), \quad (1.7)$$

with $\theta(\omega)$ the unit step function, i.e., $\theta(\omega) = 1$ for $\omega > 0$ and $\theta(\omega) = 0$ for $\omega < 0$. This power law remains valid while $\omega \ll \min\{v_F/l, E_F, D - E_F\}$. Here v_F is the Fermi velocity, E_F is the Fermi energy measured from the bottom of the conduction band, and D is the band width. The length scale l is the size of the scattering region. This is not necessarily the same length scale as the range of the qubit interaction potentials $V_\pm$, which we denote a. Consider, for instance, the system we study below, namely, a qubit placed next to a semi-infinite wire. (See Fig. 1.) Here the size of the scattering region is the distance from the tip of the wire to the point closest to the qubit, which can be much larger than the range of the potential produced by the charge of the qubit. In general, $l \geqslant a$.

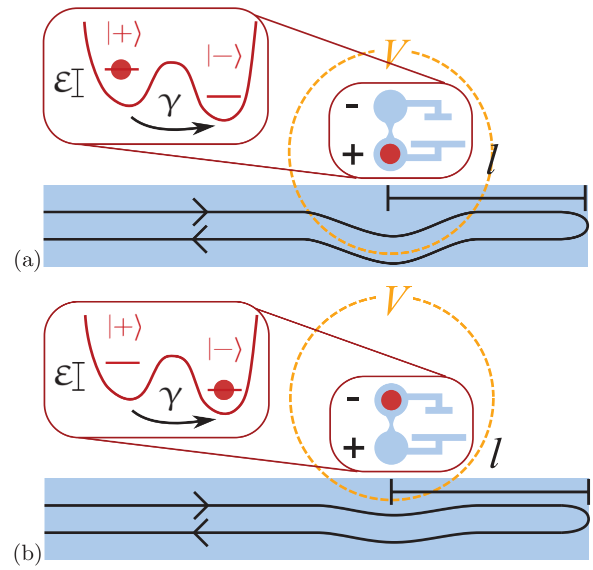

FIG. 1. (Color online) The system described by the potential $v(x)$ of Eq. (2.7). (a) The system with the qubit in the initial state $|+\rangle$ and (b) the system after the transition. The shaded rectangle represents the semi-infinite wire. The U-shaped contour indicates the single chiral channel in which electrons propagate. The diagram is only schematic. In an actual realization, the left and right propagating electrons need not be spatially separated, i.e., the U shape of the contour may be squashed into a line. The distance l between the tip of the wire and the point on the wire closest to the qubit is indicated. The qubit is represented as a double quantum dot with a single electron in it. The dashed circle indicates the range of the potential through which the qubit and the wire interact. In the exploded view, the qubit level spacing ε and the tunneling amplitude γ are indicated. While the range of the potential and the distance l are of similar size in the figure, we investigate the regime where l is much larger than the range of the potential in the text.

Since we will be investigating the rate $W(\omega)$ for ω outside the power-law regime, it is useful to understand how the conditions that determine its validity come about. The condition $\omega \ll v_F/l$ results as follows. An energy ω corresponds to density fluctuations (plasmons) in the electron gas with wavelengths of at least v_F/ω. The result of Eq. (1.7) breaks down as soon as this wavelength is short enough for these excitations to resolve the spatial structure of the scattering region. The restriction $\omega \ll \min\{E_F, D - E_F\}$ is due to the fact that Eq. (1.7) becomes invalid when particle or hole excitations are created in the electron gas close to the band edges.

An analytical expression for Λ was obtained by Tanabe and Othaka[21] in the regime $E_F \ll v_F/l$, where the wavelength of electrons near the Fermi level are too long to resolve the spatial structure of the scattering region, so that the potentials $V_\pm$ may be approximated as δ functions. (This is referred to as the limit of contact potentials.) Λ was found to be of the order of E_F.

In the same limit, approximate results for the finite ω behavior of W have been obtained. As a function of ω, these results contain features on the scale of E_F that are associated with the band structure of the model. For more detail the reader is referred to the review in Ref. 2.

We are now in a position to explain what distinguishes our work from existing studies. The previously studied regime of $E_F \ll v_F/l$ applies in a semiconductor, where it is not uncommon for the Fermi wavelength to be large compared to other length scales in the problem. However, the opposite regime, where $E_F \gg v_F/l$, which has not yet been studied in detail, also has physical relevance: In a metallic sample, the Fermi wavelength is comparable to the lattice constant, while the length scale associated with the potential may be much larger.

In this article we study the regime where $E_F \gg v_F/a \geqslant v_F/l$. There are two significant differences between this regime and the previously studied regime. First, the ultraviolet energy Λ is no longer of order E_F but, rather, is determined by the potential v_F. Second, the rate W as a function of ω starts deviating from the power law of Eq. (1.7), at energies $\sim v_F/l$ rather than at energies $\sim E_F$. The source of the deviations is no longer related to the band structure, but to excitations resolving the spatial structure of the scattering region. We confine our attention to the case of an electron gas in a single chiral channel at zero temperature. We pay particular attention to the example mentioned above, of a qubit interacting with a semi-infinite wire, where $l \gg a$. We obtain exact analytical expressions for Λ and for the closed loop factor at arbitrary times. We find that $\Lambda < v_F/a$ and that Λ depends only on the shape, and not the magnitude, of $V_\pm$; i.e., scaling $V_\pm \to c\,V_\pm$ leaves Λ unchanged. We compute the tunneling rate $W(\omega)$ away from the threshold $\omega \to 0^+$. We find that $W(\omega)$ has resonant features at an energy scale v_F/l that reveal the nature of many-body correlations induced by the qubit. Our main results are contained in Eqs. (4.20), (4.25), (5.3), and (5.10).

The method by which we obtain these results can be summarized as follows. Our starting point is Eq. (1.4) for the tunneling rate W, which follows directly from applying Fermi's golden rule to the Hamiltonian of Eq. (1.1), without any further simplifying assumptions. In order to proceed from there, we take the limit $E_F \to \infty$ and linearize the electrons' dispersion relation around the Fermi level, while still taking into account the full spatial dependence of the potentials $V_\pm$. As pointed out by Gutman *et al.* (footnote 36 in Ref. 13), special care must be taken when linearizing the dispersion relation in order to account for the anomalous contribution to $\langle F+|e^{iH_+t}e^{-iH_-t}|F+\rangle$ that is related to the Schwinger anomaly.[22] In the derivation that we present, this anomalous contribution appears quite naturally.[23] Our results are obtained by means of bosonization,[24,25] the application of which to the Fermi edge singularity was pioneered by Schotte and Schotte.[26] This method does not require any further approximations. It provides an exact mapping from our model onto an equivalent spin-boson model,[20] where a spin is coupled to a bosonic bath. In general the bath spectrum is Ohmic for sufficiently low energies. For the example of a semi-infinite wire interacting with a qubit at a point on the wire that is a distance $l \gg a$ from the tip of the wire, the bosonic bath spectrum has a nontrivial structure at higher energies. This in turn is what leads to the nontrivial finite ω behavior of the tunneling rate $W(\omega)$.

The rest of this article is structured as follows. In Sec. II we specialize to a Fermi gas consisting of a single chiral channel and introduce a model to describe a semi-infinite wire interacting with a qubit at a point a distance l from the tip of the wire. In Sec. III we use bosonization to map the system onto a spin-boson model. We also discuss Anderson's orthogonality catastrophe from the point of view provided by bosonization. In Sec. IV we derive a general and exact formula for the closed loop factor, using the mapped system. This allows us to calculate the ultraviolet energy scale Λ exactly. In Sec. V we apply the general results of Sec. IV to the specific system introduced in Sec. II, for which the tunneling rate $W(\omega)$ has nontrivial features at finite ω.

II. A SINGLE CHIRAL CHANNEL

Here we give a mathematical definition of the type of electron gas we study. Associated with the electrons in a chiral channel of length L with periodic boundary conditions are creation and annihilation operators $\psi^\dagger(x)$ and $\psi(x)$, which, respectively, create or annihilate a fermion in state $|x\rangle$ localized at position x. They obey the usual anticommutation relations

$$\{\psi(x),\psi^\dagger(x')\} = \sum_{n=-\infty}^{\infty} \delta(x - x' - nL) \tag{2.1}$$

and are periodic with period L. At the point in our derivation where it becomes convenient to do so, we send the system size L to infinity.

The noninteracting many-fermion Hamiltonians $H_\pm$ have the same linear dispersion but different external potentials. Without loss of generality (see Appendix B for details), we can set the external potential in H_+ to 0, so that

$$H_+ = \int_{-L/2}^{L/2} dx\, \psi^\dagger(x)(-i\partial_x - \mu)\psi(x), \tag{2.2}$$

while $H_- = H_+ + V$ with

$$V = \int_{-L/2}^{L/2} dx\, v(x)\rho(x), \quad \rho(x) = \psi^\dagger(x)\psi(x). \tag{2.3}$$

(Here we work in units where the Fermi velocity $v_F = 1$.) Associated with H_- is the one-dimensional (1D) scattering matrix e^{-iv_0}, where

$$v_0 = \int_{-L/2}^{L/2} dx\, v(x). \tag{2.4}$$

The ground state of H_+ is the Fermi sea

$$|F+\rangle = \prod_{k\leqslant\mu} c_k^\dagger|0\rangle, \tag{2.5}$$

where $|0\rangle$ is the state with no particles, the operator

$$c_k^\dagger = \frac{1}{\sqrt{L}}\int_{-L/2}^{L/2} dx\, e^{ikx}\psi^\dagger(x), \tag{2.6}$$

creates a fermion in a momentum eigenstate, and k is quantized in integer multiples of $2\pi/L$.

The general results we obtain are applied to the case where the electron gas resides in a semi-infinite 1D quantum wire.

In the limit of a high Fermi energy, this system is mapped onto Eqs. (1.1), (2.2), and (2.3) through the standard trick of "unfolding," so that coordinates x and $-x$ refer to the same spatial point, but to "different sides of the road," i.e., $\psi^\dagger(-x)$ and $\psi^\dagger(x)$ create electrons at the same position but moving in opposite directions.[27] Thus the potential $v(x)$ is symmetric about $x=0$. The system is depicted in Fig. 1. As a simple model for the interaction between the qubit and the electron gas, we take $v(x)=u(x-l)+u(x+l)$ with

$$u(x)=\left(\frac{av_0}{2\pi}\right)\frac{1}{x^2+a^2}. \tag{2.7}$$

Here l is the distance from the tip of the wire to the point on the wire nearest to the qubit and a is the range of the potential produced by the qubit. This choice of $u(x)$ allows us to obtain an exact analytical expression for $W(\omega)$. In the regime where $l \gg a$ we expect qualitatively similar results for any choice of $u(x)$ that is localized to a region of length ∼a.

We note here in passing that in any physical realization of our setup, the qubit is only approximately a two-level system. Its spectrum contains additional levels above the $|\pm\rangle$ states. Let the first such level occur at an energy ω_0 above the state $|+\rangle$. Provided that $\omega \ll \omega_0$, these levels can always be integrated out to obtain an effective theory with involving a two-state system. (See, for instance, Sec. II in Ref. 20.) The effect of integrating out the excited levels is to renormalize the tunneling amplitude γ from its "bare" value. The renormalization becomes significant in the regime where $\omega_0 < 1/a$. Here γ will have a strong a dependence. Thus when $\omega_0 < 1/a$, the γ that appears in our results should be interpreted as this renormalized, a-dependent tunneling amplitude.

III. BOSONIZATION

In this section we map the fermion Hamiltonians $H_\pm$ onto equivalent free boson Hamiltonians, using the technique of bosonization. Our notation closely follows Haldane's.[24] This casts the Hamiltonian H into the form of a spin-boson model with a structured environment. We also calculate the overlap $\langle F+|F-\rangle$, where, as stated after Eq. (1.3), $|F\pm\rangle$ are the many-body ground states of $H_\pm$, which will be relevant when we analyze the tunneling rate W in Sec. IV. The bosonization results that we require are stated without proof in Eqs. (3.2) and (3.4). For a derivation, we refer the reader to Refs. 24 or 25.

The free fermion Hamiltonian (2.2) together with the Fermi sea ground state $|F+\rangle$ is the starting point for the bosonization procedure. Associated with density fluctuations in the fermion system are operators

$$a_q=\left(\frac{2\pi}{Lq}\right)^{1/2}\sum_k c_k^\dagger c_{k+q}, \tag{3.1}$$

and $a_q^\dagger$, $q=2\pi n/L$, $n=1,2,3,\ldots,$ which satisfy the bosonic commutation relations

$$[a_q,a_{q'}]=0, \quad [a_q,a_{q'}^\dagger]=\delta_{q,q'}. \tag{3.2}$$

The bosonic annihilation operators a_q annihilate the Fermi sea $|F_+\rangle$, i.e.,

$$a_q|F+\rangle=0. \tag{3.3}$$

The quanta created by $a_q^\dagger$ and annihilated by a_q are called plasmons. A central (and nontrivial) result of bosonization is that, in terms of the bosonic operators, and for a fixed particle number,

$$H_+=\sum_{q>0} q\, a_q^\dagger a_q + E_+^{(0)}. \tag{3.4}$$

It follows directly fromm Eq. (3.1) that the fermion density $\rho(x)$ can be expressed in terms of the bosonic operators as

$$\rho(x)=N/L+\frac{1}{2\pi}\partial_x[\varphi(x)+\varphi^\dagger(x)], \tag{3.5}$$

where $N=\int_{-L/2}^{L/2}dx\,\rho(x)$ counts the total number of fermions and

$$\varphi(x)=-i\sum_{q>0}\left(\frac{2\pi}{Lq}\right)^{1/2}e^{iqx}a_q. \tag{3.6}$$

From Eq. (3.2) it follows that the φ operators satisfy the commutation relations

$$[\varphi(x),\varphi(x')]=0, \tag{3.7}$$

$$[\varphi(x),\varphi^\dagger(x')]=-\lim_{\eta\to0^+}\ln\left[1-e^{\frac{i2\pi}{L}(x-x')-\eta}\right]. \tag{3.8}$$

Using Eqs. (3.5) and (3.6) to express the potential V in terms of the bosonic operators, and using expression (3.4) for H_+, we find for H_-

$$H_-=E_0^{(+)}+\frac{N}{L}v_0 + \sum_{q>0} q\left\{a_q^\dagger a_q+\left(\frac{2\pi}{Lq}\right)^{1/2}\left[\frac{v_q^*}{2\pi}a_q+\frac{v_q}{2\pi}a_q^\dagger\right]\right\}, \tag{3.9}$$

where

$$v_q=\int_{-L/2}^{L/2}dx\,v(x)e^{-iqx}. \tag{3.10}$$

The Hamiltonian H_- is diagonalized by completing the square. For this purpose we define new bosonic operators,

$$b_q=a_q+\left(\frac{2\pi}{Lq}\right)^{1/2}\frac{v_q}{2\pi}, \tag{3.11}$$

which also obey the standard bosonic commutation relations. In terms of these operators the Hamiltonian H_- reads

$$H_-=E_0^{(+)}+\Delta E+\sum_{q>0} q\, b_q^\dagger b_q, \tag{3.12}$$

where

$$\Delta E=\frac{Nv_0}{L}-\sum_{q>0}\frac{2\pi}{L}\left|\frac{v_q}{2\pi}\right|^2. \tag{3.13}$$

Substitution of $H_\pm$ from Eqs. (3.4) and (3.12) into the full Hamiltonian H of Eq. (1.1) reveals that the system is described by the same Hamiltonian as the spin-boson model. (See, for instance, Ref. 20.) A quantity that plays a central role in the spin-boson model is the bosonic environment's spectral function, which, in our notation, is given by

$$J(q) = \pi q \left|\frac{v_q}{2\pi}\right|^2. \tag{3.14}$$

(Here we have implicitly taken the $L \to \infty$ limit.) In the context of dissipative quantum mechanics, spectral functions $J(q) \sim q^s$ for q smaller than some large cutoff play an important role. The case with $s = 1$ is known as an Ohmic environment. We see that a potential $v(x)$ that is peaked around $x = 0$, for instance, $v(x) = \lambda/[\pi(x^2 + \lambda^2)]$, produces an Ohmic environment. When the spectral function has a more complicated form, one talks of a structured bath. A structured bath is obtained by engineering the potential $v(x)$. As we show in Sec. V, the potential $v(x)$ of Eq. (2.7) produces an environment with an interesting structure.

The ground-state energy of H_- is $E_0^{(+)} + \Delta E$ and the ground state solves $b_q|F-\rangle = 0$, or using the definition of b_q in terms of a_q,

$$a_q|F-\rangle = -\left(\frac{2\pi}{Lq}\right)^{1/2}\frac{v_q}{2\pi}|F-\rangle. \tag{3.15}$$

From this it follows that the normalized ground state of H_- is the coherent state

$$|F-\rangle = \exp\sum_{q>0}\left(\frac{2\pi}{Lq}\right)^{1/2}\left(\frac{v_q*}{2\pi}a_q - \frac{v_q}{2\pi}a_q^\dagger\right)|F_+\rangle$$
$$= e^{-i\int_{-L/2}^{L/2}dx\, v(x)[\varphi(x)+\varphi^\dagger(x)]/2\pi}|F+\rangle. \tag{3.16}$$

For future reference we note that the overlap $\langle F+|F-\rangle$ is easily calculated from Eq. (3.16). The details of the calculation can be found in Appendix C. The result is

$$\langle F+|F-\rangle = \left(\frac{2\pi}{\Lambda L}\right)^{\alpha/2}, \tag{3.17}$$

where Λ is the energy appearing in Eq. (1.5) and, consistent with Eq. (1.6) [cf. Eq. (2.4)],

$$\alpha = (v_0/2\pi)^2. \tag{3.18}$$

For an explicit formula for Λ, see Eq. (4.25). The fact that the overlap tends to 0 as $L^{-\alpha/2}$ is known as the orthogonality catastrophe.[28] The fact that the same ultraviolet energy Λ appears in the closed loop factor and in the orthogonality catastrophe has previously been established (for a contact-type potential) by Feldkamp and Davis[29] and by Tanaka and Othabe.[21]

IV. CLOSED LOOP FACTOR

In this section our goal is to calculate the closed loop factor

$$P(t) = e^{i\Delta E t}\langle F+|Q(t)|F+\rangle, \tag{4.1}$$

where

$$Q(t) = e^{iH_+t}e^{-iH_-t}. \tag{4.2}$$

[The convenience of including the factor $\exp(i\Delta E t)$, with ΔE given by Eq. (3.13), will become apparent below.]

Having mapped the system under consideration onto a spin-boson Hamiltonian, we could simply quote the answer from the literature, namely,

$$P(t) = \exp\left[\frac{1}{\pi}\int_0^\infty dq\,\frac{J(q)}{q^2}(e^{-iqt} - 1)\right], \tag{4.3}$$

with $J(q)$ given by Eq. (3.14). [See, for instance, Eqs. (3.35) and (3.36) in Ref. 20, but note that $P(t)$ in that work refers to a different quantity than the one in Eq. (4.1) in the present work.] However, we prefer to give a self-contained derivation of this result. This derivation goes slightly farther than simply calculating $P(t)$, namely, it produces a bosonic expression for the operator $Q(t)$ that is normal ordered, i.e., in which all creation operators are to the left of all annihilation operators. This expression may prove useful in future for studying nonequilibrium effects. Readers prepared to take Eq. (4.3) as given may wish to skip to the paragraph following Eq. (4.21).

The starting point of our derivation is to consider the time derivative of Q, i.e.,

$$\partial_t Q(t) = -ie^{iH_+t}Ve^{-iH_+t}Q(t). \tag{4.4}$$

Since the Hamiltonian H_+ is also the momentum operator, $\exp(\pm iH_+t)$ is simply translation by a distance $\pm t$, so that

$$e^{iH_+t}Ve^{-iH_+t} = \int_{-L/2}^{L/2}dx\, v(x)\rho(x-t) \equiv V(t). \tag{4.5}$$

Thus we find

$$Q(t) = \mathcal{O}\exp\left[-i\int_0^t dt'V(t')\right] \tag{4.6}$$
$$= \lim_{n\to\infty} e^{-i\int_{t_{n-1}}^{t_n}dt'\,V(t')} \times \ldots \times e^{-i\int_{t_0}^{t_1}dt'\,V(t')}, \tag{4.7}$$

where $t_m = mt/n$, $m = 0, \ldots, n$.

Let us now consider one of the individual factors in the ordered exponent of Eq. (4.7). Using Eq. (3.5) to relate the density operator to the bosonic operators φ and $\varphi^\dagger$, we find

$$\int_\tau^{\tau+\Delta t}dt'\,V(t') = \int_\tau^{\tau+\Delta t}dt'\int_{-L/2}^{L/2}dx\,v(x)$$
$$\times\left\{\frac{N}{L} - \frac{1}{2\pi}\partial_{t'}[\varphi(x-t') + \varphi^\dagger(x-t')]\right\}$$
$$= \Delta t v_0 N/L - [A(\tau+\Delta t) - A(\tau)], \tag{4.8}$$

where

$$A(t) = \int_{-L/2}^{L/2}dx\,\frac{v(x)}{2\pi}[\varphi(x-t) + \varphi^\dagger(x-t)], \tag{4.9}$$

so that

$$e^{-i\int_\tau^{\tau+\Delta t}dt'\,V(t')} = e^{-i\Delta t v_0 N/L}e^{i[A(\tau+\Delta t)-A(\tau)]}. \tag{4.10}$$

Since operators $A(\tau)$ and $A(\tau+\Delta t)$ commute to a c number we have $e^{-i[A(\tau+\Delta t)-A(\tau)]} = e^{-[A(\tau+\Delta t),A(\tau)]/2}e^{-iA(\tau+\Delta t)}e^{iA(\tau)}$. Explicitly,

$$[A(\tau+\Delta t),A(\tau)] = -2i\sum_{q>0}\frac{2\pi}{Lq}\left|\frac{v_q}{2\pi}\right|^2\sin[q\Delta t] \tag{4.11}$$
$$= -2i\Delta t\sum_{q>0}\frac{2\pi}{L}\left|\frac{v_q}{2\pi}\right|^2\Delta t + \mathcal{O}(\Delta t^2). \tag{4.12}$$

We therefore find

$$e^{-i\int_{\tau}^{\tau+\Delta t} dt'\, V(t')} = e^{-i\Delta E\Delta t+\mathcal{O}(\Delta t^2)} e^{iA(\tau+\Delta t)} e^{-iA(\tau)}. \quad (4.13)$$

When substituted back into Eq. (4.7), this leads to the result

$$Q(t) = e^{-i\Delta E t} e^{iA(t)} e^{-iA(0)}. \quad (4.14)$$

We can rewrite this as

$$Q(t) = e^{-i\Delta E t} \underbrace{e^{\frac{1}{2}[A(t),A(0)]}}_{F_1} \underbrace{e^{i[A(t)-A(0)]}}_{F_2}. \quad (4.15)$$

Factor F_1 is what Gutman *et al.*[13] (see their footnote 36) calls the "anomalous" contribution to the closed loop factor. An expression involving the determinant of an operator acting on single-particle Hilbert space often appears in the literature[6,8–12] in connection with the closed loop factor. In Appendix E we show that this determinant is equal to the expectation value of factor F_2.

Below we consider factors F_1 and F_2 separately. From Eq. (4.11) we have

$$e^{\frac{1}{2}[A(t),A(0)]} = \exp\left\{-i\sum_{q>0}\frac{2\pi}{Lq}\left|\frac{v_q}{2\pi}\right|^2 \sin(qt)\right\}. \quad (4.16)$$

We write factor F_2 in boson normal-ordered form. This is done to facilitate the calculation of the expectation value with respect to $|F+\rangle$.

$$e^{i[A(t)-A(0)]} = e^{i[B^\dagger(t)-B^\dagger(0)]} e^{i[B(t)-B(0)]} e^{\frac{1}{2}[B^\dagger(t)-B^\dagger(0),B(t)-B(0)]}. \quad (4.17)$$

$$B(t) = \int_{-L/2}^{L/2} dx\, \frac{v(x)}{2\pi}\varphi(x-t). \quad (4.18)$$

Explicitly evaluating the commutator in Eq. (4.17), we find

$$e^{i[A(t)-A(0)]} = e^{i[B^\dagger(t)-B^\dagger(0)]} e^{i[B(t)-B(0)]} \times \exp\left\{\sum_{q>0}\frac{2\pi}{Lq}\left|\frac{v_q}{2\pi}\right|^2[\cos(qt)-1]\right\}. \quad (4.19)$$

Combining this with the result in Eq. (4.16) for factor F_1, we obtain

$$Q(t) = e^{-i\Delta E t} e^{i[B^\dagger(t)-B^\dagger(0)]} e^{i[B(t)-B(0)]} \times \underbrace{\exp\left\{\sum_{q>0}\frac{2\pi}{Lq}\left|\frac{v_q}{2\pi}\right|^2[e^{-iqt}-1]\right\}}_{P(t)}. \quad (4.20)$$

In the expression for $P(t)$, the infinite-system-size limit can straightforwardly be taken to obtain

$$P(t) = \exp\left\{\int_0^\infty dq\left|\frac{v_q}{2\pi}\right|^2\frac{e^{-iqt}-1}{q}\right\}, \quad (4.21)$$

in agreement with Eq. (4.3).

The large-time asymptotics of Eq. (4.21) can be extracted as follows. First, we write $\ln P(t)$ as

$$\begin{aligned}\ln P(t) &= \int_0^\infty dq\left|\frac{v_q}{2\pi}\right|^2\frac{e^{-iqt}-1}{q}\\ &= \lim_{y\to 0^+}\int_0^\infty dq\left(\left|\frac{v_q}{2\pi}\right|^2-\left|\frac{v_0}{2\pi}\right|^2+\left|\frac{v_0}{2\pi}\right|^2\right)\\ &\quad\times\frac{e^{-iqt}-1}{q}e^{-qy}.\end{aligned} \quad (4.22)$$

This expression is then split up into three terms, $\ln P(t) = \lim_{y\to 0^+}(T_1+T_2+T_3)$, where

$$\begin{aligned}T_1 &= \int_0^\infty dq\left|\frac{v_0}{2\pi}\right|^2 e^{-qy}\frac{e^{-iqt}-1}{q},\\ T_2 &= -\int_0^\infty dq\,\frac{e^{-qy}}{q}\left(\left|\frac{v_q}{2\pi}\right|^2-\left|\frac{v_0}{2\pi}\right|^2\right),\\ T_3 &= \int_0^\infty dq\,\frac{e^{-q(y-it)}}{q}\left(\left|\frac{v_q}{2\pi}\right|^2-\left|\frac{v_0}{2\pi}\right|^2\right).\end{aligned} \quad (4.23)$$

The integral in T_1 is straightforward and leads to $T_1 = -\alpha\ln(1+it/y)$, where $\alpha = (v_0/2\pi)^2$ as in Eq. (3.18) and consistent with Eq. (1.6). In Appendix D we show that $T_3 = \mathcal{O}(t^{-1})$ and hence vanishes in the large t limit. Term T_2 can be written as

$$T_2 = -\alpha\int_0^\infty dq\,\frac{e^{-qy}}{q}\left(\left|\frac{v_q}{v_0}\right|^2-1\right). \quad (4.24)$$

Thus, for large $|t|$, we obtain $P(t)\simeq(i\Lambda t)^{-\alpha}$, where

$$\Lambda = \lim_{y\to 0^+}\left[\frac{1}{y}\exp\int_0^\infty dq\,\frac{e^{-qy}}{q}\left(\left|\frac{v_q}{v_0}\right|^2-1\right)\right]. \quad (4.25)$$

This implies that Λ is determined by the shape of $v(x)$ but not by its overall magnitude: The transformation $v(x)\to c\,v(x)$ does not affect Λ.

V. SEMI-INFINITE WIRE

We apply the results of the previous section to the case of a semi-infinite wire with $v(x)$ given by Eq. (2.7) so that

$$v_q = \lambda\cos(ql)e^{-|q|a}. \quad (5.1)$$

Substitution into Eq. (4.21) then yields

$$\begin{aligned}P(t) &= C\left\{\left(1+\frac{it}{2a}\right)^2\left[\left(1+\frac{it}{2a}\right)^2+\left(\frac{l}{a}\right)^2\right]\right\}^{-\alpha/4}\\ &= C\left[1+\frac{it}{2a}\right]^{-\alpha}\left[1+\frac{(l/a)^2}{(1+it/2a)^2}\right]^{-\alpha/4},\end{aligned} \quad (5.2)$$

where $C = [1+(l/a)^2]^{\alpha/4}$. Thus in the limit of large t, $P(t)\simeq(i\Lambda t)^{-\alpha}$, where

$$\Lambda = \left\{2\sqrt{al}\left[1+\left(\frac{a}{l}\right)^2\right]\right\}^{-1}. \quad (5.3)$$

The latter result could also have been obtained using Eq. (4.25). We see that $P(t)$, and thus also the tunneling rate W, grows as $l^{\alpha/2}$ for $l\gg a$,

Some insight into the origin of this result may be obtained by considering the result in Eq. (3.17) for the overlap

$\langle F+|\,F-\rangle$, which displays the orthogonality catastrophe. For Λ given by Eq. (5.3), $|\langle F+|\,F-\rangle|^2 \propto l^{\alpha/2}$. Therefore increasing l mitigates the orthogonality catastrophe. The tunneling rate W can be written as [cf. Eq. (A1)]

$$W = 2\pi|\gamma|^2 \sum_n \delta(\varepsilon + E_0^{(+)} - E_n^{(-)})|\langle F+|n-\rangle|^2, \tag{5.4}$$

where $E_n^{(-)}$ and $|n-\rangle$ are the energies and many-body eigenstates of H_-, the Hamiltonian that describes the electrons when the qubit is in state $|-\rangle$. The results in Appendix C can be extended to show that not only the ground state–to–ground state overlap, but every overlap $|\langle F+|\,n-\rangle|^2$ where $|n-\rangle$ contains a finite number of particle hole excitations on top of $|F-\rangle$, scales like $l^{\alpha/2}$. Thus the scaling behavior of $W \sim l^{\alpha/2}$ can be understood as a consequence of the mitigation of the orthogonality catastrophe. The effect relies on the phase coherence of electrons in the section of the wire between $|x| = l$ and $x = 0$. Hence it is destroyed if the electron phase is randomized by impurity scattering or if there is inelastic scattering. Thus the increase in W with increasing l should persist until l exceeds either the elastic or the inelastic mean free path in the wire. Of course the result is also only valid as long as $\omega \lesssim 1/l$ so that excitations created by the qubit do not resolve the spatial structure of the potential. Thus, the larger l, the smaller the energy window $0 < \omega \lesssim 1/l$ in which the enhancement of W due to mitigation of the orthogonality catastrophe can be observed.

The tunneling rate W is calculated by expanding the third factor in Eq. (5.2) in a Taylor series in $(1 + it/2a)$ and Fourier transforming each term separately. Using the identities

$$\int_{-\infty}^{\infty} dt\, e^{i\omega t}(1 + it/r)^{-s} = \frac{2\pi}{\Gamma(s)} \frac{(r\omega)^s}{\omega} e^{-r\omega} \tag{5.5}$$

for $s, t > 0$, and

$$(s)_n \equiv \frac{\Gamma(s+n)}{\Gamma(s)} = \prod_{m=0}^{n-1}(s+m), \tag{5.6}$$

we obtain

$$W(\omega) = C\, W_0(\omega) \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha}{4}\right)_n}{n!(\alpha)_{2n}}[-(2l\omega)^2]^n, \tag{5.7}$$

where $W_0(\omega)$ is the $l = 0$ result

$$W_0(\omega) = \frac{2\pi|\gamma|^2}{\Gamma(\alpha)} \frac{(2a\omega)^\alpha}{\omega} e^{-2a\omega}\theta(\omega). \tag{5.8}$$

The factor $(\alpha)_{2n}$ can be rewritten

$$(\alpha)_{2n} = 2^{2n}\left(\frac{\alpha}{2}\right)_n \left(\frac{\alpha+1}{2}\right)_n. \tag{5.9}$$

Substituting this into Eq. (5.7), we identify the series as the Taylor expansion of the hypergeometric function ${}_1F_2$, yielding one of our main results:

$$W(\omega) = C\, W_0(\omega)\, {}_1F_2\left(\frac{\alpha}{4}; \frac{\alpha}{2}, \frac{\alpha+1}{2}; -(l\omega)^2\right). \tag{5.10}$$

As $\omega \to 0^+$, ${}_1F_2(\frac{\alpha}{4}; \frac{\alpha}{2}, \frac{\alpha+1}{2}; -(l\omega)^2)$ tends to 1, so that $W(\omega)$ indeed has the expected power law singularity for small ω [cf. Eq. (1.7)], with Λ given by Eq. (5.3). When ω becomes of the order $1/l$, excitations in the wire are able to resolve the spatial structure of the potential $v(x)$ and $W(\omega)$ starts deviating from simple power law behavior. In the weak coupling limit, i.e., small α, the hypergeometric function ${}_1F_2(\frac{\alpha}{4}; \frac{\alpha}{2}, \frac{\alpha+1}{2}; -(l\omega)^2)$ reduces to $\cos(l\omega)^2$, and W therefore shows oscillations with period π/l as a function of ω. These oscillations are due to the resonant creation of a single plasmon (consisting of an electron-hole pair) with an energy that satisfies the resonance condition $\omega = \pi n/l$. This condition maximizes the single-particle matrix element $\langle h|\,V\,|p\rangle$, where h and p refer to the single-particle orbitals of the hole and the excited particle constituting the plasmon. In plasmon language the equivalent explanation is the following: at weak coupling (small α) the qubit creates a single plasmon. The amplitude to create a left-moving plasmon at $x = l$ is $\langle F+|\,a_{q=\omega}\rho(l)\,|F+\rangle \sim e^{i\omega l}$, while the amplitude for creating a right-moving plasmon is $\langle F+|a_{q=\omega}\rho(-l)|F+\rangle \sim e^{-i\omega l}$. These amplitudes interfere constructively when $\omega = \pi n/l$, maximizing the probability for plasmon creation. Plasmon creation is accompanied by qubit decay. Thus when the probability to create a plasmon is high, so too is the qubit decay rate.

As shown in Fig. 2, the oscillations become damped as α is increased. The damping is a signature of a phenomenon known as Fermi sea shake-up. At strong coupling (large α), rather than creating a single plasmon, with energy ω, many low-energy plasmons are created with energies $\omega_i > 0$ such that their combined energy is $\sum_i \omega_i = \omega$. They have wavelengths $1/\omega_i \gg 2l$. As a result, the constructive interference between left- and right-moving components is impossible; the individual plasmon wavelengths are too large. Hence there is no clear resonance any more, and the oscillations in $W(\omega)$ are washed out.

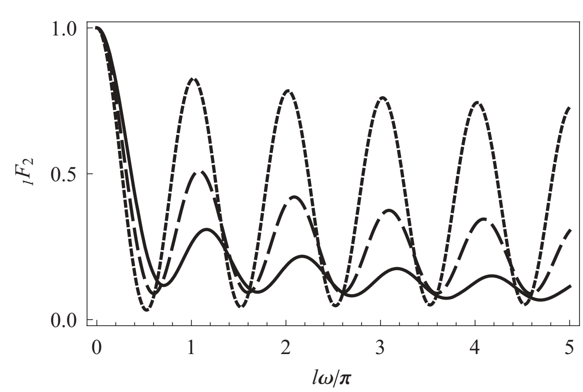

FIG. 2. The function ${}_1F_2(\frac{\alpha}{4}; \frac{\alpha}{2}, \frac{\alpha+1}{2}; -(l\omega)^2)$, which determines the finite ω behavior of the tunneling rate W, as a function of ω for various couplings α, for the potential $v(x)$ of Eq. (2.7). The dotted curve corresponds to $\alpha = 1/8$; the dashed curve, to $\alpha = 1/2$; and the solid curve, to $\alpha = 1$. Oscillations with period π/l result from the resonant creation of a single-particle hole pair by the qubit. At larger α the oscillations are washed out due to Fermi sea shake-up, i.e., the excitation by the qubit of a multitude of particle hole pairs with a broad distribution of energies.

It is also instructive to investigate the regime $1/l \ll \omega$. Here the asymptotic behavior of the hypergeometric function is

$$ {}_1F_2\left(\frac{\alpha}{4};\frac{\alpha}{2},\frac{\alpha+1}{2};-(l\omega)^2\right) \simeq \frac{\Gamma(\alpha)}{\Gamma(\alpha/2)}(2l\omega)^{-\alpha/2}. \tag{5.11} $$

If we substitute this into the expression [Eq. (5.10)] for W, assuming $l \gg a$, we obtain

$$ W(\omega)|_{\omega\gg 1/l} = \frac{2\pi|\gamma|^2}{\Gamma(\alpha/2)}\frac{(2a\omega)^{\alpha/2}}{\omega}e^{-2a\omega}\theta(\omega). \tag{5.12} $$

This is the same rate as would be obtained from a closed loop factor,

$$ P(t) = \left[\left(1+\frac{it}{2a}\right)^{(v_0/4\pi)^2}\right]^2. \tag{5.13} $$

Such a closed loop factor could also be obtained by coupling the qubit to two independent chiral channels, where the potential the qubit produces in either channel equals $u(x)$ of Eq. (2.7), i.e., one of the peaks in the full potential $v(x) = u(x+l) + u(x-l)$. This means that, in the single-channel semi-infinite wire, at energies $\omega \gg 1/l$, the potential $u(x-l)$ experienced by right-moving electrons and the potential $u(x+l)$ experienced by left-moving electrons contribute incoherently to the rate W, as if left-movers and right-movers belong to separate channels. Could this indicate that electrons reflected at the tip of the wire have lost all memory of their in-bound encounter with the qubit by the time that they again reach the qubit on their out-bound journey? Since the electrons undergo no relaxation between encounters with the qubit, the answer is "no." Rather, what Eq. (5.12) indicates is that processes in which an individual electron wave packet with width $1/\omega \ll l$ is scattered twice, once while incident on the tip and once after being reflected at the tip, are rare and make a vanishingly small contribution to the rate $W(\omega)$.

As stated in the introduction, W corresponds to the exponential decay rate of the probability to find the qubit in state $|+\rangle$, provided that $1 \gg W(\omega)/\omega$. We conclude this section by investigating when this inequality holds. For $\alpha < 2$, W/ω diverges when $\omega \to 0^+$, and the criterion for exponential decay is violated at small ω. From the small ω asymptotics $W(\omega)/\omega \sim |\gamma|^2(\omega/\Lambda)^\alpha/\omega^2$, we conclude that, for $\alpha < 2$, exponential decay at rate W occurs when

$$ \omega \gg |\gamma||\gamma/\Lambda|^{\alpha/(2-\alpha)}. \tag{5.14} $$

When $\alpha > 2$, on the other hand, $W(\omega)/\omega$ no longer diverges but, rather, reaches a maximum value of order $|a\gamma|^2$ at $\omega \sim 1/a$. Thus, for $\alpha > 2$, exponential decay at rate W occurs for all $\omega > 0$, provided that

$$ 1 \gg |a\gamma|^2. \tag{5.15} $$

The same regime for exponential decay as in Eqs. (5.14) and (5.15) was identified more rigorously by Legget *et al.*[20] in the context of the spin-boson Hamiltonian with an unstructured Ohmic bath (corresponding to $l = 0$ in our system). [See their Sec. VII.B and, in particular, their Eq. (7.17a). Note that the quantity that we denote α is twice the quantity that they denote α.] The fact that our qubit is immersed in a structured bath does not affect the result because the bath spectral function $J(q)$ still has the same large and small q asymptotics as in the case of an Ohmic bath.

VI. SUMMARY AND CONCLUSIONS

In this paper we have studied the quantity $\langle F+|e^{iH_+t}e^{-iH_-t}|F+\rangle$, known in the language of the Fermi edge singularity as the closed loop factor, for the case where the Hamiltonians $H_\pm$ describe electrons in a single chiral channel. We investigated the regime $E_F \gg v_F/a$, where a is the typical length scale on which the potentials $V_\pm$ associated with $H_\pm$ vary, a regime not studied before. We investigated a system where the Fourier transform of the closed loop factor gives the tunneling rate of a two-level system (charge qubit) coupled to the electron gas. Under the assumption of linear dispersion [cf. Eq. (2.2)] we obtained the exact expression for the closed loop factor valid for arbitrary $V_\pm$ and arbitrary times. We studied its large-time asymptotics and obtained an exact formula for the ultraviolet energy Λ that appears in Eq. (1.5). Unlike in the previously studied regime where $\Lambda \sim E_F$, here we find $\Lambda < 1/a \ll E_F$. Furthermore, it turns out that Λ is determined by the shape, but not the overall magnitude of the potentials $V_\pm$, i.e., scaling $V_\pm \to c\,V_\pm$ leaves Λ invariant.

We applied our general results to the example of a semi-infinite wire. The qubit interacts with the wire at a point that is a distance l from the tip of the wire. In this system we found that the tunneling rate W could be enhanced without increasing either γ or α. Λ decreases like $l^{-1/2}$ so that W grows like $l^{\alpha/2}$. Thus the tunneling rate W becomes higher the farther the qubit is from the tip of the wire. This effect is due to a mitigation of the orthogonality catastrophe. It holds as long as l is less than the phase-coherence length of electrons in the wire and for level splittings $0 < \omega \lesssim 1/l$.

Armed with an expression for the closed loop factor that is valid also for small times, we obtain an exact expression for W for the semi-infinite wire system. The finite ω features of W probe the spatial profile of the potential $V_\pm$ at length scales $1/\omega$ (in units where $v_F = 1$). We study how the ω dependence of W changes as the coupling α between the qubit and the electron gas grows. At weak coupling (small α), we find that the rate W oscillates as a function of ω and that the period is π/l. This is due to the resonant creation of a single plasmon (density fluctuation consisting of an electron-hole pair). The resonance condition $\omega = \pi n/l$ ensures that the amplitude for creating a left-moving plasmon in the vicinity of the qubit interferes constructively with the amplitude for creating a right-moving plasmon.

At strong coupling (large α), on the other hand, the oscillations of the rate W as a function of ω are overdamped. The reason for this is that, at strong coupling, many low-energy plasmons are created when qubit decays. The energies ω_i of the individual plasmons are too low to meet the resonance condition $\omega_i = n\pi/l$. The large number of plasmons corresponds to a large number of electron-hole pairs created when the qubit decays. This is known as Fermi sea shake-up. One of our main results [Eq. (5.10)] is an exact formula for this damping of $W(\omega)$ by means of Fermi sea shake-up. The result is illustrated in Fig. 2.

We also analyzed rate W in the $\omega \gg 1/l$ limit and saw that here the left- and right-moving electrons contribute to the rate W as if they belong to independent channels. This happens despite the fact that each electron incident on the tip encounters the qubit twice, once before being reflected at the tip and once afterwards, and no electron relaxation occurs between qubit encounters. The result therefore indicates that processes in which an individual electron wave packet of width $1/\omega \ll l$ is scattered twice, once while incident on the tip and once after being reflected at the tip, make a vanishingly small contribution to the rate $W(\omega)$.

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APPENDIX A: OBTAINING W FROM FERMI's GOLDEN RULE

In this Appendix we apply Fermi's golden rule to obtain the expression in Eq. (1.4) for the transition rate W. The initial state for the transition is $|F+\rangle \otimes |+\rangle$ with energy $E_0^{(+)} + \varepsilon$. Possible final states are of the form $|n-\rangle \otimes |-\rangle$, where $|n-\rangle$ is an eigenstate of H_- and has energy $E_n^{(-)}$. We have to sum over all eigenstates of H_-. Thus

$$\begin{aligned} W_{-+} &= 2\pi|\gamma|^2 \sum_n \delta(\varepsilon + E_0^{(+)} - E_n^{(-)})\,|\langle F+|\, n-\rangle|^2 \\ &= |\gamma|^2 \sum_n \int_{-\infty}^{\infty} dt\, e^{i(\varepsilon+E_0^{(+)}-E_n^{(-)})t}\,|\langle F+|\, n-\rangle|^2 \\ &= |\gamma|^2 \sum_n \int_{-\infty}^{\infty} dt e^{i\varepsilon t} < F+|e^{iH_+t}|n-\rangle < n-|e^{-iH_-t}|F+\rangle \\ &= |\gamma|^2 \int_{-\infty}^{\infty} dt\, e^{i\varepsilon t}\,\langle F+|\, e^{iH_+t} e^{-iH_-t}\,|F+\rangle\,. \end{aligned} \tag{A1}$$

APPENDIX B: GAUGING AWAY V_+

In the main text we chose the potential V_+ as 0 and stated that this does not involve any loss of generality. Here we prove this claim. Suppose that

$$H_\pm = \int_{-L/2}^{L/2} dx\, \psi^\dagger(x)(-i\partial_x - \mu + v_\pm(x))\psi(x). \tag{B1}$$

Now define position-dependent phases

$$\lambda_\pm(x) = -\int_{-L/2}^{x} dx'\, v_\pm(x') \tag{B2}$$

and total phase shifts $\lambda_\pm = \lambda_\pm(L/2)$. Then define a new set of fermion operators $\bar\psi(x)$ related to $\psi(x)$ by

$$\bar\psi(x) = e^{i[\lambda_+(x)-\lambda_+ x/L]}\psi(x). \tag{B3}$$

The operators $\bar\psi^\dagger(x')$ and $\bar\psi(x)$ obey the same anticommutation relations as $\bar\psi^\dagger(x')$ and $\bar\psi(x)$ and are also periodic with period L.

In terms of $\bar\psi(x)$ and $\bar\psi^\dagger(x)$, the Hamiltonian H_+ has the form

$$H_+ = \int_{-L/2}^{L/2} dx\, \bar\psi^\dagger(x)\,(-i\partial_x - \mu - \lambda_+/L)\,\bar\psi(x), \tag{B4}$$

while

$$\begin{aligned} H_+ = \int_{-L/2}^{L/2} dx\, \bar\psi^\dagger(x)(&-i\partial_x - \mu - \lambda_+/L \\ &+ v_-(x) - v_+(x))\bar\psi(x). \end{aligned} \tag{B5}$$

Thus, in terms of the new fermion operators, H_+ and H_- are of the same form as in Eqs. (2.2) and (2.3), with $v(x) \to v_-(x) - v_+(x)$ and $\mu \to \mu + \lambda_+/L$.

APPENDIX C: ANDERSON's ORTHOGONALITY CATASTROPHE

Anderson[28] states that the overlap $\langle F+|\, F-\rangle$ vanishes as a power law $L^{-\alpha/2}$ as the system size grows. For the present system we can calculate this overlap exactly for arbitrary potentials $v(x)$. Our starting point is Eq. (3.16) and the operator identity $e^{A+B} = e^A e^B e^{-[A,B]/2}$, provided that $[A,[A,B]] = [B,[A,B]] = 0$.

$$\begin{aligned} &\langle F+|F-\rangle \\ &\quad = \langle F+|\exp \sum_{q>0} \left(\frac{2\pi}{Lq}\right)^{1/2} \left(\frac{v_q}{2\pi} a_q - \frac{v_q *}{2\pi} a_q^\dagger\right)|F+\rangle \\ &\quad = \exp\left\{-\frac{1}{2}\sum_{q>0}\frac{2\pi}{Lq}\left|\frac{v_q}{2\pi}\right|^2\right\}. \end{aligned} \tag{C1}$$

In the large-L limit, we rewrite this as

$$\begin{aligned} \langle F+|\, F-\rangle = \lim_{y\to 0^+} \Bigg[& \underbrace{\frac{1}{y}\exp\left[\int_0^\infty dq\, e^{-qy}\left(\left|\frac{v_q}{v_0}\right|^2 - 1\right)\right]}_{1} \\ & \times \underbrace{y\exp\left(\sum_q^\infty \frac{2\pi}{Lq} e^{-qy}\right)}_{2} \Bigg]^{-\alpha/2}, \end{aligned} \tag{C2}$$

where $\alpha = |v_0/2\pi|^2$ as in Eq. (3.18). The $y \to 0^+$ limit of the two factors marked 1 and 2 can be taken separately. Referring back to Eq. (4.25), we identify the factor marked 1 as the energy Λ that appears in Eqs. (1.5). The sum in the exponent of the factor marked 2 is the Taylor expansion of the logarithm function, and hence

$$y\exp\left(\sum_q^\infty \frac{2\pi}{Lq} e^{-qy}\right) = y[1 - \exp(-2\pi y/L)]^{-1} \simeq L/2\pi. \tag{C3}$$

This leads to the result

$$\langle F+|F-\rangle = \left(\frac{2\pi}{\Lambda L}\right)^{\alpha/2}. \tag{C4}$$

APPENDIX D: ASYMPTOTICS OF $P(t)$

In the main text, in the derivation of the asymptotic form of $P(t)$, we stated that T_3 in Eq. (4.23) vanishes like $1/t$ in the large-$|t|$ limit. Here we give a proof. By Fourier transforming from v_q to $v(x)$ we obtain

$$T_3 = \int dx \int dx' \frac{v(x)}{2\pi}\frac{v(x')}{2\pi} \underbrace{\int_0^\infty dq\, \frac{e^{-iq(t-iy)}}{q}[e^{iq(x-x')}-1]}_{I}. \tag{D1}$$

The integral I can be performed to obtain

$$I = \ln\left(1+\frac{it}{y}\right) - \ln\left(1+\frac{i(t-x+x')}{y}\right). \tag{D2}$$

Expanding in $1/t$ we find $I = (x-x')/t + \mathcal{O}(t^{-2})$.

APPENDIX E: DETERMINANTAL FORMULA RELATED TO THE CLOSED LOOP CONTRIBUTION

Here we show that the expectation value of the factor F_2 in Eq. (4.15) with respect to $|F+\rangle$ equals a determinant of an operator acting on single-particle Hilbert space. The proof relies on the following general result for fermionic systems. Let $B = \{|n\rangle\, |n = 1,2,\ldots\}$ be a set of orthonormal single-particle orbitals and let $c_n^\dagger$ and c_n be the associated fermionic creation and annihilation operators. Let F be a subset of B. Without loss of generality, we may take $F = \{|n\rangle\, |n = 1,2,\ldots N\}$. Let $|F\rangle$ be the many-fermion state

$$|F\rangle = \prod_{m=1}^{N} c_m^\dagger\, |0\rangle\,. \tag{E1}$$

Let H be the operator

$$H = \sum_{m,n=1}^{\infty} h_{mn} c_m^\dagger c_n. \tag{E2}$$

Then $e^{iH}\,|F\rangle = \prod_{m=1}^N \tilde{c}_m^\dagger\,|0\rangle$, where the fermionic operator $\tilde{c}_m^\dagger$ creates a particle in the orbital $|\tilde{n}\rangle = e^{ih}\,|n\rangle$, where

$$h = \sum_{m,n=1}^{\infty} h_{mn}\,|m\rangle\,\langle n| \tag{E3}$$

is an operator acting on single-particle Hilbert space. This implies that $\langle F|e^{iH}|F\rangle = \det e^{ih}_{FF}$ is a Slater determinant where e^{ih}_{FF} is an $N\times N$ matrix with entries

$$\left[e^{ih}_{FF}\right]_{mn} = \langle m|\,e^{ih}\,|n\rangle\,. \tag{E4}$$

The operator $f = \sum_{n=1}^N |n\rangle\,\langle n|$ projects onto the subspace spanned by the set F, and hence the matrix e^{ih}_{FF} has the same determinant as the operator $1 - f + e^{ih} f$, leading to the result

$$\langle F|\,e^{iH}\,|F\rangle = \det(1 - f + e^{ih} f). \tag{E5}$$

We derived this result for a state $|F\rangle$ containing a finite number of particles. We postulate that a similar result holds for state $|F+\rangle$, representing an infinitely deep Fermi sea.

In order to apply the above result to $F_2 = e^{-i[A(t)-A(0)]}$, we have to show that $A(t) - A(0)$ is quadratic in fermion creation and annihilation operators. This is indeed so, as is seen by referring to Eqs. (3.5) and (4.9) to obtain

$$A(t) - A(0) = \int_0^t dt' \int_{-L/2}^{L/2} dx \left[v(x-t') - \frac{v_0}{L}\right]\psi(x)^\dagger\psi(x). \tag{E6}$$

Thus we may use Eq. (E5) to write

$$\langle F+|\,e^{-i[A(t)-A(0)]}\,|F+\rangle = \det(1 - n + e^{-i\delta} n), \tag{E7}$$

where n and δ are the single-particle operators

$$n = \sum_{k<\mu} |k\rangle\,\langle k|\,, \qquad \delta = \int_0^t dt' \int_{-L/2}^{L/2}\left[v(x-t') - \frac{v_0}{L}\right]|x\rangle\,\langle x|\,. \tag{E8}$$

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