Direct access to quantum fluctuations through cross-correlation measurements

Iurii Chernii and Eugene V. Sukhorukov

Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland
(Received 23 December 2011; published 22 May 2012)

Detection of the quantum fluctuations by conventional methods meets certain obstacles, since it requires high-frequency measurements. Moreover, quantum fluctuations are normally dominated by classical noise and are usually further obstructed by various accompanying effects such as a detector backaction. In the present work, we demonstrate that these difficulties can be bypassed by performing the cross-correlation measurements. We propose to use a pair of two-level detectors, weakly coupled to a collective mode of an electric circuit. Fluctuations of the current source accumulated in the collective mode induce stochastic transitions in the detectors. These transitions are then read off by quantum-point contact (QPC) electrometers and translated into two telegraph processes in the QPC currents. Since both detectors interact with the same collective mode, this leads to a certain fraction of the correlated transitions. These correlated transitions are fingerprinted in the cross correlations of the telegraph processes, which can be detected at zero frequency, i.e., with long-time measurements. Concerning the dependance of the cross correlator on the detectors' energy splittings ε_1 and ε_2, the most interesting region is at the degeneracy points $\varepsilon_1 = \pm\varepsilon_2$, where it exhibits a sharp nonlocal resonance, that stems from higher-order processes. We find that at certain conditions, the main contribution to this resonance comes from the quantum noise. Namely, while the resonance line shape is weakly broadened by the classical noise, the height of the peak is directly proportional to the square of the quantum component of the noise spectral function.

I. INTRODUCTION

It has been recently understood that the noise phenomena are not necessarily something detrimental in physical experiments, but instead, they may carry a useful new information about the underlaying processes.[1,2] This information may be difficult to extract from the measurements of average quantities, therefore properties of the fluctuations become themselves a valuable subject of research. Particularly, one may study higher irreducible moments (or cumulants) of physical quantities.[3] It is especially interesting to detect essentially quantum properties of noise, such as nonsymmetrized correlators, reflecting the quantum noncommutativity.[4]

To motivate this interest, it is useful to introduce an example of a quantum-mechanical operator $j(t)$, representing a fluctuating electric current. Suppose this current is measured by a classical ammeter of certain bandwidth, and the raw measurement data are stored as a sequence of numbers $\langle j(t_k) \rangle$. The classical properties related to the symmetrized correlators, such as the noise power $S_{\text{sym}}(\omega) = \int d\tau \exp(i\omega\tau)\langle\{\delta j(t), \delta j(t+\tau)\}\rangle$, or higher-order symmetrized correlators, can be extracted from the raw data in postprocessing. By contrast, the quantum (antisymmetrized) parts of the correlators can not be found, in principle, from postprocessing of such raw data, since this information is lost in the measurement of the average values $\langle j(t_k) \rangle$. Instead, these quantum properties may be inferred from the measurement of more complex quantities and systems, occurring naturally, or engineered on purpose. However, in this kind of measurements, it may not always be immediately clear which particular quantity is measured. Specifically, there is certain ambiguity concerning the questions of the operator ordering in the complex quantities and of the process of their reduction to the classical values. Thus a careful approach requires the knowledge of the detailed model of the detector and of the process of measurement.

Hence one arrives at the notion of a mesoscopic on-chip detector.[5–9] It is a part of the measurement apparatus, which interacts directly with the system and is responsible for transforming the quantum information into a classical signal. As an example, the two-level detectors have been studied theoretically and implemented in experiments,[10–15] and their operation is now quite well understood. Such a detector consists of a double quantum dot, or a similar structure, which has two energetically relevant quantum states (see Fig. 1), that have different spatial charge distributions. Then a noninvasive QPC electrometer[16–24] may be used to read out the state of the two-level detector, and the resulting signal can then be amplified by conventional means. In a properly adjusted operating regime, the two-level detector is weakly coupled to the mesoscopic system, and at short times, they evolve together quantum mechanically. Due to the weakness of coupling, fluctuations in the system induce rare stochastic (nonadiabatic) transitions in the detector. Because of the noisy nature of these fluctuations, the state of the detector becomes decohered to a statistical mixture.[17,25–27] In this case, the QPC electrometer effectively senses already classical state of the two-level detector, and thus it actually satisfies our definition of an on-chip detector.

While the on-chip detector approach should at least clarify as what exactly is measured, the extraction of the information about the quantum fluctuations may still be challenging for some general reasons. The quantum effects often appear as small corrections to classical contributions, thus a high relative accuracy may be needed. Another source of complication is the fact that the system of interest is subjected to the perturbations induced by the measuring device itself,[28] and other extrinsic sources of noise. Therefore some advanced techniques have to be employed to carefully extract the useful information. One such technique that demonstrated certain success at isolating the properties of the measured
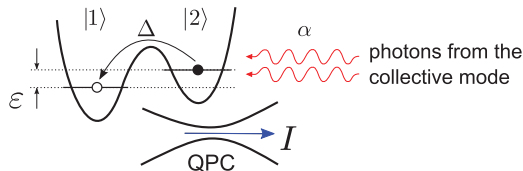
FIG. 1. (Color online) A schematic representation of the two-level detector as a double well structure with the level splitting ε. Stochastic transitions in the detector are induced by the tunneling coupling Δ and assisted by photons coming from the collective mode. The state of the detector is monitored by the QPC electrometer, located nearby one of the wells.

FIG. 3. (Color online) A diagram of the hierarchy of the time scales in the measurement process: the noise correlation time τ_c, the decoherence time τ_d, switching time of the detectors τ_s, and the proposed measurement time τ_m.

system is a so-called cross-correlation technique.[29,30] The main idea is that two (or probably more) detectors are used to measure certain fluctuating signal from the same system. Then, since the detectors are meant to be independent, any local processes at one of the detectors should not lead to cross correlations in the fluctuations measured by different detectors. Therefore the measurement of the cross correlations of the detectors' outputs gives certain level of protection from the local unwanted sources of noise, and from the detectors' backaction to the system, and thus may enhance the accuracy of the experiments.[31–36]

In the present work, we consider using the cross-correlation technique to gain access to quantum fluctuations of current. We propose the following measurement setup: a mesoscopic system, source of current noise, is incorporated into an electric circuit, so the fluctuations of its current are accumulated on a capacitor (see Fig. 2). An electric charge on the capacitor plays the role of a so-called collective mode to which a pair of two-level detectors are coupled. We assume the weak-coupling regime, when the evolution of the detectors' state can be described by the master equation for the average state occupations. Then fluctuations of the collective mode induce rare stochastic transitions in the detectors. These transitions generate two telegraph processes in the outputs of corresponding QPC electrometers, (see Fig. 4) and the cross correlator of these outputs has to be measured in the Markovian (long-time) limit.

Throughout the paper we rely on the concept of the time scale separation. In our model, there are several independent small parameters (such as the coupling constant α, and the tunneling amplitude Δ) that produce the following grid of time scales (see Fig. 3). The smallest scale is the noise correlation time τ_c at which the coherent quantum-mechanical evolution of the joint system of the noise source, the circuit, and the detector takes place. However, starting from the decoherence time $\tau_d \gg \tau_c$, the noise source plays role of the heat bath for the detector. Then the evolution of the average occupations of
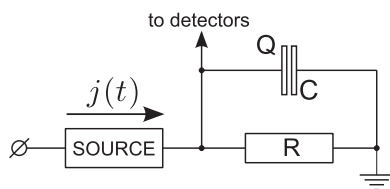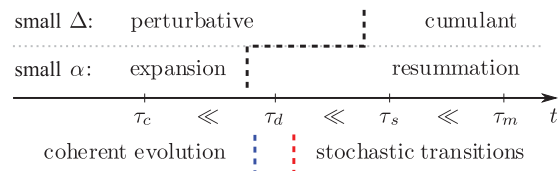
the detector states is described by the master equation. These occupations vary on the characteristic time $\tau_s \gg \tau_d$ of the order of the inverse transition rates. Finally, the longest time scale $\tau_m \gg \tau_s$ is the Markovian limit of the telegraph processes, where the cross correlator should be measured.

To describe the quantum mechanical evolution of the joint system of the collective mode ϕ and the two-level detectors on short-time scales $t < \tau_d$, it is rather more adequate to think of the two detectors in the space of four states $|11\rangle, |12\rangle, |21\rangle, |22\rangle$. A calculation of the reduced density matrix for the detectors involves averaging over the fluctuations of the collective mode ϕ. To the nth order of the perturbation expansion with respect to tunneling Δ, this requires finding averages of the Keldysh ordered products of the corresponding number of exponential phase operators $e^{i\alpha\phi}$ (so-called vertex operators). In the weak-coupling regime $\alpha \ll 1$, we use the cumulant expansion, since every next cumulant enters with one extra power of the small coupling constant, and we limit ourselves to the third cumulant. The main contribution comes from the times as long as the decoherence time τ_d where the cumulants can be expressed in terms of the zero-frequency expansion of the noise spectral functions. Quantum corrections to the classical long-time asymptotic of the cumulants are small, and thus they can be taken into account perturbatively in the coupling constant α.

On the time scales $t > \tau_d$, we find the transition rates perturbatively in Δ. We show that the most interesting effects do not appear on the level of the standard $P(E)$ theory,[37,38] which accounts tunneling to the lowest (second) order, only. Notice that some of the transitions between these states, such as $|11\rangle \rightleftarrows |22\rangle$ or $|12\rangle \rightleftarrows |21\rangle$, that arise in the fourth order in Δ, directly correspond to simultaneous, correlated switching of the detectors and lead to cross correlations at long times (see Fig. 4). However, computation of the transition rates in the energy representation encounters well-known divergences in higher orders of the perturbation theory. We avoid those divergences by considering directly the time evolution of the detector states. Namely, at times longer than the decoherence time $t > \tau_d$, but much shorter than the switching time $t \ll \tau_s$, the master equation description suggests linear in time drift of the occupations probabilities from the initial distribution. On the other hand, in the fourth order, we find quadratic in time terms. We identify them with the reducible contribution generated in the perturbation expansion and, accordingly, find the transition rates by extracting the irreducible part from the long-time asymptotic of the occupation probabilities. As typical for the perturbation theory in higher orders, immense

FIG. 2. The equivalent scheme of the measurement electric circuit coupling the noise source $j(t)$ to detectors via the collective mode Q.
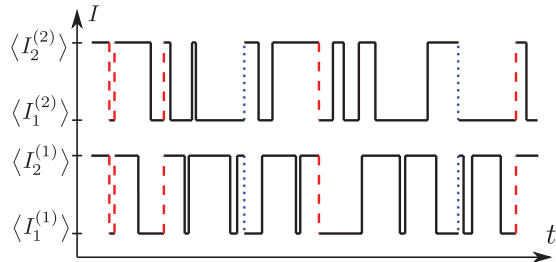
FIG. 4. (Color online) An example of the two telegraph processes generated by (partially) correlated switching of the two detectors. One of the mechanisms of cross correlation is illustrated by correlated transitions $|11\rangle \rightleftarrows |22\rangle$ (dashed lines) and anticorrelated $|12\rangle \leftrightarrows |21\rangle$ (dotted lines), both governed by the antidiagonal elements of the transition matrix. The net cross correlator may be estimated simply by counting the number of dashed lines minus the number of dotted lines.

numbers of terms are generated. Nevertheless, we managed to find all our results analytically.[40]

Note that the transition rates may be measured from the time-resolved observation of the telegraph processes.[18–24] Then, by fitting their dependence on the controllable parameters of the system, such as energy splittings and coupling constants, one may try to infer some of the noise properties. The drawback of this approach is that large amounts of real-time data need to be recorded and analyzed. This also limits the possible measurement pace since the real-time switching resolution is required to extract the transition rates. Instead, we expect that by measuring directly the cross correlator of the two telegraph processes on the time scale τ_m much longer than the switching time τ_s, one can considerably simplify the implementation of experiments. To evaluate the resulting cross correlator of the telegraph processes, we generalize for the case of two detectors the approach that has been proposed in Refs. 24 and 41 to study the statistics of bistable systems. We present an exact general result, which is convenient to use if the transition matrix is symmetric, i.e., for the classical noise. For the case of quantum noise, we also develop a perturbative in tunneling calculation that is better suitable for analytical computations.

Finally, we analyze the physically different contributions to the cross correlator from classical and quantum noise. Note that although the two detectors do not interact directly, an effective nonlocal interaction emerges between the detectors due to their coupling to the collective mode [see Eq. (8)]. The effective interaction does not depend on the properties of the noise, but it leads to the trivial second-order contribution to the cross correlator. This contribution is not of much interest, since it can be activated by subjecting both, or even any one of the detectors to local classical noise. However, since it is proportional to the strength of the effective interaction, it can be minimized by tuning the parameters of the circuit. By contrast, the more interesting kind of contributions arise as a consequence of the correlated response of the detectors to the fluctuations of the collective mode in the higher (fourth) order processes. The distinctive feature of these contributions is a sharp peak of the nonlocal resonance at the degeneracy point, where the detectors' level splittings satisfy $\varepsilon_1 \pm \varepsilon_2 = 0$.

We note that the nonlocal resonance contains both classical and quantum parts of noise. Remarkably though, the classical part in the fourth order is also proportional to the effective interaction strength and thus may be rendered small, while the purely quantum contribution to the cross correlator survives if the effective interaction is "switched off." The corresponding conditions (see Sec. VI C) can be achieved, basically, if the noise temperature is sufficiently high and the coupling constants are made small enough.

The paper is organized as follows. We present the model of two-level detectors in Sec. II. Then, in Sec. III, we give an overview of the counting statistics approach, generalize it for the cross correlations, and present two methods of evaluating the cross correlator, usable for classical and quantum noise, respectively. In Secs. IV and V, we show how the cumulant expansion is applied to calculate the averages needed to find the transition rates. Finally, in Sec. VI, we present the results for the classical and quantum noise cases and analyze the conditions needed to access the required measurement regime.

II. TWO-LEVEL DETECTORS

An electric circuit may be modeled[37] by a set of bosonic fields, ϕ_k, and their conjugated "charges" q_k. We wish to single out one of these fields, a so-called collective mode Q and its phase ϕ, which are linearly coupled to the rest of the fields and enter quadratically to the corresponding Hamiltonian of the circuit. One can show that any such Hamiltonian may be transformed to the form, where all the couplings are carried by the phase ϕ only:

$$\mathcal{H}_c = \frac{Q^2}{2C} + \mathcal{H}_n(\phi, q_k, \phi_k). \tag{1}$$

This model is sufficiently general to describe any circuit with the usual linear elements, such as resistors, capacitors, and inductances, and can also include such mesoscopic elements as tunnel junctions by adding nonquadratic potentials to the Hamiltonian.

Let us assume that the collective mode Q is linearly coupled, with a dimensionless coupling strength α, to a two-level detector represented by the Hamiltonian

$$\mathcal{H}_0 = \frac{\varepsilon}{2}\sigma_z + \Delta\sigma_x + \frac{\alpha}{2C}\sigma_z Q, \tag{2}$$

where ε is the energy level splitting, $\Delta \ll \varepsilon$ is a weak level mixing and we use the units $e = \hbar = 1$ throughout the paper. The constant Δ can also be understood as the tunneling amplitude between the levels, or as the quantum level broadening. This type of quantum detectors have been considered in a number of works and have been implemented experimentally.[10–15]

In what follows, we treat the term $\Delta\sigma_x$ as a smallest perturbation. We therefore render the total Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_c$ in a more convenient form by performing the following transformation:

$$\mathcal{H}' = e^{i\frac{\alpha}{2}\sigma_z\phi}\mathcal{H}e^{-i\frac{\alpha}{2}\sigma_z\phi}. \tag{3}$$

This transformation affects only those terms that do not commute with σ_z or ϕ. Since $[\phi, Q] = i$, it shifts the charge Q by $-\alpha\sigma_z/2$, canceling the linear coupling term $\alpha\sigma_z Q/2C$ and

bringing interactions in the form of operators $e^{\pm i\alpha\phi}$,

$$\mathcal{H}' = \frac{\varepsilon}{2}\sigma_z + \Delta \begin{pmatrix} 0 & e^{i\alpha\phi} \\ e^{-i\alpha\phi} & 0 \end{pmatrix} - \frac{\alpha^2}{8C} + \mathcal{H}_c, \qquad (4)$$

where the energy is also shifted by a constant $-\alpha^2/8C$. We switch to the interaction picture with the time-dependent tunneling Hamiltonian

$$\mathcal{H}_I(t) = \Delta \begin{pmatrix} 0 & e^{i[\phi(t)+\varepsilon t]} \\ e^{-i[\phi(t)+\varepsilon t]} & 0 \end{pmatrix}. \qquad (5)$$

This suggests a perturbative expansion in powers of Δ. To justify this expansion, we assume that the Δ has to be the smallest energy scale in the system. Particularly, it must be smaller than the level broadening introduced by noise.

A. Pair of two-level detectors

Now, let us consider two such detectors, both coupled to the same circuit via the charge Q,

$$\mathcal{H}_0 = \sum_{j=1,2} \mathcal{H}_0^{(j)} = \sum_{j=1,2} \left[\frac{\varepsilon_j}{2}\sigma_z^{(j)} + \Delta_j\sigma_x^{(j)} + \frac{\alpha_j}{2C}\sigma_z^{(j)}Q \right], \qquad (6)$$

where we denote quantities belonging to the different detectors with an additional index $j = 1,2$. In this case, the transformation analogous to Eq. (3),

$$\mathcal{H}' = e^{\frac{i}{2}\sum_j \alpha_j\sigma_z^{(j)}\phi} \mathcal{H} e^{-\frac{i}{2}\sum_j \alpha_j\sigma_z^{(j)}\phi} \qquad (7)$$

leads to

$$\mathcal{H}' = \sum_{j=1,2} \left[\frac{\varepsilon_j}{2}\sigma_z^{(j)} + \Delta_j \begin{pmatrix} 0 & e^{i\alpha_j\phi} \\ e^{-i\alpha_j\phi} & 0 \end{pmatrix} - \frac{\alpha_j^2}{8C} \right]$$
$$+ \mathcal{H}_c + \frac{E_c}{2}\sigma_z^{(1)}\sigma_z^{(2)}. \qquad (8)$$

Thus, besides changing the energy by a constant, it also generates the cross term with

$$E_c = \alpha_1\alpha_2/2C. \qquad (9)$$

The cross term represents the effective nonlocal interaction between the detectors mediated by the circuit collective mode. At the degeneracy points $\varepsilon_1 = \pm\varepsilon_2$, this interaction leads to the quantum avoided-crossing level splitting of the value

$$\Delta\varepsilon = 4\Delta_1\Delta_2 E_c/\varepsilon_1^2. \qquad (10)$$

Finally, after switching to the interaction picture, the tunneling Hamiltonian for two detectors takes the form

$$\mathcal{H}_I = \begin{pmatrix} 0 & \Delta_2 e^{i\alpha_2\phi + i(\varepsilon_2 - E_c)t} & \Delta_1 e^{i\alpha_1\phi + i(\varepsilon_1 - E_c)t} & 0 \\ \Delta_2 e^{-i\alpha_2\phi - i(\varepsilon_2 - E_c)t} & 0 & 0 & \Delta_1 e^{i\alpha_1\phi + i(\varepsilon_1 + E_c)t} \\ \Delta_1 e^{-i\alpha_1\phi - i(\varepsilon_1 - E_c)t} & 0 & 0 & \Delta_2 e^{i\alpha_2\phi + i(\varepsilon_2 + E_c)t} \\ 0 & \Delta_1 e^{-i\alpha_1\phi - i(\varepsilon_1 + E_c)t} & \Delta_2 e^{-i\alpha_2\phi - i(\varepsilon_2 + E_c)t} & 0 \end{pmatrix}. \qquad (11)$$

Note that if $E_c = 0$ the tunneling Hamiltonian (11) reduces to a tensor sum $\mathcal{H}_I = \mathcal{H}_I^{(1)} \otimes \mathbb{E} + \mathbb{E} \otimes \mathcal{H}_I^{(2)}$. We will see that the presence of E_c leads to a trivial mechanism of cross correlations even in the presence of only local noise. However, these cross correlations vanish with small E_c, and thus may become dominated by some more interesting phenomena, such as effects of quantum fluctuations in higher-order processes.

B. Time evolution

To study cross correlations in the detectors' output, we wish to consider the long-time limit, where evolution of the detectors' states can be described by the master equation

$$\dot{P} = \hat{M}P \qquad (12)$$

for the occupation probabilities $P = (p_1, \ldots, p_4)$. Such description is valid if the decoherence time $\tau_d = (\alpha^2 R^2 S)^{-1}$ is much shorter, $\tau_d \ll \Delta^{-1}$, than the quantum time scale associated with the quantum level repulsion of a single detector, or $\tau_d \ll (\Delta\varepsilon)^{-1}$ in the degeneracy point. These conditions are equivalent to a requirement that the classical level broadening due to the noise is always stronger then the quantum avoided-crossing level repulsion. Or, in other words,

one may say that the stochastic switching time $\tau_s = 1/(\Delta^2\tau_d)$ is longer than the dephasing time, $\tau_s \gg \tau_d$.

The time dependence of the reduced density matrix of the detectors is

$$\rho(t) = \text{Tr}_c[U\tilde{\rho}(0)U^\dagger], \qquad (13)$$

where

$$U = \hat{T}\exp\left[-i\int_0^t dt' \mathcal{H}_I(t')\right] \qquad (14)$$

is the interaction picture evolution operator, the initial condition is represented by the full density matrix of the system of detectors together with the circuit $\tilde{\rho}(0) = \rho(0) \times \rho_c(0)$, and Tr_c means averaging over the circuit degrees of freedom.

As soon as $\rho(t)$ is found, one observes that the off-diagonal elements of ρ decay exponentially over the time τ_d. Therefore we can concentrate our attention on the probabilities

$$p_k(t) = \rho_{kk}(t) = \text{Tr}[\rho(0) \times \rho_c(0)\, U^\dagger|k\rangle\langle k|U], \qquad (15)$$

where the trace is over all the degrees of freedom of the system. Equation (15) may be recast in the form

$$P(t) = [\mathbb{E} + \hat{\mathcal{M}}(t)]P(0), \qquad (16)$$

where the elements of the time-dependent transition probability matrix $\hat{\mathcal{M}}(t)$ read

$$\{\hat{\mathcal{M}}(t)\}_{kl} = \mathrm{Tr}_c[\rho_c(0)\,\langle k|U^\dagger|l\rangle\langle l|U|k\rangle] - \delta_{kl}. \qquad (17)$$

They may be found from Eq. (15) as the perturbative expansion $\hat{\mathcal{M}}(t) = \sum_n \hat{\mathcal{M}}_n(t)$ with respect to the tunneling Hamiltonian (11). Comparing the expression (16) with a solution of Eq. (12), $\mathrm{P}(t) = \exp\{\hat{M}t\}\mathrm{P}(0)$, one finds that the transition matrix $\hat{M}$ can be obtained from the long-time asymptotics of the irreducible part of $\hat{\mathcal{M}}(t)$ as

$$\hat{M} = \lim_{t\to\infty} \frac{1}{t} \ln[\mathbb{E} + \hat{\mathcal{M}}(t)]. \qquad (18)$$

Note, that due to the specific structure of (11), only even powers of Δ_j are present in the diagonal elements of the perturbation series for the density matrix (13). Therefore, to the fourth order in Δ_j, one can write $\hat{\mathcal{M}}(t) = \hat{\mathcal{M}}_2(t) + \hat{\mathcal{M}}_4(t)$. The time dependence of an arbitrary element of the $\hat{\mathcal{M}}_2(t)$ appears to be quite simple:

$$\{\hat{\mathcal{M}}_2(t)\}_{kl} = C_0(t) + C_1 t, \qquad (19)$$

where C_1 is a constant and the first term $C_0(t)$ is a decaying oscillating function of time. This decaying term only represents the artefact of the chosen decomposition of the initial condition $\tilde{\rho} = \rho \times \rho_c$, which, due to the detector-circuit interaction, does not describe a stationary state. However, the stationary, linear in time behavior is restored after the decoherence time τ_d, as illustrated by the decaying oscillations in the Fig. 5.

The elements of the next term, $\hat{\mathcal{M}}_4(t)$, may have a more complex structure:

$$\{\hat{\mathcal{M}}_4(t)\}_{kl} = C_0'(t) + [C_1' + D_1'(t)]t + C_2' t^2, \qquad (20)$$

where the functions $C_0'(t)$ and $D_1'(t)$ are analogous to $C_0(t)$, the constant C_1' is analogous to C_1, and there is a quadratic in time term $C_2' t^2$. This last term is a reducible part of the $\hat{\mathcal{M}}_4(t)$, which is exactly eliminated in the logarithm in Eq. (18). Particularly, to the fourth order, one has

$$\hat{M} = \lim_{t\to\infty} \frac{1}{t}\left[\hat{\mathcal{M}}_2(t) + \hat{\mathcal{M}}_4(t) - \frac{1}{2}\hat{\mathcal{M}}_2^2(t)\right]. \qquad (21)$$

This expression provides a conclusive point in the calculation of the transition matrix $\hat{M}$ from the explicit time dependence of the occupations $\mathrm{P}(t)$.
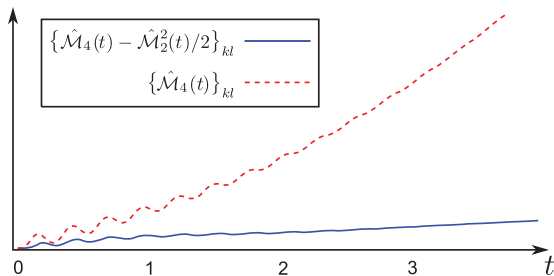

FIG. 5. (Color online) Typical quadratic in time dependence (dashed line) of the elements of the bare matrix $\hat{\mathcal{M}}_4(t)$, and linear in time (solid line) for the corrected matrix $\hat{\mathcal{M}}_4(t) - \hat{\mathcal{M}}_2^2(t)/2$. Time scale in the units of the decoherence time τ_d.

III. FULL COUNTING STATISTICS OF THE QPC CURRENTS

In this section, we present the method for calculating the statistics of a QPC current using the generating functions approach. We first recall the simpler case of a single detector[24,41] and later generalize it to the case of two detectors. Consider a QPC as a charge detector tuned to detect the state of the two-level system (see Fig. 1). The statistics of the current passing through the corresponding conductance levels $k = 1,2$ of the QPC is described at long times by the moment generating function

$$g_k(\lambda,t) = \sum_n \exp(\lambda n) f_k(n) = e^{H_k(\lambda)t}, \qquad (22)$$

where $f_k(n)$ is the probability that n electrons are transferred, while the level k is occupied, t is the total time of the measurement, and $H_k(\lambda)$ is the function generating current cumulants:

$$\langle\!\langle I_k^m\rangle\!\rangle = \frac{\partial^m H_k}{\partial \lambda^m}\bigg|_{\lambda\to 0}. \qquad (23)$$

One can see that $g_k(\lambda,t)$ satisfy the equations

$$\dot{g}_k(\lambda,t) = H_k(\lambda)g_k(\lambda,t). \qquad (24)$$

These equations may be modified in order to take into account the mixing of the current channels induced by switching of the detector. Introducing $\mathrm{G}(\lambda,t) \equiv (g_1,g_2)$, the extended master equation reads

$$\dot{\mathrm{G}}(\lambda,t) = \hat{W}\mathrm{G}(\lambda,t), \qquad (25)$$

where the matrix $\hat{W} = \hat{H} + \hat{M}$ is a sum of the cumulant generating functions matrix

$$\hat{H} = \begin{pmatrix} H_1(\lambda) & 0 \\ 0 & H_2(\lambda) \end{pmatrix}, \qquad (26)$$

and the transition matrix

$$\hat{M} = \begin{pmatrix} -\Gamma_+ & \Gamma_- \\ \Gamma_+ & -\Gamma_- \end{pmatrix}, \qquad (27)$$

where the transition rates are denoted as $\Gamma_\pm$. In the long-time limit, the statistics of the QPC current is given by the generating function

$$H(\lambda) = \lim_{t\to\infty} \frac{1}{t} \ln \sum_k g_k(\lambda,t), \qquad (28)$$

which is the largest eigenvalue of the matrix $\hat{W}$,

$$H(\lambda) = H_s - \Gamma_s + \sqrt{(H_d + \Gamma_d)^2 + \Gamma_-\Gamma_+}, \qquad (29)$$

where $H_{s,d} = \frac{1}{2}(H_1 \pm H_2)$, and $\Gamma_{s,d} = \frac{1}{2}(\Gamma_+ \pm \Gamma_-)$. Current cumulants are then found as derivatives of this generating function:

$$\langle\!\langle I^m\rangle\!\rangle = \frac{\partial^m H(\lambda)}{\partial \lambda^m}\bigg|_{\lambda\to 0}. \qquad (30)$$

Taking into account that

$$\frac{\partial H(\lambda)}{\partial \lambda}\bigg|_{\lambda \to 0} = \sum_k \frac{\partial H}{\partial H_k} \frac{\partial H_k}{\partial \lambda}\bigg|_{\lambda \to 0}$$

$$= \sum_k \langle I_k \rangle \frac{\partial H}{\partial H_k}\bigg|_{H_k \to 0}, \qquad (31)$$

we obtain the expected result for the average current:

$$\langle I \rangle = \frac{\Gamma_- \langle I_1 \rangle}{2\Gamma_s} + \frac{\Gamma_+ \langle I_2 \rangle}{2\Gamma_s} = \bar{p}_1 \langle I_1 \rangle + \bar{p}_2 \langle I_2 \rangle, \qquad (32)$$

and similarly, for the noise,

$$\langle\!\langle I^2 \rangle\!\rangle = \frac{\bar{p}_1 \bar{p}_2}{\Gamma_s}(\langle I_2 \rangle - \langle I_1 \rangle)^2 + \bar{p}_1 \langle\!\langle I_1^2 \rangle\!\rangle + \bar{p}_2 \langle\!\langle I_2^2 \rangle\!\rangle. \qquad (33)$$

Here, $\langle I_k \rangle$ is the average current at the conduction level k of the QPC, $\bar{p}_{2,1} = \Gamma_\pm/(\Gamma_+ + \Gamma_-)$ are the stationary level occupations, and $\langle\!\langle I_k^2 \rangle\!\rangle$ is the corresponding zero-frequency noise power of the QPC.

A. Cross correlator

The above scheme can be easily generalized to a pair of two such detectors. Without level mixing, the joint probability distribution for the numbers of electrons passed through each QPC factorizes: $f_{kl}(n_1, n_2) = f_k(n_1) f_l(n_2)$, therefore the corresponding generating functions acquire the form

$$g_{kl}(\lambda, \eta, t) = g_k^{(1)}(\lambda, t) g_l^{(2)}(\eta, t), \qquad (34)$$

with different counting variables λ and η for the two QPCs, respectively, and satisfies the equation

$$\dot{g}_{kl}(\lambda, \eta, t) = \left[H_k^{(1)}(\lambda) + H_l^{(2)}(\eta) \right] g_{kl}(\lambda, \eta, t), \qquad (35)$$

where the upper index (j) indicates the properties of the detector j.

Switching of the detectors is accounted for in the extended master equation, $\dot{G} = \hat{W}G$, with the matrix that has the same structure as before: $\hat{W} = \hat{M} + \hat{H}$, where $\hat{M}$ is the transition matrix. In the basis of states $(|11\rangle, |12\rangle, |21\rangle, |22\rangle)$, the diagonal matrix $\hat{H}$ of the cumulant generating functions is

$$\hat{H} = \hat{H}_1 \otimes \mathbb{E} + \mathbb{E} \otimes \hat{H}_2, \qquad (36)$$

where $\hat{H}_j$ are the single detector matrices (26),

$$\hat{H}_j = \begin{pmatrix} H_1^{(j)} & 0 \\ 0 & H_2^{(j)} \end{pmatrix}, \qquad (37)$$

as functions of corresponding counting variables λ or η.

As before, current cumulants may be found as derivatives of the largest eigenvalue $H(\lambda, \eta)$ of the matrix $\hat{W}$. Particularly, the current cross correlator reads

$$\langle\!\langle I_1 I_2 \rangle\!\rangle = \partial_\lambda \partial_\eta H(\lambda, \eta)|_{\lambda, \eta = 0}. \qquad (38)$$

Since there is only one derivative with respect to each counting variable, the cumulant generators $H_k^{(j)}$ can be replaced by their corresponding average currents $H_k^{(1)}(\lambda) \approx \lambda \langle I_k^{(1)} \rangle$ and $H_k^{(2)}(\eta) \approx \eta \langle I_k^{(2)} \rangle$. Thus the problem of finding the cross correlator of QPC currents can be reformulated as finding a second-order correction to the largest eigenvalue of the matrix $\hat{M}$ from the perturbation $\hat{H}$.

Note that due to the conservation of the total probability $\sum_j \hat{M}_{jk} = 0$, the transition matrix is degenerate, i.e., one of its eigenvalues $m_0 = 0$, and all the others are negative, $m_{i \neq 0} < 0$. Therefore we can use the general result of the perturbation theory that $H(\lambda, \eta)$ can be found as a second-order correction to the largest eigenvalue m_0:

$$H(\lambda, \eta) = \sum_{i \neq 0} \frac{\langle m_0 | \hat{H} | m_i \rangle \langle m_i | \hat{H} | m_0 \rangle}{m_0 - m_i}$$

$$= -\langle m_0 | \hat{H} \hat{M}^* \hat{H} | m_0 \rangle, \qquad (39)$$

where $\hat{M}^* = \sum_{i \neq 0}(1/m_i)|m_i\rangle\langle m_i|$ is also known as a Moore-Penrose pseudoinverse of the matrix $\hat{M}$; $|m_i\rangle$ are the eigenvectors[42] of $\hat{M}$.

To find the cross correlator, we need to calculate the derivative in Eq. (38). Since the counting variables only enter the matrix $\hat{H}$, it is convenient to introduce the following operator:

$$\hat{\mathcal{D}} = \frac{\partial^2}{\partial_\lambda \partial_\eta}(\hat{H}|m_0\rangle\langle m_0|\hat{H})|_{\lambda, \eta \to 0}, \qquad (40)$$

so that the cross correlator may be expressed as

$$\langle\!\langle I_1 I_2 \rangle\!\rangle = -\text{Tr}(\hat{\mathcal{D}} \cdot \hat{M}^*). \qquad (41)$$

It appears also, that the cross correlator is always proportional to the differences of the current levels $\Delta I_1 = \langle I_2^{(1)} \rangle - \langle I_1^{(1)} \rangle$ and $\Delta I_2 = \langle I_2^{(2)} \rangle - \langle I_1^{(2)} \rangle$, therefore it is natural to define a normalized cross correlator that has the dimensionality of time:

$$X = \frac{\langle\!\langle I_1 I_2 \rangle\!\rangle}{\Delta I_1 \Delta I_2} = -\frac{\text{Tr}(\hat{\mathcal{D}} \cdot \hat{M}^*)}{\Delta I_1 \Delta I_2}. \qquad (42)$$

Finally, we note that in practice, for systems with sufficiently general transition matrix $\hat{M}$, the approach described above typically leads to cumbersome expressions, in particular, for the eigenvalues and the eigenvectors of $\hat{M}$. Therefore we present a different way to evaluate the cross correlator for a particular class of transition matrices.

B. Perturbative approach

We wish to consider a particular case, when the effective interaction E_c in the Hamiltonian (11) can be neglected, $E_c \to 0$. In this case, one can easily find the cross correlator perturbatively in tunneling amplitude Δ. The transition matrix can be represented as

$$\hat{M} = \hat{M}_0 + \delta\hat{M}, \qquad (43)$$

where $\hat{M}_0$ is evaluated to the second order in tunneling and represents completely uncorrelated detectors, while $\delta\hat{M}$ is a small correction of the fourth order. The matrix $\hat{M}_0 = \hat{M}_1 \otimes \mathbb{E} + \mathbb{E} \otimes \hat{M}_2$ is a tensor sum of the single-detector transition matrices $\hat{M}_1$ and $\hat{M}_2$, defined according to Eq. (27) as

$$\hat{M}_j = \begin{pmatrix} -\Gamma_+^{(j)} & \Gamma_-^{(j)} \\ \Gamma_+^{(j)} & -\Gamma_-^{(j)} \end{pmatrix}. \qquad (44)$$

The extended master equation matrix then reads

$$\hat{W} = \hat{M} + \hat{H} = \hat{W}_0 + \delta\hat{M}, \qquad (45)$$

where, the matrix $\hat{W}_0 = \hat{M}_0 + \hat{H}$ retains the same tensor structure: $\hat{W}_0 = \hat{W}_1 \otimes \mathbb{E} + \mathbb{E} \otimes \hat{W}_2$, and $\hat{W}_1$ and $\hat{W}_2$ are the single detector matrices as in Eq. (25). The matrix $\hat{W}_0$ can be diagonalized explicitly, and its largest eigenvalue is, obviously, the sum of the corresponding eigenvalues (29) for each detector separately, $H_0(\lambda,\eta) = H^{(1)}(\lambda) + H^{(2)}(\eta)$. Thus, as expected for the uncorrelated detectors,

$$\langle\langle I_1 I_2 \rangle\rangle_0 = \partial_\lambda \partial_\eta H_0(\lambda,\eta) = 0, \tag{46}$$

where the index 0 stands for the lowest-order contribution. We see that any cross correlations may only be generated by the correction matrix $\delta\hat{M}$, which brakes the tensor sum structure of $\hat{W}$.[44]

As before, we need to find the largest eigenvalue $H(\lambda,\eta) = H_0(\lambda,\eta) + \delta H(\lambda,\eta)$ of $\hat{W}$, where $\delta H(\lambda,\eta)$ is a correction due to the perturbation $\delta\hat{M}$. Note that in contrast to the general case, the counting variables are not contained in the perturbation $\delta\hat{M}$, but are absorbed into the matrix $\hat{W}_0$ (which makes $\hat{W}_0$ nondegenerate). Therefore it is sufficient to use the first-order perturbation theory:

$$\delta H(\lambda,\eta) = \langle m_0 | \delta\hat{M} | m_0 \rangle, \tag{47}$$

where $|m_0\rangle$ is the eigenvector of $\hat{W}_0$ corresponding to the eigenvalue $H_0(\lambda,\eta)$.

Finally, the cross correlator is given by

$$\langle\langle I_1 I_2 \rangle\rangle = \partial_\lambda \partial_\eta \delta H(\lambda,\eta)|_{\lambda,\eta=0}, \tag{48}$$

and analogously to Eq. (31), we find that

$$\langle\langle I_1 I_2 \rangle\rangle = \sum_{k,l} \frac{\partial^2 \delta H}{\partial H_k^{(1)} \partial H_l^{(2)}}\bigg|_{H_k^{(j)}=0} \langle I_k^{(1)}\rangle\langle I_l^{(2)}\rangle. \tag{49}$$

Although the full expression for δH is somewhat cumbersome, it has an important property that the derivatives with respect to $H_k^{(1)}$ and $H_l^{(2)}$ are all proportional to the same quantity X_p,

$$\frac{\partial^2 \delta H}{\partial H_k^{(1)} \partial H_l^{(2)}}\bigg|_{\lambda,\eta=0} = (-1)^{k+l} X_p. \tag{50}$$

Therefore the cross correlator (49) acquires the same form as in Eq. (42) and can be found using one of the relations (50):

$$X_p = \frac{2\Gamma_-^{(1)}\Gamma_+^{(1)}\Gamma_-^{(2)}\Gamma_+^{(2)}}{(\Gamma_-^{(1)} + \Gamma_+^{(1)})^3 (\Gamma_-^{(2)} + \Gamma_+^{(2)})^3} \times \sum_{ij} \delta\hat{M}_{ij}\hat{K}_{ij}, \tag{51}$$

where to present a compact expression for X_p, we have introduced the matrix of the coefficients $\hat{K}_{ij}$:

$$\hat{K} = \begin{pmatrix} 1 & \frac{\Gamma_+^{(2)}}{\Gamma_-^{(2)}} & \frac{\Gamma_+^{(1)}}{\Gamma_-^{(1)}} & \frac{\Delta\Gamma^{(1)}\Delta\Gamma^{(2)}}{2\Gamma_-^{(1)}\Gamma_-^{(2)}} - 1 \\ -\frac{\Gamma_-^{(2)}}{\Gamma_+^{(2)}} & -1 & \frac{\Delta\Gamma^{(1)}\Delta\Gamma^{(2)}}{2\Gamma_-^{(1)}\Gamma_+^{(2)}} + 1 & -\frac{\Gamma_+^{(1)}}{\Gamma_-^{(1)}} \\ -\frac{\Gamma_-^{(1)}}{\Gamma_+^{(1)}} & \frac{\Delta\Gamma^{(1)}\Delta\Gamma^{(2)}}{2\Gamma_+^{(1)}\Gamma_-^{(2)}} + 1 & -1 & -\frac{\Gamma_+^{(2)}}{\Gamma_-^{(2)}} \\ \frac{\Delta\Gamma^{(1)}\Delta\Gamma^{(2)}}{2\Gamma_+^{(1)}\Gamma_+^{(2)}} - 1 & \frac{\Gamma_-^{(1)}}{\Gamma_+^{(1)}} & \frac{\Gamma_-^{(2)}}{\Gamma_+^{(2)}} & 1 \end{pmatrix}, \tag{52}$$

and the notation $\Delta\Gamma^{(j)} = \Gamma_+^{(j)} - \Gamma_-^{(j)}$ is used, with $\Gamma_\pm^{(j)}$ being the transition rates to the second-order of the perturbation theory.

IV. CUMULANT EXPANSION

To find the transition rates (43), we expand the transition probabilities (17) in the perturbation series with respect to the tunneling Hamiltonian (11). We will henceforth use simple angled brackets to denote the weighted average $\langle \ldots \rangle \equiv \mathrm{Tr}_c[\rho_c \ldots]$, which is used in Eq. (17). By substituting the evolution operator (14), expanded perturbatively with respect to the tunneling Hamiltonian (11), into the expression (17) for the transition probability matrix, one can show that the elements of $\hat{\mathcal{M}}(t)$ may be expressed in terms of time integrals of the Keldysh ordered correlation functions of the following type:

$$\langle e^{i\gamma_1\phi_1} \ldots e^{i\gamma_n\phi_n} \rangle = e^{F_n[\eta(t)]}|_{\eta(t)=\sum_{j=1}^n i\gamma_j\delta(t-t_j)}, \tag{53}$$

where $\phi_j \equiv \phi(t_j)$ for simplicity, the γ_j take values of $\pm\alpha_1$ or $\pm\alpha_2$, and the generating function

$$F_n[\eta(t)] = \sum_k \frac{1}{k!} \int d^k t \prod_{j=1}^k \eta(t_j) \, \mathrm{T}_K\langle\langle \phi_1 \ldots \phi_k \rangle\rangle \tag{54}$$

is an expansion in terms of the Keldysh ordered cumulants $\mathrm{T}_K\langle\langle \phi_1 \ldots \phi_k \rangle\rangle$ of the order k.

We do not need to know all the correlators that appear in the expansion of the density matrix, but only those that contribute to the master equation through the matrices $\hat{\mathcal{M}}_2(t)$ and $\hat{\mathcal{M}}_4(t)$ from Eq. (21). In the second-order term $\hat{\mathcal{M}}_2(t)$, due to the structure of the tunneling Hamiltonian (11), we find only the terms proportional to Δ_1^2 or Δ_2^2, thus "originating" separately from each detector. In this case, we need the correlator of the form

$$\langle e^{i\gamma\phi_1} e^{-i\gamma\phi_2} \rangle = e^{-J_2^{(2)} - iJ_3^{(2)}}, \tag{55}$$

where the second and third cumulants, $J_2^{(2)}(t_1,t_2)$ and $J_3^{(2)}(t_1,t_2)$, are found by expanding both sides of the equation (53) in ϕ_j:

$$J_2^{(2)}(t_1,t_2) = \frac{\gamma^2}{2}\langle\phi_1^2 - 2\phi_1\phi_2 + \phi_2^2\rangle, \tag{56}$$

$$J_3^{(2)}(t_1,t_2) = \frac{\gamma^3}{6}\langle\phi_1^3 - 3\phi_1^2\phi_2 + 3\phi_1\phi_2^2 - \phi_2^3\rangle. \tag{57}$$

By contrast, in the fourth-order term $M_4(t)$, there is more diversity. First of all, there are terms proportional to $\Delta_1^2\Delta_2^2$.

Such terms are of our main interest, since they represent the nonlocal processes, involving both detectors. Thus we are interested in correlators

$$\langle e^{i\gamma_1\phi_1}\dots e^{i\gamma_4\phi_4}\rangle = e^{-J_2^{(4)}(t_1,\dots,t_4)-iJ_3^{(4)}(t_1,\dots,t_4)}, \quad (58)$$

where the four parameters γ_j take four different values of $\pm\alpha_1$ and $\pm\alpha_2$. Again, expanding both sides of Eq. (58), one finds

$$J_2^{(4)} = \frac{1}{2}\left\langle \sum_k \gamma_k^2\phi_k^2 + 2\sum_{k<l}\gamma_k\gamma_l\phi_k\phi_l \right\rangle, \quad (59)$$

and, in contrast with the result (57), the third-order cumulant

$$J_3^{(4)} = \frac{1}{6}\left\langle \sum_i \gamma_i^3\phi_i^3 + 3\sum_{i<j}\gamma_i\gamma_j\phi_i(\gamma_i\phi_i+\gamma_j\phi_j)\phi_j \right.$$
$$\left. + 6\sum_{i<j<k}\gamma_i\gamma_j\gamma_k\phi_i\phi_j\phi_k \right\rangle \quad (60)$$

contains the three-point correlators $\langle\phi_i\phi_j\phi_k\rangle$ taken at three different times.

V. CURRENT AND PHASE CORRELATION FUNCTIONS

From the equation of motion $\dot\phi = Q/C$ generated by the Hamiltonian (1), it follows that the phase ϕ is an integral of the voltage on the capacitor. Since the constant component of voltage can be absorbed into the level splitting energy ε, we include only the fluctuations δV,

$$\phi(t) = \int_{-T}^{t} dt'\delta V(t'), \quad (61)$$

where instead of integrating over time from minus infinity, we have introduced the regularization with some finite time $-T$, which is larger than any other time scale in the system.

The voltage fluctuations are related to the current fluctuations by the circuit impedance $Z(\omega)$,

$$\delta V(\omega) = Z(\omega)j(\omega). \quad (62)$$

Combining the last two equations together, one obtains

$$\phi(t) = \int \frac{d\omega}{2\pi} Z(\omega)j(\omega)\frac{e^{-i\omega t}-e^{i\omega T}}{(-i\omega)}, \quad (63)$$

where, due to the regularization, a pole at $\omega = 0$ is canceled by a zero in the numerator. We further assume the limit of fast circuit,[45] i.e., the main contribution to the phase correlators comes from times much longer than the circuit response time $\tau_{RC} = RC$, and the impedance may be considered a constant $Z(\omega) = R$.

We define the current spectral functions $S_2(\omega)$ and $S_3(\omega_1,\omega_2)$ through nonsymmetrized two- and three-point current correlation functions:

$$\langle j(\omega_1)j(\omega_2)\rangle = 2\pi\delta(\omega_1+\omega_2)S_2(\omega_1), \quad (64)$$

$$\langle j(\omega_1)j(\omega_2)j(\omega_3)\rangle = 2\pi\delta(\omega_1+\omega_2+\omega_3)S_3(\omega_1,\omega_2), \quad (65)$$

where the δ-function are the manifestation of the time translation invariance, since we consider a stationary process. Using these definitions, the phase correlators are expressed as following:

$$\langle\phi(t_1)\phi(t_2)\rangle = R^2\int \frac{d^2\omega}{(2\pi)^2}\delta(\omega_1+\omega_2)S_2(\omega_1)$$
$$\times \prod_{j=1,2}\left(\frac{e^{-i\omega_j t_j}-e^{i\omega_j T}}{-i\omega_j}\right), \quad (66)$$

and analogously,

$$\langle\phi(t_1)\phi(t_2)\phi(t_3)\rangle = R^3\int \frac{d^3\omega}{(2\pi)^3}\delta(\omega_1+\omega_2+\omega_3)S_3(\omega_1,\omega_2)$$
$$\times \prod_{j=1..3}\left(\frac{e^{-i\omega_j t_j}-e^{i\omega_j T}}{-i\omega_j}\right). \quad (67)$$

Taking the integrals over ω, we find the long-time asymptotic behavior of the correlator (66) as

$$\langle\phi(t_i)\phi(t_j)\rangle = R^2\{S[T+\min(t_i,t_j)] - iS'\,\mathrm{sign}(t_i-t_j)\}. \quad (68)$$

Here, only the zero-frequency components of the spectral function contribute to the correlator:

$$S_2(\omega) \simeq S + \omega S', \quad (69)$$

where $S \equiv S_2(0)$ is the classical noise power and $S' \equiv dS_2(\omega)/d\omega|_{\omega=0}$ is the quantum part, representing noncommutativity of the current operator.

The correlator (68) has a divergent term proportional to T, which, however, does not contribute to any physically meaningful quantity. For instance, the cumulants (56) and (59) read

$$J_2(t_1,t_2) = \frac{\alpha_j^2 R^2}{2}[S|t_1-t_2| - iS'\,\mathrm{sign}(t_1-t_2)], \quad (70)$$

while the cumulant from the fourth order can be rewritten in the similar form:

$$J_2^{(4)}(t_1\dots t_4) = \frac{R^2}{2}\sum_{i<j}\alpha_i\alpha_j[S|t_i-t_j| - iS'\,\mathrm{sign}(t_i-t_j)]. \quad (71)$$

A weak-coupling regime, considered throughout the paper, implies that either the detector to circuit coupling is weak (small $\alpha_j \ll 1$) or the impedance R is small. Then the entire J_2 is small by the factor $\alpha_i\alpha_j R^2$, and the contribution to the time integrals of correlators (55) and (58) that enter the perturbation expansion for $\hat{\mathcal{M}}(t)$, see Eq. (17), comes from large differences of times of order $|t_i - t_j| \sim (\alpha_i\alpha_j R^2 S)^{-1}$. This justifies the use of asymptotic expressions of the type (70) and (71) and clarifies the assumption of the fast circuit $\tau_{RC} \ll \tau_d = (\alpha^2 R^2 S)^{-1}$.

Analogously, for the three-point correlator (67), we expand the spectral function around zero frequencies,

$$S_3(\omega_1,\omega_2) = S_3 + S_3^{(1)}\omega_1 + S_3^{(2)}\omega_2, \quad (72)$$

and obtain

$$\langle \phi(t_1)\phi(t_2)\phi(t_3)\rangle = -R^3 S_3[T + \min(t_j)]$$
$$- i R^3 \sum_{j=1,2} S_3^{(j)} \left\{ \theta\left[\min_{k\neq j}(t_k) - t_j \right] - \frac{1}{2} \right\}. \tag{73}$$

However, the parameters $S_3^{(1)}$ and $S_3^{(2)}$ are not independent. From the definition (65), one can find that $(S_3^{(1)})^* = S_3^{(1)} = 2\mathrm{Re}S_3^{(2)}$, therefore they are parameterized by two real numbers: the real and imaginary parts of $S_3^{(2)}$. Again, although these correlators diverge with T, the third cumulant, such as Eq. (57), is regular:

$$J_3^{(2)}(t_1,t_2) = -\frac{\gamma^3 R^3}{6} \left\{ S_3(t_1 - t_2) \right.$$
$$\left. + \frac{3i}{2}\left[\mathrm{Re}S_3^{(2)} + i\,\mathrm{Im}S_3^{(2)}\mathrm{sign}(t_1 - t_2)\right]\right\}. \tag{74}$$

Note, that in contrast to the second cumulant (70), which depends on the absolute value $|t_1 - t_2|$, the classical part of the third cumulant depends on the difference $(t_1 - t_2)$ and because of this, it enters the transition rates the same way as the energies ε_j. This leads to a specific shift of the energies, discussed later. A similar expression is obtained for the third cumulant (60) to the fourth order, but it is not displayed here, because it is somewhat cumbersome.

VI. THE CROSS CORRELATOR: RESULTS

Using the cumulant expansions such as Eqs. (70), (71), and (74), we can find the correlators (55) and (58), which we need to calculate the transition probability matrix (17). From the latter, we extract the master equation matrix (21) and, finally, find the cross correlator using one of the results (42) or (51). We assume that the cross correlator is a sufficiently smooth function of the parameters of the system, and thus consider separately different contributions, each being suppressed by some small parameter. This allows us to estimate ratios of these contributions and determine regions of the parameters where one or another contribution is dominating. We start with the case when the effective interaction E_c in the Hamiltonian is neglected, and the noise is classical. Then the detector outputs become completely uncorrelated. We consecutively "turn on" the effective interaction and the quantum parts of the noise cumulants to investigate separately each contribution.

A. Classical noise

For the classical noise, the current operator $j(t)$ may be replaced by a classical variable, which commutes with itself at different moments of time. Therefore the spectral function $S_2(\omega)$ becomes a symmetric function, and its derivative at zero frequency is $S' = 0$. In the same way, the three-point spectral function (65) has zero slope $S_3^{(1)} = S_3^{(1)'} = 0$, and we set $S_3(\omega_1,\omega_2) = S_3$.

The transition matrix $\hat{M}$ for the classical noise is symmetric. Taking into account that the sum over columns of the elements of $\hat{M}$ is zero, one can show that the stationary solution of the master equation, corresponding to the largest eigenvalue,

is $|m_0\rangle = \frac{1}{4}(1,1,1,1)$ with equal occupations of $p_j = 1/4$ for all the four states, which can be regarded as a limit of high temperature. The matrix (40) in this case is a constant independent on the parameters of the system, and the pseudoinverse $\hat{M}^*$ can be found explicitly. Note that since the matrix $\hat{M}$ is symmetric, it is parametrized by six elements of its upper-right triangle. Then, finally, the classical contribution to the cross correlator is expressed in terms of those elements:

$$X_{\mathrm{cl}} = \frac{1}{32\sum_2}\left[(m_{12} - m_{34})(m_{13} - m_{24}) - (m_{14} - m_{23})\sum_1\right], \tag{75}$$

where $\sum_1 = m_{12} + m_{13} + m_{24} + m_{34}$ and the sum in the denominator $\sum_2 = \sum m_{ij}m_{kl}m_{nm}$ is over products of all combinations of three different elements from the six independent elements of $\hat{M}$.

The expression (75) gives certain interpretation of how different combinations of the transition rates contribute to the cross correlator, particularly, if the transition rates in one detector depend on the current state of the other detector. For example, if $m_{12} > m_{34}, m_{13} > m_{24}$, then the positive cross correlations are induced. One important observation is that if the effective interaction term in the Hamiltonian is neglected, then the transition matrix acquires additional symmetry $m_{ij} = m_{kl}$ for $i + l = j + k = 5$ (symmetry relative to the antidiagonal), and $m_{14} = m_{23}$. Then the cross correlator is identically equal to zero. In other words, the cross correlator for classical noise vanishes together with the effective interaction strength E_c.

If the transition rates are calculated only to the second order in Δ_j, the antidiagonal elements of $\hat{M}$ are zero, and the result (75) simplifies further. Substituting the corresponding transition rates, we obtain the classical contribution, shown in the Fig. 6:

$$X_{\mathrm{cl}}^{(2)} = \frac{1}{4R^2 S} \frac{\varepsilon_1\varepsilon_2 E_c^2}{\Delta_2^2\alpha_2^2(\varepsilon_1^2 + E_c^2 + \Omega_1^2) + \Delta_1^2\alpha_1^2(\varepsilon_2^2 + E_c^2 + \Omega_2^2)}, \tag{76}$$

where we introduce the classical level broadening,
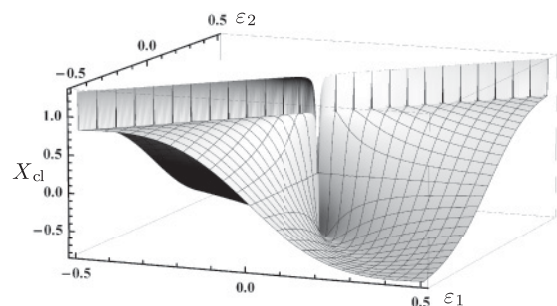
$$\Omega_j = \alpha_j^2 R^2 S/2, \tag{77}$$

FIG. 6. The classical contribution $X_{\mathrm{cl}} = X_{\mathrm{cl}}^{(2)} + X_{\mathrm{cl}}^{(4)}$ to the cross correlator in units of the dephasing time $1/\Omega = (\alpha_1\alpha_2 RT_{\mathrm{eff}})^{-1}$ as a function of the detector level splittings ε_1 and ε_2 in units of the effective temperature T_{eff} at $\Delta_j = 0.07\Omega, \alpha_j = 0.55 \pm 0.05, R = 0.12$, and $C = 0.8/\Omega$. Note that the second-order part $X_{\mathrm{cl}}^{(2)}$ does not exhibit any singularities at the nonlocal resonances $\varepsilon_1 \pm \varepsilon_2 \to 0$, and the sharp peaks come from the fourth-order part $X_{\mathrm{cl}}^{(4)}$.

and recall that E_c is the effective nonlocal interaction energy (9). The typical maximum magnitude of this quantity, that is acquired at $\varepsilon_1/\varepsilon_2 \sim \pm \alpha_1 \Delta_1 / \alpha_2 \Delta_2$, can be estimated as

$$X_{\mathrm{cl}}^{(2)} \approx \pm \frac{E_c^2}{8\alpha_1\alpha_2\Delta_1\Delta_2 R^2 S} = \pm \frac{E_c^2}{16\Delta_1\Delta_2\sqrt{\Omega_1\Omega_2}}. \quad (78)$$

It contains competing small parameters: Δ_j that are considered the smallest energy scales in the system, and effective interaction strength E_c. It is especially remarkable that E_c is a property of the circuit itself and does not depend on the source of noise. In fact, the quantity $X_{\mathrm{cl}}^{(2)}$ reflects rather the influence of the effective nonlocal coupling, than of the common noise source. If we replace the common noise source with two independent local sources, or even only one local source of noise, the cross correlations will persist. For the same reason, the classical contribution (76) is a well bounded function of the energies ε_1 and ε_2, and it does not exhibit singularities at the degeneracy points $\varepsilon_1 = \pm\varepsilon_2$.

By contrast, if the transition rates are calculated up to the fourth order in Δ_j, then one obtains a correction $X_{\mathrm{cl}}^{(4)}$, which shows resonance peaks at $\varepsilon_2 = \pm\varepsilon_1$ that become more pronounced as the coupling constants approach each other. Naturally, this is the most interesting region of parameters. Using notations $\alpha_{1,2} = \alpha \pm \delta\alpha/2$, $\varepsilon = (\varepsilon_1 + \varepsilon_2)/2$, and $\Omega_{1,2} \sim \Omega = \alpha^2 R^2 S/2$, we present the asymptotic form of the fourth-order classical contribution to the cross correlator:

$$X_{\mathrm{cl}}^{(4)} \approx \frac{\Omega_{12} E_c^2}{16\Omega^2\left[(\varepsilon_2 - \varepsilon_1)^2 + \Omega_{12}^2\right]}, \quad (79)$$

where

$$\Omega_{12} = \tfrac{1}{2}(\alpha_1 - \alpha_2)^2 R^2 S \quad (80)$$

is the classical broadening of the nonlocal resonance. Here, we assumed that Ω_{12} is larger than the avoided-crossing energy splitting $\Delta\varepsilon$ and the classical level broadenings due to the local sources of noise. The height of the peak is equal to

$$\max X_{\mathrm{cl}}^{(4)} \approx \frac{E_c^2}{16\Omega_{12}\Omega^2}, \quad \text{at} \quad \varepsilon_1 = \varepsilon_2. \quad (81)$$

B. Quantum noise

In this section, we concentrate on the effects of quantum noise, i.e., we now take into account the antisymmetric parts of the spectral functions $S_2(\omega)$ and $S_3(\omega_1, \omega_2)$. At the same time, we set $E_c = 0$, so the tunneling Hamiltonian (11) becomes a tensor sum as well does the second-order part of the transition matrix, $\hat{M}_2$, but not the fourth-order correction $\hat{M}_4$. This allows us to obtain analytic results using the approach described in the Sec. III B where we associate $\hat{M}_2$ with $\hat{M}_0$ in Eq. (43) and the fourth-order part $\hat{M}_4$ with the perturbation $\delta\hat{M}$.

In the limit of weak coupling, the main contribution from the correlator (66) to the transition rates comes from long times. This means that the first term in the cumulant, $S|t_1 - t_2|$, is much larger then the second term $iS'\mathrm{sign}(t_1 - t_2)$. This allows to include the quantum corrections perturbatively. Since the third cumulant contains one more coupling constant than the second cumulant, we expect the main contribution to come from Gaussian fluctuations. Surprisingly, however, the first-order contribution of S' vanishes, and we have to keep
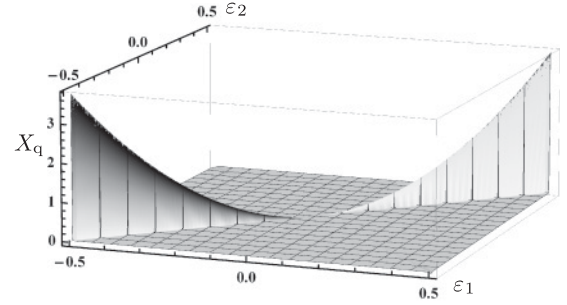

FIG. 7. Cross correlations due to the quantum Gaussian component of noise over the classical background displayed in Fig. 6, in the same units, as a function of the energies ε_1 and ε_1 at $\alpha_1, \alpha_2 > 0$, develop a sharp resonance peak on the diagonal, the height of which is proportional to the energy squared.

the expansion at least up to S'^2. The fact that the Gaussian fluctuations do not contribute to the cross correlator in the lowest possible order with respect to coupling constant means that we have to check if the contribution of third-order correlations may enter with a lower power of the coupling constant. Unfortunately, this is not the case, and non-Gaussian correlations only make a small correction to the Gaussian contribution, as both of them enter quadratically into the cross correlator (82). As a function of energy splittings, X_{q} exhibits the nonlocal resonances at $\varepsilon_1 = \pm\varepsilon_2$. The exact expression is somewhat cumbersome, thus we do not display it, but concentrate on one of the resonances and find the asymptotic form:

$$X_{\mathrm{q}} \approx \frac{\varepsilon^2\Omega_{12}}{64S^2} \frac{16(S')^2 + [4\alpha^2 + 15(\delta\alpha)^2]R^2\left(\mathrm{Re}\,S_3^{(2)}\right)^2}{(\varepsilon_1 - \varepsilon_2)^2 + \Omega_{12}^2}. \quad (82)$$

Note that in contrast to Eq. (81), the height of the resonance in the present case,

$$\max X_{\mathrm{q}} \approx \frac{\varepsilon^2(S')^2}{4\Omega_{12}S^2}, \quad (83)$$

is proportional to the square of the energy ε, as can be seen in Fig. 7.

To complete this section, let us emphasize that the resonant quantum contribution to the cross correlator is an, essentially-nonlocal effect. In the limit when the effective interaction E_c is rendered vanishingly small, the quantum contribution becomes the only one present that carries the manifestation of the nonlocal quantum correlation.

C. Analysis of physical conditions
for the quantum noise detection

One may notice that in the results presented in this section above, we have not included the classical non-Gaussian noise S_3. This is because the effect of this component of noise has been discussed earlier[43] and it is easy to describe: it consists in a uniform shift of the energies $\varepsilon_j \to \tilde{\varepsilon}_j = \varepsilon_j + \alpha_j^3 R^3 S_3/6$ and, in addition to that, a small shift of $3\alpha_1\alpha_2\delta\alpha R^3 S_3$ applies to the energy position of the nonlocal resonance $\tilde{\varepsilon}_2 - \tilde{\varepsilon}_1$. Since the coupling constants $\alpha_j \ll 1$, the displacements of the resonances are much smaller than their widths Ω_j. Moreover, we are interested in the regions of larger energies $\varepsilon_j \gg \Omega_j$ where these shifts clearly become irrelevant.
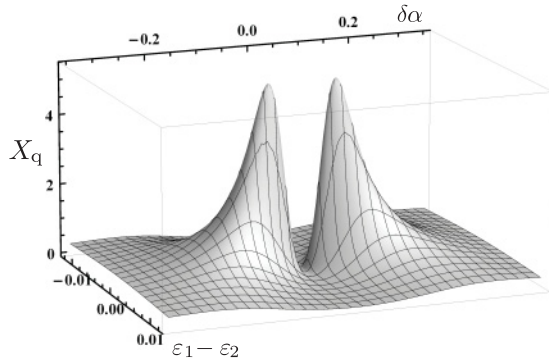
FIG. 8. The cross section of the resonant peak of the quantum contribution to the cross correlator as a function of $\delta\alpha = \alpha_2 - \alpha_1$ and the energy difference $\varepsilon_2 - \varepsilon_1$ illustrates how the peak becomes thinner and, finally, disappears as $\delta\alpha \to 0$. The gap width is determined by the local source noise, that also limits the height of the resonance. Same units as in previous figures are used.

Another effect that we have studied, but that was for the sake of simplicity omitted until now, is the influence of the local noise sources. These local sources may be modeled by additional fluctuating phases $\beta_j \phi_j$ added to the collective mode phase $\alpha_j \phi$, including respective coupling constants β_j. Introducing corresponding noise powers $\beta_j^2 S_{\mathrm{loc}}^{(j)}$, one can find that at the local resonances, these will simply add up to the classical level broadening $\Omega_j \to \Omega_j + \beta_j^2 S_{\mathrm{loc}}^{(j)}$. More important, however, is their contribution to the classical level broadening of the nonlocal resonance: $\Omega_{12} \to \Omega_{12} + \sum_j \beta_j^2 S_{\mathrm{loc}}^{(j)}$. Thus, if we wish that this does not significantly wash out the resonance, then, besides of the requirement of weakness of the local noise $\beta_j^2 S_{\mathrm{loc}}^{(j)} \ll \Omega_j$, also the difference $\delta\alpha = \alpha_2 - \alpha_1$ must be kept large enough. The effect of local noise is best seen in the Fig. 8 as a finite width gap between the two peaks at $\delta\alpha > 0$ and at $\delta\alpha < 0$. Certain optimal value of the coupling constants may be chosen that maximizes the magnitude of the nonlocal resonance.

We have to determine if there is a range of physical parameters where the quantum contribution (83) to the cross correlator is greater then the classical resonance contribution (81) and the classical background (78). In order to do this, we concentrate again on the nonlocal resonance $\varepsilon_j = \varepsilon$ where the quantum effects are maximal. We recall the expression for the effective nonlocal interaction strength (9) and our assumptions made throughout the paper: (i) the level mixing constant is smaller than the the classical level broadening $\Delta \ll \Omega_j$, (ii) the circuit is fast $\tau_{\mathrm{RC}} \ll (\Omega_j)^{-1}$, and (iii) the zero-frequency expansion for the spectral functions is valid for energies smaller than the noise temperature $\varepsilon < T_{\mathrm{eff}}$. Demanding that the quantum contribution is larger then the classical background, $X_q > X_{\mathrm{cl}}^{(2)}$, and using (iii), we obtain the inequalities

$$T_{\mathrm{eff}} > \varepsilon > \frac{\delta\alpha}{\alpha} \frac{E_c}{\Delta} T_{\mathrm{eff}}, \tag{84}$$

where we have substituted $S/S' = 2T_{\mathrm{eff}}$. These inequalities are consistent when $E_c/\Delta < 1$ or

$$\alpha^2 R < \Delta \tau_{\mathrm{RC}} \ll 1, \tag{85}$$

i.e., if the coupling is made sufficiently weak. However, since the background is smooth, one can demand that the quantum contribution is larger than only the resonant part of the classical contribution, $X_q > X_{\mathrm{cl}}^{(2)}$. This requirement leads to the condition

$$T_{\mathrm{eff}} > \varepsilon > \frac{E_c}{\Omega} T_{\mathrm{eff}}, \tag{86}$$

which is weaker than Eq. (85), since $\Delta \ll \Omega$. This condition may also be reformulated as

$$T_{\mathrm{eff}} \tau_{\mathrm{RC}} > 1. \tag{87}$$

Hence one can see that, although $\tau_{\mathrm{RC}} \ll \Omega_j^{-1} = 1/(\alpha^2 R T_{\mathrm{eff}})$, the above inequality can be easily satisfied, in the weak-coupling limit $1/\alpha^2 R \gg 1$.

Finally, we can also compare the cross correlator with one of the autocorrelators $A_j = \langle\langle I_j^2 \rangle\rangle/(\Delta I_j)^2$, which can be found from Eq. (33) and estimated as inverse of the typical transition rate far from the local resonance $A_j \approx \varepsilon_j^2/(\Delta_j^2 \Omega_j)$. Thus, even in the resonance, the cross correlator to autocorrelator ratio, $X_q/A_j \sim \alpha^6 R^2/(\delta\alpha)^2 \ll 1$, is still much smaller than one. In other words, correlated transitions make only a small fraction of all transitions.

VII. SUMMARY AND CONCLUSIONS

Motivated by the challenges on the way of measuring the properties of quantum fluctuations, we have presented in detail one possible approach to the problem, based on the cross-correlation technique. We use a concept of the collective mode, through which fluctuations of the current of the noise source are transferred to the on-chip detector. The latter consists of a pair of the two-level systems, monitored by QPC charge detectors. Thus noise-induced transitions generate two telegraph processes in the QPC currents, the cross correlator of which is supposed to be measured. The cross-correlation technique allows one to reduce the effects of local noise sources and can provide direct access to the quantum component of the collective mode noise.

In the weak tunneling regime, the switching of the detectors can be described in terms of the master equation. The cross correlator, on the other hand, is found using the extended master equation formalism, which describes the evolution of the probability distribution in the mixed space: for the charge, transmitted through the QPCs and the occupations of the two detectors' states. The corresponding generalized transition matrix includes both the rates of the transitions of the detectors and the cumulant generators of the currents in the QPCs. The cross correlator is then obtained from the largest eigenvalue of the generalized transition matrix. We propose two methods of the evaluation of this eigenvalue, each method having its own advantages for different symmetries of the transition matrix in the cases of the classical and quantum noise.

To find the transition rates, we calculate directly the time evolution of the detectors state to the fourth order of the perturbation theory. While, typically, this leads to well-known divergences originating from the continuum of the environmental states, we avoid those divergences in a well controlled way by using the idea of the separation of time scales. We consider a time interval much shorter than a characteristic time of the

evolution of the detectors, and much longer than the correlation time of the noise that induces transitions. During this time interval, the solution of the master equation suggests a linear in time variation of the occupation probabilities, while the formal perturbation expansion of the transition probabilities, with respect to the tunneling Hamiltonian, gives rise to the quadratic in time terms. We observe that the quadratic in time terms are reducible, and thus cancel by taking the logarithm of the time-dependent transition matrix. This leads to the final results for the elements of the transition matrix.

By turn, perturbative calculation of the time evolution of the transition probabilities requires averaging the products of two and four vertex operators over the fluctuations of the collective mode. In the limit of weak coupling of the detectors to the collective mode, the cumulant expansion has been used, and the cumulants are expressed through the components of the zero-frequency expansion of the current noise spectral functions. Such an expansion is justified in the limit of weak coupling, because the main contribution to time integrals for the elements of the transition matrix comes from long times. Note that the weak coupling resummed in this way, leads to the classical broadening of the quantum resonances at $\varepsilon_j = 0$ in the transition rates of each detector separately. But due to the local nature of these resonances, they do not contribute to the cross correlator.

However, interaction of the detectors with the collective mode leads to the effective nonlocal coupling between the detectors. Already to the second order in tunneling, this effective coupling contributes to the cross correlator a smooth background, which is induced by the classical noise, and thus does not represent interest.

Finally, a more interesting effect appears in the fourth order of the perturbation theory with respect to tunneling Hamiltonian. Namely, at the degeneracy points $\varepsilon_1 = \pm\varepsilon_2$ of the two detectors, a narrow nonlocal resonance arises. We show that when the effective coupling between two detectors is reduced, the main contribution to this nonlocal resonance arises from the quantum component of the noise source current. We estimate this effect, compare it to classical contribution to the cross correlator and propose a range of parameters where the quantum contribution dominates. We show that the observation of quantum noise is feasible in the regime of weak coupling. Moreover, we argue that a sharp nonlocal resonance from the quantum noise may be well visible over the smooth background of the classical contribution, and has a characteristic energy dependence that can be used to distinguish it from the classical resonance.

[1] C. W. J. Beenakker and C. Schonenberger, Phys. Today **56**, 37 (2003).

[2] Ya. M. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).

[3] L. S. Levitov, H. Lee, and G. B. Lesovik, J. Math. Phys. **37**, 4845 (1996).

[4] A. A. Clerk, M. H. Devoret, S. M. Girvin, Florian Marquardt, and R. J. Schoelkopf, Rev. Mod. Phys. **82**, 1155 (2010).

[5] T. T. Heikkilä, P. Virtanen, G. Johansson, and F. K. Wilhelm, Phys. Rev. Lett. **93**, 247005 (2004).

[6] J. Ankerhold and H. Grabert, Phys. Rev. Lett. **95**, 186601 (2005).

[7] E. V. Sukhorukov and A. N. Jordan, Phys. Rev. Lett. **98**, 136803 (2007).

[8] A. N. Jordan and E. V. Sukhorukov, Phys. Rev. B **72**, 035335 (2005).

[9] E. B. Sonin, Phys. Rev. B **70**, 140506 (2004).

[10] T. Fujisawa, T. H. Oosterkamp, W. G. van der Wiel, B. W. Broer, R. Aguado, S. Tarucha, and L. P. Kouwenhoven, Science **282**, 932 (1998).

[11] R. Aguado and L. P. Kouwenhoven, Phys. Rev. Lett. **84**, 1986 (2000).

[12] R. J. Schoelkopf, A. A. Clerk, S. M. Girvin, K. W. Lehnert, and M. H. Devoret, in *Quantum Noise in Mesoscopic Physics*, edited by Y. V. Nazarov (Kluwer, Dordrecht, 2003).

[13] E. Onac, F. Balestro, L. H. Willems van Beveren, U. Hartmann, Y. V. Nazarov, and L. P. Kouwenhoven, Phys. Rev. Lett. **96**, 176601 (2006).

[14] V. S. Khrapai, S. Ludwig, J. P. Kotthaus, H. P. Tranitz, and W. Wegscheider, Phys. Rev. Lett. **97**, 176803 (2006).

[15] S. Gustavsson, M. Studer, R. Leturcq, T. Ihn, K. Ensslin, D. C. Driscoll, and A. C. Gossard, Phys. Rev. Lett. **99**, 206804 (2007).

[16] M. Field, C. G. Smith, M. Pepper, D. A. Ritchie, J. E. F. Frost, G. A. C. Jones, and D. G. Hasko, Phys. Rev. Lett. **70**, 1311 (1993).

[17] E. Buks, R. Schuster, M. Heiblum, D. Mahalu, and V. Umansky, Nature (London) **391**, 871 (1998).

[18] J. M. Elzerman, R. Hanson, J. S. Greidanus, L. H. Willems van Beveren, S. De Franceschi, L. M. K. Vandersypen, S. Tarucha, and L. P. Kouwenhoven, Phys. Rev. B **67**, 161308(R) (2003).

[19] T. Fujisawa, T. Hayashi, Y. Hirayama, H. D. Cheong, and Y. H. Jeong, Appl. Phys. Lett. **84**, 2343 (2004).

[20] R. Schleser, E. Ruh, T. Ihn, K. Ensslin, D. C. Driscoll, and A. C. Gossard, Appl. Phys. Lett. **85**, 2005 (2004).

[21] L. M. K. Vandersypen, J. M. Elzerman, R. N. Schouten, L. H. Willems van Beveren, R. Hanson, and L. P. Kouwenhoven, Appl. Phys. Lett. **85**, 4394 (2004).

[22] L. DiCarlo, H. J. Lynch, A. C. Johnson, L. I. Childress, K. Crockett, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Phys. Rev. Lett. **92**, 226801 (2004).

[23] T. Fujisawa, T. Hayashi, R. Tomita, and Y. Hirayama, Science **312**, 1634 (2006).

[24] E. V. Sukhorukov, A. N. Jordan, S. Gustavsson, R. Leturcq, Th. Ihn, and K. Ensslin, Nat. Phys. **3**, 243 (2007).

[25] S. A. Gurvitz, Phys. Rev. B **56**, 15215 (1997).

[26] Y. Levinson, Europhys. Lett. **39**, 299 (1997).

[27] L. Stodolsky, Phys. Rep. **320**, 51 (1999).

[28] S. Gustavsson, R. Leturcq, M. Studer, I. Shorubalko, T. Ihn, K. Ensslin, D. C. Driscoll, and A. C. Gossard, Surf. Sci. Rep. **64**, 191 (2009).

[29] M. Reznikov, M. Heiblum, H. Shtrikman, and D. Mahalu, Phys. Rev. Lett. **75**, 3340 (1995).

[30] A. Kumar, L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. **76**, 2778 (1996).

[31] C. B. Doiron, B. Trauzettel, and C. Bruder, Phys. Rev. B **76**, 195312 (2007).

[32] B. Küng, O. Pfäffli, S. Gustavsson, T. Ihn, K. Ensslin, M. Reinwald, and W. Wegscheider, Phys. Rev. B **79**, 035314 (2009).

[33] U. Gasser, S. Gustavsson, B. Küng, K. Ensslin, T. Ihn, D. C. Driscoll, and A. C. Gossard, Phys. Rev. B **79**, 035303 (2009).

[34] A. N. Jordan and M. Büttiker, Phys. Rev. Lett. **95**, 220401 (2005).

[35] C. B. Doiron, B. Trauzettel, and C. Bruder, Phys. Rev. Lett. **100**, 027202 (2008).

[36] S. Walter and B. Trauzettel, Phys. Rev. B **83**, 155411 (2011).

[37] G.-L. Ingold and Y. V. Nazarov, in *Single Charge Tunneling*, edited by H. Grabert and M. H. Devoret (Plenum, New York, 1992), Chap. 2.

[38] For a generalization beyond Gaussian noise, see Refs. 39 and 43.

[39] E. B. Sonin, Phys. Rev. Lett. **98**, 030601 (2007).

[40] We use the Wolfram MATHEMATICA package for analytical computing.

[41] A. N. Jordan and E. V. Sukhorukov, Phys. Rev. Lett. **93**, 260604 (2004).

[42] Note that since in general case the transition matrix $\hat{M}$ is not symmetric, the left and right eigenvectors, $\langle m_j |$ and $| m_j \rangle$, are not Hermitian conjugated.

[43] E. V. Sukhorukov and J. Edwards, Phys. Rev. B **78**, 035332 (2008).

[44] Note that elements of $\delta\hat{M}$ acquire a simple physical interpretation. For example, $\delta M_{14}, \delta M_{23}, \delta M_{32}, \delta M_{41}$ represent the rates of correlated transitions $|11\rangle \leftrightarrow |22\rangle$ and $|12\rangle \leftrightarrow |21\rangle$, while $\delta M_{12}, \delta M_{13}, \delta M_{21}, \delta M_{31}$ characterize the asymmetry of transitions $|11\rangle \leftrightarrow |21\rangle$ and $|12\rangle \leftrightarrow |22\rangle$, or, in other words, how the transition rates in one detector depend on the state of another.

[45] For a general situation, see the analysis in Ref. 43.