Quantum mechanical picture of the coupling between interlayer electronic excitations and infrared active phonons in bilayer cuprate superconductors

Jiří Vašátko and Dominik Munzar

Department of Condensed Matter Physics, Faculty of Science, and Central European Institute of Technology, Masaryk University, Kotlářská 2, 61137 Brno, Czech Republic

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The formula frequently used to describe the c-axis infrared response of the coupled electron-phonon system of bilayer cuprate superconductors and providing important insights into the physics of these materials has been originally obtained at the level of the phenomenological multilayer model. Here we derive it using diagrammatic perturbation theory.

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The phase diagram of underdoped high-T_c cuprate superconductors is divided into three regions: the superconducting one below T_c, the pseudogap phase in the temperature range $T_c < T < T^*$, where T^* is the onset temperature of the (high-energy) pseudogap, and the anomalous normal state above T^*.[1,2] There is an interesting and important class of physical properties that begin to develop/whose temperature dependence changes its character at an onset temperature much lower than T^* but at the same time considerably higher than T_c.[3] These include, for example, the NMR relaxation rate $(1/T_1T)$,[4] amplitude of the transverse plasma mode (also referred to as the 400 cm^{-1} mode) and some phonon structures in the c-axis infrared conductivity of $YBa_2Cu_3O_{7-\delta}$ (Y-123),[5–10] in-plane scattering rate and in-plane conductivity below $\sim$700 cm^{-1},[3,10] amplitude of the neutron resonance,[11] Nernst signal,[12] diamagnetic response,[13] specific heat,[14] some features of the excitation gap,[15,16] spectral weight at the chemical potential,[17] magnetic field dependence of the in-plane resistivity,[18] THz conductivity,[19,20] and the quasiparticle interference.[21] The values of the onset temperature range from $\sim$15 K above T_c (THz conductivity) to more than 100 K above T_c (transverse plasmon and the phonon structures, Nernst). Some of the phenomena (increase of the THz conductivity and quasiparticle interference) are almost certainly related to superconducting fluctuations. Some others with a much higher onset temperature (e.g., the transverse plasmon and the phonon structures, Nernst) have also been attributed to a precursor superconducting state (PSC) lacking the long-range phase coherence. This exciting issue, however, is fairly controversial and no consensus has been reached thus far. There are two prerequisites for an ultimate understanding: (a) A detailed knowledge of both doping (p) and temperature dependence of the relevant quantities. (b) An understanding of the physical contents of these quantities, in particular, the relation to superconductivity.

Dubroka *et al.*[10] have studied in detail the temperature dependence of the phonon structures in the infrared c-axis conductivity of Y-123 for a broad range of p of $0.03 < p < 0.17$ and determined the p dependence of the corresponding onset temperature T^{ons} (for the precise definition, see Ref. 10). The relation between the phonons and the electronic degrees of freedom has been understood in terms of the phenomenological, essentially classical, multilayer model,[22,23] involving the local conductivity of the bilayer units (denoted here as σ_{bl}) and that of the spacing layers (σ_{int}), extended[24,25] by including the phonons. For a schematic representation of the electronic part of the model and for definitions of the relevant quantities, see Fig. 1(a). The model, as formulated in Refs. 24 and 25, involves a set of self-consistent equations that allows one to express the local quantities j_{bl}, j_{int}, E_{bl}, and E_{int} in terms of E_{ext} and the local conductivities σ_{bl} and σ_{int}. The total conductivity is given by $\langle j\rangle/\langle E\rangle$, where $\langle j\rangle$ is the volume averaged current density $\langle j\rangle = (d_{bl}j_{bl} + d_{int}j_{int})/d$ and $\langle E\rangle$ is the volume averaged electric field $\langle E\rangle = (d_{bl}E_{bl} + d_{int}E_{int})/d$. The lattice vibrations enter the phenomenological model in two different ways: (a) The local field driving the Lorentz oscillator representing a particular phonon mode depends on the phonon eigenvector. Due to the presence of the charged planes, different ions feel different local fields. (b) The displacements of the ions, on the other hand, modify the fields E_{bl} and E_{int} (a feedback of the phonons). For a careful discussion and for the model equations, see Ref. 25. The model has allowed the authors of Ref. 10 to conclude that at T^{ons} the spectral weight of the low-energy component of σ_{bl} (the Drude peak in the analysis of Fig. 1 of Ref. 10) begins to increase, signaling an increase of coherence of the response. Several arguments have been advanced suggesting that this is due to the formation of a PSC.

The importance of the topic of PSC and of other interesting fields of application of the multilayer model[27–30] calls for developing a fully quantum mechanical description of the response of the coupled electron-phonon system that would support the phenomenological approach. Chaloupka *et al.* have already constructed (see Ref. 26, CBM in the following) a microscopic gauge-invariant theory of the c-axis infrared response of bilayer cuprate superconductors, such as Y-123, providing both a justification and limits of the electronic part of the multilayer model and of a related earlier approach.[31] The phonons, however, have not yet been incorporated. Here we provide an extension of the theory of CBM, where they are included at the Green's function level. In contrast with the phenomenological approach, both electronic and phonon current densities are treated as operators and expressed in terms of the quasiparticle creation and annihilation operators, the conductivity is calculated with the help of the Kubo formula, and the current-current correlator of the formula expressed using diagrammatic perturbation theory. A formally similar approach has been recently used to explain the electric field dependence of the infrared spectra of bilayer graphene.[32] The

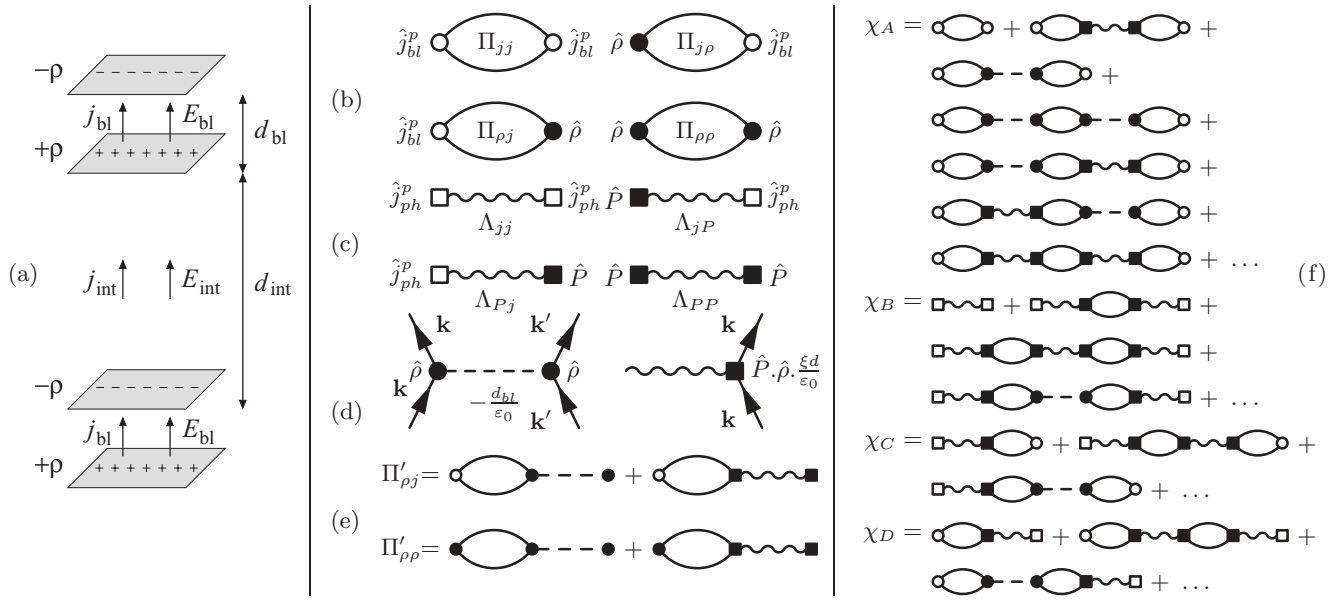

FIG. 1. (a) Schematic representation of the multilayer model, reproduced from Ref. 26. The shaded planes represent the copper-oxygen planes. The distance between the closely spaced planes forming a bilayer (between the bilayers) is denoted by $d_{\rm bl}$ ($d_{\rm int}$), the c-axis lattice parameter d is given by $d = d_{\rm bl} + d_{\rm int}$. If an ac electric field $E_{\rm ext}$ is applied perpendicular to the planes, currents flow between the planes. The corresponding current densities are denoted by $j_{\rm bl}$ and $j_{\rm int}$. They are not equal and the planes thus become charged. The resulting charge density alternates from one plane to the other (ρ and $-\rho$). As a consequence, the average electric field of the bilayer region $E_{\rm bl}$ differs from that of the interbilayer region (spacing layer) $E_{\rm int}$. The current densities are related to the fields via $j_{\rm bl} = \sigma_{\rm bl} E_{\rm bl}$ and $j_{\rm int} = \sigma_{\rm int} E_{\rm int}$ with the local conductivities $\sigma_{\rm bl}$ and $\sigma_{\rm int}$. Feynman diagrams representing the correlators defined in the text [(b) and (c)], the interaction vertices corresponding to H_C of Eq. (13) and H_{e-p} of Eq. (16) [(d)], and two segments that appear in the series of part (f) [(e)]. (f) Series of the diagrams corresponding to the quantities χ_A, χ_B, χ_C, and χ_D defined in the text. The sum of the contributions of the diagrams is to be multiplied by $d_{\rm bl}^2/d^2$ in case of χ_A and by $d_{\rm bl}/d$ in case of χ_C and χ_D.

final formula for the c-axis conductivity will be derived and it will be shown to coincide, for a natural choice of the electron-phonon coupling term, with that of the phenomenological model. The essential aspects of the theory are formulated in the points (i)–(viii).

For the sake of simplicity we neglect the hopping through the spacing layer ($t'_\perp$ in the notation of CBM, $\sigma_{\rm int} = 0$),[33] focus, as usual, on the long-wavelength limit of $\mathbf{q} = 0$,[34] and limit ourselves to the case of one infrared active phonon.

(i) *Relevant equations of CBM*. The (electronic) conductivity σ_{bl} of the bilayer unit is given by

$$\sigma_{\rm bl}(\omega) = -i\omega\varepsilon_0\chi_{\rm bl}(\omega) = \frac{\kappa_{\rm dia-bl} + d_{\rm bl}\Pi_{jj}(\omega)}{i(\omega + i\delta)}, \qquad (1)$$

where $\chi_{\rm bl}$ is the bilayer susceptibility, $\kappa_{\rm dia-bl}$ is the term describing the diamagnetic contribution, $d_{\rm bl}$ is the distance between the closely spaced planes, and $\Pi_{jj}(\omega)$ is the current-current correlation function. The term $\kappa_{\rm dia-bl}$ is given by

$$\kappa_{\rm dia-bl} = -\frac{e^2 d_{\rm bl}}{\hbar^2 N a^2}\sum_{\mathbf{k}s} t_\perp(\mathbf{k})\langle c^\dagger_{B\mathbf{k}s}c_{B\mathbf{k}s} - c^\dagger_{A\mathbf{k}s}c_{A\mathbf{k}s}\rangle, \qquad (2)$$

where N is the number of unit cells per copper-oxygen plane, a is the in-plane lattice parameter, the sum runs over all values of the in-plane quasimomentum $\mathbf{k}$ and spin s, $t_\perp(\mathbf{k})$ is the intrabilayer hopping parameter, and $c^\dagger_{B\mathbf{k}s}$, $c_{B\mathbf{k}s}$ ($c^\dagger_{A\mathbf{k}s}$, $c_{A\mathbf{k}s}$) are the quasiparticle operators corresponding to the bonding (antibonding) band. The correlator Π_{jj} is given by

$$\Pi_{jj}(\omega) = \frac{iNa^2}{\hbar}\int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle[\hat{j}^{\rm p}_{\rm bl}(t), \hat{j}^{\rm p}_{\rm bl}(0)]\rangle\theta(t), \qquad (3)$$

where $\hat{j}^{\rm p}_{\rm bl}$ is the operator of the intrabilayer paramagnetic current density,

$$\hat{j}^{\rm p}_{\rm bl} = \frac{ie}{\hbar N a^2}\sum_{\mathbf{k}s} t_\perp(\mathbf{k})(c^\dagger_{B\mathbf{k}s}c_{A\mathbf{k}s} - c^\dagger_{A\mathbf{k}s}c_{B\mathbf{k}s}). \qquad (4)$$

In the absence of interlayer Coulomb interaction and any phonon contribution, the total (volume averaged) c-axis current density would be equal to $\sigma^*(\omega)E^{\rm ext}(\omega)$ with $\sigma^*(\omega) = (d_{\rm bl}/d)\sigma_{\rm bl}(\omega)$ and $E^{\rm ext}$ is the external field, d is the c-axis lattice parameter.

(ii) *Response of a c-axis polarized infrared active phonon mode*. The well-known formulas describing the one-phonon absorption (see, e.g., Ref. 35) are rewritten here in terms of the phonon current density operator. This will allow us to construct the perturbation expansion needed to describe the interacting case.

In the absence of any electronic contribution, the (volume averaged) current density can be expressed as $\sigma^*(\omega)E^{\rm ext}(\omega)$ with

$$\sigma^*(\omega) = \sigma_{\rm ph}(\omega) = -i\omega\varepsilon_0\chi = \frac{\kappa_{\rm dia-ph} + d\Lambda_{jj}(\omega)}{i(\omega + i\delta)}, \qquad (5)$$

where χ is the phonon polarizability, $\kappa_{\rm dia-ph}$ is the term describing the diamagnetic contribution, and $\Lambda_{jj}(\omega)$ is the

phonon current-current correlator.[36] The term $\kappa_{\mathrm{dia-ph}}$ is equal to $-\varepsilon_0\omega_P^2$, where ω_P is the phonon plasma frequency,

$$\omega_P^2 = \frac{1}{(a^2 d)\varepsilon_0}\left\{\sum_\mu [e_\mu(\mathbf{e})_{\mu z}/\sqrt{m_\mu}]\right\}^2. \tag{6}$$

Here $a^2 d$ is the volume of a unit cell and e_μ, $(\mathbf{e})_{\mu z}$, and m_μ are the effective charge, the c-axis component of the polarization vector, and the mass of the μth ion, respectively. The correlator is given by

$$\Lambda_{jj}(\omega) = \frac{iNa^2}{\hbar}\int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle[\hat{j}_{\mathrm{ph}}^{\mathrm{p}}(t), \hat{j}_{\mathrm{ph}}^{\mathrm{p}}(0)]\rangle\theta(t). \tag{7}$$

Here $\hat{j}_{\mathrm{ph}}^{\mathrm{p}}$ is the phonon paramagnetic current-density operator

$$\hat{j}_{\mathrm{ph}}^{\mathrm{p}} = \frac{i}{\sqrt{N}a^2 d}\sqrt{\frac{\hbar\omega_0}{2}}\sum_\mu \frac{e_\mu(\mathbf{e})_{\mu z}}{\sqrt{m_\mu}}(a^\dagger - a), \tag{8}$$

ω_0 is the phonon frequency, and $a^\dagger$ and a are the phonon creation and annihilation operators, respectively. The correlator can be further expressed in terms of the retarded phonon propagator $D(\omega)$, that is, the Fourier component of $D(t) = -i\langle[\varphi(t),\varphi^+(0)]\rangle\Theta(t)$ with $\varphi = a + a^\dagger$. We obtain

$$d\Lambda_{jj} = \varepsilon_0\omega_0^2\chi = -\frac{\varepsilon_0\omega_P^2\omega_0}{2}D(\omega). \tag{9}$$

In the noninteracting case, $D(\omega) = D_0(\omega) = 2\omega_0/(\omega^2 - \omega_0^2 + i\omega\delta)$, the phonon conductivity is given by

$$\sigma_{\mathrm{ph}}(\omega) = -i\varepsilon_0\omega\frac{\omega_P^2}{\omega_0^2 - \omega^2 - i\omega\delta}, \tag{10}$$

and the phonon contribution to the dielectric function by $\Delta\varepsilon(\omega) = \omega_P^2/(\omega_0^2 - \omega^2 - i\omega\delta)$ and $\Delta\varepsilon_2(\omega) = \pi(\omega_P^2/2\omega_0)\delta(\omega - \omega_0)$. These are the familiar formulas describing the one-phonon absorption in the absence of any final state interactions.[35] Our expression for the phonon plasma frequency [the right-hand side of Eq. (6)] coincides with that appearing in the equations of Ref. 35.

(iii) *Case of the electronic contribution coexisting with the phonon one*. The total current density is given by $\sigma^*(\omega)E^{\mathrm{ext}}$ with

$$\sigma^*(\omega) = \frac{\kappa'_{\mathrm{dia-bl}} + \kappa_{\mathrm{dia-ph}} + d\,\Xi_{jj}(\omega)}{i(\omega + i\delta)}, \tag{11}$$

where $\kappa'_{\mathrm{dia-bl}} = (d_{\mathrm{bl}}/d)\kappa_{\mathrm{dia-bl}}$,

$$\Xi_{jj}(\omega) = \frac{iNa^2}{\hbar}\int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle[\hat{j}_{\mathrm{bl}}^{\mathrm{p}}(t) + \hat{j}_{\mathrm{ph}}^{\mathrm{p}}(t), \hat{j}_{\mathrm{bl}}^{\mathrm{p}}(0) + \hat{j}_{\mathrm{ph}}^{\mathrm{p}}(0)]\rangle\theta(t), \tag{12}$$

and $\hat{j}_{\mathrm{bl}}^{\mathrm{p}} = (d_{\mathrm{bl}}/d)\hat{j}_{\mathrm{bl}}^{\mathrm{p}}$. In the absence of interlayer Coulomb interaction and electron-phonon interaction, the mixed correlators (involving both $\hat{j}_{\mathrm{bl}}^{\mathrm{p}}$ and $\hat{j}_{\mathrm{ph}}^{\mathrm{p}}$) would vanish and we would obtain simply $\sigma^*(\omega) = (d_{\mathrm{bl}}/d)\sigma_{\mathrm{bl}}(\omega) + \sigma_{\mathrm{ph}}(\omega)$ with σ_{bl} from Eq. (1) and σ_{ph} from Eq. (5). The interactions will be shown to lead both to a renormalization of σ_{bl} and σ_{ph} and to nonzero values of the mixed correlators.

(iv) *Formulas used to describe the final state interactions*. Let us begin our considerations with the simple scheme of Fig. 1(a). The difference between the local conductivities σ_{bl} and σ_{int} leads to a difference between the current densities j_{bl} and j_{int} resulting in the excess charge density ρ [see Fig. 1(a)]. The electrostatic energy due to ρ is given by $Na^2 d_{\mathrm{bl}}\rho^2/(2\varepsilon_0)$ and the corresponding contribution to the Hamiltonian by Eq. (25) of CBM, that is,

$$H_{\mathrm{C}} = \frac{Na^2 d_{\mathrm{bl}}}{2\varepsilon_0}\hat{\rho}^2, \tag{13}$$

where $\hat{\rho}$ is the density operator that can be expressed in terms of the quasiparticle operators as

$$\hat{\rho} = \frac{e}{2Na^2}\sum_{\mathbf{k}s}(c_{A\mathbf{k}s}^\dagger c_{B\mathbf{k}s} + c_{B\mathbf{k}s}^\dagger c_{A\mathbf{k}s}). \tag{14}$$

Next we derive the formula for the coupling term $H_{\mathrm{e-p}}$ describing the interaction between the charge densities of the planes ρ and $-\rho$ and a phonon. We begin with the textbook formula for the energy U of a polarized medium in a homogeneous electric field: $U = -V\cdot P\cdot E$, where V is the volume of the medium, P is the polarization, and E is the field. In the present case, V is the volume of N unit cells $V \to Na^2 d$, P is the polarization due to the phonon $P \to \hat{P}$,

$$\hat{P} = \frac{1}{\sqrt{N}a^2 d}\sqrt{\frac{\hbar}{2\omega_0}}\sum_\mu \frac{e_\mu(\mathbf{e})_{\mu z}}{\sqrt{m_\mu}}(a^\dagger + a), \tag{15}$$

and E is the electric field due to ρ and $-\rho$, $E \to \hat{\rho}/\varepsilon_0$. In addition, the product has to be multiplied by a factor denoted by ξ, representing the degree of overlap of E and the oscillating microscopic dipoles. The reason is that E is nonzero only in the region between the closely spaced planes, that is, inside the bilayer unit [see Fig. 1(a)]. For the sake of simplicity, only three types of phonons are considered: those associated with vibrations inside (outside) the bilayer unit, with $\xi = 1$ (0), and phonons involving mainly the oxygens of the CuO_2 planes. In the latter case, $\xi \approx 1/2$, as explained in simple terms below. Consider the scheme of Fig. 1(a). The charge density of the planar oxygens can be assumed to be distributed partially above the fixed metallic planes and partially below them (for a related scheme, see Fig. 5 of Ref. 25). The vibrations thus occur partially inside and partially outside the bilayer unit. As a result, the degree of overlap of E and P is intermediate between the two types and $\xi \approx 1/2$. By combining the expressions for V, P, and E we obtain

$$H_{\mathrm{e-p}} = -\xi\frac{Na^2 d}{\varepsilon_0}\hat{P}\hat{\rho}. \tag{16}$$

(v) *Modification of the response by H_{C} and $H_{\mathrm{e-p}}$*. The function Ξ_{jj} defined by Eq. (12) can be written as the sum of four contributions: χ_A [the term involving $\hat{j}_{\mathrm{bl}}^{\mathrm{p}}(t)$ and $\hat{j}_{\mathrm{bl}}^{\mathrm{p}}(0)$], χ_B [$\hat{j}_{\mathrm{ph}}^{\mathrm{p}}(t)$ and $\hat{j}_{\mathrm{ph}}^{\mathrm{p}}(0)$], χ_C [$\hat{j}_{\mathrm{bl}}^{\mathrm{p}}(t)$ and $\hat{j}_{\mathrm{ph}}^{\mathrm{p}}(0)$], and χ_D [$\hat{j}_{\mathrm{ph}}^{\mathrm{p}}(t)$ and $\hat{j}_{\mathrm{bl}}^{\mathrm{p}}(0)$]. These correlators can be expressed, using perturbation theory, in terms of those referring to the case of $H_{\mathrm{C}} = 0$ and $H_{\mathrm{e-p}} = 0$: Π_{jj}, $\Pi_{j\rho}$, $\Pi_{\rho j}$, $\Pi_{\rho\rho}$, Λ_{jj}, Λ_{jP}, Λ_{Pj}, and Λ_{PP}. Two of the latter, Π_{jj} and Λ_{jj}, have been defined above, see Eqs. (3) and (7), the others are defined similarly, for example, $\Lambda_{jP}(\omega) = (iNa^2/\hbar)\int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle[\hat{j}_{\mathrm{ph}}^{\mathrm{p}}(t), \hat{P}(0)]\rangle\theta(t)$. Figure 1 shows the Feynman diagrams representing the correlators [in (b) and (c)] and the interaction vertices [in (d)]. Also shown are

the factors to be assigned to the vertices when calculating the contributions of the diagrams. Figure 1(f) shows the series of the diagrams corresponding to χ_A, χ_C, χ_C, and χ_D. The series can be summed up by standard techniques[37] and the results are

$$\begin{aligned}\left(\frac{d}{d_{\rm bl}}\right)^2\chi_A &= \Pi_{jj}+\Pi_{j\rho}\frac{1}{1-\Pi'_{\rho\rho}}\Pi'_{\rho j},\\ \chi_B &= \Lambda_{jj}+\frac{\xi^2d^2}{\varepsilon_0^2}\Lambda_{jP}\frac{\Pi_{\rho\rho}}{1-\Pi'_{\rho\rho}}\Lambda_{Pj},\\ \frac{d}{d_{\rm bl}}\chi_C &= \frac{\xi d}{\varepsilon_0}\Pi_{j\rho}\frac{1}{1-\Pi'_{\rho\rho}}\Lambda_{Pj},\\ \frac{d}{d_{\rm bl}}\chi_D &= \frac{\xi d}{\varepsilon_0}\Lambda_{jP}\frac{1}{1-\Pi'_{\rho\rho}}\Pi_{\rho j},\end{aligned} \tag{17}$$

where $\Pi'_{\rho\rho/\rho j}$ is the expression corresponding to the repeating segment of the diagrams shown in Fig. 1(d),

$$\Pi'_{\rho\rho/\rho j}=\frac{d_{\rm bl}}{\varepsilon_0}\left(\frac{\xi^2d^2}{d_{\rm bl}\varepsilon_0}\Lambda_{PP}-1\right)\Pi_{\rho\rho/\rho j}. \tag{18}$$

(vi) *Simplifications resulting from the gauge invariance*. The sum in the numerator on the right-hand side of Eq. (11) containing $\kappa_{\rm dia-bl}$, $\kappa_{\rm dia-ph}$, and the four correlators of Eq. (17) can be considerably simplified by using the relations

$$\begin{aligned}\Pi_{\rho j}&=\frac{i\omega\varepsilon_0}{d_{\rm bl}}\chi_{\rm bl}=-\Pi_{j\rho},\quad \Pi_{\rho\rho}=\frac{\varepsilon_0}{d_{\rm bl}}\chi_{\rm bl},\\ \Lambda_{Pj}&=\frac{i\omega\varepsilon_0}{d}\chi=-\Lambda_{jP},\quad \Lambda_{PP}=\frac{\varepsilon_0}{d}\chi\end{aligned} \tag{19}$$

that follow from the requirement of the charge conservation formulated in different gauges of the electromagnetic potentials or equivalently from the requirement of the gauge invariance.[26,38] We obtain

$$\begin{aligned}\sigma^*(\omega)=&-i\omega\varepsilon_0[(d_{\rm bl}/d)\chi_{\rm bl}+\chi+(1-2\xi)\chi_{\rm bl}\chi]\\ &\times\frac{1}{1-\chi_{\rm bl}[(d/d_{\rm bl})\xi^2\chi-1]}.\end{aligned} \tag{20}$$

(vii) *Total electric field, from $\sigma^*(\omega)$ to $\sigma(\omega)$*. The total electric field E consists of the external field $E^{\rm ext}$ and the induced field $E^{\rm ind}$ due to the charge fluctuations described by ρ and the phonon polarization described by P, $E^{\rm ind}=(d_{\rm bl}/d)(\rho/\varepsilon_0)-(P/\varepsilon_0)=j/(i\omega\varepsilon_0)$. We have used the continuity equation $i\omega\rho=j_{\rm bl}$ and the relation between P and the total phonon current density $j_{\rm p}$, $-i\omega P=j_{\rm p}$, j is the total current density. The c-axis conductivity $\sigma(\omega)$ defined by $\sigma(\omega)=j(\omega)/E(\omega)$ is equal to $\sigma^*(\omega)[E^{\rm ext}(\omega)/E(\omega)]$, that is,

$$\sigma(\omega)=\frac{\sigma^*(\omega)}{1-i\sigma^*/(\omega\varepsilon_0)}=-i\omega\varepsilon_0\frac{(d_{\rm bl}/d)\chi_{\rm bl}+\chi+(1-2\xi)\chi_{\rm bl}\chi}{1+[1-(d_{\rm bl}/d)]\chi_{\rm bl}-\chi-[1-2\xi+(d_{\rm bl}/d)\xi^2]\chi_{\rm bl}\chi}. \tag{21}$$

(viii) *Considerations of screening by high-frequency processes* lead to a slightly modified formula:

$$\begin{aligned}\sigma(\omega)\ =\ &\text{rhs of Eq. (21)}[\varepsilon_0\to\varepsilon_0\varepsilon_\infty,\\ &\chi_{\rm bl}\to\chi_{\rm bl}/\varepsilon_\infty,\ \chi\to\chi/\varepsilon_\infty],\end{aligned} \tag{22}$$

where ε_∞ is the interband dielectric function. The total dielectric function is given by $\varepsilon(\omega)=\varepsilon_\infty+i\sigma(\omega)/(\omega\varepsilon_0)$.

It is an easy exercise to show that the final formula [(21) or (22)] is precisely the same as the one resulting from the single phonon version of the phenomenological multilayer model.[39] The important point is that is has been derived here on microscopic grounds, using perturbation theory, with the electron-phonon coupling constant determined by simple electrostatics. Within the phenomenological model, the response of a phonon to the applied field is governed by the eigenvector-dependent local electric field. Here the eigenvector enters the equations through the coupling constant. Our result adds credibility to the central claim of Ref. 10, that at $T^{\rm ons}$ the spectral weight of the low-energy component of the (local) intrabilayer conductivity $\sigma_{\rm bl}$ begins to increase, possibly a manifestation of precursor superconductivity. Future studies involving the present formalism in conjunction with the local conductivities resulting from microscopic calculations such as those reported in CBM (rooted in the Fermi liquid theory) or those of the kind of Ref. 40 based on a phenomenological ansatz[41] related to the resonating valence bond picture are needed to achieve further progress.

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