# Penetration of the magnetic field into the twinning plane in type-I and -II superconductors

S. V. Mironov[1] and A. Buzdin[2]
[1]*Institute for Physics of Microstructures, Russian Academy of Sciences, 603950, Nizhny Novgorod, GSP-105, Russia*
[2]*Institut Universitaire de France and University Bordeaux, LOMA UMR-CNRS 5798, F-33405 Talence Cedex, France*
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It is demonstrated that in the type-I and -II superconductors with weakly transparent twinning planes (TP) the penetration of the external parallel magnetic field into the region of the twinning plane can be energetically favorable. In the type-I superconductors the twinning planes become similar to Josephson junctions and the parallel magnetic field penetrates into the TP in the form of Josephson-like vortices. This leads to an increase in the critical magnetic field values. The corresponding phase diagram in the parameter plane "temperature-magnetic field" essentially differs from the one obtained without taking the finite value of the magnetic field near the TP into account. A comparison between the obtained phase diagrams and experimental data for different type-I superconductors can allow to estimate the value of the TP transparency, which is the only fitting parameter in our theory.

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The phenomenon of the twinning plane superconductivity (TPS) has been the subject of intensive investigations during the last three decades (see Ref. 1 for a review). A twinning plane (TP) may produce more favorable conditions for the superconducting nucleation compared with a bulk crystal, and a superconducting layer localized on the TP can appear even above the bulk critical temperature $T_c$. Recently interest to the physics of twins in superconductors was renewed since it was shown experimentally that TP affects the properties of many relatively new superconductors which belong to the pnictide family.[2–7] In these superconductors, twinning planes can enhance locally the superfluid density[5,6] or influence the vortex pinning.[7,8] In particular, it was demonstrated[8] that in $Ba(Fe_{1-x}Co_x)_2As_2$ twinning planes repulse vortices and act as strong barriers for vortex motion. Thereupon, the theoretical investigations of the magnetic properties of the TP in superconductors are of current importance.

Twinning planes can effectively screen a parallel magnetic field which leads to the increase in the value of the critical field as a function of temperature. The corresponding phase diagrams $H_c(T)$ for the absolutely transparent TP were studied theoretically within the phenomenological Ginzburg-Landau formalism both for the type-I[9–13] and the type-II superconductors.[9] In particular, for the ultra type-I superconductors it was shown[11–13] that for small but finite values of the Ginzburg-Landau parameter $\kappa$ the magnetic field penetration into the superconducting area leads to the corrections to the TPS free energy proportional to $\kappa^{(n+1)/2}$ ($n = 0,1,\ldots$) and in practice only the term $\propto\kappa^{1/2}$ plays an important role while the terms of the order $\kappa^{3/2}$ and higher can be neglected since they weakly contribute to the resulting $H_c(T)$ diagram. The resulting dependencies $H_c(T)$ do not contain any fitting parameters, which allowed their quantitative experimental verification for concrete superconductors.[14,15] In some cases[16,17] the local enhancement of superconductivity may occur near the sample surface. The upper critical field for this situation was considered in Ref. 18. However, for Sn (the type-I superconductor) the superconductivity in a magnetic field should appear as the first order transition (except the very narrow region near the critical temperature) and this field corresponds to the overcooling of the normal phase.

Practically, for TP with finite electron transparency the standard Ginzburg-Landau free energy functional should be generalized by inserting an additional term which breaks the requirement of the order parameter continuity at TP (Refs. 19 and 20). The influence of the finite TP transparency on the upper critical magnetic field in the type-II superconductors was analyzed in Ref. 21. At the same time for superconductors of the I type it was found that in the case of weakly transparent TP the essentially asymmetric distributions of the order parameter relative to the TP can become energetically more favorable than the symmetric ones.[22] The striking prediction of this paper is that under certain conditions the order parameter is nonzero only at one side of the TP while at another side it should be zero.

We would like to point out that all theoretical results which have been obtained up to now are based on the assumption that the superconducting nucleus localized near TP is screening the magnetic field effectively: It was believed that in the type-I superconductors the magnetic field can penetrate only into the region which is far from the TP (the magnetic field value at the TP is exponentially small) while in the type-II superconductors the magnetic field value has its minimum at the TP.

In the present paper we show that for both type-I and -II superconductors with weakly transparent TP the parallel magnetic field can fully penetrate into the twinning plane region and the corresponding state is energetically favorable. For the type-II superconductors this fact results in small corrections to the magnetic susceptibility only. At the same time for the type-I superconductors the magnetic field penetration into the center of the TP leads also to essential changes in the dependence of the critical magnetic field on temperature due to the negative contribution to the free energy, which has the order of $\kappa$. Note that the obtained solutions have lower energy than the ones found in Ref. 22. At the same time the corresponding profiles of the order parameter are symmetric relative to the TP.

Let us consider a bulk superconducting sample with a single twinning plane at $z = 0$. The external magnetic field $\mathbf{H}$ is assumed to have only the $y$ component. We choose the corresponding vector potential in the form $A_x(z) = Hz$ so that the order parameter $\psi$ depends only on $x$ and $z$. We will use the standard Ginzburg-Landau free energy functional to describe the local enhancement of the superconductivity on the TP[11,21]

$$G = \int dxdz\Bigg\{\frac{\hbar^2}{4m}\left|\left(\nabla - \frac{2ie}{\hbar c}\mathbf{A}\right)\psi\right|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{(\mathbf{B}-\mathbf{H})^2}{8\pi} + \frac{\hbar^2}{4m}\left[\frac{8}{\rho}|\psi_+ - \psi_-|^2 - \frac{1}{2\xi_s}(|\psi_+|^2 + |\psi_-|^2)\right]\delta(z)\Bigg\}, \quad (1)$$

where $a = \alpha(T - T_c)$, $\mathbf{B} = \mathrm{rot}\mathbf{A}$, $\rho$ is a phenomenological constant describing the finite transparency of the TP, $\psi_\pm = \psi(x,y,\pm 0)$ and the value $\xi_s$ will be defined below. Let us also introduce the temperature-dependent coherence length as $\xi(T) = \hbar/\sqrt{4m\alpha(T-T_c)}$. The last two terms in the functional (1) correspond to the energy of the TP: (i) The term with $\xi_s$ describes the change in the superconducting coupling constant near the TP which leads to the local enhancement of the critical temperature and (ii) the term with $\rho$ describes the Josephson-like coupling between the two sides of the TP.

For further analysis it is convenient to rewrite the functional (1) in dimensionless variables

$$t = \frac{T - T_c}{T_s - T_c}, \quad H_s = H_c(t=-1), \quad \xi_s = \xi(t=-1),$$
$$\psi_s = \psi_0(t=-1), \quad \tilde{\psi} = \frac{\psi}{\psi_s}, \quad \tilde{x} = \frac{x}{\xi_s}, \quad \tilde{z} = \frac{z}{\xi_s}, \quad (2)$$
$$h = \frac{H}{H_s}, \quad \tilde{\mathbf{A}} = \frac{\mathbf{A}}{\kappa H_s \xi_s}, \quad r = \frac{\rho}{\xi_s}, \quad G_s = \frac{\xi_s H_s^2}{8\pi}.$$

Here $T_s$ is the critical temperature of the superconductor with the twinning plane ($T_s > T_c$), $\psi_0(T) = \sqrt{|a|/b}$ and $H_c(T) = \sqrt{4\pi a^2/b}$ are the values of the order parameter wave function and the critical filed for the bulk superconductor without twinning plane; $\kappa = mc\sqrt{b}/\sqrt{2\pi}e\hbar$ is the Ginzburg-Landau parameter. In what follows we will omit the tildes since we will consider only dimensionless expressions. Then the functional for the free-energy per unit length along the TP reads as

$$G = G_s \int dxdz\Bigg\{2\left|\frac{\partial\psi}{\partial z}\right|^2 + 2\left|\left(\frac{\partial}{\partial x} - \frac{iA}{\sqrt{2}}\right)\psi\right|^2 + 2t|\psi|^2 + |\psi|^4 + \left[\frac{2}{r}|\psi_+ - \psi_-|^2 - 2(|\psi_+|^2 + |\psi_-|^2)\right]\delta(z) + \left(\kappa\frac{dA}{dz} - h\right)^2\Bigg\}. \quad (3)$$

Let us start with the case of the absolutely opaque TP ($r = \infty$) in the ultra type-I superconductor ($\kappa \ll 1$) and consider the order parameter wave function in the form $\psi(x,z) = \varphi(z)\exp\{i\theta(x)\}$. Varying the functional (3) by $\psi^*$ and $A$ for $z \neq 0$ we obtain the equations

$$-\partial_z^2\varphi + t\varphi + \varphi^3 + \varphi(A/\sqrt{2} - \partial_x\theta)^2 = 0,$$
$$\partial_z^2 A = (\sqrt{2}\varphi^2/\kappa^2)(A/\sqrt{2} - \partial_x\theta) \quad (4)$$

with the boundary conditions $\partial_z\varphi_+ = -\varphi_+$ and $\partial_z\varphi_- = \varphi_-$. Note that the system (4) has the first integral

$$(\partial_z\varphi)^2 - t\varphi^2 - \frac{\varphi^4}{2} - \varphi^2\left(\frac{A}{\sqrt{2}} - \partial_x\theta\right)^2 + \frac{\kappa^2}{2}(\partial_z A)^2 = \frac{h^2}{2}. \quad (5)$$

The modulus of the order parameter on the TP is given by[11] $\varphi_0^2 = (1-t) + \sqrt{(1-t)^2 - h^2}$. In what follows we will assume that the magnetic field penetration into the TP weakly affects the order parameter modulus near the TP. The validity of this assumption will be discussed below. Also we would like to mention that the order parameter decay length $l$ is proportional to $t^{-1/2}$ while the magnetic field penetration length is proportional to $\kappa/\sqrt{1-t}$ and for $\kappa \ll 1$ and $(1-t) \gg \kappa^2$ it is much less than $l$. This allows one to consider the constant order parameter value in the region near the TP where the local magnetic field $b = \kappa\partial_z A$ is nonzero. We will assume that $b(z = \pm\infty) = h$ and $b(z=0) = h_0$, where $h_0 \leqslant h$. Then from the solution of Eq. (4) on the vector potential we obtain

$$b(z) = h_0\exp\left\{-\frac{\varphi_0}{\kappa}|z|\right\}, \quad \partial_x\theta = \frac{\mathrm{sign}z}{\sqrt{2}}\frac{h_0}{\varphi_0}. \quad (6)$$

The resulting correction to the free energy value per unit length along the $x$ axis has the form $G = G_0 + G_s(2\kappa/\varphi_0)(h_0^2 - 2hh_0)$, where $G_0$ is the free energy obtained without taking the penetration of the magnetic field to the TP into account[11,13]

$$\frac{G_0}{G_s} = \left[4\sqrt{2}\int_0^{\varphi_0}\sqrt{\varphi^4 + 2t\varphi^2 + h^2}d\varphi - 4\varphi_0^2 - 2.06\sqrt{\kappa h^3}\right].$$

From the obtained dependence $G(h_0)$ one can see that its minimum corresponds to the case $h_0 = h$ and the resulting free energy value is

$$G = G_0 - G_s(2\kappa/\varphi_0)h^2. \quad (7)$$

Thus it is energetically favorable for the magnetic field to penetrate fully into the TP. Then solving numerically the inequality $G \leqslant 0$ we obtain the temperature dependence of the critical magnetic field $h_c(t)$, which is shown in Fig. 1 (red solid curve).

Note that the magnetic field (6) does not affect the value of the order parameter at the TP. Indeed substituting the corresponding vector potential at $z = 0$ to the first integral (5) and one can obtain that at the TP the order parameter $\varphi^2(z=0) = 2(1-t) - h^2/\varphi_0^2 \equiv \varphi_0^2$.

The expression for the local magnetic field (6) allows one to calculate the correction to the magnetic moment of the TP, which has the form [here we consider only the correction due to the magnetic field (6) and restore the dimension of the expression]

$$\Delta M_{(I)} = \int_{-\infty}^{\infty}\frac{B(z)}{4\pi}dz = \frac{\kappa H\xi_s}{2\pi\varphi_0}. \quad (8)$$

Comparing this value in the limit $h \to 0$ with the diamagnetic moment $M_{d(I)}$ (Ref. 1) due to the expulsion of the magnetic field from the superconducting region we obtain that the ratio $\eta_{(I)} = |\Delta M_{(I)}/M_{d(I)}| = \kappa(\ln\kappa^{-1})^{-1}\sqrt{t/2(1-t)}$. Practically, $\eta_{(I)} \ll 1$ for all temperatures where the TPS has the I type. Indeed, for example, for tin with $\kappa = 0.13$ the condition $\eta_{(I)} \sim 1$ gives $(1-t) < 2 \times 10^{-3}$. For such small values of $(1-t)$ the obtained results are not applicable since for $(1-t) < 2.7 \times 10^{-2}$ the superconducting phase transition is of the II type.[1]

Now let us generalize the obtained results for the case of TP with finite but small transparency (we will assume that $r^{-1} \ll 1$). For finite $r$ the TP is similar to the Josephson junction with

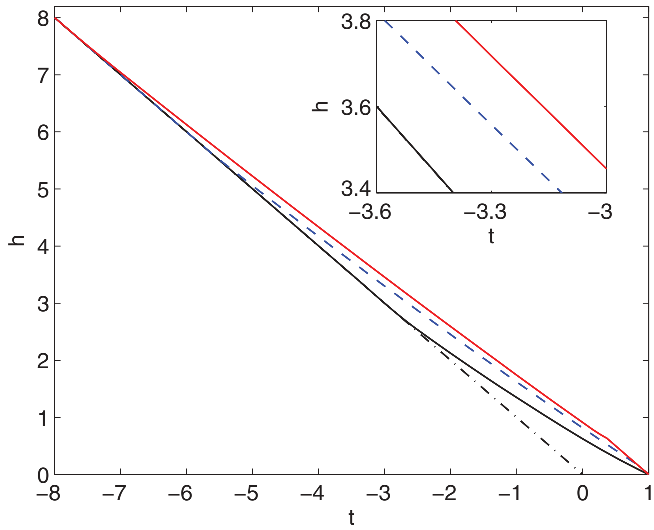


FIG. 1. (Color online) Phase diagram of the type-I superconductors with absolutely opaque TP in the parallel magnetic field. Penetration of the magnetic field into the TP leads to increase in the critical magnetic field $h_c(t)$ (red solid curve) compared with the one calculated with the assumption of exponentially small magnetic field at the TP (Ref. 11) (blue dashed curve). In the inset we show the fragment of these dependencies in more detail. In our calculations we took $\kappa = 0.13$ corresponding to tin. Also critical magnetic fields of the superconductor with $\kappa = 0$ (black solid curve) and for a bulk superconductor (black dash dotted curve) are shown.

corresponding Gibbs free energy $G_r/G_s \propto r^{-1} + O(r^{-2})$. Note that the boundary conditions for the order parameter $\psi$ at $z = 0$ should be modified to describe the transparent TP[21] so that

$$\begin{aligned}\partial_z\psi_+ &= -\psi_+ + r^{-1}(\psi_+ - \psi_-),\\ \partial_z\psi_- &= \psi_- + r^{-1}(\psi_+ - \psi_-).\end{aligned} \tag{9}$$

Note that the corrections to the order parameter modulus $|\psi|$ due to the changes in the boundary conditions have the order of $r^{-1}$. The corresponding corrections to the free energy have at least the order of $r^{-2}$ and can be neglected. Thus in what follows we will assume that for weakly transparent TP the spatial distribution of the order parameter module $\varphi(z)$ is the same as in case of the absolutely opaque TP. Then it is easy to obtain the expression for the Josephson free energy of the TP, which has the form $G_r/G_s = (4\varphi_0^2/r)\int(1-\cos\Delta\theta)\,dx$, where $\Delta\theta = \theta_+ - \theta_-$ and $\theta_\pm = \theta(x,\pm 0)$.

It is natural to expect that the magnetic field can penetrate into the TP in the form of Josephson-like vortices. Indeed the magnetic field at the TP is defined by the phase difference on the two sides of the TP and has the form $h_0(x) = (\varphi_0/\sqrt{2})\partial_x\Delta\theta$ while the dimensionless Josephson current through the TP can be written as

$$j_z(x) = \frac{\sqrt{2}\varphi_0^2}{\kappa r}\sin\Delta\theta \tag{10}$$

(here we use the value $cH_s/4\pi\xi_s$ as the unit of current). Then substituting $h_0(x)$ and $j_z(x)$ into the Maxwell equations one can obtain the analog of the Ferrell-Prange equation $\partial_x^2\Delta\theta = \lambda_J^{-2}\sin\Delta\theta$, where $\lambda_J = (\kappa r/2\varphi_0)^{1/2}$ is the Josephson penetration depth. It is convinient to introduce the new dimensionless coordinate $x' = x\sqrt{2\varphi_0/\kappa r}$. Then the magnetic part of the Helmholtz free energy $F = G + 2BH$ per unit length can be represented in the standard form

$$F = G_s\frac{4\varphi_0^2}{r}\int\left\{(1-\cos\Delta\theta) + \frac{1}{2}(\partial_{x'}\theta)^2\right\}dx'. \tag{11}$$

From the expression (11) it is easy to obtain the value of the Josephson critical field $h_{cJ}$ which is the minimal field of the vortex penetration into the junction.[23] The expression for $h_{cJ}$ reads as

$$h_{cJ}(t) \approx \frac{4}{\pi}\frac{1}{\sqrt{\kappa r}}[2(1-t)]^{3/4}. \tag{12}$$

To describe the temperature dependence of the critical magnetic field which corresponds to field penetration into the TP we use the results of Ref. 23, where the averaged magnetization $M_J$ of the Josephson junction and the corresponding Helmholtz energy $F_J$ are calculated as implicit functions of the external magnetic field $h$. Substituting these dependencies into the Gibbs free energy $G$ of the TP and performing numerical calculations we obtain the dependence of the critical magnetic field on temperature $h_c(t)$ which is shown in Fig. 2 (red solid curve). Obviously this dependence exceeds the dependence $h_{c0}(t)$ corresponding to the condition $G_0 = 0$ (see the blue dashed curve in Fig. 2) only for temperatures where $h_{c0}(t) > h_{cJ}(t)$. Note that the obtained dependence $h_c(t)$ depends only on one fitting parameter $r$. In Fig. 3 we plot the value $\Delta h_c(t) = h_c(t) - h_{c0}(t)$ for different transparencies $r^{-1}$ of the TP. This value describes the increase in the critical

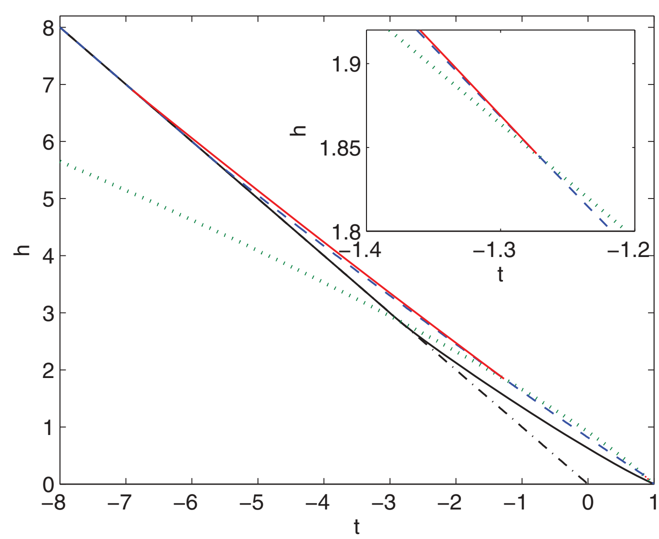


FIG. 2. (Color online) Phase diagram of the type-I superconductors with weakly transparent TP ($r = 25$) in the parallel magnetic field. Penetration of the magnetic field into the TP is energetically favorable for temperatures below the point where the dependence of the critical Josephson magnetic field $h_{cJ}(t)$ (green dotted curve) crosses the critical field dependence calculated with the assumption of the exponentially small magnetic field at the TP (Ref. 11) (blue dashed curve). In this temperature region the resulting dependence of the critical magnetic field is shown with red solid curve. In our calculations we took $\kappa = 0.13$ (tin).

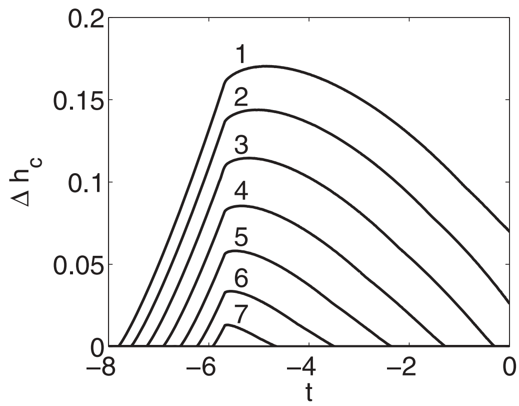


FIG. 3. The increase $\Delta h_c$ in the critical magnetic field of the type-I superconductors due to the field penetration into the TP. The number $n$ of the curve corresponds to different transparencies of the TP: $r^{-1} = 0.01n$. In our calculations we took $\kappa = 0.13$ (tin).

magnetic field due to the field penetration into the TP. We hope that the high-accuracy measurements of the critical magnetic fields can allow to estimate the values of the twinning plane transparencies for different type-I superconductors.

Note also that the magnetic field penetration into the TP can be detected experimentally in the Josephson current measurements. Let us consider the pair of contacts which are positioned parallel to the TP at a distance which is much less than a width of superconducting region $l \propto t^{-1/2}$ near the TP. Then the Josephson current through these contacts would be extremely sensitive to the magnetic field at the TP. Indeed without magnetic field the averaged over the TP length Josephson current can be nonzero if $\Delta\theta \neq 0$ [see Eq. (10)]. Otherwise in the case of the magnetic field penetration the phase difference would increase with the increase in $x$ and the corresponding averaged Josephson current through the TP would be negligibly small.

For the type-II superconductors the fact of full penetration of the magnetic field into the center of the TP does not lead to any substantial consequences since the TP in this case weakly screens the external magnetic field. To calculate the correction to the magnetic susceptibility one can use the approach from Ref. 9. We will restrict ourselves for the case of ultra type-II superconductors with $\kappa \gg 1$, $r = \infty$ and the temperatures in the range $0 < t < 1$. In this case the magnetic field $b(z)$ slightly differs from the external field $h$ which allows one to consider the field profile in the form $b(z) = h + \delta b(z)$, where $|\delta b(z)| \ll h$. Then the value $\delta b(z)$ satisfies the equation $(u^2 - 1)\partial_u^2 \delta b = 2h/\kappa^2$, where $u = \coth(\sqrt{t}|z| + p/2)$ and $p = \ln[(1+\sqrt{t})/(1-\sqrt{t})]$. The boundary conditions in the case of full penetration of the magnetic field into the TP are $\delta b(u=1) = 0$ and $\delta b(u = t^{-1/2}) = 0$. Note that under the assumption that the magnetic field has its minimum at the TP the last condition should be replaced with[9] $\partial_u \delta b(u = t^{-1/2}) = 0$. The exact solution of the equation for $\delta b$ allows one to obtain the correction to the magnetic susceptibility due to the penetration of the magnetic field into the TP. This correction has the form (in dimensional units)

$$\Delta M_{(II)} = \frac{H\xi_s}{\kappa^2}\frac{1}{\pi(1-\sqrt{t})}\ln^2\left(\frac{1+\sqrt{t}}{2\sqrt{t}}\right). \tag{13}$$

Note that at $t \to 0$ the correction $\Delta M_{(\mathrm{II})} \propto \ln^2(t^{-1})$ and is negligibly small since it has weak singularity compared to the full diamagnetic moment $M_{d(\mathrm{II})}$ of the TP, which diverges like $t^{-1/2}\ln^2(t^{-1})$ (see Ref. 9). At $t \to 1$ the correction $\Delta M_{(\mathrm{II})} \propto (1-t)$ and is also small.

Thus we have shown that in superconductors with twinning planes with low transparency the penetration of the parallel magnetic field into the twinning plane is energetically favorable. For the type-I superconductors this leads to the essential increase of the critical magnetic field and to the broadening of the temperature range where the TPS can exist. Our theory does not contain any fitting parameters except $r$, so this can provide an opportunity to estimate the $r$ values for different type-I superconductors on the basis of the critical magnetic field measurements.

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