High-order perturbation corrections to the density of states of disordered metals in a magnetic field

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We study the effect of electron-electron interaction on one-particle density of states (DOS) $\rho^{(d)}(\epsilon,T,\mathbf{B})$ of weakly disordered metals in magnetic field (B), employing the conventional impurity diagram technique. The geometric resummation of the most singular self-energy corrections via the Dyson equation is examined. Around the Fermi level ($\epsilon = 0$), we obtain that the DOS is linearly dependent on energy, $\rho^{(2)}(\epsilon,T=0,\mathbf{B}) \sim |\epsilon|/B$, in two-dimensional systems while in three dimensions it acquires a power-law behavior, $\rho^{(3)}(\epsilon,T=0,\mathbf{B}) \sim |\epsilon|^{\frac{3}{4}}/B$. It is revealed that in both dimensions the DOS depends inversely on magnetic field strength at low energies and vanishes at the Fermi energy.

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I. INTRODUCTION

To understand the behavior of interacting electrons in weakly disordered metallic systems,[1] much effort has been made both theoretically[2–5] and experimentally.[6] In these systems, where $k_F l \gg 1$ with k_F being the Fermi wave number and l the elastic mean free path, the Coulomb interaction was shown to modify the physical properties of the metals. An impurity scattering mechanism due to the disorder becomes crucial at low temperatures, $k_B T < \frac{\hbar}{\tau}$ where T is the temperature, τ is the impurity scattering time, and k_B is the Boltzmann's constant. The amount of disorder was found to enhance the effects of electron-electron (*ee*) interaction.[3,7,8] Examinations of the interference of electron impurity and *ee* scatterings have found nontrivial quantum corrections to the density of states (DOS) and conductivity.[4,9]

In tunneling experiments[10] a strong suppression of the tunneling conductance at low biases was observed, commonly referred to as a zero-bias anomaly. It reflects the suppression of the DOS near the Fermi level.[3,11,12] Theoretically, the suppression of the single-particle DOS at the Fermi surface was first discussed by Altshuler and Aronov (AA)[9] for the diffusive systems under short range interactions. Quantum corrections to the DOS strongly depend on the dimension of the system. In two dimensions (2D), the effect of them on the DOS was revealed to be logarithmic,[2] $\rho^{(2)}(\epsilon) \sim \ln(|\epsilon|\tau)$, where ϵ denotes the energy of the electron measured relative to the Fermi level. This result is related to the enhancement of the *ee* interaction due to disorder. On the other hand, in three dimensions (3D), it was obtained that there was no singular contribution,[13] $\rho^{(3)}(\epsilon) \sim \sqrt{|\epsilon|\tau}$, due to the interference effects which are not as strong as in low-dimensional systems.

The experimental discovery of a metal-insulator transition (MIT)[14] in 2D semiconductor devices has shown that the current theoretical understanding of *ee* interactions in disordered electronic systems is incomplete. Thus it has reopened the problems.[15] In the presence of screening, two-dimensional DOS tuned through the MIT was realized,[16] where the DOS was found to be linearly dependent on energy near the Fermi level for different magnetic fields and electron densities. The suppression revealed in Ref. 16 was deeper and wider near the Fermi level with an increasing magnetic field. It was partially different from the prior results.[2]

The effect of a magnetic field on the physical properties of systems has been studied for a long time.[17–21] A strong magnetic field gives rise to the quantization of electronic states (Landau orbits). However, a weak magnetic field has a small effect ($\approx \omega_c^2\tau^2$, where $\omega_c = \frac{eB}{mc}$ is the cyclotron frequency, and e and m are the electron charge and mass, respectively) on the physical properties of electronic systems at high temperatures, $k_B T > \frac{\hbar}{\tau}$. An arbitrary external weak magnetic field destroys the electron localization and, as a consequence, gives rise to an increasing conductivity with an external magnetic field. This effect, known as *negative magnetoresistance*, was studied theoretically by Kawabata[22] for 3D disordered systems and by Altshuler *et al.*[18] for a 2D electron gas. The role of both the magnetic field and *ee* interactions in impure systems was considered by AA.[17] In their work, only first-order quantum contribution to the DOS was taken into account which revealed an essential energy and magnetic field dependence of the DOS in classically weak fields.

The purpose of the present paper is a detailed study of the DOS around the Fermi level (in the vicinity of $\epsilon = 0$) in 2D and 3D weakly disordered systems within the diagrammatic perturbation theory in the presence of both a weak magnetic field and *ee* interactions. This study is an extension of our previous work[23] where, in the absence of a magnetic field, we obtained a nondivergent solution for the DOS at low energies using the Dyson equation[1] in low-dimensional systems, and AA results were recovered at high energies. In contrast to the AA results[3] through the first-order perturbation theory, nonperturbative calculations led to a nondivergent solution as well for the DOS at low energies with a power-law behavior.[24,25] Here, we show that the behavior of the DOS at low energies [$\rho^{(2)}(\epsilon,\mathbf{B}) \sim |\epsilon|/B$ and $\rho^{(3)}(\epsilon,\mathbf{B}) \sim |\epsilon|^{\frac{3}{4}}/B$] is evidently distinct from that at high energies [$\rho^{(2)}(\epsilon,\mathbf{B}) \sim B^2/\epsilon^2$ and $\rho^{(3)}(\epsilon,\mathbf{B}) \sim B^2/|\epsilon|^{\frac{3}{2}}$] in the presence of a finite magnetic field. In both dimensions we reveal that at zero temperature, the DOS vanishes at the Fermi energy and depends inversely on magnetic field strength at low energies. AA results[17] in both dimensions are revised in order to give a systematical analysis for the DOS. The reanalysis is done by taking into account geometric resummation of the most singular self-energy corrections through the Dyson equation. At high energies, when $\epsilon \gg \hbar\omega_c$, AA results in Ref. 17 [$\rho^{(2)}(\epsilon,\mathbf{B}) \sim B^2/\epsilon^2$ and $\rho^{(3)}(\epsilon,\mathbf{B}) \sim B^2/|\epsilon|^{\frac{3}{2}}$] are recovered in both dimensions. In the present study the obtained nondivergent solution for the DOS in both dimensions at low energies originates from the summation of high-order diagrammatic contributions. In Sec. II, employing the high-order perturbation corrections, the calculation of the DOS is presented and conclusions are given in Sec. III.

II. HIGH-ORDER PERTURBATION CORRECTIONS

The DOS $\rho^{(d)}(\epsilon,T)$ can be evaluated explicitly in terms of Green's function (GF)

$$\rho^{(d)}(\epsilon,T) = -\frac{2}{\pi}\mathrm{Im}\int \frac{d^{\mathrm{d}}p}{(2\pi)^{\mathrm{d}}} G^{\mathrm{R}}(\mathbf{p},i\epsilon_n)_{i\epsilon_n\to\epsilon}, \qquad (1)$$

where $G^{\mathrm{R}} = G^{\mathrm{R}}(\mathbf{p},i\epsilon_n)$ is the finite temperature impurity averaged total retarded GF in momentum space with $\mathbf{p}$ being the electron momentum. The temperature-dependent GF coincides with the retarded GF at discrete points on the positive imaginary semiaxis, i.e., $G(\epsilon_n) = G^{\mathrm{R}}(i\epsilon_n)$ at $\epsilon_n > 0$. Here $\epsilon_n = \pi T(2n+1)$ stands for the Matsubara frequency[1] at temperature T. For a classically weak magnetic field, satisfying the condition $\omega_c\tau \ll 1$ or $l_{\mathrm{H}} \gg 1$, where $l_{\mathrm{H}}^2 = \frac{\hbar c}{eB}$ is the radius of magnetic orbit (Larmour radius), the electron's wave function acquires an additional phase. In this case, G^{R} involves an additional phase and is given, through a quasiclassical approximation, by

$$G^{\mathrm{R}}(\mathbf{r},\mathbf{r}',i\epsilon_{\mathrm{n}}) = \exp\left\{\mp i\frac{e\hbar}{c}\int_{\mathbf{r}}^{\mathbf{r}'}\mathbf{A}(\mathbf{l})d\mathbf{l}\right\}G^{\mathrm{R}}(\mathbf{r}-\mathbf{r}',i\epsilon_{\mathrm{n}}), \qquad (2)$$

where $G^{\mathrm{R}} = G^{\mathrm{R}}(\mathbf{r},\mathbf{r}',i\epsilon_n)$ is the real space representation of GF and $\mathbf{A}$ denotes the vector magnetic potential.

In the weak localization theory,[3] the main contributions to the physical properties of disordered systems originate from two singularities, known as *diffusion pole* characterizing an electron-hole pair and *Cooper pole* representing the propagation of an electron-electron pair. For the former (latter) one difference (summation) of momenta is small, and for both the energy difference is small. The Cooper pole is significantly influenced by the external magnetic field as this field breaks down the time invariance. Hence, significant correction to the DOS comes from the diagrams containing the Cooper pole, which is more sensitive to an external magnetic field[18] than is the diffusion pole.

In calculating the DOS the contemplations for the choice of appropriate diagrams were discussed by AA.[3] Here we follow the well-known route first developed by them for the electron motion in disordered systems under a weak magnetic field.[17] One can utilize the diagrams in Fig. 1, where the first order in Coulomb interaction contributions to the self-energy $\Sigma(\mathbf{r},\mathbf{r}',\epsilon)$ are illustrated. High order in Coulomb interaction corrections to the GF can be taken into account by means of the Dyson equation, upon involving the most singular self-energy contributions. In the presence of a magnetic field, $G^{\mathrm{R}}(\mathbf{r},\mathbf{r}',i\epsilon_n)$ given in the coordinate space in Eq. (2) can be utilized for the calculation of the Dyson equation. In real space, as depicted in Fig. 1(a), the Dyson equation reads

$$G^{\mathrm{R}}(\mathbf{r},\mathbf{r}',i\epsilon_n) = G_0^{\mathrm{R}}(\mathbf{r},\mathbf{r}',i\epsilon_n) + \int G_0^{\mathrm{R}}(\mathbf{r},\mathbf{r}_1,i\epsilon_n) \times \Sigma(\mathbf{r}_1,\mathbf{r}_2,i\epsilon_n)G^{\mathrm{R}}(\mathbf{r}_2,\mathbf{r}',i\epsilon_n), \qquad (3)$$

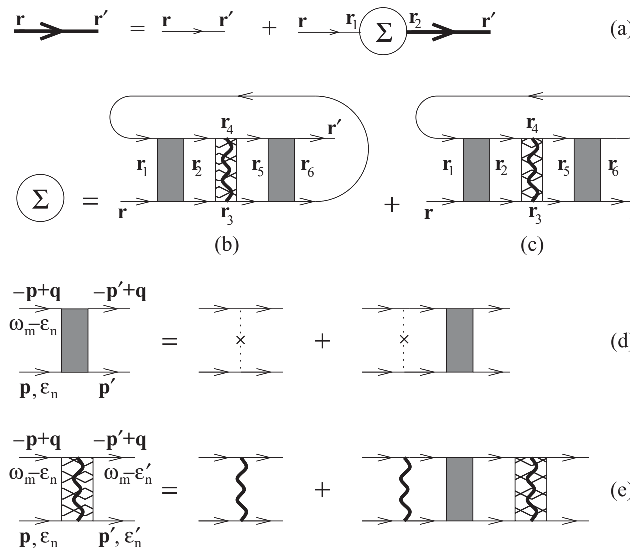

FIG. 1. (a) Real space representation of Dyson equation in quasiclassical approximation. Real space representation of (b) exchange and (c) Hartree diagrams for the calculation of $\Sigma(\mathbf{r},\mathbf{r}',\epsilon)$. (d) Diagram equation giving the Cooper pole and (e) that involving effective interaction. Here the thick wavy lines denote the effective Coulomb interaction and the dashed line with cross represents the impurity scattering.

where G_0^{R} is the bare retarded GF. GF decreases exponentially in a range of mean free path l in the coordinate space. Therefore the crucial part of the integration is $|\mathbf{r}-\mathbf{r}'| \approx l$ and one may set $\mathbf{r} = \mathbf{r}'$. The self-energy $\Sigma(\mathbf{r},\mathbf{r}',i\epsilon_n)$ in Fig. 1(b) is expressed as

$$\begin{aligned}\Sigma(\mathbf{r},\mathbf{r}',i\epsilon_n) &= \int d\mathbf{r}_1\int d\mathbf{r}_2\int d\mathbf{r}_3\int d\mathbf{r}_4\int d\mathbf{r}_5\int d\mathbf{r}_6 \\ &\times C_\omega(\mathbf{r}_1,\mathbf{r}_2)G_0^{\mathrm{R}}(\mathbf{r}_2,\mathbf{r}_3,i\epsilon_n)G_0^{\mathrm{R}}(\mathbf{r}_3,\mathbf{r}_5,i\epsilon_n)C_\omega(\mathbf{r}_5,\mathbf{r}_6) \\ &\times G_0^{\mathrm{A}}(\mathbf{r}_2,\mathbf{r}_4,i\epsilon_n - i\omega_m)\lambda_c(\mathbf{r}_3,\mathbf{r}_4)G_0^{\mathrm{A}}(\mathbf{r}_4,\mathbf{r}_5,i\epsilon_n - i\omega_m) \\ &\times G_0^{\mathrm{A}}(\mathbf{r}_6,\mathbf{r}_1,i\epsilon_n - i\omega_m), \end{aligned} \qquad (4)$$

where G_0^{A} is the bare advanced GF, C_ω denotes the *cooperon* representing a diagram equation for the Cooper pole [see Fig. 1(d)], λ_c stands for the effective Coulomb interaction, and $\omega_m = 2\pi Tm$ is the Matsubara frequency. The magnetic field dependence of GFs in Eq. (4) can be achieved using the quasi-classical approximation in Eq. (2). Therefore, in the equation above the main issue is the calculation of the cooperon in the magnetic field. Here we are not going to dwell on the derivation of the cooperon in the magnetic field. The calculation of C_ω in real space was outlined and the detailed derivation was presented in Ref. 18. Using the procedures in Ref. 18 together with Ref. 17, we obtain

$$C_\omega(\mathbf{r},\mathbf{r}') = 2\pi\rho_0^{(d)}\sum_{n,\alpha}^{(\frac{L_{\mathrm{H}}}{l})^2}\int\frac{dq_z}{2\pi} \times \frac{\psi_{n,\alpha}(\mathbf{r})\psi_{n,\alpha}^*(\mathbf{r}')}{-i\omega + Dq_z^2 + \frac{D}{L_{\mathrm{H}}^2}(2n+1)}, \qquad (5)$$

where $\psi_{n,\alpha}(\mathbf{r})$ is the eigenfunction of an electron in a quantized magnetic field, with n and α being Landau orbit and spin index, respectively. $\sum \psi_{n,\alpha}(\mathbf{r})\psi^*_{n,\alpha}(\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')$ satisfies the orthogonality condition and $D = \frac{v_F^2\tau}{d}$ stands for the diffusion coefficient of a d-dimensional system with v_F being the velocity at the Fermi level. $\rho_0^{(d)}$ is the pure DOS corresponding to a noninteracting electron gas. In 2D and 3D it is given as $\rho_0^{(2)} = \frac{m}{2\pi\hbar^2}$ and $\rho_0^{(3)} = \frac{mp_F}{2\pi^2\hbar^2}$, respectively. In the equation above, the applied field is taken along the z direction and the gauge is chosen to be $\mathbf{A} = (0,Bx,0)$. The upper limit of the summation over n is taken as $(\frac{L_H}{l})^2 \gg 1$ with a magnetic length L_H of $2e$ charge. Note that the expression of the cooperon in Eq. (5) can be derived from the equation

$$\left[-i\omega + D\left(-i\frac{\partial}{\partial\mathbf{r}} - \frac{2e}{c}\mathbf{A}\right)^2\right]C_\omega(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'), \quad (6)$$

where $C_\omega(\mathbf{r},\mathbf{r}')$ can be expanded over $\psi_{n,\alpha}(\mathbf{r})$ as shown in Ref. 18.

The Coulomb interaction λ_c is screened by the cooperon in Eq. (5), which is represented by the diagrammatic equation in Fig. 1(e). In the presence of a magnetic field, $\lambda_c = \lambda_c(q_z,2\epsilon-\omega,\mathbf{B})$ has the form[17]

$$\lambda_c(q_z,2\epsilon-\omega,\mathbf{B}) = \frac{1}{\lambda_0^{-1} - \ln\left[\frac{-i|2\epsilon-\omega|+\omega_c(n+\frac{1}{2})+Dq_z^2}{\epsilon_0}\right]}, \quad (7)$$

where λ_0 is the bare interaction constant and $\epsilon_0 = \epsilon \to 0$ for the Coulomb repulsion. Through the cooperon in Eq. (5) and the effective Coulomb interaction in Eq. (7), we can write the self-energy expression in Eq. (4) in momentum space under an external field

$$\Sigma(\mathbf{p},i\epsilon_n,\mathbf{B},T) = -\frac{2i}{\tau^2}\frac{2eB}{\hbar c}G_0^A(\mathbf{p},i\epsilon_n)D\int_0^\infty \frac{d\omega}{2\pi}\int\frac{dq_z}{2\pi} \times \sum_{n=0}^{(\frac{L_H}{l})^2}\lambda_c(q_z,2\epsilon-\omega,\mathbf{B}) \times \frac{\tanh\left(\frac{\omega+\epsilon}{2T}\right)+\tanh\left(\frac{\omega-\epsilon}{2T}\right)}{\left[-i\omega+Dq_z^2+\frac{D}{L_H^2}(2n+1)\right]^2}, \quad (8)$$

where for a 2D system integration over q_z disappears. According to the Dyson equation the total retarded GF in momentum space, $G^R(\mathbf{p},i\epsilon_n)$, is defined as

$$G^R(\mathbf{p},i\epsilon_n) = \sum_{n=0}^{\infty}\left[G_0^R(\mathbf{p},i\epsilon_n)\right]^{n+1}[\Sigma(\mathbf{p},i\epsilon_n,\mathbf{B},T)]^n. \quad (9)$$

Substituting Eq. (8) into Eq. (9) and plugging the latter into Eq. (1) one can find the following expression for the DOS, $\rho^{(d)} = \rho^{(d)}(\epsilon,\mathbf{B},T)$, in the presence of a magnetic field:

$$\rho^{(d)}(\epsilon,\mathbf{B},T) = \rho_0^{(d)} - \frac{2}{\pi}\mathrm{Im}\int\frac{d^dp}{(2\pi)^d}\sum_{n=0}^{\infty}[\beta(\epsilon,\mathbf{B},T)]^n \times \left[G_0^R(\mathbf{p},i\epsilon_n)\right]^{n+1}\left[G_0^A(\mathbf{p},i\epsilon_n)\right]^n = \rho_0^{(d)} - \frac{2}{\pi}\mathrm{Im}\sum_{n=0}^{\infty}A_n[\beta(\epsilon,\mathbf{B},T)]^n. \quad (10)$$

In the equation above, the quantity $\beta(\epsilon,\mathbf{B},T)$ has the same form as the self-energy in Eq. (8) without $G_0^A(\mathbf{p},i\epsilon_n)$ and A_n is described by

$$A_n = \int\frac{d^dp}{(2\pi)^d}\left[G_0^R(\mathbf{p},i\epsilon_n)\right]^{n+1}\left[G_0^A(\mathbf{p},i\epsilon_n)\right]^n. \quad (11)$$

This integration yields

$$A_n = -\rho_0^{(d)}2\pi i\,\tau^{2n}\frac{n(2n-1)!}{(n!)^2}. \quad (12)$$

In order to evaluate the sum in Eq. (10), one can employ the definition $\ln[1+\sqrt{1+x^2}] = \ln 2 - \sum_{n=1}^{\infty}(-1)^n\frac{(2n-1)!}{(n!)^2 2^{2n}}x^{2n}$. Plugging Eq. (12) into Eq. (10) and performing the sum over n gives rise to the following contribution, arising from the Cooper pole, to the DOS in d dimensions:

$$\rho^{(d)}(\epsilon,\mathbf{B},T) = \rho_0^{(d)}\left[1-\mathrm{Im}\left\{\frac{4\alpha}{(1-4i\,\alpha)+\sqrt{1-4i\,\alpha}}\right\}\right], \quad (13)$$

where $\alpha = -i\,\tau^2\beta(\epsilon,\mathbf{B},T)$. Equation (13) is the main result, which is found from the Dyson equation by involving most singular self-energy contributions in a weak magnetic field. In the case of point-like interactions (i.e., static Coulomb interactions) λ_c does not depend on either $\mathbf{q}$ or ω (λ_c becomes a constant). Using this fixed interaction, the calculation of $\alpha = \alpha(\epsilon,\mathbf{B},T)$ can be simplified as the integration in Eq. (8) becomes straightforward. In 2D and 3D, it is calculated to be

$$\alpha(\epsilon,\mathbf{B},T) = \begin{cases} -iC_1\lambda_c\frac{B^2}{\max\{\epsilon^2,T^2\}}, & (d=2) \\ -iC_2\lambda_c\frac{B^2}{\max\{|\epsilon|^{3/2},T^{3/2}\}}, & (d=3) \end{cases} \quad (14)$$

where C_1 and C_2 are constants. Substituting Eq. (14) into Eq. (13), we readily obtain the DOS in both dimensions as

$$\rho^{(2)}(\epsilon,T,\mathbf{B}) = \rho_0^{(2)}\frac{1}{\sqrt{1+4C_1\lambda_c\frac{B^2}{\max\{\epsilon^2,T^2\}}}},$$
$$\rho^{(3)}(\epsilon,T,\mathbf{B}) = \rho_0^{(3)}\frac{1}{\sqrt{1+4C_2\lambda_c\frac{B^2}{\max\{|\epsilon|^{3/2},T^{3/2}\}}}}. \quad (15)$$

This result gives a deviation from the free electron DOS. It yields a dip (zero DOS) formed at the Fermi level, $\epsilon = 0$, in 2D and 3D systems due to correlation effects in the presence of an arbitrary magnetic field. The zero DOS at the Fermi level (a signature of the Coulomb gap) can be ascribed to the fact that the contribution of localized states dominates over that of the metallic ones. Equation (15) reveals that at absolute zero, around the Fermi level (i.e, at low energies) the DOS $\rho^{(d)}(\epsilon,T=0,\mathbf{B}) = \rho^{(d)}(\epsilon,\mathbf{B})$ depends linearly on energy, $\rho^{(2)}(\epsilon,\mathbf{B}) \sim |\epsilon|/B$, in 2D; while in 3D it is given by a power-law behavior, $\rho^{(3)}(\epsilon,\mathbf{B}) \sim |\epsilon|^{\frac{3}{4}}/B$, under a finite magnetic field. Note that in the presence of magnetic field, the logarithmic suppression of the DOS disappears in 2D. It originates from the fact that the magnetic field destroys the interference of wave functions. Power-law behavior was found through the nonperturbative calculations in earlier studies.[24,25] For instance, in Ref. 24, high-order Coulomb potential contributions were summed up through the renormalization group theory, giving power-law behavior. In the present study the obtained power-law behavior originates from the summation

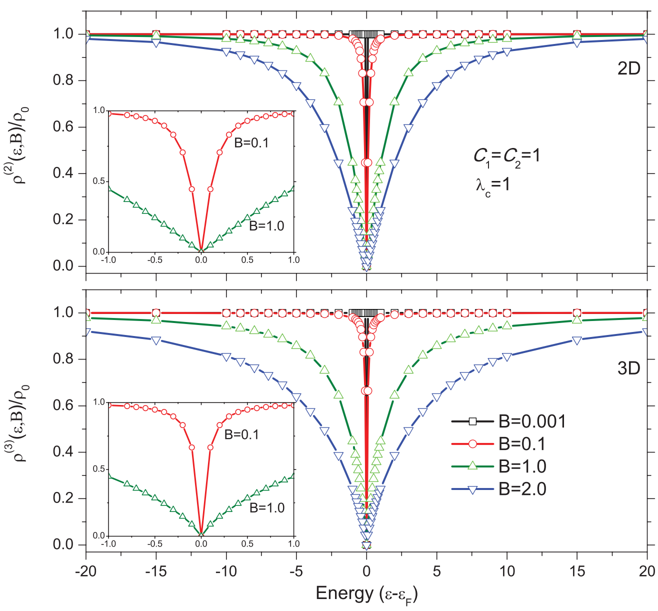

FIG. 2. (Color online) Correlation effects on $\rho^{(2)}(\epsilon,\mathbf{B})$ and $\rho^{(3)}(\epsilon,\mathbf{B})$ for various magnetic fields through fixed Coulomb interaction strength. In the inset, the variation of the DOS is shown around the Fermi level.

of high-order diagrammatic contributions. At high energies, when $\epsilon \gg \hbar\omega_c$, we recover the AA results in Ref. 17 where $\rho^{(2)}(\epsilon,\mathbf{B}) \sim B^2/\epsilon^2$ and $\rho^{(3)}(\epsilon,\mathbf{B}) \sim B^2/|\epsilon|^{\frac{3}{2}}$. The DOS as a function of energy in each dimension is presented for various magnetic fields, at $T = 0$, in Fig. 2. We see that in the vicinity of the Fermi level, in both dimensions, the DOS becomes reduced by increasing the magnetic field. It can also be reduced by increasing the impurity strength, i.e., decreasing τ, through Eq. (13). The suppression around the Fermi energy and vanishing of tunneling DOS in 2D at $\epsilon = 0$ was observed in Ref. 16 in the presence of screening under a magnetic field. In Ref. 16 it was also found that the suppression becomes deeper with increasing magnetic field at low energies, which agrees well with our findings. For various magnetic fields, the absence of zero bias tunneling conductance has also been demonstrated experimentally[26] in agreement with our results.

III. CONCLUSIONS

In the present paper we provide a diagrammatic perturbation treatment on the disordered interacting problem. We attempt to describe the role of the magnetic field and *ee* interaction in weakly doped metallic disordered systems. Through the Dyson equation, we examine the most singular self-energy contributions to the DOS in 2D and 3D disordered electronic systems influenced by an external magnetic field. Incorporating high order in Coulomb interaction scattering corrections does not change the variation of the DOS given in Ref. 17 at high energies. However, at low energies it is drastically modified in 2D and 3D, resulting in a strong energy and magnetic field dependence of the DOS at energies close to the Fermi energy. We reveal that, in both dimensions, the DOS depends inversely on magnetic field strength in the vicinity of the Fermi level and becomes zero at the Fermi energy. One may argue that, upon employing the Dyson equation, the interplay of *ee* interaction and magnetic field in weakly disordered metallic systems substantially modifies the form of the DOS in the vicinity of the Fermi level.

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[1] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloskinskii, *Methods of Quantum Field Theory in Statistical Physics* (Pergamon, New York, 1965).

[2] B. L. Altshuler, A. G. Aronov, and P. A. Lee, Phys. Rev. Lett. **44**, 1288 (1980).

[3] B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (Elsevier, North-Holland, 1985), Ch. 1.

[4] H. Fukuyama, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (Elsevier, North-Holland, 1985), Ch. 2.

[5] S. Caliskan, E. Sasioglu, M. Kumru, E. P. Nakhmedov, O. Cakiroglu, and B. Karaoglu, Phys. Status Solidi B **229**, 1205 (2002).

[6] M. Nissim, Y. Lereah, and R. Rosenbaum, Phys. Rev. B **40**, 6351 (1989); S. Y. Hsu and J. M. Valles Jr., *ibid.* **49**, 16600 (1994); A. Vaknin, Z. Ovadyahu, and M. Pollak, Phys. Rev. Lett. **81**, 669 (1998).

[7] M. Lee, J. G. Massey, V. L. Nguyen, and B. I. Shklovskii, Phys. Rev. B **60**, 1582 (1999).

[8] E. Helgren, G. Gruner, M. R. Ciofalo, D. V. Baxter, and J. P. Carini, Phys. Rev. Lett. **87**, 116602 (2001).

[9] B. L. Altshuler and A. G. Aronov, Sov. Phys. JETP **50**, 968 (1979).

[10] E. Bielejec, J. Ruan, and W. Wu, Phys. Rev. Lett. **87**, 036801 (2001); A. Bachtold, M. S. Fuhrer, S. Plyasunov, M. Forero, E. H. Anderson, A. Zettl, and P. L. McEuen, *ibid.* **84**, 6082 (2000).

[11] R. Egger and A. O. Gogolin, Phys. Rev. Lett. **87**, 066401 (2001).

[12] L. Bartosch and P. Kopietz, Eur. Phys. J. B **28**, 29 (2002).

[13] B. L. Altshuler and A. G. Aronov, Solid State Commun. **36**, 115 (1979).

[14] S. V. Kravchenko, W. E. Mason, G. E. Bowker, J. E. Furneaux, V. M. Pudalov, and M. DIorio, Phys. Rev. B **51**, 7038 (1995).

[15] E. P. Nakhmedov, V. Prigodin, S. Caliskan, and E. Sasioglu, Phys. Rev. B **66**, 233105 (2002).

[16] H. B. Chan, P. I. Glicofridis, R. C. Ashoori, and M. R. Melloch, Phys. Rev. Lett. **79**, 2867 (1997).

[17] B. L. Altshuler and A. G. Aronov, Solid State Commun. **38**, 11 (1981).

[18] B. L. Altshuler, D. E. Khmelnitzkii, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).

[19] P. Jiang, I. Yang, W. Kang, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **96**, 126804 (2006).

[20] I. L. Aleiner, H. U. Baranger, and L. I. Glazman, Phys. Rev. Lett. **74**, 3435 (1995).

[21] W. Teizer, F. Hellman, and R. C. Dynes, Phys. Rev. Lett. **85**, 848 (2000).
[22] A. Kawabata, J. Phys. Soc. Jpn. **49**, 628 (1980); Solid State Commun. **34**, 431 (1980).
[23] E. Sasioglu, S. Caliskan, and M. Kumru, Phys. Rev. B **79**, 035123 (2009).
[24] A. M. Finkelshtein, Sov. Phys. JETP **57**, 97 (1983).
[25] D. Belitz and T. R. Kirkpatrick, Phys. Rev. B **48**, 14072 (1993); P. Kopietz, Phys. Rev. Lett. **81**, 2120 (1998); A. Kamenev and A. Andreev, Phys. Rev. B **60**, 2218 (1999).
[26] L. Bokacheva, W. Teizer, F. Hellman, and R. C. Dynes, Phys. Rev. B **69**, 235111 (2004).