Work fluctuation theorem for a classical circuit coupled to a quantum conductor

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We propose a setup for a quantitative test of the quantum fluctuation theorem. It consists of a quantum conductor, driven by an external voltage source, and a classical inductor-capacitor circuit. The work done on the system by the voltage source can be expressed by the classical degrees of freedom of the LC circuit, which are measurable by conventional techniques. In this way, the circuit acts as a classical detector to perform measurements of the quantum conductor. We prove that this definition is consistent with the work fluctuation theorem. The system under consideration is effectively described by a Langevin equation with non-Gaussian white noise. Our analysis extends the proof of the fluctuation theorem to this situation.

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I. INTRODUCTION

The degrees of freedom of physical systems usually fluctuate, with strength which in thermal equilibrium is related to the transport properties by the fluctuation-dissipation theorem. Out of equilibrium, the fluctuation theorem (FT) imposes universal constraints on the probability distributions of the fluctuating parameters.[1–6] The FT has been studied in a variety of contexts, including applications to electron transport in mesoscopic systems.[7–11] There exist several equivalent versions of the FT, all derived from two main assumptions, namely, (i) equilibrium Gibbs form of the initial statistical distribution, and (ii) time reversibility of the microscopic evolution equations. In the following, we will focus on one of these versions: the work FT.[12,13] It is formulated in terms of the distribution $P(W;B)$ of the work W done on the system by the external force during time τ in the presence of a magnetic field B. In its simplest form, it reads as

$$\frac{P(W;B)}{P(-W;-B)} = e^{\beta W}, \tag{1}$$

where $\beta = 1/k_{\mathrm{B}}T$ is the inverse temperature. It is valid both for classical and for quantum systems,[1] where the magnetic field helps revealing interference effects.

The interpretation of the identity (1) for a classical system is straightforward, and there exist no fundamental obstacles to its experimental verification. Let us briefly discuss the corresponding experimental procedure. One basically needs to switch on an external force at time zero and switch it off at time τ. During this time interval, one continuously monitors the change of the relevant system parameters. The work W is usually related to these parameters in a simple way and can be computed. Repeating this experiment many times, one can determine the distribution $P(W;B)$. In this way, the fluctuation theorem has been confirmed in various systems ranging from colloidal particles in a solution[14] and RNA molecules[15] to quantum dots in the regime of strong Coulomb blockade where individual tunneling electrons can be counted.[16,17]

The experimental protocol is more subtle when the object under consideration is a quantum system. In this case, the work W is defined as a difference between the final and initial energies of the quantum system.[1,18,19] Thus, in order to recover the distribution $P(W;B)$, one should perform two projective measurements at the beginning and end of every experimental run. While this procedure might work, e.g., for qubits and ultracold atoms,[1] it becomes difficult to realize if one deals with more conventional quantum mesoscopic objects such as Aharonov-Bohm interferometers or quantum dots. This is one of the reasons why the FT (1) has not yet been fully tested in such systems. So far, only the relations between the nonlinear transport coefficients, which follow from the identity (1) for low bias voltages, have been verified.[20]

In order to overcome this problem, we propose a different scheme to measure the work W. On one hand, it should be applicable in systems involving small mesoscopic conductors. On the other hand, as we will show, it is still consistent with the FT (1). Our approach is motivated by the theory of Nazarov and Kindermann,[21] who have proposed to measure the full counting statistics[22,23] (FCS) of the charge transferred through a quantum conductor with the aid of a classical system coupled to it. Extending this idea, we propose to couple the conductor to a classical oscillator consisting of an inductor L and a capacitor C. To ensure its classical behavior, we require the oscillator frequency to be small,

$$\hbar/\sqrt{LC} \ll \max\{k_{\mathrm{B}}T, eV\}, \tag{2}$$

where V is the voltage drop across the quantum conductor. Since the LC oscillator is a classical system, one can in principle continuously measure the fluctuating voltage $V(t)$ by a sensitive amplifier, from which the work W is obtained by classical arguments [see Eq. (6)]. This definition of the work would be exact if both the LC circuit and the conductor were classical. However, it differs from the standard definition of the work in the quantum regime,[1,18,19] and hence the standard proof of the quantum FT (1) does not apply any longer. We will show that, nevertheless, the FT (1) remains valid as long as the condition (2) is satisfied.

Finally, we note that in our model the dynamics of the classical LC oscillator is described by a Langevin equation

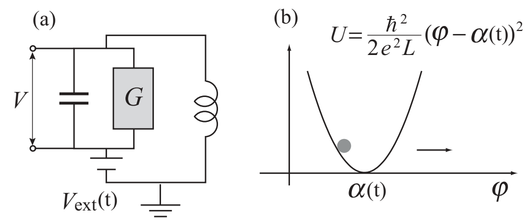

FIG. 1. (a) Schematics of the system, consisting of a coherent quantum conductor (denoted by G) connected to an inductor and a capacitor. The voltage drop across the quantum conductor V is measured by a voltmeter. (b) A Brownian particle in a driven harmonic potential.

with white non-Gaussian noise generated by the quantum conductor. As far as we know, the FT has not yet been proven for such conditions. Its derivation constitutes a further motivation for our work.

The outline of the paper is as follows: In Sec. II we define the model; in Sec. III we derive the probability distribution of the work, show how it is related to the FCS, and how one can treat the back action of the LC circuit on the conductor; in Sec. IV we show that under constant bias voltage, the FCS of the work W is equivalent to the FCS of the charge transferred through the quantum conductor; in Sec. V we prove the work FT (1) for coupled quantum and classical system; and in Sec. VI we apply our theory to a quantum-dot Aharonov-Bohm interferometer. Finally, we will summarize our results.

II. MODEL

We consider the system depicted in Fig. 1(a), i.e., a quantum conductor with conductance G coupled in parallel to a capacitor C and in series to an inductor L. A bias voltage $V_{\rm ext}$ is applied from the external voltage source. The system is described by the Hamiltonian

$$\hat{H} = \hat{H}_G(\hat{p}_j,\hat{x}_j;\hat{\varphi}) + \hat{H}_{LC}(\hat{q},\hat{\varphi};\alpha), \tag{3}$$

where $\hat{H}_G(\hat{p}_j,\hat{x}_j;\hat{\varphi})$ refers to the conductor, with $\hat{p}_j,\hat{x}_j$ being its microscopic degrees of freedom (e.g., the momenta and coordinates of the tunneling electrons, or degrees of freedom of the electromagnetic environment, etc.). Our analysis is applicable to a wide range of quantum conductors, and we do not further specify $\hat{H}_G$. The Hamiltonian of the LC circuit reads as

$$\hat{H}_{LC}(\hat{q},\hat{\varphi};\alpha) = \frac{\hat{q}^2}{2C} + \left(\frac{\hbar}{e}\right)^2 \frac{(\hat{\varphi}-\alpha)^2}{2L}, \tag{4}$$

where $\hat{q}$ is the operator of the charge stored in the capacitor, related to the voltage drop across the conductor $\hat{V}$ in a usual way $\hat{q} = C\hat{V}$, while $\hat{\varphi}(t) = \int^t dt' e\hat{V}(t')/\hbar$ is the operator of the phase[24] associated with the voltage drop $\hat{V}$. In analogy to the mechanical quantum system, the phase $\hbar\varphi/e$ plays the role of the coordinate and the charge q that of the momentum with appropriate commutation relations. Finally, $\alpha(t) = \int^t dt' eV_{\rm ext}(t')/\hbar$ characterizes the external voltage bias. As discussed, we will assume the LC oscillator to be a classical system. Hence, we can replace the operators $\hat{q},\hat{\varphi}$ by the classical charge and phase variables q and φ.

We emphasize that the Hamiltonian (3) takes into account the back-action of the detector, i.e., the LC circuit, on the quantum conductor. This back-action manifests itself through the dependence of the Hamiltonian $\hat{H}_G$ on the coordinate of the LC oscillator $\hat{\varphi}$. Finally, we note that our setup is the electric analog of a colloidal particle dragged by a harmonic optical trap with a velocity $\dot{\alpha}(t)$ (Ref. 14) [see Fig. 1(b)]. However, our system possesses two striking differences as compared to the driven colloidal particle. First, as we will show the noise is non-Gaussian. Second, we can break time-reversal symmetry by applying a magnetic field.[25] In general, the probability distribution of noise depends on the direction of this field.

Next, we define the work done by the external voltage source on the whole system, i.e., the conductor and the LC circuit, for a given realization of the fluctuating time-dependent voltage $V(t)$ (Ref. 26):

$$W[\varphi;\alpha] = \int_{-\tau/2}^{\tau/2} dt\, \dot{\alpha} \frac{\partial H_{LC}(q,\varphi;\alpha)}{\partial \alpha} \tag{5}$$

$$= \int_{-\tau/2}^{\tau/2} dt\, V_{\rm ext}(t) \int_{-\tau/2}^{t} dt' \,\frac{V_{\rm ext}(t') - V(t')}{L}. \tag{6}$$

Since the LC circuit is classical, the fluctuating voltage $V(t)$, in principle, can be measured. Hence, the work $W[\varphi;\alpha]$ can be measured as well. Experimentally, one should first record the fluctuating voltage $V(t)$ during a time interval τ. Afterwards, the work (6) can be computed. Repeating these measurements many times, one obtains the probability distribution of the work $P(W;B)$ and then can test the relation (1). These kinds of measurements may be challenging, but with the development of low-invasive and wide-band on-chip electrometers, quantum point contacts, or single-electron transistors,[27,28] such measurements should be possible.

The problem of measuring the probability distribution of the work W is equivalent to that of measuring the distribution of the charge transferred through the conductor, i.e., to the problem of measuring its FCS.[21] The key point here is that the work done on the LC circuit turns into Joule heat, which is dissipated in the quantum conductor. It suggests that the fluctuation properties of the work, the Joule heat, and the transmitted charge are the same. One can demonstrate this property from the equations of motion of the circuit

$$\frac{\hbar}{e}\dot{\varphi} = \frac{\partial H_{LC}(\varphi,q;\alpha)}{\partial q}\,, \tag{7}$$

$$\dot{q} = -\frac{e}{\hbar}\frac{\partial H_{LC}(\varphi,q;\alpha)}{\partial \varphi} - I(t)\,, \tag{8}$$

where $I(t)$ is the fluctuating current flowing through the quantum conductor. The work (6) is related to the charge $Q = \int_{-\tau/2}^{\tau/2} dt' I(t')$ transmitted through the conductor via $W = V_{\rm ext} Q - V_{\rm ext}[q(\tau/2) - q(-\tau/2)]$. In the long-time limit $\tau \to \infty$, which is relevant under the condition (2), the second term in this expression becomes much smaller than the first one, which proves our statement.

Equations (7) and (8) are equivalent to the Langevin equation

$$C\frac{\hbar\ddot{\varphi}}{e} + \frac{\hbar\varphi}{eL} = \frac{V_{\rm ext}t}{L} - I(t), \tag{9}$$

where we have assumed V_{ext} to be constant. In this equation, we included a current $I(t)$ with nonzero average value $\bar{I}$ and fluctuations $\delta I(t) = I(t) - \bar{I}$ which act as noise. Correlators of the fluctuations $\langle \delta I(t_1)\delta I(t_2)\rangle$ decay quickly in time for $|t_1 - t_2| \gg \min\{\hbar/eV,\hbar/T\}$. Since the LC oscillator is slow [see Eq. (2)], we may consider the currents at different times as uncorrelated and treat $\delta I(t)$ as white noise.

In contrast to conventional models, the fluctuations of the current $I(t)$ are not Gaussian. We characterize their statistical properties by the probability $p(t,Q,V;B)$ that the charge $Q = \int_0^t dt'\, I(t')$ is transferred through the conductor biased by the voltage V during time t in the presence of a magnetic field B. It is convenient to introduce the characteristic function (CF) of current fluctuations

$$\mathcal{Z}_G(\lambda,V;B) = \sum_{Q/e} e^{i\lambda Q/e} p(t,Q,V;B). \tag{10}$$

In the white-noise approximation considered here, the time dependence of the CF reduces to a simple exponent

$$\mathcal{Z}_G(\lambda,V;B) \approx e^{t\mathcal{F}_G(\lambda,V;B)}, \tag{11}$$

where $\mathcal{F}_G(\lambda,V;B)$ is the cumulant generating function (CGF) of the conductor, which satisfies the FT (Refs. 2, 4, and 5)

$$\mathcal{F}_G(\lambda,V;B) = \mathcal{F}_G(-\lambda + i\beta eV, V; -B). \tag{12}$$

III. PROBABILITY DISTRIBUTION OF THE WORK

We define the CF of the work distribution

$$\mathcal{Z}(\xi;B) = \int dW\, e^{i\xi W} P(W;B), \tag{13}$$

and the corresponding CGF

$$\mathcal{F}(\xi) = \lim_{\tau\to\infty} \frac{1}{\tau} \ln \mathcal{Z}(\xi)\,. \tag{14}$$

In order to evaluate the CF (13), we follow the method proposed in Refs. 21 and 29. We split the measurement time interval $[-\tau/2,\tau/2]$ into $N = \tau/\Delta t$ pieces. The time step Δt should lie in the range $1/\max\{eV,T\} \lesssim \Delta t \lesssim \sqrt{LC}$, i.e., be sufficiently short to accurately describe the dynamics of the LC circuit and sufficiently long for the long-time approximation for the CF of the quantum conductor (11) to be valid. In this case, at each time t_i the LC circuit and the quantum conductor are not entangled, and the system's density matrix factorizes:

$$\rho(t_j) \approx \rho_{LC}(t_j) \otimes \rho_G[V(t_j)]. \tag{15}$$

Here, $t_j = j\Delta t$ are the discretized times, $\rho_{LC}(t_j)$ and $\rho_G[V(t_j)]$ are the reduced density matrices of the oscillator and of the quantum conductor, respectively, and $V(t_j)$ is the value of the bias voltage during the interval $t_j < t < t_{j+1}$. This voltage drop is introduced as the back action of the classical LC circuit.

Next, following Ref. 21, we express the reduced density matrix at time t_j in the form

$$\begin{aligned}\rho_{LC}(\varphi_j^+,\varphi_j^-) &= \mathrm{Tr}[\langle\varphi_j^+|\rho(t_j)|\varphi_j^-\rangle] \\ &\approx \int d\varphi_{j-1}^+ d\varphi_{j-1}^-\, \pi_{\Delta t}(\varphi_j^+,\varphi_j^-|\varphi_{j-1}^+,\varphi_{j-1}^-;\alpha_j) \\ &\quad \times \rho_{LC}(\varphi_{j-1}^+,\varphi_{j-1}^-),\end{aligned} \tag{16}$$

where the propagator for one time step $\pi_{\Delta t}$ reads as

$$\begin{aligned}&\pi_{\Delta t}(\varphi_j^+,\varphi_j^-|\varphi_{j-1}^+,\varphi_{j-1}^-;\alpha_j) \\ &\quad = \int \frac{dq_j^+}{2\pi e}\frac{dq_j^-}{2\pi e} e^{iq_j^+(\varphi_j^+-\varphi_{j-1}^+)/e - iq_j^-(\varphi_j^- - \varphi_{j-1}^-)/e} \\ &\quad\times e^{-i[H_{LC}(\varphi_j^+,q_j^+;\alpha_j) - H_{LC}(\varphi_j^-,q_j^-;\alpha_j)]\Delta t/\hbar} \\ &\quad\times \mathrm{Tr}[e^{-iH_G(\varphi_j^+;B)\Delta t/\hbar}\rho_G[V(t_{j-1})]e^{iH_G(\varphi_j^-;B)\Delta t/\hbar}],\end{aligned} \tag{17}$$

with $\alpha_j = \alpha(t_j)$, etc. The operator of the current through the conductor is related to its Hamiltonian via $\hat{I} = (e/i\hbar)\partial H_G(\varphi;B)/\partial\varphi|_{\varphi=0}$.

In the Keldysh formalism, $\varphi = (\varphi^+ + \varphi^-)/2$ and $q = (q^+ + q^-)/2$ are related to classical dynamical variables, which are measurable, while $\tilde{\varphi} = \varphi^+ - \varphi^-$ and $\tilde{q} = q^+ - q^-$ are "quantum" variables, which are small in the classical limit. We perform a first-order expansion in $\tilde{\varphi},\tilde{q}$, approximating the difference of the Hamiltonians as $H_{LC}(\varphi^+,q^+;\alpha) - H_{LC}(\varphi^-,q^-;\alpha) \approx (\tilde{\varphi}\partial_\varphi + \tilde{q}\partial_q)H_{LC}(\varphi,q;\alpha)$. Furthermore, we define the free energy of the classical LC circuit

$$F_{LC}(\alpha) = -k_{\mathrm{B}}T \ln \int \frac{d\varphi\, dq}{2\pi e} \exp[-\beta H_{LC}(\varphi,q;\alpha)]\,. \tag{18}$$

Finally, the CF (13) may be transformed to the form

$$\begin{aligned}\mathcal{Z} = \langle e^{i\xi\, W[\varphi;\alpha]}\rangle &\equiv \lim_{N\to\infty} \int \frac{d\varphi_0 dq_0}{2\pi e} e^{-\beta[H_{LC}(\varphi_0,q_0;\alpha_0) - F_{LC}(\alpha_0)]} \prod_{j=1}^N \int d\varphi_j d\tilde{\varphi}_j e^{i\xi(\alpha_j - \alpha_{j-1})\,\partial H_{LC}(\varphi_j,q_j;\alpha_j)/\partial\alpha} \\ &\quad\times \pi_{\Delta t}(\varphi_j + \tilde{\varphi}_j/2, \varphi_j - \tilde{\varphi}_j/2|\varphi_{j-1} + \tilde{\varphi}_{j-1}/2, \varphi_{j-1} - \tilde{\varphi}_{j-1}/2;\alpha_j)\end{aligned} \tag{19}$$

$$= \lim_{N\to\infty} \int \frac{d\varphi_0 dq_0}{2\pi e} e^{-\beta[H_{LC}(\varphi_0,q_0;\alpha_0) - F_{LC}(\alpha_0)]} \left(\prod_{j=1}^N \int \frac{dq_j d\varphi_j}{2\pi e} \int \frac{d\tilde{q}_j d\tilde{\varphi}_j}{2\pi e}\right) e^{iS_\xi/\hbar}. \tag{20}$$

Equation (19) can be interpreted as follows: As in a real experiment, the classical phase φ_j is supposed to be measured at every time t_j. Then, the derivative $\partial H_{LC}/\partial\alpha$, which is independent of the charge q, may be computed. Next, the exponent $\exp[i\xi(\alpha_j - \alpha_{j-1})\partial H_{LC}(\varphi_j,q_j;\alpha_j)/\partial\alpha]$ is constructed and averaged over all possible realizations of the current fluctuations. The latter are described by the propagators $\pi_{\Delta t}$ coming from the evolution of the quantum conductor.

In Eqs. (20), we have introduced the action of the whole system S_t, which is composed of three parts:

$$S_t = \xi\hbar W + S_{LC} + S_G. \tag{21}$$

Here, W is the discretized version of the work (5),

$$W[\{\varphi_j,\alpha_j\}] = \sum_{j=1}^{N}(\alpha_j - \alpha_{j-1})\frac{\partial H_{LC}(\varphi_j,q_j;\alpha_j)}{\partial\alpha}, \tag{22}$$

and S_{LC} is the discrete form of the Martin-Siggia-Rose action[30] of the LC circuit

$$\begin{aligned} S_{LC} = {} & \sum_{j=1}^{N}\tilde{q}_j\left(\frac{\hbar}{e}\frac{\varphi_j-\varphi_{j-1}}{\Delta t} - \frac{\partial H_{LC}(\varphi_j,q_j;\alpha_j)}{\partial q}\right)\Delta t \\ & + \tilde{\varphi}_j\left(-\frac{\hbar}{e}\frac{q_{j+1}-q_j}{\Delta t} - \frac{\partial H_{LC}(\varphi_j,q_j;\alpha_j)}{\partial\varphi}\right)\Delta t \\ & + \frac{\hbar}{e}(q_{N+1}\tilde{\varphi}_N - q_1\tilde{\varphi}_0). \end{aligned} \tag{23}$$

In what follows, we will omit unimportant boundary terms in the last line of this expression. Finally, the action of the conductor takes the form

$$i\frac{S_G}{\hbar} = \sum_{j=1}^{N}\Delta t\,\mathcal{F}_G\left(-\tilde{\varphi}_j,\frac{\hbar(\varphi_j-\varphi_{j-1})}{e\Delta t};B\right), \tag{24}$$

where

$$\begin{aligned} \mathcal{F}_G(\lambda,V;B) = \lim_{t\to\infty}\frac{1}{t}\ln \mathrm{Tr}[& e^{-iH_G(-\lambda/2;B)t/\hbar} \\ & \times\rho_q(V)\,e^{iH_G(\lambda/2;B)t/\hbar}] \end{aligned} \tag{25}$$

is the standard CGF of a quantum conductor.[22,23]

In order to demonstrate the equivalence of the abstract formulation of the problem in terms of the CF (20) to the Langevin equation approach [Eqs. (7) and (8)], we evaluate the integrals over $\tilde{q}_j$ and $\tilde{\varphi}_j$ in Eq. (20) and with the help of Eqs. (10), (11), and (22)–(24) transform it to the form

$$\begin{aligned} \mathcal{Z} = {} & \lim_{N\to\infty}\int\frac{d\varphi_0 dq_0}{2\pi e}e^{-\beta[H_{LC}(\varphi_0,q_0;\alpha_0)-F_{LC}(\alpha_0)]} \\ & \times\left(\prod_{j=1}^{N}\int dq_j d\varphi_j\sum_{\Delta Q_j/e}\right)e^{i\xi W} \\ & \times\Bigg[\prod_{j=1}^{N}\delta\left(\varphi_j-\varphi_{j-1}-\frac{e}{\hbar}\frac{\partial H_{LC}(\varphi_j,q_j;\alpha_j)}{\partial q}\Delta t\right) \\ & \times\delta\left(-q_{j+1}+q_j-\Delta Q_j-\frac{e}{\hbar}\frac{\partial H_{LC}(\varphi_j,q_j;\alpha_j)}{\partial\varphi}\Delta t\right) \\ & \times p[\Delta t,\Delta Q_j,\hbar(\varphi_j-\varphi_{j-1})/(e\Delta t);B]\Bigg], \end{aligned} \tag{26}$$

where p has been introduced in Eq. (10). This expression is nothing but the representation of the discrete Langevin equation in the presence of the non-Gaussian white noise ΔQ. It is easy to see that these equations become equivalent to Eqs. (7) and (8) in the limit $N\to\infty$, $\Delta t\to 0$.

Equation (20) is the main result of this section providing an exact formal expression for the characteristic function (CF) of a system governed by Langevin equations with non-Gaussian white noise [see Eqs. (7), (8), and (9)]. The quasistationary approximation, which we have used above, has been used earlier to analyze properties of Josephson junction threshold detectors.[31] It is also very similar in spirit to the stochastic path-integral approach.[32]

IV. SADDLE-POINT APPROXIMATION UNDER CONSTANT BIAS VOLTAGE

Let us consider the effect of a constant bias voltage $V_{\rm ext} = $ const. In the limit of sufficiently long measurement time τ, we may use the saddle-point approximation to evaluate the integral (20). Considering the limit $N\to\infty$, we solve the equations $\delta S_t/\delta\varphi(t) = \delta S_t/\delta\tilde{\varphi}(t) = \delta S_t/\delta q(t) = \delta S_t/\delta\tilde{q}(t) = 0$. The corresponding solution reads as $\tilde{q}(t) = 0$, $\dot{q}(t) = 0$, $\varphi(t) = eV_{\rm ext}t/\hbar + \varphi(0)$, and $\tilde{\varphi}(t) = -\xi eV_{\rm ext}$. In this approximation, the CGF of the work (14) acquires a simple form

$$\begin{aligned} \mathcal{F}(\xi) & \approx \frac{i}{\hbar}(\tilde{\varphi}+\xi eV_{\rm ext})\frac{\partial H_{LC}}{\partial\varphi} + \mathcal{F}_G(\xi eV_{\rm ext},V_{\rm ext};B) \\ & = \mathcal{F}_G(\xi eV_{\rm ext},V_{\rm ext};B). \end{aligned} \tag{27}$$

It is interesting that in this regime the contributions S_{LC} and $\hbar\xi W$ in the total action (21) cancel each other. Thus, we have proven that the statistical properties of the work done on the classical LC circuit and those of the current flowing through the quantum conductor are the same. This interesting conclusion remains valid only in the saddle-point approximation, which works well as long as $\sqrt{L/(CR^2)} \ll 1$, where R is the resistance of the quantum conductor.

By virtue of the FT (12), which is valid for an isolated conductor, the CGF of the work satisfies

$$\mathcal{F}(\xi;B) = \mathcal{F}(-\xi+i\beta;-B). \tag{28}$$

This identity is equivalent to the work FT (1).

V. WORK FLUCTUATION THEOREM FOR COUPLED CLASSICAL AND QUANTUM SYSTEMS

In this section, we show that the FT (28) holds even beyond the saddle-point approximation as long as one uses the quasistationary approximation introduced in Sec. III. The basis of our proof is the FT as given by Eq. (12) for the charge transport through the quantum conductor. As a first step, we apply the FT (12) N times for every time interval $t_j < t < t_{j+1}$. Since the quantum phase $-\tilde{\varphi}_j$ and the combination $\hbar(\varphi_j-\varphi_{j-1})/e\Delta t$ play the same role in the action (24) as the counting field λ and the bias voltage V in Eq. (12), respectively, the transformation $\lambda\to-\lambda+i\beta V$ in Eq. (12) translates into the replacement $-\tilde{\varphi}_j\to\tilde{\varphi}_j+i\beta\hbar(\varphi_j-\varphi_{j-1})/e\Delta t$. Similarly, we should replace the quantum charge $-\tilde{q}_j$ with $\tilde{q}_j+i\beta\hbar(q_j-q_{j-1})/e\Delta t$. At the next step, we invert the signs of the quantum phase and charge. By combining these two operations, we arrive at the following transformation in Eq. (20):

$$\begin{aligned} \tilde{\varphi}_j & \to \tilde{\varphi}_j - i\beta\hbar(\varphi_j-\varphi_{j-1})/\Delta t, \\ \tilde{q}_j & \to \tilde{q}_j - i\beta\hbar(q_j-q_{j-1})/\Delta t \end{aligned} \tag{29}$$

($j = 1,\ldots,N$). One can show that its Jacobian equals to 1. Under the transformation (29), the action of the quantum conductor (24) acquires the form

$$S_G \to -i\hbar \sum_{j=1}^{N} \Delta t\, \mathcal{F}_G\left(\tilde{\varphi}_j, \frac{(\varphi_j - \varphi_{j-1})\hbar}{e\Delta t}; -B\right). \tag{30}$$

Likewise, the action for the LC circuit (23) becomes

$$S_{LC} \to S_{LC} + i\beta\hbar\, Q_h\,, \tag{31}$$

$$Q_h = \sum_{j=1}^{N}\left[(q_j - q_{j-1})\frac{\partial H_{LC}(\varphi_j,q_j;\alpha_j)}{\partial q} + (\varphi_j - \varphi_{j-1})\frac{\partial H_{LC}(\varphi_j,q_j;\alpha_j)}{\partial \varphi}\right], \tag{32}$$

where we neglected irrelevant terms. The combination Q_h may be interpreted as the heat absorbed by the quantum conductor. With its aid, the first law of thermodynamics, or energy conservation, may be written in the form

$$H_{LC}(\varphi_N,q_N;\alpha_N) - H_{LC}(\varphi_0,q_0;\alpha_0) \approx Q_h + W\,,$$

and thus we find

$$S_{LC} \to S_{LC} + i\beta\hbar\, [H_{LC}(\varphi_N,q_N;\alpha_N) - H_{LC}(\varphi_0,q_0;\alpha_0) - W]\,. \tag{33}$$

We rewrite it as

$$iS_{LC}/\hbar - \beta H_{LC}(\varphi_0,q_0;\alpha_0) \to iS_{LC}/\hbar - \beta[H_{LC}(\varphi_N,q_N;\alpha_N) - W]. \tag{34}$$

It indicates how a part of the exponent in Eq. (20), the action and the exponent of the initial density matrix of the LC circuit, transforms after the variable transformations (29).

Next, we perform the time-reversal operation[33] $t \to -t$, $q \to -q$, and $\tilde{q} \to -\tilde{q}$ without changing the sign of φ and $\tilde{\varphi}$. Under this transformation, the external driving is reversed and the phase $\alpha(t)$ is replaced by a time-reversed one $\alpha_R(t) = \alpha(-t)$. In the discrete form, this transformation reads as

$$\tilde{\varphi}_j \to \tilde{\varphi}_{N-j}\,, \quad \varphi_j \to \varphi_{N-j}, \tag{35}$$

$$\tilde{q}_j \to -\tilde{q}_{N-j+1}\,, \quad q_j \to -q_{N-j+1}, \tag{36}$$

and $\alpha_{N-j+1} = \alpha_{R\,j}$. Here, we choose the subscripts of the charge variables to be $N-j+1$ instead of $N-j$ in order to satisfy our convention that the subscripts of q are not smaller than the subscripts of φ. Keeping in mind the properties of the Hamiltonian, $H_{LC}(\varphi,q;\alpha) = H_{LC}(\varphi,-q;\alpha)$ and $\partial H_{LC}(\varphi,q;\alpha)/\partial q = -\partial H_{LC}(\varphi,-q;\alpha)/\partial q$, we arrive at the following transformations, up to $\mathcal{O}(1)$ in Δt:

$$W \to -W_R, \tag{37}$$

$$S_G \to S_{G,R}. \tag{38}$$

Applying the transformations (35) and (36) to the right-hand side of the expression (34), we obtain

$$iS_{LC}/\hbar - \beta H_{LC}(\varphi_0,q_0;\alpha_0) \to iS_{LC,R}/\hbar - \beta H_{LC}(\varphi_0,q_0;\alpha_{R,0}) - \beta\, W_R, \tag{39}$$

where we used the fact that $\alpha_j - \alpha_{j-1} \propto \Delta t$ and kept only terms of zeroth order in Δt. The transformed work and the action of the LC circuit, W_R and $S_{LC,R}$, are obtained from W [Eq. (22)] and S_{LC} [Eq. (23)] by means of the replacement $\alpha \to \alpha_R$. Finally, after the transformations (35) and (36), the action of the conductor becomes

$$S_{G,R} = -i\hbar \sum_{j=1}^{N} \Delta t\, \mathcal{F}_G\left(\tilde{\varphi}_j, -\frac{(\varphi_j - \varphi_{j-1})\hbar}{e\Delta t}; -B\right). \tag{40}$$

Note that the second argument of the CGF, i.e., the voltage drop, changes its sign. It indicates, in turn, that the source and drain electrodes of the quantum conductor are effectively interchanged after the time reversal.

After all these manipulations, we can derive the following identity:

$$\begin{aligned}\langle e^{i\xi W}\rangle &= \lim_{N\to\infty}\int \frac{d\varphi_0 dq_0}{2\pi e} e^{-\beta[H_{LC}(\varphi_0,q_0;\alpha_{R0}) - F_{LC}(\alpha_{R0})]} \\ &\quad\times\left(\prod_{j=1}^{N}\int\frac{dq_j d\varphi_j}{2\pi e}\int\frac{d\tilde{q}_j d\tilde{\varphi}_j}{2\pi e}\right) \\ &\quad\times e^{i(-\xi+i\beta)W_R + i[S_{LC,R}+S_{G,R}]/\hbar} \\ &\quad\times e^{-\beta[F_{LC}(\alpha_N) - F_{LC}(\alpha_0)]} \end{aligned} \tag{41}$$

$$= \langle e^{i(-\xi+i\beta)W_R}\rangle_R\, e^{\beta[F_{LC}(\alpha_0) - F_{LC}(\alpha_N)]}\,, \tag{42}$$

which can be written in an equivalent form as

$$\mathcal{Z}(\xi) = e^{-\beta\{F_{LC}[\alpha(\tau/2)] - F_{LC}[\alpha(-\tau/2)]\}}\,\mathcal{Z}_R(-\xi + i\beta). \tag{43}$$

After Fourier transformation, we arrive at the work FT

$$\frac{P(W)}{P_R(-W)} = e^{\beta\{F_{LC}[\alpha(-\tau/2)] - F_{LC}[\alpha(\tau/2)]\} + \beta W}. \tag{44}$$

This form is more general than the form (1) quoted in the Introduction and is valid also for time-dependent bias voltages $V_{\mathrm{ext}}(t)$. The subscript R in Eqs. (43) and (44) indicates the time-reversal operation. The latter consists of three steps: (i) interchanging of the source and drain electrodes of the quantum conductor, (ii) replacement of $\alpha(t)$ with $\alpha_R(t) = \alpha(-t)$, and (iii) reversal of the magnetic field $B \to -B$. This completes the proof of the FT in general case.

The general time-reversal operation described above may be difficult to realize in experiment. Fortunately, it may be simplified in many cases. Consider, for example, the model introduced in Sec. II. Since the Hamiltonian of the LC circuit has the symmetry $H_{LC}(\varphi,q;\alpha) = H_{LC}(-\varphi,-q;-\alpha)$, one can perform an additional transformation $\varphi_j \to -\varphi_j$, $\tilde{\varphi}_j \to -\tilde{\varphi}_j$, $q_j \to -q_j$, $\tilde{q}_j \to -\tilde{q}_j$ in Eq. (42), which results in the following identity:

$$\mathcal{Z}[\tau,\xi,B;\alpha(\tau')] = \mathcal{Z}[\tau,-\xi+i\beta,-B;\,-\alpha(-\tau')]. \tag{45}$$

Here, we have also used the fact that the free energy of the LC oscillator does not depend on α and hence $F_{LC}[\alpha(-\tau/2)] - F_{LC}[\alpha(\tau/2)] \equiv 0$. Next, if the external bias voltage is constant, then $\alpha(\tau') = -\alpha(-\tau') = eV_{\mathrm{ext}}\tau'$, and the FT (45) becomes equivalent to Eq. (1). Thus, in order to perform the time reversal in this system experimentally, one just needs to change the sign of the magnetic field.

The simplified version of the FT (1) is also valid if the quantum conductor has an antisymmetric I-V curve, $I(-V) = -I(V)$. More precisely, it is valid when the CGF of the conductor satisfies the symmetry

$$\mathcal{F}_G(\lambda,V;B) = \mathcal{F}_G(-\lambda,-V;B), \tag{46}$$

and Eq. (42) reduces to Eq. (45) regardless of the symmetries of the Hamiltonian H_{LC}.

As usual, from the FT (45), one can derive various other relations including the fluctuation-dissipation theorem and Onsager's relations in presence of magnetic field. Some of them were analyzed, e.g., in Refs. 3–5.

We conclude this section with two remarks. First, we would like to emphasize once again that our approach takes into account the back action of the LC circuit on the quantum conductor. Moreover, this back action is essential to ensure the validity of the FT. Second, our analysis may also be interpreted as the proof of the FT for a Langevin equation with non-Gaussian white noise (9), thus extending the existing proof of the FT for the Langevin equation with Gaussian noise.[34]

VI. QUANTUM-DOT AHARONOV-BOHM INTERFEROMETER

So far, our analysis has been general and we did not specify the nature of the quantum conductor. In this section, we consider a specific system, an Aharonov-Bohm (AB) interferometer replacing the quantum conductor in the setup shown in Fig. 1(a). This system is suited for our purposes since its conductance is sensitive to the magnetic field. The latter is an additional control parameter, which allows one to test the predictions of the FT in more detail. Furthermore, the first experimental test of the FT in the quantum regime has been carried out with an AB ring,[20] although it was restricted to testing the universal relations between the linear response of the noise and second nonlinear conductance, which were derived from the FT.[4,5] Based on this success, we expect that a complete verification of the FT in this system is also possible. It would require the measurement of the probability distribution of the work $P(W;B)$ using the setup shown in Fig. 1(a) and following the method described in Sec. II. Once the probability distribution is obtained, it can be first compared to the theoretical prediction shown in Fig. 3(a) and, second, the work FT (1) can be directly tested. The expected results of such tests under various configurations of the magnetic field are shown in Fig. 3(b). This figure clearly illustrates the importance of the magnetic field inversion for the validity of the FT.

To simplify the analysis, we consider an Aharonov-Bohm (AB) interferometer with a quantum dot (QD) embedded in one of its arms[35] (Fig. 2). The magnetic flux threading the ring

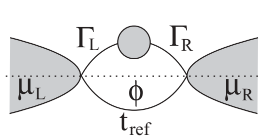

FIG. 2. Aharonov-Bohm interferometer with a quantum dot embedded in one arm. The magnetic flux Φ threads the ring, and the electron wave function acquires the AB phase ϕ for an electron traveling in clockwise direction. The energy parameters $\Gamma_{L/R}$ characterize the strength of the tunnel couplings between the quantum dot and the left/right leads. An electron can also be transmitted through the lower reference arm, characterized by the tunneling amplitude $t_{\rm ref}$. The left/right chemical potentials are $\mu_{L/R}$.

induces an additional phase difference between the electron waves traveling through the upper and the lower arms. We also assumed that the mutual inductance is small so that the magnetic flux induced by the external inductance is much smaller than the applied magnetic flux. This regime may easily be achieved in the experiment if one makes the area of the AB ring sufficiently small.

In this setup, the Coulomb interaction and the magnetic field induce an asymmetry in the nonequilibrium current distribution.[25] The microscopic theory of this system based on an extended Anderson model has been developed in Ref. 35. Here, we briefly summarize its key points.

The S matrix of the QD AB ring[35–37]

$$\boldsymbol{S}(E;\phi) = \begin{pmatrix} S_{LL}(E;\phi) & S_{LR}(E;\phi) \\ S_{RL}(E;\phi) & S_{RR}(E;\phi) \end{pmatrix} \tag{47}$$

satisfies the microreversibility $S_{rr'}(E;\phi) = S_{r'r}(E;-\phi)$. Its four components read as

$$S_{LL/RR} = 1 - \frac{i\Gamma_{LL/RR} + t_{\rm ref}\sqrt{\Gamma_L\Gamma_R}\cos\phi + t_{\rm ref}^2\,E/2}{\Delta(E;\phi)}, \tag{48}$$

$$S_{RL/LR} = -i\,\frac{e^{\pm i\phi}t_{\rm ref}\,E + \sqrt{\Gamma_L\Gamma_R}}{\Delta(E;\phi)}, \tag{49}$$

$$\Delta = \frac{t_{\rm ref}\sqrt{\Gamma_L\Gamma_R}\cos\phi}{2} + \left(1 + \frac{t_{\rm ref}^2}{4}\right)E + i\frac{\Gamma}{2}, \tag{50}$$

where we set the dot energy level as $\epsilon_{\rm D} = 0$. Here, $\Gamma_{L/R}$ are the tunnel couplings between the quantum dot and the left/right leads $\Gamma = \Gamma_L + \Gamma_R$. An electron can also be transmitted through the lower reference arm, characterized by the tunneling amplitude $t_{\rm ref}$. The AB phase $\phi = 2\pi\,\Phi/\Phi_0$ is given by the ratio of the magnetic flux Φ threading the ring and the flux quantum $\Phi_0 = hc/e$. It changes sign when the magnetic field is reversed, $\phi(B) = -\phi(-B)$.

Within the mean-field approximation for the onsite Coulomb interaction U, the CGF of the AB interferometer is given by the following expression:[35]

$$\mathcal{F}_G(\lambda,V;B) = \mathcal{F}_{\rm AB}(\lambda,V,v_c,v_q;B) - Mv_cv_q/U. \tag{51}$$

Here, M denotes the full degeneracy including channel and spin degeneracies, while v_c and v_q are classical and quantum Keldysh components, respectively, of the fluctuating dot potentials multiplied by the electron charge. The CGF for the QD AB ring is given by the usual formula applicable to small conductors[22]

$$\mathcal{F}_{\rm AB} = \frac{M}{h}\int dE\,\ln\det\left[\mathbf{1} + \boldsymbol{f}\boldsymbol{K}\right], \tag{52}$$

$$\boldsymbol{K} = e^{i\boldsymbol{\lambda}/2}\,\boldsymbol{S}(E - v_c - iv_q/2)^\dagger\,e^{-i\boldsymbol{\lambda}}\,\boldsymbol{S}(E - v_c - iv_q/2) \times e^{i\boldsymbol{\lambda}/2} - \mathbf{1}, \tag{53}$$

where $\mathbf{1}$ is a 2×2 unit matrix, $\boldsymbol{\lambda} = {\rm diag}(\lambda,0)$, and $\boldsymbol{f} = {\rm diag}[f(E-\mu_L), f(E-\mu_R)]$ is the matrix of the Fermi distribution function $f(E) = 1/[\exp(\beta E)+1]$ and $\mu_{L/R} = \kappa_{L/R}eV$ are the left/right chemical potentials. In the limit of large number of conducting channels $M\to\infty$, one can use mean-field theory while taking the integral over the fluctuating parameters v_c, v_q. In this case, they are determined by two

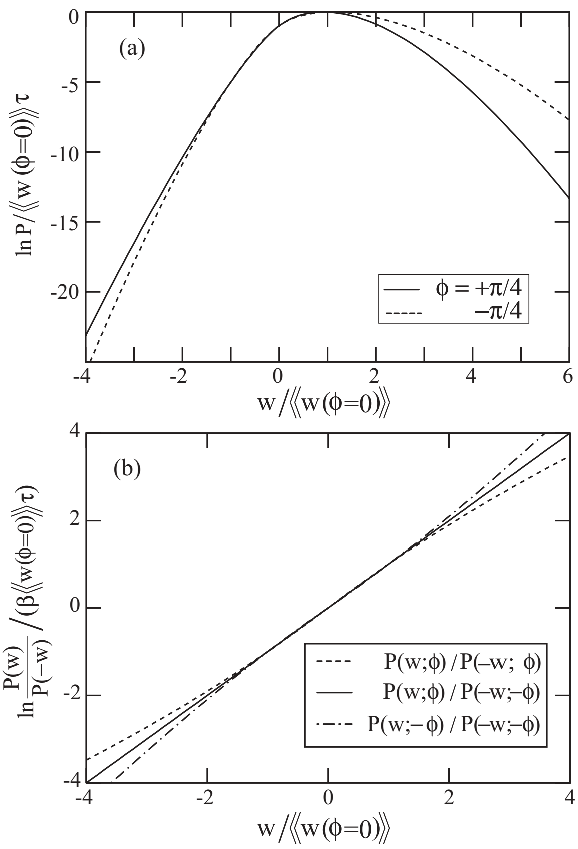

FIG. 3. (a) Probability distributions of the work for $\phi = \pm\pi/4$. (b) Ratio between the positive and negative work probability distributions. When the direction of magnetic field is also reversed, the steady-state work fluctuation theorem is satisfied. The parameters are $U/\Gamma = 4$, $V = \Gamma$, $k_B T = 0.2\Gamma$, $\Gamma_L = 0.25\Gamma$, $\Gamma_R = 0.75\Gamma$, and $t_{\rm ref} = 0.25$ and $\kappa_L = -\kappa_R = 0.5$. The average is $\langle\langle w(\phi = 0)\rangle\rangle/(MeV_{\rm ext}) \approx 2.9 \times 10^{-2}\Gamma/\hbar$. The plots are not sensitive to the specific value of Γ since it is canceled after the work w is normalized to $\langle\langle w(\phi = 0)\rangle\rangle$.

coupled saddle-point equations

$$v_q = \frac{U}{M}\frac{\partial \mathcal{F}_{\rm AB}}{\partial v_c}, \quad v_c = \frac{U}{M}\frac{\partial \mathcal{F}_{\rm AB}}{\partial v_q}. \quad (54)$$

Substituting the solution of these equations back in the Eq. (51), we obtain the CGF of the current flowing though the AB ring. Afterwards, we apply Eq. (27) and derive the CGF of the work from it.

Returning to the work fluctuation theorem, we note that in the limit of long measurement time τ, it is more convenient to express the result in terms of the power $w = W/\tau$ instead of the work W. Performing the inverse Fourier transformation of CF of the work, we write its probability distribution in the form

$$P(w) \approx \frac{1}{2\pi}\int d\xi\, e^{-i\tau w\xi + \tau\mathcal{F}(\xi)}, \quad (55)$$

with $\mathcal{F}$ given by Eq. (27). Within the saddle-point approximation, the distribution function $P(w)$ acquires the form

$$\ln P(w) \approx \tau[\mathcal{F}_G(\xi^* eV_{\rm ext}, V_{\rm ext}; B) - i\,\xi^* w], \quad (56)$$

where ξ^* is the saddle-point value of ξ to be found from the equation

$$w = \frac{\partial}{\partial(i\xi^*)}\mathcal{F}_G(\xi^* eV_{\rm ext}, V_{\rm ext}; B). \quad (57)$$

Equations (54) and (57) can be easily solved numerically, which allows one to compute the distribution function $P(w)$ given by Eq. (56).

Figure 3(a) shows the probability distributions of the work for negative and positive values of the AB phase. For the chosen parameters, they are both non-Gaussian and differ significantly when the direction of the magnetic field is reversed. Figure 3(b) shows the ratio between the probability distributions for positive and negative work. The solid line, obtained with appropriate change of the sign of the magnetic field, satisfies the work FT. For comparison, we also show the ratios when the magnetic field is not reversed, in which case the work FT would not be satisfied (dashed and dotted-dashed lines).

VII. SUMMARY

We have proposed an experimental setup which may be used to test the quantum fluctuation theorem. It consists of a quantum conductor coupled to a classical LC circuit. We note that the usual definition of the work done by an external force on a quantum system[1,18,19] is not convenient when applied to transport experiments in mesoscopic structures. Therefore, we propose an alternative definition of the work (6) by expressing it through the degrees of freedom of a classical LC oscillator, which may be measured by conventional techniques. Our approach takes into account the back action of the LC circuit on the quantum conductor. We have proven the work fluctuation theorem for this system and have shown that under constant bias voltage and with properly chosen parameters of the LC circuit, the probability distribution of the work is directly related to the probability distribution of current flowing through the quantum conductor. We applied our theory also to a quantum-dot Aharonov-Bohm interferometer and demonstrated the magnetic field induced asymmetry in the work distribution. We expect that the probability distribution of the work can be measured with currently developed ultrafast and ultrasensitive on chip electrometers, such as single-electron transistors or quantum point contacts. Finally, we noted that the classical measurement system coupled to the quantum conductor is effectively described by a Langevin equation with non-Gaussian white noise. Therefore, our analysis also extends the proof of the fluctuation theorem to this situation.

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