Shubnikov–de Haas oscillations in GaAs quantum wells in tilted magnetic fields

A. T. Hatke,[1] M. A. Zudov,[1,*] L. N. Pfeiffer,[2] and K. W. West[2]

[1]*School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*

[2]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*

We report on quantum magneto-oscillations in an ultrahigh mobility GaAs/AlGaAs quantum well at very high ($\geqslant 87°$) tilt angles. Unlike previous studies, we find that the spin and cyclotron splittings become equal over a *continuous range* of angles, but only near certain, angle-dependent filling factors. At high enough tilt angles, Shubnikov–de Haas oscillations reveal a prominent beating pattern, indicative of *consecutive* level crossings, *all* occurring at the same angle. We explain these unusual observations by an in-plane field-induced increase of the carrier mass, which leads to accelerated, filling-factor-driven crossings of spin sublevels in tilted magnetic fields.

When a two-dimensional electron system (2DES) is subject to a perpendicular magnetic field $B_\perp$, its low-temperature magnetoresistivity exhibits well-known Shubnikov–de Haas oscillations (SdHO). SdHO originate from the magnetic quantization of the energy spectrum

$$E_{N,\mp} = \hbar\omega_c(N + 1/2) \pm \Delta_s/2 , \qquad (1)$$

where $\hbar\omega_c = \hbar e B_\perp/m^\star$ is the cyclotron energy, $m^\star$ is the effective mass, N is the Landau level index, and Δ_s is the sum of the single-particle Zeeman energy Δ_Z and the exchange energy Δ_X. The resistivity of the 2DES senses the density of electron states at the Fermi level ε_F and will show a minimum whenever ε_F falls in the middle between the energy levels [see Eq. (1)], i.e., when the filling factor $\nu = 2\varepsilon_F/\hbar\omega_c$ acquires an integer value.

The strength of the SdHO at odd and even ν is determined by *spin splitting* $\Delta_{\text{odd}} = \Delta_s$ and *cyclotron splitting* $\Delta_{\text{even}} = \hbar\omega_c - \Delta_s$, respectively. Furthermore, the exchange contribution to the spin splitting becomes important only when there is an appreciable population difference between $(N,+)$ and $(N,-)$ sublevels. The latter turns off rather abruptly at some critical, disorder-dependent filling factor ν_s. The associated disappearance of the odd-ν SdHO minima is referred to as a critical collapse of the exchange-enhanced spin splitting.[1–5] At odd $\nu \gg 1$, but considerably lower than ν_s, the exchange energy scales with the cyclotron energy $\Delta_X = \alpha_X\hbar\omega_c$,[1,6] while at even ν, one can set $\Delta_X \approx 0$ and $\Delta_s \approx \Delta_Z = \alpha_Z\hbar\omega_c$. Since in GaAs $\alpha_Z \ll 1$, $\alpha_X < 1$ (Ref. 7), and, most importantly, since there are no critically collapsing contributions to Δ_{even}, the SdHO at even ν persist to filling factors much larger than ν_s.[8]

Adding an in-plane magnetic field will increase the Zeeman energy since the latter scales with the total field. On the other hand, one usually assumes that both the cyclotron and the exchange energies depend only on $B_\perp$.[2,4,9] As a result, when the magnetic field is tilted by angle θ away from the sample normal, for any given ν, the cyclotron splitting $\Delta_{\text{even}} \propto 1 - \alpha_Z/\cos\theta$ will decrease with θ and eventually approach the increasing spin splitting $\Delta_{\text{odd}} \propto \alpha_X + \alpha_Z/\cos\theta$. This approach forms a basis of the *coincidence method*,[10] which was successfully used to study the exchange contribution.[2,11] The "coincidences," manifested as equally strong even-ν and

odd-ν SdHO, will occur when[12] $\Delta_{\text{even}} = \Delta_{\text{odd}}$ or

$$\gamma_i \equiv \frac{1}{\cos\theta_i} = \frac{i - \alpha_X}{2\alpha_Z}, \quad i = 1,3,5,\dots . \qquad (2)$$

Equation (2),[13] dictating that the coincidences should occur *simultaneously* for all ν, was confirmed experimentally[2,11] for $i = 1$, $\theta_1 \approx 87.2°$ ($\gamma_1 \approx 20$).

In this Rapid Communication, we report on spin-resolved SdHO in an ultrahigh mobility GaAs/AlGaAs quantum well at very high tilt angles ($\theta \geqslant 87°$). We find that the evolution of the SdHO waveform depends sensitively not only on θ, as predicted by Eq. (2), but also on the filling factor. More specifically, we find that coincidence conditions $\Delta_{\text{even}} = \Delta_{\text{odd}}$ can be satisfied *over a range of tilt angles*, but only at certain filling factors. These filling factors monotonically increase with θ and eventually diverge at $\theta = \theta_i$. At $\theta > \theta_1$, SdHO reveal a beating pattern indicating *consecutive* ($i = 3,5$), filling-factor-driven coincidences, occurring at the same θ. These findings are in contrast with Eq. (2) and previous experiments.[2,11] To explain this unusual behavior, we propose a model which takes into account an in-plane field-induced increase of the effective mass due to the finite width of our 2DES. This increase leads to a nonmonotonic dependence of the cyclotron splitting on ν and its eventual collapse, resulting in accelerated crossings of spin sublevels.

Our sample is a Hall bar (width 200 μm) fabricated from a 29-nm-wide GaAs-Al$_{0.24}$Ga$_{0.76}$As quantum well, grown by molecular beam epitaxy, having density $n_e \approx 2.85 \times 10^{11}$ cm^{-2} and mobility $\mu \approx 2.4 \times 10^7$ cm^2/Vs . The resistivity was recorded using a standard low-frequency (a few hertz) lock-in technique at temperature $T \approx 0.3$ K in sweeping magnetic field. The tilt angle θ, controlled *in situ* by a precision rotator, was held constant during each magnetic field sweep.

In Fig. 1, we present the oscillatory part of the resistivity $\delta\rho$ (left axes), obtained from the total resistivity by subtracting a slowly varying background, versus ν at different θ, as marked. The corresponding amplitudes $\delta\rho_{\text{max}}$ at even and odd filling factors are shown on the right (logarithmic) axes as circles and squares, respectively. These amplitudes were evaluated from $\delta\rho$ at the minima and the average of the neighboring maxima.

At the lowest θ [Fig. 1(a)], we observe that at $\nu \gtrsim 45$ the SdHO minima occur only at even ν. The corresponding (even-ν) amplitude follows roughly exponential dependence,
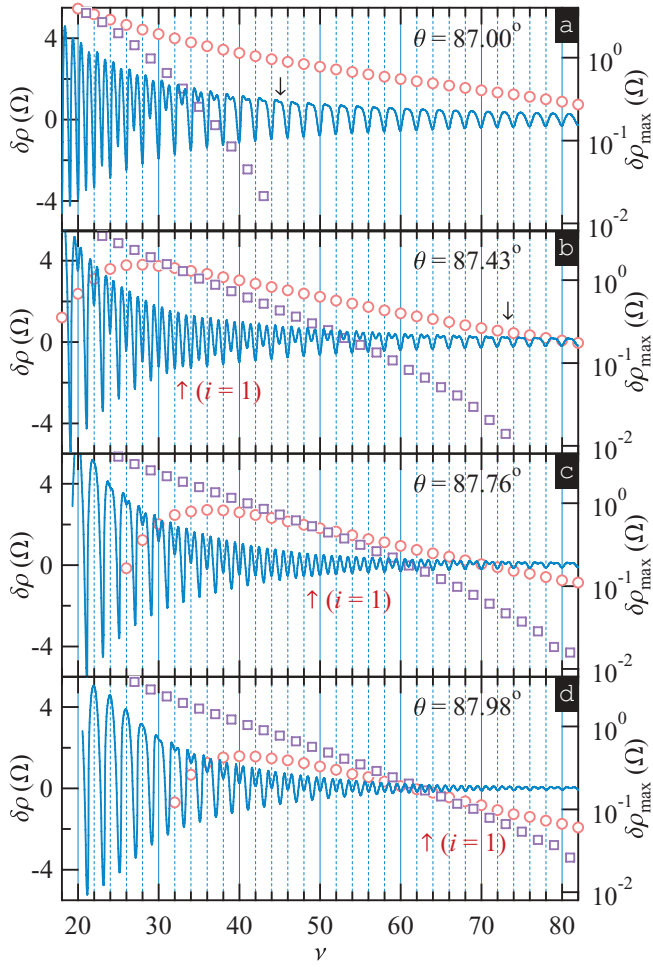
FIG. 1. (Color online) Oscillatory part of the resistivity $\delta\rho$ (left axis) and amplitude $\delta\rho_{max}$ (right axis) versus the filling factor ν measured at tilt angles (a) $\theta = 87.00°$ ($\gamma = 19.1$), (b) $\theta = 87.43°$ ($\gamma = 22.3$), (c) $\theta = 87.76°$ ($\gamma = 25.6$), and (d) $\theta = 87.98°$ ($\gamma = 28.4$). Even-ν (circles) and odd-ν (squares) amplitudes become equal at $\nu = \nu_1$, as marked by $\uparrow$.

as expected from the Lifshitz-Kosevich formula.[14] The minima at odd filling factors first appear at $\nu = \nu_s \approx 45$ (cf. $\downarrow$), which marks the onset of the (exchange-enhanced) spin splitting. At $\nu < \nu_s$, the odd-ν amplitude increases rapidly and approaches the even-ν amplitude. Overall, the data in Fig. 1(a) are very similar to the data obtained at all lower θ, including $\theta = 0$, the only difference being that ν_s monotonically increases with θ. Such an increase of ν_s is a result of accelerated opening of the spin gap due to the growing single-particle Zeeman energy[15] and is well established experimentally.[2,4]

At the next tilt angle [Fig. 1(b)], in addition to the onset of spin splitting moving to much higher filling factors $\nu_s \approx 73$, the data reveal other interesting features, some of which are rather unexpected. First, the amplitudes at odd-ν and even-ν become equal at $\nu_1 \approx 33$ (cf. $\uparrow$). Second, at lower filling factors, $\nu < \nu_1$, the odd-ν amplitude continues to grow but the even-ν amplitude first saturates and then starts to *decrease*. As evidenced by the data at higher θ [Fig. 1(c)], this decrease leads to a complete disappearance of the even-ν amplitude at $\nu = \nu_c \approx 24$. The data at still higher tilt [Fig. 1(d)], confirm this

observation and let us conclude that *all* characteristic filling factors ν_s, ν_1, and ν_c increase with θ.

The observed amplitude crossings indicate that at $\nu = \nu_1$ the spin splitting becomes equal to the cyclotron splitting. On the other hand, the vanishing of the even-ν amplitude at $\nu = \nu_c$ signals a collapse of the cyclotron gap, which is a precursor of crossing of spin sublevels from the neighboring Landau levels. Such crossing would occur when $\Delta_{even} = \hbar\omega_c - \Delta_Z = 0$, so the observed collapse indicates that, with increasing $1/\nu$, Δ_Z is growing faster than $\hbar\omega_c$. As we show in the following, the collapse of Δ_{even} can be explained by a $B_\parallel$-induced increase of the electron effective mass, which leads to a sublinear dependence of $\hbar\omega_c$ on $1/\nu$ and its eventual saturation.

We further notice that, since $\nu_s - \nu_1 \gg 1$, the exchange energy at odd filling factors should be close to its maximum value $\alpha_X \hbar\omega_c$.[1,2,4,6] This conclusion is also supported by the observations that, at $\nu \lesssim \nu_1$, the separation between spin-split maxima is close to unity $\delta\nu \approx 1$, and that the odd-ν amplitude shows excellent exponential dependence. Finally, at all $\theta \gtrsim 87°$, $\gamma\alpha_Z$ is no longer a small fraction and Δ_Z gives a significant (if not dominant) contribution to the spin splitting.

SdHO measured at higher tilt angles (as marked) are shown in Fig. 2, which has the same layout as Fig. 1. Compared to the data at lower θ discussed above, Fig. 2 reveals further significant changes of the SdHO waveform. Here, we observe that the even-ν amplitude remains considerably smaller than the odd-ν amplitude over the whole range of ν, and the crossing never takes place. Furthermore, we observe in Fig. 2(c) that the even-ν SdHO disappear completely, indicating that the condition for the first level crossing is approximately satisfied for all filling factors under study. However, closer examination
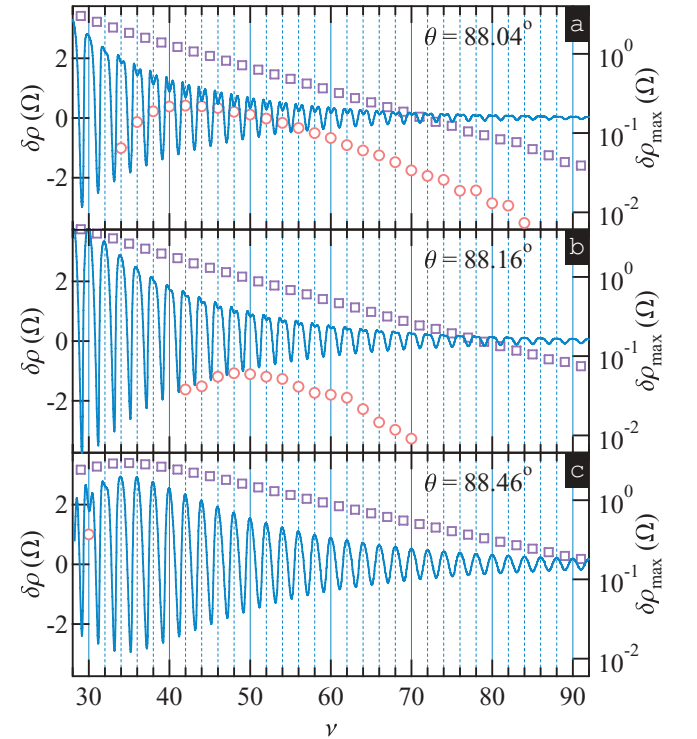

FIG. 2. (Color online) Same as Fig. 1 (a) $\theta = 88.04°$ ($\gamma = 29.2$), (b) $\theta = 88.16°$ ($\gamma = 31.1$), (c) $\theta = 88.46°$ ($\gamma = 37.2$).
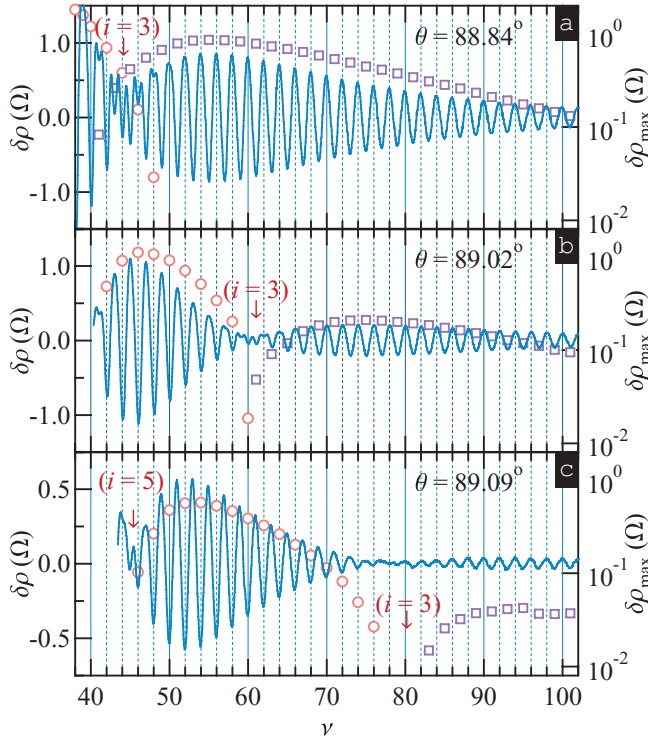
FIG. 3. (Color online) Same as Figure 1, 2 (a) $\theta = 88.84°$ ($\gamma = 49.6$), (b) $\theta = 89.02°$ ($\gamma = 58.1$), (c) $\theta = 89.09°$ ($\gamma = 62.7$).

of Fig. 2(c) reveals a maximum of the odd-ν amplitude at $\nu \approx 35$, and the reemergence of the even-ν SdHO at lower ν, indicating that at these filling factors the crossing of spin sublevels has already occurred.

In Fig. 3, we present the data obtained at still higher tilt angles. The most striking feature of all three data sets is a node, which occurs at $\nu = \nu_3$ (cf. ↓). Similar to the "coincidence" at $\nu = \nu_1$, this node moves to progressively higher ν with increasing θ. Passing through the node from high to low filling factors, odd-ν SdHO decay away while the even-ν SdHO set in. As a result, at $\nu \approx \nu_3$ the amplitudes of the odd-ν and even-ν SdHO again become close to each other, reflecting equal gaps, similar to the situation at $\nu \approx \nu_1$ (cf. Fig. 1).

While at $\nu < \nu_3$, even-ν SdHO dominate the response, the oscillation amplitude is not monotonic and after passing through a maximum again decays away [cf. Fig. 3(b)]. As shown by the data at the highest tilt angle [Fig. 3(c)], this decay gives rise to another node, at $\nu = \nu_5 \approx 45$, which signals yet another crossing of the even-ν and odd-ν amplitudes. While the oscillation patterns shown in Fig. 3 might appear very different from those shown in Figs. 1 and 2, as we show next, all observations can be attributed to subsequent level crossings. Clearly, such crossings can not be described by Eq. (2), which is independent of ν.

To explain our findings, we propose a model based on finite thickness effects which, in combination with $B_\parallel$, lead to an increase of the effective mass. This increase was studied theoretically[16–21] and confirmed in experiments examining the temperature damping of the SdHO amplitude[22] and the shift of magnetoplasmon resonances.[23,24] Following Ref. 24, we use the expression for a parabolic confining potential[16,21] and

substitute the intersubband splitting of our quantum well, ε_{10}, for the intersubband spacing in the parabolic well. At $\gamma \gg 1$, the effective mass increases with respect to its value at $\theta = 0$ by a factor $1/\alpha_\gamma$, which is given by[16,21]

$$\frac{1}{\alpha_\gamma} = \frac{m_\theta^\star}{m_0^\star} \approx \sqrt{1 + \left(\frac{\beta\gamma}{\nu}\right)^2}, \tag{3}$$

where $\beta = 2\varepsilon_F/\varepsilon_{10}$ and ε_F is evaluated at $\theta = 0$. Since β is of the order of unity,[25] a significant mass increase is expected at $\nu \lesssim \gamma$. This increase will result in a sublinear dependence of the cyclotron energy on $1/\nu$ and its eventual saturation at $\hbar\omega_c = \varepsilon_{10}/\gamma \ll \varepsilon_{10}$, in the limit of $\nu \ll \gamma$. As a result, at $\theta \neq 0$, the cyclotron splitting $\Delta_{\text{even}} \equiv \hbar\omega_c - \Delta_Z \propto [\alpha_\gamma(\nu) - \alpha_Z\gamma]/\nu$ is no longer a monotonic function of ν and, after reaching a maximum,[26] will decrease and eventually collapse. This nonmonotonic behavior of the cyclotron splitting translates to the ν dependence of the even-ν SdHO amplitude, which is observed in Figs. 1(b)–1(d) and 2(a) and 2(b).

Coincidence conditions, which take into account the mass increase [Eq. (3)], can now be obtained by replacing i with $i\alpha_\gamma$ in Eq. (2) and solving for ν, which results in

$$\nu_i \approx \frac{\beta\gamma}{\sqrt{i^2/(2\alpha_Z\gamma + \alpha_X)^2 - 1}}. \tag{4}$$

In the limit of large ν, $\alpha_\gamma = 1$ and Eq. (4) reduces to Eq. (2), which defines ν-independent coincidence angles.

Tilting the sample with respect to the magnetic field is also known to affect α_Z due to the anisotropy of the g factor. Recent experiments on electron-spin resonance in a symmetric quantum well, similar to ours, have shown that $g_0 = \sqrt{g_\perp^2 \cos^2\theta + g_\parallel^2 \sin^2\theta}$, where $g_\perp = 0.414$ and $g_\parallel = 0.340$.[27] At $\gamma \gg 1$, we obtain $g_0 \approx g_\parallel = 0.340$. It is also known that, at high magnetic fields, quadratic-in-B corrections to the Zeeman energy become important.[27–29] These terms can be incorporated into the g factor, which acquires linear B dependence $g = g_0 - aB$. At $\theta \neq 0$, $a \approx (\cos\theta/g)(g_\perp a_\perp \cos^2\theta + g_\parallel a_\parallel \sin^2\theta)$, where $a_\perp \sim a_\parallel \sim 0.01$ T^{-1}.[27] At $\gamma \gg 1$, $a \approx a_\parallel/\gamma \ll a_\parallel$,

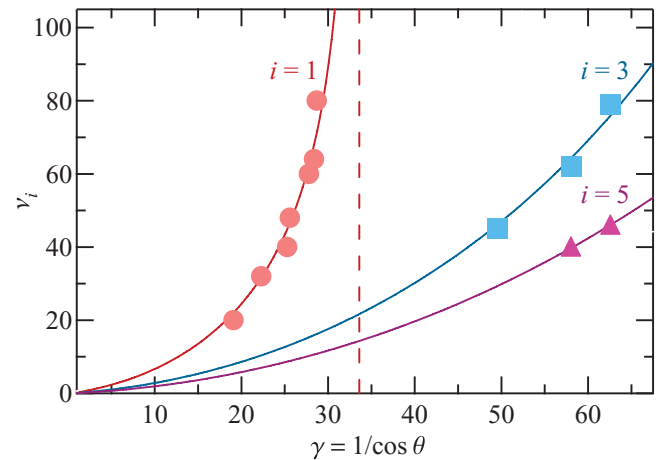FIG. 4. (Color online) Filling factors ν_1 (circles), ν_3 (squares), and ν_5 (triangles) corresponding to crossings of the even-ν and odd-ν amplitudes as functions of γ. Solid lines are fits to the data (see text). Dashed vertical line is the asymptote of the $i = 1$ coincidence at large ν.

and the correction to the g factor, aB, remains less than 1% in our experiment even at the highest magnetic fields. We will therefore assume a B- and θ-independent g factor $g = g_0 = 0.34$ in our analysis.

To demonstrate that our experimental findings can be described by Eq. (4), we present in Fig. 4 the filling factors ν_1 (circles), ν_3 (squares), and ν_5 (triangles), corresponding to crossings of the even-ν and odd-ν amplitudes as functions of γ. Solid lines represent fits to the data using Eq. (4) with $\alpha_Z \equiv g_0 m^\star/2m = 0.0114$, $\alpha_X = 0.234$,[7] and β used as a fitting parameter. For ν_1, the fit generates $\beta \approx 1.3$, which is close to our estimated value of $\beta = 1.1$.[25] The vertical dashed line is drawn at $\theta = \theta_1$, calculated using Eq. (2). For ν_3 and ν_5, the fits produce $\beta \approx 1.8$ and 2.1, respectively. Although not unreasonable, these values are higher than our estimate, which likely reflects the limitations of our model.

In summary, we have demonstrated that in a typical ultrahigh mobility GaAs/AlGaAs quantum well, the evolution of Shubnikov–de Haas oscillations in tilted magnetic fields can not be described by a simple series of level crossings characterized by a set of angles defined by Eq. (2). Instead, multiple level crossings can be realized for a given angle, but only near some characteristic filling factors [see Eq. (4) and Fig. 4]. These characteristic filling factors depend sensitively on θ, diverging near $\theta = \theta_i$ given by Eq. (2). Both multiple level crossings and filling-factor-driven collapses of the energy gaps can be explained by finite thickness effects, which give rise to the effective mass increase induced by in-plane magnetic field.

*Corresponding author: zudov@physics.umn.edu

[1] M. M. Fogler and B. I. Shklovskii, Phys. Rev. B **52**, 17366 (1995).

[2] D. R. Leadley, R. J. Nicholas, J. J. Harris, and C. T. Foxon, Phys. Rev. B **58**, 13036 (1998).

[3] B. A. Piot, D. K. Maude, M. Henini, Z. R. Wasilewski, K. J. Friedland, R. Hey, K. H. Ploog, A. I. Toropov, R. Airey, and G. Hill, Phys. Rev. B **72**, 245325 (2005).

[4] B. A. Piot, D. K. Maude, M. Henini, Z. R. Wasilewski, J. A. Gupta *et al.*, Phys. Rev. B **75**, 155332 (2007).

[5] W. Pan, K. W. Baldwin, K. W. West, L. N. Pfeiffer, and D. C. Tsui, Phys. Rev. B **84**, 161307 (2011).

[6] I. L. Aleiner and L. I. Glazman, Phys. Rev. B **52**, 11296 (1995).

[7] According to Ref. 6, $\alpha_X = \ln(k_F a_B)/\pi k_F a_B$, where k_F is the Fermi wave number, and a_B is the effective Bohr radius. In our 2DES, we estimate $\alpha_X \approx 0.23$.

[8] While odd-ν minima disappear abruptly at $\nu = \nu_s$, the even-ν oscillations decay roughly exponentially with ν.

[9] While the fully enhanced Δ_X scales with $B_\perp$, ν_s will increase with $\Delta_Z \propto \gamma$, making exchange contribution relevant at progressively higher ν with increasing θ (Refs. 2 and 4).

[10] F. F. Fang and P. J. Stiles, Phys. Rev. **174**, 823 (1968).

[11] R. J. Nicholas, R. J. Haug, K. v. Klitzing, and G. Weimann, Phys. Rev. B **37**, 1294 (1988).

[12] Equation (2) for $i = 1$ coincides with Eq. (11) of Ref. 2 if α_X is replaced by $2\alpha_X$. The extra factor appears because in Ref. 2 Δ_X is included in both Δ_{odd} and Δ_{even}.

[13] Equation (2) should also be approximately valid at even i corresponding to level crossings if one takes the exchange part to be only partially ($\simeq 50\%$) enhanced (Ref. 11).

[14] A. Isihara and L. Smrčka, J. Phys. C: Solid State Phys. **19**, 6777 (1986).

[15] Zeeman energy is also expected to smear the phase transition associated with the critical collapse of the exchange contribution (Ref. 1).

[16] A. Khaetskii and B. Shklovskii, ZhETF **85**, 721 (1983) [Sov. Phys.–JETP **58**, 421 (1983)].

[17] J. C. Maan, in *Two Dimensional Systems, Heterostructures, and Superlattices*, edited by G. Bauer, F. Kuchar, and H. Heinrich (Springer, Berlin, 1990).

[18] R. Merlin, Solid State Commun. **64**, 99 (1987).

[19] H. Tang and P. N. Butcher, J. Phys. C: Solid State Phys. **21**, 3313 (1988).

[20] L. Smrcka, J. Phys.: Condens. Matter **2**, 8337 (1990).

[21] L. Smrcka and T. Jungwirth, J. Phys.: Condens. Matter **6**, 55 (1994).

[22] L. Smrčka, P. Vašek, J. Koláček, T. Jungwirth, and M. Cukr, Phys. Rev. B **51**, 18011 (1995).

[23] E. Batke and C. W. Tu, Phys. Rev. B **34**, 3027 (1986).

[24] V. Kozlov, S. Gubarev, and I. Kukushkin, Pis'ma v ZhETF **94**, 429 (2011) [JETP Lett. **94**, 397 (2011)].

[25] Using infinite square well approximation for our 2DES gives $E_{10} \approx 225$ K and $\beta \approx 1.1$ ($E_F \approx 125$ K).

[26] The maximum occurs at $\nu = \beta\gamma[(\alpha_Z\gamma)^{-2/3} - 1]^{-1/2}$.

[27] Y. A. Nefyodov, A. V. Shchepetilnikov, I. V. Kukushkin, W. Dietsche, and S. Schmult, Phys. Rev. B **84**, 233302 (2011).

[28] M. Dobers, K. von Klitzing, and G. Weimann, Phys. Rev. B **38**, 5453 (1988).

[29] Y. A. Nefyodov, A. V. Shchepetilnikov, I. V. Kukushkin, W. Dietsche, and S. Schmult, Phys. Rev. B **83**, 041307 (2011).